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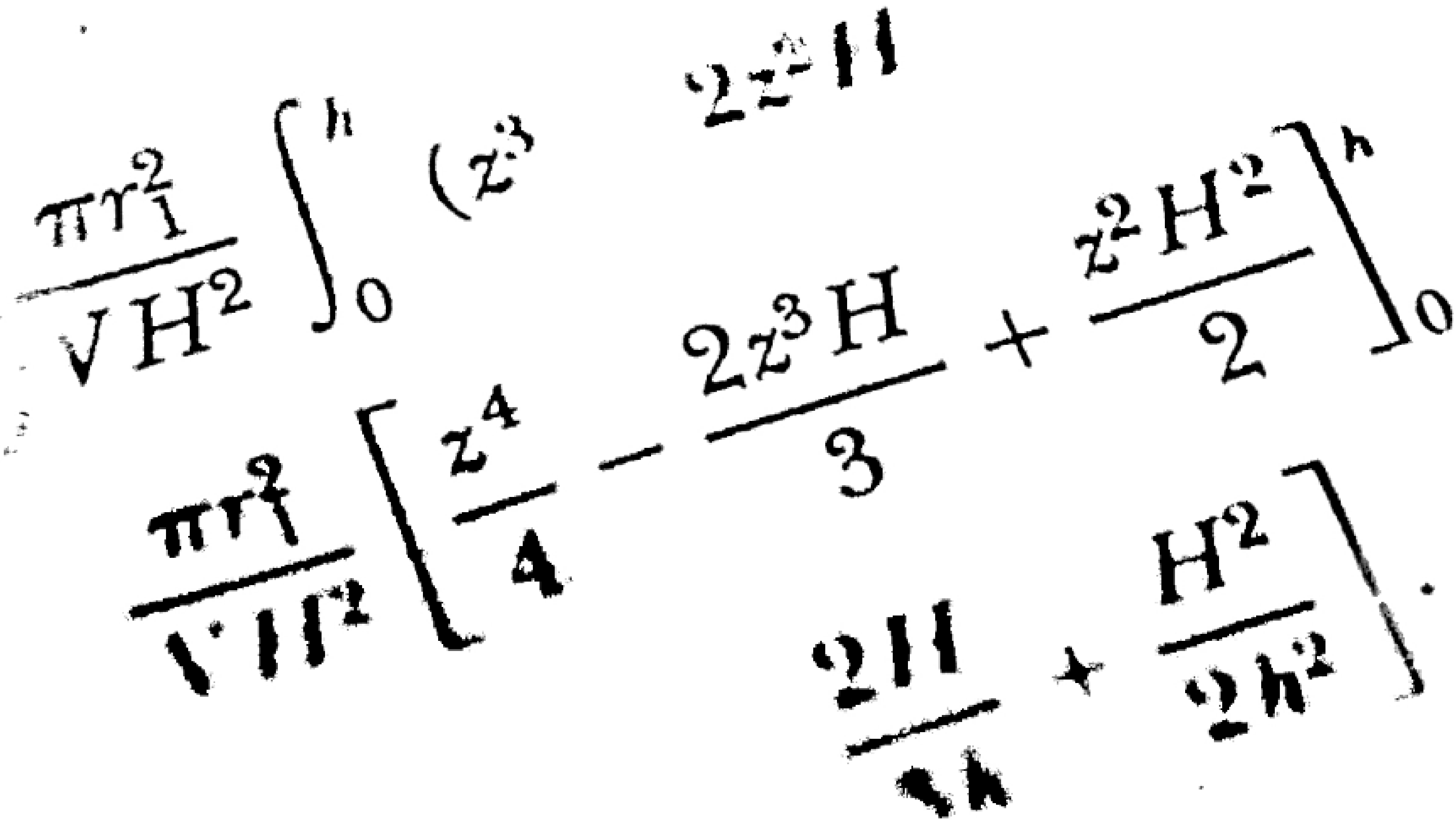


O'ZBEKISTON RESPUBLIKASI OLIY VA O'RTA MAXSUS TA'LIM VAZIRLIGI

NIZOMIY NOMIDAGI TOSHKENT DAVLAT PEDAGOGIKA UNIVERSITETI

Saparboyev Jamoladdin Yuldashevich, Davletov Davronbek Egamberganovich,
Toxirova Malika

MATEMATIKADAN MISOL VA MASALALAR YECHISH METODIKASIDAN MASALALAR TO'PLAMI

(Metodik qo'llanma)

Qayta ishlangan va to'ldirilgan 2-nashr

Toshkent – 2019

Mázkur qo'llanma matematika o'qitish metodikasi yo'nalishiga matematikadan misol va masalalar yechish metodikasi fanidan amaliy mashg'ulotlarda foydalanish uchun tayyorlandi. Qo'llanmaning maqsadi talabalarning matematikadan olgan nazariy bilimlarini o'rta maktab matematikasi bilan bog'lash, ularda masala va misollar yechish malakasini takomillashtirish hamda rivojlantirishda iborat.

Qo'llanmadan, shuningdek, matematika o'qituvchilari va matematika bilan qiziqqanlar ham foydalanishlari mumkin. Qo'llanma amaldagi dasturga to'la mos bo'lib, mavzularga doir qisqacha nazariy ma'lumotlar hamda misol va masalalar keltirilgan.

Taqrizchilar:

To'raqulov N.– TTESI AL matematika o'qituvchisi

Raximov I. – Umumiy matematika kafedrasi katta o'qituvchisi

Ushbu qo'llanma Nizomiy nomidagi TDPU O'quv-uslubiy Kengashi yig'ilishining 2018- yil ______________dagi ___-raqamli qarori bilan nashrga tavsiya etilgan.

Kirish

«Matematikadan misol va masalalar yechish metodikasi» matematikaning fundamental bo'limlaridan bo'lib, uning poydevori hisoblanadi. Matematikadan misol va masalalar yechish metodikasi matematikaning turli bo'limlari (algebra, geometriya) asosida o'rganilib, hamda boshqa sohalarni (fizika, astronomiya va h.k.) o'rganishda, ularning masalalarini yechishda muhim ahamiyatga ega.

Ushbu uslubiy qo'llanma "Matematikadan misol va masalalar yechish metodikasi" fanining dasturiga mos yozilgan bo'lib, talabalarni matematikaning zaruriy ma'lumotlari majmuasi (tushunchalar, tasdiqlar va ularning isboti, amaliy masalalarni yechish usullari va boshqalar) bilan tanishtirish hamda matematika yo'nalishlarining uzviy bog'liqliklarini o'rganishga bag'ishlangan. Ayni paytda u talabalarni mantiqiy fikrlashga, to'g'ri xulosa chiqarishga, matematik madaniyatini oshirishga xizmat qiladi.

Masalalar yechish jarayoni alohida didaktik funksiyani bajarishni ta'kidlash lozim. Qo'llanma talabalarga umumiy o'rta ta'lim maktab, akademik litsey qaralayotgan yechilishi murakkabroq matematik masalalarni, ularni yechish usullarini chuqurroq o`rgatish, talabalarda matematik masalalar yechish bo'yicha umumlashgan ko`nikma va malakalarni shakllantirish va rivojlantirishga yordam beradi.

1.Turli sanoq sistemalarida amallar bajarishga oid misollar yechish. Bo'linish belgilariga oid misollar yechish.

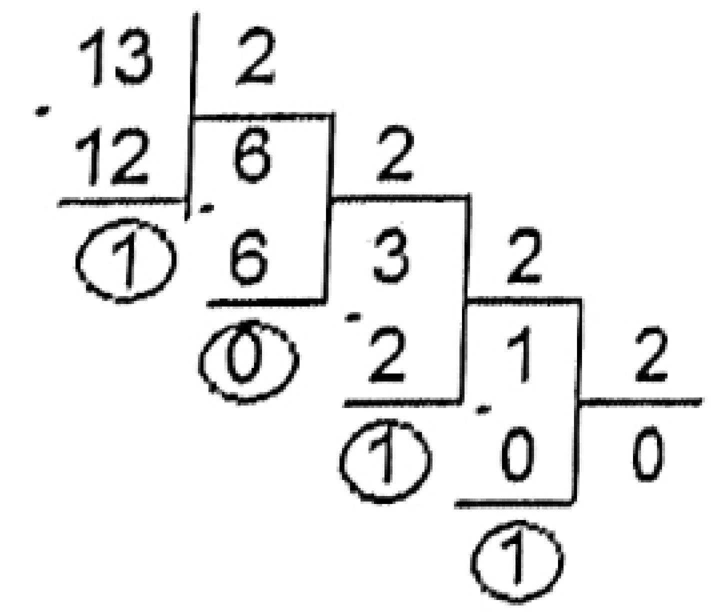
Sonlarni yozish usuliga sanoq sistemasi deb ataladi. Sonlarni yozish uchun har bir sanoq sistemasida o'ziga xos turli belgilar to'plamidan foydalaniladi. Foydalanilgan to'plamdagi belgilar ularning soni, sanoq sistemasini xarakterlovchi asosiy kattaliklardir. Sanoq sistemasida foydalaniladigan belgilar soni sanoq sistemasining asosini tashkil etadi. Berilgan sanoq sistemasida sonlarni yozishdagi foydalanilgan belgilar soniga qarab, o'nlik, ikkilik, sakkizlik, o'n oltilik va boshqa sanoq sistemalarni kiritish mumkin. Shu bilan birga sanoq sistemalarini *pozitsion* va *nopozitsion* turlarga ajratish mumkin. Pozitsion sanoq sistemasida berilgan sonning qiymati sonni tasvirlovchi raqamlarning egallagan o'rniga bog'liq bo'ladi. Misol sifatida, 0,1,2,3,. . . ,9 arab raqamlaridan tashkil topgan o'nlik sanoq sistemani qarash mumkin. Nopozitsion sanoq sistemalarida, belgining qiymati uning egallagan o'rniga bog'liq emas. Misol sifatida rim raqamlari sanoq sistemasini keltirish mumkin. Masalan, XX sonida X raqami, qayerda joylashganiga qaramasdan o'nlik sanoq sistemasidagi 10 qiymatini anglatadi. Quyidagi jadvalda o'nlik sanoq sistemasida berilgan 1 dan 16 gacha sonlarning ikkilik, sakkizlik va o'n oltilik sanoq sistemalaridagi ko'rinishi keltirilgan.

SANOQ SISTEMALARI							
2	**3**	**4**	**5**	**6**	**8**	**10**	**16**
0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1
10	2	2	2	2	2	2	2
11	10	3	3	3	3	3	3

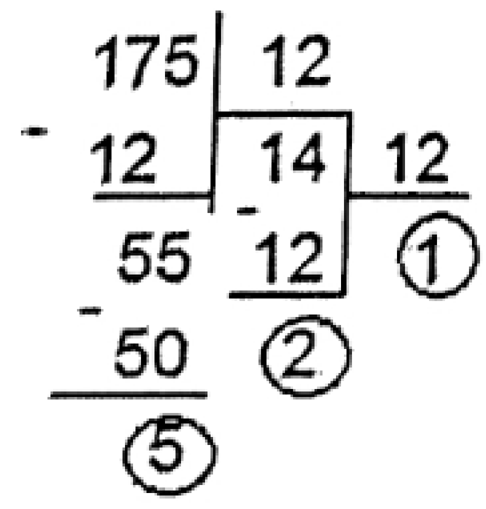
100	11	10	4	4	4	4	4
101	12	11	10	5	5	5	5
110	20	12	11	10	6	6	6
111	21	13	12	11	7	7	7
1000	22	20	13	12	10	8	8
1001	100	21	14	13	11	9	9
1010	101	22	20	14	12	10	?
1011	102	23	21	15	13	11	?
1100	110	30	22	20	14	12	?
1101	111	31	23	21	15	13	?
1110	112	32	24	22	16	14	?
1111	120	33	30	23	17	15	F
10000	121	100	31	24	20	16	10

Bu jadval bo'yicha bir sanoq sistamasidan ikkinchisiga o'tish masalasini ko'rib o'taylik. Masalan: 10 lik sanoq sistemasidagi 13 soniga 8 lik sanoq sistemasida 15 soni mos keladi va u 13 ni 8 ga bo'linganda hosil bo'lgan butun son 1 va qoldiq 5 lardan tashkil topgan. Xuddi shuningdek 13 ni 6 ga bo'lganda hosil bo'luvchi butun son 2 va qoldiq 1 lar 21 sonini hosil qiladi. Bu son 13 sonining 6 lik sanoq sistemasidagi qiymatidir.

Odatda biror X sonining qaysi sanoq sistemasiga tegishliligini ko'rsatish uchun uning pastida indeks sifatida zarur sanoq sistemasining asosi ko'rsatiladi. Masalan, X_6 – X sonining 6 lik sanoq sitemasiga tegishli ekanligini ko'rsatadi.



X_{10}=13 sonining X_2-ikkilik sanoq sistemasidagi ko'rinishini topaylik. Yuqoridagidek, 13 ni ketma-ket 2 ga bo'lamiz va bo'lishni to butun qismida nol hosil bo'lguncha davom ettiramiz. O'ngdan chapga tartibida yozilgan qoldiqlar, ya'ni 1101 soni $X_{10}=13_{10}$ sonining ikkilik sanoq sistemasidagi ko'rinishi bo'ladi.



Endi 8 lik sanoq sistemasidan 10 lik sanoq sistemasiga bo'lish yo'li bilan o'tishga doir *misol*lar ko'raylik. Masalan, jadval bo'yicha 15_8 ga 13_{10} mos keladi. Endi uni topib kuraylik, buning uchun 15_8 ni 10 lik sanoq sistemasining asosi–10 ning 8 lik sanoq sistemasidagi ko'rinish – 12 ga bo'lish kerak bo'ladi. 158 ni 128 ga bo'lsa butun qismida 1 va qoldiqda 3, ya'ni 13_{10} – hosil bo'ladi. Bunga jadval orqali ishonch hosil qilish ham mumkin.

Ikkinchi *misol*: 175_8 sonini 10 lik sanoq sistemasidagi ko'rinishini topish talab qilingan bo'lsin. Xuddi yuqoridagidek 175_8 ni 12_8 ga ketma-ket bo'lamiz. Eslatib o'tamiz, bo'lish amali 8 sonlik sanoq sistemasida olib boriladi.

R sanoq sistemasida berilgan sonni Q sanoq sistemasiga o'tkazish uchun, R sanoq sistemasidagi X soni Q sanoq sistemasining asosiga, ya'ni Q ga ketma-ket, to butun qismida 0 hosil bo'lguncha davom ettirish kerak. Qoldiqlar o'ngdan chapga karab ketma-ket yozilsa, R sanoq sistemasida berilgan Xr sonining Q sanoq sistemasidagi Xq ko'rinishi hosil bo'ladi. Bo'lish amali berilgan R sanoq sistemasida amalga oshiriladi.

Ba'zi bir sanoq sistemalaridan ikkinchisiga qulayroq, osonroq holda o'tish imkoniyatlari mavjud. Xususiy holda, 2 ga karrali sonlarning biridan 2 ikkinchisiga o'tish qoidasini ko'rib o'tamiz.

Masalan, 8 lik sanoq sistemasida berilgan X_8=5361 sonidan X_2 ga bo'lish uchun, X_8 ning har bir raqamini 2 likdagi ko'rinishi-triadalar (2^3=8) bilan almashtirib chiqamiz:

$$X_2 = \underbrace{101}_{5}\,\underbrace{011}_{3}\,\underbrace{110}_{6}\,\underbrace{001}_{1}$$

D8A2$_{16}$ni 2 lik sanoq sistemasiga o'tkazish uchun uning har bir raqamini 2 lik sanoq sistemasidagi to'rtliklar-tetradalar bilan

almashtiramiz:

$$X_2 = \underbrace{1101}_{D}\,\underbrace{1000}_{8}\,\underbrace{1010}_{A}\,\underbrace{0010}_{2}$$

Ikkilik sanoq sistemasida berilgan sondan 8 lik sanoq sistemasiga o'tish uchun, uning o'ng tomonidan boshlab har bir uchliklarni (triadalarni) 8 likdagi mos raqamlar bilan almashtiramiz. Masalan

$$X_2 = \underbrace{001}_{1}\,\underbrace{010}_{2}\,\underbrace{010}_{2}\,\underbrace{101}_{5} = 1225_8$$

Yuqoridagi X_2 sonini 16 lik sanoq sistemasiga o'tkazish uchun X_2 ni o'ng tomondan boshlab to'rtliklar (tetradalar) bilan almashtiramiz.

$$X_2 = \underbrace{0010}_{2}\,\underbrace{1001}_{9}\,\underbrace{0101}_{5} = 295_8$$

Endi, ixtiyoriy sanoq sistemasidan o'nlik sanoq sistemasiga o'tishning xususiy qoidasini ko'rib o'tamiz.

Sakkizlik sanoq sistemasida berilgan sonning 175_8o'nlik sanoq sistemasidagi ko‘rinishini X_{10} topish talab etilsin. Buning uchun berilgan sonning 8 lik sanoq sistemasidagi yoyilmasini yozib olamiz.

$$X_8 = 175_8 = (1 \cdot 10^2 + 7 \cdot 10^1 + 5 \cdot 10^0)_8$$

va 8 lik sanoq sistemasida $10_8 = 8$ ekanligini hisobga olib topamiz.

$$X_8 = (1 \cdot 8^2 + 7 \cdot 8^1 + 5 \cdot 8^0) = (64 + 56 + 5)_{10} = 125_{10}$$

Xuddi yuqoridagilardek, quyidagi *misol*larni ham qurish mumkin:

$$X_{16} = AB_{16} = (A \cdot 10^1 + B \cdot 10^0)_{16} = (10 \cdot 16^1 + 11 \cdot 16^0)_{10} = (160 + 11)_{10} = 171_{10}$$
$$X_6 = 154_6 = (1 \cdot 10^2 + 5 \cdot 10^1 + 4 \cdot 10^0)_6 = (1 \cdot 6^2 + 5 \cdot 6^1 + 4 \cdot 6^0)_{10} = 70_{10}$$

Shu paytgacha biz butun sonlarni bir sanoq sistemasidan boshqasiga o‘tkazish bilan shug‘ullandik. Kasr sonlarni bir sanoq sistemasidan ikkinchisiga o‘tkazish uchun, uning butun qismi yuqorida keltirilgan qoida, ya’ni bo‘lish asosida amalga oshiriladi. Kasr qismini R sanoq sistemasidan Q sanoq sistemasiga o‘tkazish uchun kasr sonni Q ga ketma-ket ko‘paytirishda hosil bo‘lgan sonning butun kismlari ketma-ketligi, berilgan son kasr qismining Q sanoq sistemasidagi ko‘rinishini hosil qiladi. Misol sifatida o‘nlik sanoq sistemasida berilgan X_{10}=25,205 sonini 8 lik sanoq sistemasiga o‘tkazaylik. Berilgan sonning butun qismi- 25_{10} sakkizlik sanoq sistemasida 41_8 ga teng. Endi kasr qismi 0,205 ni 8 lik sanoq sistemasiga o‘tkazamiz. Buning uchun uni ketma-ket 8 ga ko‘paytiramiz va hosil bo‘lgan butun qismini chiziqning chap tomoniga o‘tkazamiz.

0	0.205 8
1	0,640 8
5	0,040 8
0	0,320 8
2	0,560

0,205 ni 8 ga ko‘paytirganimizda 1,640 hosil bo‘ladi va uning butun qismini chiziqning chap tomoniga o‘tkazamiz. Keyin 0,640 yana 8 ga ko‘paytiramiz va hosil bo‘lgan 5,040 sonining butun qismini chiziqning chap tomoniga o‘tkazamiz.

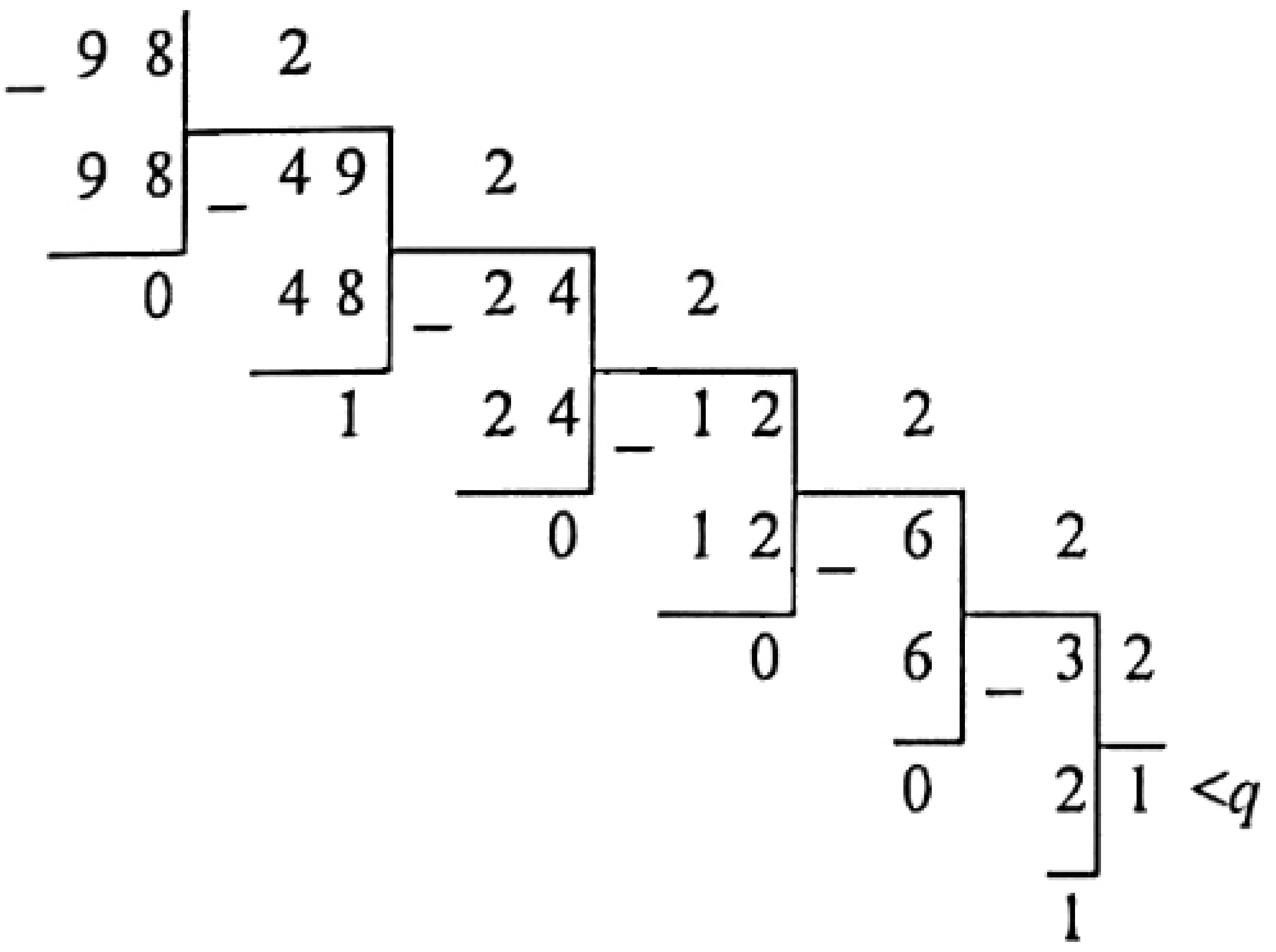
Ko'paytirishni shu tarzda davom ettiramiz natijada 0,15028 sonini hosil qilamiz va butun qismini 41_8 ni hisobga olib, berilgan X_{10}=25,205 sonini 8 lik sanoq sistemasidagi ko'rinishini topamiz:

O'nlik sanoq sistemadan ikkilikga , sakkizlikga, o'n oltilikga o'tkazish.

Butun o'nlik sonni ikkilik sanoq sistemaga o'tkazish uchun berilgan soni ikkiga bo'lib birinchi qoldiq topiladi.qeyin bo'linmani yana ikkiga bo'lib ikkinchi qoldiq va shu tariqa bo'lish jarayoni ikkidan kichik bo'lgan birinchi qoldikgacha davom etadi va bu qoldiq sonning ikkilik sanoq sistemaga o'tgan eng kata raqamini aniqlaydi, teskari tartibda yozilgan qoldiqlar esa ikkilik sonning qolgan razryadlarini aniqlaydi. Shu tartibda butun sonni sakkizlikga,o'n oltilikka o'tkaziladi. Bu xolatlarda sonni sakkiz va o'n oltiga bo'lish zarur bo'ladi.

To'g'ri kasrni boshqa sanoq sistemaga o'tkazish quyidagicha amalga oshiriladi: Sonning kasr qismini yangi sanoq sistema asosiga ko'paytiriladi, Hosil bo'lgan kasr qismining katta xonasini beradi, ko'paytmaning kasr qismini yana yangi sanoq sistema asosiga ko'paytirish kerak. Hosil bo'lgan ko'paytmaning butun qismi qidirilayotgan son kasr qismining keyingi xonasini beradi. Shu tariqa jarayon kasr sonning butun qismining *p* raqami topilguncha takrorlanaveradi. Natija raqamlarni yuqoridan pastga qarab o'qish orqali olinadi.

1- Misol: 98_{10} sonni ikkili sanoq sistemasiga quydagicha o'tkaziladi.



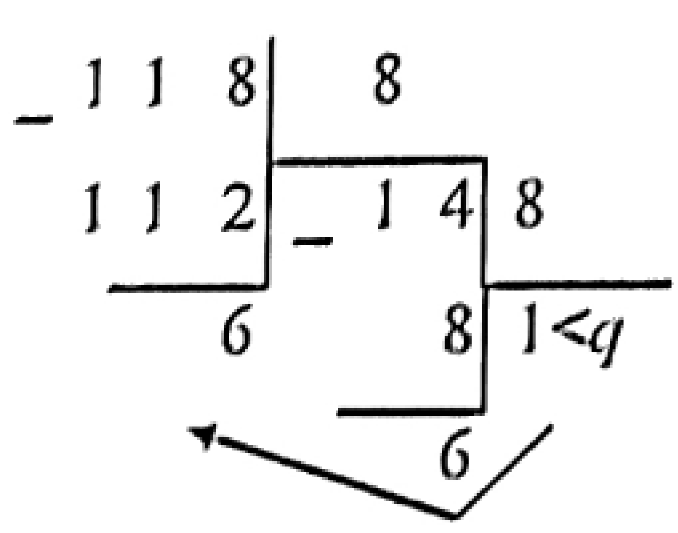
Javob: $98_{10} = 1100010_2$

2-Misol : 0,41 o‘nli soni ikkilik sanoq sistemasiga quyidagicha o‘tkaziladi.

0,41
× 2
0,82
× 2
1,64
× 2
1,28
× 2
0,56
× 2
1,12
× 2
0,24
× 2
0,48

$0{,}41_{10} = 0{,}0110100_2$

3-Misol: 118 o‘nlik soni sakkizlik sanoq sistemasiga quyidagicha o‘tkaziladi.



$1\ 1\ 8_{10} = 1\ 6\ 6_8$

4-misol. O'nlik sanoq sistemasida berilgan 54 sonini ikkilik sanoq sistemasiga o'tkazing.

Yechilishi:

54 | 2   Javob: $54 = 110110_2$

54   27 | 2

0   26   13 | 2

1   12   6 | 2

1   6   3 | 2

0   2   1

1

5 – misol. $111101100111_2 = X_{10}$; X=?

Buning uchun 111'101'100'111 ga ajratib jadvaldan foydalanamiz.

$111_2=7_{10}$

$100_2=4_{10}$

$101_2= 5_{10}$

$111_2= 7_{10}$

Demak, $111101100111_2=7547_{10}$

6 – misol. O'nlik sanoq sistemasida berilgan $875_{(10)}$ sonini o'n oltilik sanoq sistemasiga o'tkazing.

Yechilishi:

875 | 16

864   54 | 16

11   48   3

6

Demak, $875_{(10)}=36B_{(16)}$

Kasr sonlarni bir sanoq sistemasidan ikkinchisiga o'tkazish uchun, berilgan sonni o'tkaziladigan sanoq sistemasining asosiga ko'paytiramiz. Natijada hosil bo'lgan butun sonlar o'tkaziladigan sanoq sistemasidagi songa teng bo'ladi.

7 – misol. O'nlik sanoq sistemasida berilgan $0,624_{(10)}$ sonini ikkilik sanoq sistemasiga o'tkazing.

Yechilishi:

0, 624 x 2=

1| 248 x 2=

0| 946 x 2=

0| 992...

Demak, $0,624_{(10)}=0,100_{(2)}$

8– misol. O'nlik sanoq sistemasida berilgan 0,546 sonini sakkizlik sanoq sistemasiga o'tkazing.

Yechilishi:

0, 546 x 8 =

4 | 368 x 8=

2 | 944 x 8=

7 | 552...

Demak, $0,546_{(10)}=0,427_{(8)}$

9 – misol. O'nlik sanoq sistemasida berilgan 0,29 sonini o'n oltilik sanoq sistemasiga o'utkazing.

Yechilishi:

4 | 64 x 16=

1 | 0,29 x 16=

0 | 24 x 16=

3 | 84 ...

Demak, $0{,}29_{(10)}=0{,}4A3_{(16)}$

Sakkizlik sanoq sistemasida berilgan sonni ikkilik sanoq sistemasiga o'tkazish uchun har bir sakkizlik son unga ekvivalent bo'lgan uchta ikkilik son (triada)ga almashtiriladi

10 – misol. Sakkizlik sanoq sistemasida berilgan $50721{,}621_{(8)}$ sonini ikkilik sanoq sistemasiga o'tkazing.

Yechilishi:

5 0 7 2 1, 6 2 $1_{(8)}$

| | | | | | | |

101 000 111 010 001, 110 010 $001_{(2)}$

Demak, $50721{,}621_{(8)}$=101 000 111 010 001,110 010 $001_{(2)}$

Ikkilik sanoq sistemasida berilgan sonni sakkizlik sanoq sistemasiga o'tkazish uchun har bir uchta ikkilik son (triada) unga ekvivalent bo'lgan bitta sakkizlik songa almashtiriladi Aralash sonlarda (noto'g'ri kasrlarda) triada uchun sonlar yetishmasa, unung chap (butun qismining oldini) va o'ng (kasr qismining oxirini) tomonlarini nollar bilan to'ldiramiz.

11–misol. Ikkilik sanoq sistemasida berilgan

$1110111010100101{,}1101110010_{(2)}$ sonini sakkizlik sanoq sistemasiga o'tkazing.

Yechilishi:

Triadaga to'ldirilgan nollar

001 110 111 010 100 101 , 110 111 001 $000_{(2)}$

| | | | | | | | | |

1 6 7 2 4 5 , 6 7 1 $0_{(8)}$

Demak, $1110111010100101{,}1101110010_{(2)}=167245{,}6710_{(8)}$

[illegible] kasrni ikkilik sanoq sistemasiga o'tkazish uchun har bir o'n oltilik son unga ekvivalent bo'lgan to'rtta ikkilik son (tetradaga) almashtiriladi.

12-misol. O'n oltilik sanoq sistemasida berilgan $15S,16D_{(16)}$ sonini ikkilik sanoq sistemasiga o'tkazing.

Yechilishi:

1 5 C , 1 6 $D_{(16)}$

| | | | | |

0001 0101 1100, 0001 0110 $1101_{(2)}$

Demak, $15S,16D_{(16)}=101011100,000101101101_{(2)}$

Ikkilik sanoq sistemasida berilgan noto'g'ri kasrni o'n oltilik sanoq sistemasiga o'tkazish uchun har bir (o'n oltilik songa ekvivalent bo'lgan) to'rtta ikkilik son (tetrada) unga ekvivalent bo'lgan bitta o'n oltilik songa almashtiriladi. Agar berilgan sonlarda tetrada uchun sonlar yetishmasa, uning chap (butun qismining oldini) va o'ng (kasr qismining oxirini) tomonlarini nollar bilan to'ldiramiz.

13-misol. Ikkilik sanoq sistemasida berilgan $10111111000111,111100101_{(2)}$ sonini o'n oltilik sanoq sistemasiga o'tkazing.

0010 1111 1100 0111, 1111 0010 $1000_{(2)}$

| | | | | | |

2 F C 7, F 2 $8_{(16)}$

Demak, $10111111000111,111100101_{(2)}=2FC7,F28_{(16)}$

Har qanday sanoq sistemasida berilgan sonni o'nli sanoq sistemasiga o'tkazish uchun yuqorida aytib o'tilgan polinomdan foydalanamiz. Masalan,

$175,61_{(8)}=1\cdot 8^2+7\cdot 8^1+5\cdot 8^0+6\cdot 8^{-1}+1\cdot 8^{-2}==64+56+5+0,75+0,015625=125,765625_{(10)}$

$1101,11_{(2)}=1\cdot 2^3+1\cdot 2^2+0\cdot 2^1+1\cdot 2^0+1\cdot 2^{-1}+1\cdot 2^{-2}= =8+4+0+1+0,5+0,25=13,75_{(10)}$

$A1F,96_{(16)}=10\cdot16^{2}+1\cdot16^{1}+15\cdot16^{0}+9\cdot16^{-1}+6\cdot16^{-2}$

$==2560+16+1+0,625+0,0234375=2591,6484_{(10)}$

Mashqlar

1. Ikkilik sanoq sistemasida berilgan sonlar ustida qo'shish amalini bajaring.

a) 101+111	b) 1011+110	g) 10,101+11,111
c) 1101+110	d) 1010+1111	h) 110,01+11,0101
e) 1111+1011	f) 11,011+101,01	i) 111,01+111

2. Ikkilik sanoq sistemasida berilgan sonlar ustida ayirish amalini bajaring.

a) 1010-110	b) 1110-101	g) 10101-111,11
c) 11011,11-101,01	d) 1011-11,11	h) 110,01-11,0101
e) 10010,01-111,1	f) 11011-101,01	i) 1000,01-111

3. Ikkilik sanoq sistemasida berilgan sonlar ustida ko'paytirish amalini bajaring.

a) 101×110	b) 110×101	g) 101×11
c) 1101×10,01	d) 1011,01×101	h) 110,01×1,101
e) 10010,01×111,1	f) 111×11,101	i) 10101×101,011

2. Bajarilgan amallardan qaysi biri noto'g'ri?

a) 101-11=11	b) 111010+10=111100
d) 11100+11=100111	e) 11×11=1001
f) 1001-11=100	g) 11111×1010=100110110
h) 110011,001-1,011=111110,1	i) 1110,01+1,01=111110
j) 11001,1-110,11=10010,11	k) 101×1110=10101100
l) 100,101-1,010=11,011	m) 110100-1101=100

5. O'tkazishni bajaring:

a) $101111101_2\rightarrow?_{10}$ b) $1110000_2\rightarrow?_{10}$ c) $6317_{10}\rightarrow?_2$

d) $1190_{10}\to ?_2$ e) $909_{10}\to ?_2$ f) $1236_{10}\to ?_2$

g) $11011_{10}\to ?_2$ h) $11011_2\to ?_{10}$ i) $101011011_2\to ?_{10}$

j) $6702_{10}\to A_8\to A_2\to A_{16}$ k) $1101110_2 > \to A16\to A8$,

6.Amallarni bajaring.

a)6508+638, b)111112+1012, c) 1A9B16+52C316, в)CEA16-9EC16.

2.Qoldiqli bo`lish. EKUB va EKUK. Arifmetikaning asosiy teoremasi. Natural sonlarning kanonik yoyilmasi.

Butun sonlarning bo'linishi deganda biz qoldiqli va qoldiqsiz bo'lishni tushunamiz.

a va b butun sonlar berilgan bo'lsin. Agar ularning birini ikkinchisiga bo'lsak, $a=bq+r;\ 0\le r<b$ hosil bo'ladi, bu yerda a - bo'linuvchi, b - bo'luvchi, q - bo'linma, r - qoldiq deyiladi. Agar $r\ne 0$ bo'lsa, qoldiqli bo'lishga, agar $r=0$ bo'lsa, qoldiqsiz bo'lishga ega bo'lamiz. 2, 3, 4, 5, 9, 10 ga bo'linish belgilari (alomatlari) mavjud bo'lib, ulardan masala yoki misollarni yechishda foydalaniladi.

a sonni q ga bo'lganda r_1 qoldiq, b ni q ga bo'lganda r_2 qoldiq qolib, $r_1=r_2$ bo'lsa, u holda a va b sonlar teng qoldiqli sonlar deb ataladi.

Bizga $a,b\in Z$ sonlar berilgan bo'lsa,

$$(a+b)^2=a^2+2ab+b^2=aA_2+b^2;\ A_2=a+2b,$$

$$(a+b)^3=aA_3+b^3,\ \ (a+b)^4=aA_4+b^4,\ldots$$

tengliklardan $(a+b)^n=aA_n+b^n$ ni yoza olamiz.

Agar $b=1$ bo'lsa, $(a+1)^n=aA_n+1$,

agar $n=2k,\ b=-1$ bo'lsa, $(a-1)^n=aA_n+1$, agar $n=2k+1,\ b=-1$ bo'lsa, $(a-1)^n=aA_n-1$ larni hosil qilamiz.

1-teorema. *Agar a son b ga qoldiqsiz bo'linib,* $|b|>|a|$ *bo'lsa, u holda* $a=0$ *bo'ladi.*

2-teorema. *a butun sonning b songa qoldiqsiz bo'linishi uchun* $|a|\ \vdots\ |b|$ *bo'lishi zarur va yetarlidir.*

3-teorema. *Agar* $a_i\ \vdots\ b,\ i=\overline{1,n},\ a_i\in N$ *bo'lsa, u holda*

$\sum_{i=1}^{n} a_i \vdots b$ bo'ladi.

1-misol. 5^{10} ni 4 ga bo'lgandagi qoldiqni toping.

Yechish. $5^{10} = (4+1)^{10} = 4A_{10}+1$, demak, qoldiq $r=1$ bo'lar ekan.

2-misol. $(3^{198}-7^{17})$ ayirmani 2 ga bo'lgandagi qoldiqni toping.

Yechish. $3^{198}-7^{17} = (2+1)^{198}-(6+1)^{17} = 2A_{198}+1-6A_{17}-1 = 2A_{198}-6A_{17}$, bundan qoldiq $r=0$ ga teng ekani kelib chiqadi.

1-masala. Yig'indini 7 ga bo'lgandagi qoldiqni toping 1995 + 1996 + 1997 + 1998 + 1999

Yechish.

1995 = 7•285, unda 1995≡0 (mod 7), demak, 1995 + 1996 + 1997 + 1998 + 1999≡0 + 1 + 2 + 3 + 4 = 10≡3 (mod 7).

Qoldiq 3 ga teng.

2-masala. Ko'paytmani 7 ga bo'lgandagi qoldiqni toping. 1995•1996•1997•1998•1999.

Yechish. 1995≡0 (mod 7), unda 1995•1996•1997•1998•1999≡0•1•2•3•4=0(mod 7).

Qoldiq 0 ga teng ekan.

3-masala. Ko'paytmani 7 ga bo'lgandagi qoldiqni toping. 1996•1997•1998•1999•2000•2001.

Yechish. 1996•1997•1998•1999•2000•2001≡ 1•2•3•4•5•6 = 720≡20≡6 (mod 7).

Qoldiq 7 ga teng.

1•2•3•4, 1•2•3•4•5, 1•2•3•4•5•6, . . . ko'rinishdagi ko'paytmalar tez-tez uchrab turadi.

Ular 1•2•3•4=4!, 1•2•3•4•5=5! Ko'rinishda yoziladi. O'qilishi: 4-faktorial, 5-faktorial va h.k. Umuman, 1•2•3•...•n = n! (n-faktorial).

Ta'kidlash lozimki, berilgan sonni 10 ga bo'lgandagi qoldiq shu sonning oxirgi raqamidir.

Masalan. $21\equiv 1 \pmod{10}$, $134\equiv 4 \pmod{10}$.

Talabalarga oxirgi raqamni topishga doir masalalarni yechishda quyidagi foydali "jadvalni" tavsiya qilish mumkin:

$1^k\equiv 1 \pmod{10}$	$5^k\equiv 5 \pmod{10}$	$6^k\equiv 6 \pmod{10}$	
$4^{2k}\equiv 6 \pmod{10}$	$4^{2k+1}\equiv 4 \pmod{10}$	$9^{2k}\equiv 1 \pmod{10}$	$9^{2k+1}\equiv 9 \pmod{10}$
$2^{4k}\equiv 6 \pmod{10}$	$3^{4k}\equiv 1 \pmod{10}$	$7^{4k}\equiv 1 \pmod{10}$	$8^{4k}\equiv 6 \pmod{10}$
$2^{4k+1}\equiv 2 \pmod{10}$	$3^{4k+1}\equiv 3 \pmod{10}$	$7^{4k+1}\equiv 7 \pmod{10}$	$8^{4k+1}\equiv 8 \pmod{10}$
$2^{4k+2}\equiv 4 \pmod{10}$	$3^{4k+2}\equiv 9 \pmod{10}$	$7^{4k+2}\equiv 9 \pmod{10}$	$8^{4k+2}\equiv 4 \pmod{10}$
$2^{4k+3}\equiv 8 \pmod{10}$	$3^{4k+3}\equiv 7 \pmod{10}$	$7^{4k+3}\equiv 3 \pmod{10}$	$8^{4k+3}\equiv 2 \pmod{10}$

Misollarda bu taqqoslamalardan kelib chiqadigan xulosalarni ko'rish mumkin:

$7\equiv 7 \pmod{10}$;

$7^2 = 49\equiv 9 \pmod{10}$;

$7^3 = 7^2\cdot 7\equiv 9\cdot 7 = 63\equiv 3 \pmod{10}$;

$7^4 = 7^3\cdot 7\equiv 3\cdot 7 = 21\equiv 1 \pmod{10}$.

Bularni davom ettirsak qoldiqlar takrorlanadi: 7 sonining 4 ga karrali darajalarida qoldiq 1; 7 ning 4 ga bo'lganda 1 qoldiq qoladigan darajalarida qoldiq 7; 7 ning 4 ga bo'lganda 2 qoldiq qoladigan darajalarida qoldiq 9; 7 ning 4 ga bo'lganda 3 qoldiq qoladigan darajalarida qoldiq 3.

Masala – 1. 137^{100} ning oxirgi raqamini toping?

Yechilishi. $137\equiv 7 \pmod{10}$, $137^{100}\equiv 7^{100} = 7^{25\cdot 4}\equiv 1 \pmod{10}$ larni yozish mumkin.

Oxirgi raqami 1 ekan.

Masala – 2. Quyidagi sonlarning oxirgi raqamlarini toping: 7^7, 77^{77}, 2^{100}, 3^{1999}, 19^{100}, 1999^{1999}.

Yechilishi.

$7^7 = 7^{4\cdot 1+3}\equiv 3 \pmod{10}$,

$77^{77}\equiv 7^{77} = 7^{4\cdot 19+1}\equiv 7 \pmod{10}$,

$2^{100} = 2^{4\cdot25} \equiv 6 \pmod{10}$,

$3^{1999} = 3^{4\cdot499+3} \equiv 7 \pmod{10}$,

$19^{100} \equiv 9^{100} \equiv 1 \pmod{10}$.

$1999^{1999} \equiv 9^{1999} \equiv 9 \pmod{10}$.

Demak, mos ravishda oxirgi raqamlari 3, 7, 6, 7, 1, 9 bo'lar ekan.

Masala – 3.

a) $1998^{1998} + 1999^{1999}$ yig'indini 3 ga bo'lgandagi qoldiqni toping.
b) $1998^{1998} + 1999^{1999}$ yig'indining oxirgi raqamini toping.

Yechilishi.

a) $1998^{1998} + 1999^{1999} \equiv 0^{1998} + 1^{1999} = 0 + 1 \equiv 1 \pmod 3$. Қолдиқ 1 га тенг экан.
б) $1998^{1998} + 1999^{1999} \equiv 8^{1998} + 9^{1999} = 8^{4\cdot499+2} + 9^{2\cdot999+1} \equiv 4 + 9 = 13 \equiv 3 \pmod{10}$.

Oxirgi raqami 3 ekan.

Boshqacharoq misollar ko'raylik.

Masala – 4. $n^3 - n$ ifoda n ning barcha natural qiymatlarida 6 ga bo'linishini isbotlang.

Yechilishi. Tasdiqning to'g'riligini n ning ba'zi qiymatlarida tekshirib ko'raylik. Masalan,

$n = 1$ da $1^3 - 1 = 0 \vdots 6$,

$n = 2$ da $2^3 - 2 = 8 - 2 = 6 \vdots 6$,

$n = 10$ da $10^3 - 10 = 1000 - 10 = 990 \vdots 6$.

Endi tasdiqni isbotlaymiz. Sonni 6 ga bo'lganda qolishi mumkin bo'lgan qoldiqlar: 0, 1, 2, 3, 4, 5.

U holda

$n \equiv 0 \pmod 6$, bundan $n^3 - n = 0^3 - 0 = 0 \pmod 6$,

$n \equiv 1 \pmod 6$, bundan $n^3 - n = 1^3 - 1 = 0 \pmod 6$,

$n \equiv 2 \pmod 6$, bundan $n^3 - n = 2^3 - 2 = 6 \equiv 0 \pmod 6$,

$n \equiv 3 \pmod 6$, bundan $n^3 - n = 3^3 - 3 = 24 \equiv 0 \pmod 6$,

$n \equiv 4 \pmod 6$, bundan $n^3 - n = 4^3 - 4 = 60 \equiv 0 \pmod 6$,

$n \equiv 5 \pmod 6$, bundan $n^3 - n = 5^3 - 5 = 120 \equiv 0 \pmod 6$.

Bulardan $n^3 - n$ ifoda n ning ixtiyoriy natural qiymatida 6 ga bo'linar ekan

Masala – 5. $n^2 + 3n$ ko'rinishdagi sonni 7 ga bo'lganda qolishi mumkin bo'lgan barcha qoldiqlarni yozing.

Yechilishi. 7 ga bo'lganda 0, 1, 2, 3, 4, 5, 6 qoldiq bo'lishi mumkin. U holda:

$n \equiv 0 \pmod 7$, $n^2 + 3n = 0^2 + 3 \cdot 0 = 0 \pmod 7$,

$n \equiv 1 \pmod 7$, $n^2 + 3n = 1^2 + 3 \cdot 1 = 4 \pmod 7$,

$n \equiv 2 \pmod 7$, $n^2 + 3n = 2^2 + 3 \cdot 2 = 10 \equiv 3 \pmod 7$,

$n \equiv 3 \pmod 7$, $n^2 + 3n = 3^2 + 3 \cdot 3 = 18 \equiv 4 \pmod 7$,

$n \equiv 4 \pmod 7$, $n^2 + 3n = 4^2 + 3 \cdot 4 = 16 + 12 = 28 \equiv 0 \pmod 7$,

$n \equiv 5 \pmod 7$, $n^2 + 3n = 5^2 + 3 \cdot 5 = 25 + 15 = 40 \equiv 5 \pmod 7$,

$n \equiv 6 \pmod 7$, $n^2 + 3n = 6^2 + 3 \cdot 6 = 36 + 18 = 54 \equiv 5 \pmod 7$.

Qolishi mumkin bo'lgan qoldiqlar: 0, 3, 4, 5.

Mustaqil yechish uchun taklif qilinadigan misollar:

1. Oxirgi raqamni toping:

a) $11^6 + 14^5 + 16^6$;

b) $11 \cdot 13 \cdot 15 \cdot 17 \cdot 19$.

2. 4 ga bo'lgandagi qoldiqni toping:

a) $11^6 + 14^5 + 16^6$;

б) $11 \cdot 13 \cdot 15 \cdot 17 \cdot 19$.

3. $n^3 + 2n$ ifoda n ning ixtiyoriy natural qiymatida 3 ga bo'linishini isbotlang.

Yechilishi.

1.

a) $11^6 + 14^5 + 16^6 \equiv 1^6 + 4^5 + 6^6 \equiv 1 + 6 + 6 = 13 \equiv 3 \pmod{10}$;

б) $11 \cdot 13 \cdot 15 \cdot 17 \cdot 19 = 1 \cdot 3 \cdot 5 \cdot 7 \cdot 9 \equiv 5 \pmod{10}$.

2.

а) $11^6 + 14^6 + 16^6 \equiv 3^6 + 2^6 + 0^6 = 3^4 \cdot 3^2 + 64 \equiv 1 \cdot 9 + 0 = 9 \equiv 1 \pmod 4$;

б) $11 \cdot 13 \cdot 15 \cdot 17 \cdot 19 \equiv 3 \cdot 1 \cdot 3 \cdot 1 \cdot 3 = 27 \equiv 3 \pmod 4$.

3.

$n \equiv 0 \pmod 3$, $n^3 + 2n \equiv 0^3 + 2 \cdot 0 \equiv 0 \pmod 3$,

$n \equiv 1 \pmod 3$, $n^3 + 2n \equiv 1^3 + 2 \cdot 1 = 3 \equiv 0 \pmod 3$,

$n \equiv 2 \pmod 3$, $n^3 + 2n \equiv 2^3 + 2 \cdot 2 = 8 + 4 = 12 \equiv 0 \pmod 3$.

Bulardan ko'rinadiki, $n^3 + 2n$ ifoda n ning ixtiyoriy natural qiymatida 3 ga bo'linar ekan.

Talabalarning bilimini tekshirish uchun misollar.

Variant 1 Variant 2

Oxirgi raqamni toping:

1.	666^{666}	
2.	1999^{1998}	
3.	5!	

1.	444^{444}	
2.	9991^{8991}	
3.	6!	

Bo'lishdagi qoldiqni toping:

4.	13n+5 ni	13	
5.	20n+23 ni	4	

4.	29n+5 ni	29	
5.	20n+23 ni	5	

Talabalar bu misollarga javob yozib topshirishadi. Bularni o'qituvchi tekshirguncha talabalar quyidagi misollarni yechib turishadi.

Variant 1

$1998^{1999} + 1999^{1998}$ ni 9 ga bo'lgandagi qoldiqni toping.

Bu son 3 ga to'linadimi? Aks holda 3 ga bo'lgandagi qoldiqni toping?

Variant 2

$1998^{1999} + 1999^{1998}$ ning oxirgi raqamini toping.

Bu son 5 ga bo'linadimi? 2ga-chi?

Umumiy fikrlarni birlashtirib ba'zi misollarni yechamiz.

1. $n(n+1) \vdots 2$ ni isbotlang.

I usul.

$n \equiv 0 \pmod 2$, $n(n+1) \equiv 0(0+1) = 0 \pmod 2$;
$n \equiv 1 \pmod 2$, $n(n+1) \equiv 1(1+1) = 2 \equiv 2 \pmod 2$.

II usul. Ikkita ketma-ket kelgan natural sonlardan n va $(n+1)$ bittasi aniq juft, bundan, ularning ko'paytmasi $n(n+1)$ juft ekanini kelib chiqadi.

2. $n(n+1)(n+2) \vdots 3$ ni isbotlang.

Ikkita talaba yuqoridagi usullarda isbotlashadi. O'qituvchi bu misollarga analog misollar tuzishni topshiradi. Talabalar javobi:

Quyidagilarni isbotlang:

$n(n+1)(n+2)(n+3) \vdots 4$;

$n(n+1)(n+2)(n+3)(n+4) \vdots 5$;

$n(n+1)(n+2)(n+3)(n+4)(n+5) \vdots 6$.

Oxirida umumlashtirsak: $n(n+1)(n+2)...(n+m) \vdots (m+1)$.

Xulosa o'rnida $n(n+1) \vdots 2$ va $n(n+1)(n+2) \vdots 3$, 2 va 3 – o'zaro tub sonlar, demak, $n(n+1)(n+2) \vdots 1 \cdot 2 \cdot 3 = 3! = 6$.

$n(n+1)(n+2)(n+3) \vdots 4! = 24$ ni isbotlang.

$n(n+1)(n+2)(n+4) \vdots 5!$ ni isbotlang;

$n(n+1)(n+2)(n+3)(n+4)(n+5) \vdots 6!$ ni isbotlang.

Mustaqil yechish uchun

1. Yig'indining oxirgi raqamini toping:

a) $1^2 + 2^2 + ... + 10^2$;

б) $1^2 + 2^2 + ... + 100^2$.

2. Isbotlang:

а) $n^2 + n \vdots 2$;

б) $n^2 - n \vdots 2$.

3. Toq sonning kvadratini 8 ga bo‘lganda 1 qoldiq qolishini isbotlang.

4. Ikkita ketma-ket natural sonlarning kvadratlari yig‘indisini 4 gu bo‘lganda 1 qoldiq qolishini isbotlang.

Yechilishi.

1.

а) $1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2 = 1 + 4 + 9 + 16 + 25 + 36 + 49 + 64 + 81 + 100 \equiv$

$\equiv 1 + 4 + 9 + 6 + 5 + 6 + 9 + 4 + 1 + 0 = 45 \equiv 5 \pmod{10}$. Охирги рақам 5.

б) $1^2 + 2^2 + .. + 100^2 \equiv 5 \cdot 10 = 50 \equiv 100 \pmod{10}$. Охирги рақам 0.

2.

а) $n \equiv 0 \pmod 2$, $n^2 + n \equiv 0^2 + 0 = \pmod 2$, $n \equiv 1 \pmod 2$, $n^2 + n \equiv 1^2 + 1 = 2 \equiv 0 \pmod 2$.

б) Mustaqil isbotlang.

3. $(2n + 1)^2 \equiv 1 \pmod 8$ ni isbotlang. Quyidagilarni yozamiz:

$n \equiv 0 \pmod 8$, $(2 \cdot 0 + 1)^2 = 1 \equiv 1 \pmod 8$,

$n \equiv 1 \pmod 8$, $(2 \cdot 1 + 1)^2 = 9 \equiv 1 \pmod 8$,

$n \equiv 2 \pmod 8$, $(2 \cdot 2 + 1)^2 = 25 \equiv 1 \pmod 8$,

$n \equiv 3 \pmod 8$, $(2 \cdot 3 + 1)^2 = 49 \equiv 1 \pmod 8$,

$n \equiv 4 \pmod 8$, $(2 \cdot 4 + 1)^2 = 81 \equiv 1 \pmod 8$,

$n \equiv 5 \pmod 8$, $(2 \cdot 5 + 1)^2 = 121 \equiv 1 \pmod 8$,

$n \equiv 6 \pmod 8$, $(2 \cdot 6 + 1)^2 = 169 \equiv 1 \pmod 8$,

$n \equiv 7 \pmod 8$, $(2 \cdot 7 + 1)^2 = 225 \equiv 1 \pmod 8$.

4. Quyidagi sistemani natural qiymatlarda yeching.

$$\begin{cases} x+y=150, \\ D(x,y)=30; \end{cases} \begin{cases} D(x,y)=45, \\ x:y=11:7; \end{cases} \begin{cases} xy=8400, \\ D(x,y)=20; \end{cases}$$

$$\begin{cases} x:y=5:9, \\ D(x,y)=28; \end{cases} \begin{cases} xy=20, \\ K(x,y)=10. \end{cases}$$

3.Butun koeffitsiyentli aniqmas tenglamalarga oid misollar yechish.

Ta'rif. Tarkibida bittadan ortiq o'zgaruvchisi bor tenglama aniqmas tenglama deyiladi.

Ta'rif. Birinchi tartibli ikki x, y o'zgaruvchili aniqmas tenglama $mx + ny = k$, bunda $m, n, k, x, y \in Z$ $k \neq 0$ ko'rinishda bo'ladi.

Tasdiq 1. Agar $mx + ny = k$ tenglamadagi ozod had k m va n larning EKUB iga bo'linmasa, u holda $mx + ny = k$ tenglama butun yechimga ega emas.

Misol: 34x – 17y = 3.

(34; 17) = 17, 3 soni 17 ga bo'linmaydi, demak, tenglamaning butun yechimi yo'q.

Tasdiq 2.

Agar $mx + ny = k$ tenglamada m va n lar o'zaro tub bo'lsa, u holda tenglama kamida bitta yechimga ega.

Tasdiq 3.

Agar $mx + ny = k$ tenglamada m va n lar o'zaro tub bo'lsa, u holda tenglama cheksiz ko'p yechimga ega.

$\begin{cases} x = x_1 + nt, \\ y = y_1 - mt \end{cases}$ bunda $(x_1; y_1)$ – juftlik $mx + ny = k$ tenglamaning biror yechimi.

$t \in Z$

Misollar. 1) 9x – 18y = 5

(9;18)=9

5 soni 9 ga bo'linmaydi. Demak, tenglamaning butun yechimi yo'q.

2) $x + y = xy$

Tanlash orqali ham yechish mumkin.

Javob: (0;0), (2;2)

Tenglamani butun sonlarda yeching $3x - 4y = 1$.

$3x = 4y + 1$ ko'rinishda yozsak. Bu tenglamaning chap tomoni 3 ga bo'linadi, bundan o'ng tomoni ham bo'linishini yozaylik. Uchta hol bo'lishi mumkin:

1. Agar $y = 3m$, $m \in Z$, u holda $4y + 1 = 4 \cdot 3m + 1 = 12m + 1$ ifoda 3 ga bo'linmaydi.
2. Agar $y = 3m + 1$, u holda $4y + 1 = 4 \cdot (3m + 1) + 1 = 12m + 5$ ifoda 3 ga bo'linmaydi.
3. Agar $y = 3m + 2$, u holda $4y + 1 = 4 \cdot (3m + 2) + 1 = 12m + 9$ ifoda 3 ga bo'linadi, shuning uchun ham $3x = 12m + 9$ dan $x = 4m + 3$ kelib chiqadi, bundan $y = 3m + 2$ bo'ladi.

Javob: $\begin{cases} x = 4m+3, \\ y = 3m+2 \end{cases}$, bunda $m \in Z$.

2-darajali aniqmas tenglama

Barcha tipdagi bunday tenglamalarni kvadrat lar ayirmasi yoki ko'paytuvchilarga ajratish yo'li bilan yechiladi.

Misol: Tenglamani butun sonlarda yeching.

$x^2 - 4y^2 = 13$

$(x - 2y)(x + 2y) = 13$

13 – tub son, shuning uchun ham: $13 = 13 \cdot 1 = 1 \cdot 13 = (-1)(-13) = (-13)(-1)$ hollarnigina qarash mumkin

Bu hollarni qarab chiqamiz:

а) $\begin{cases} x-2y=-13, \\ x+2y=1 \end{cases} \Rightarrow \begin{cases} x_1=7 \\ y_1=-3 \end{cases}$

б) $\begin{cases} x-2y=1, \\ x=2y=13 \end{cases} \Rightarrow \begin{cases} x_2=7 \\ y_2=3 \end{cases}$

в) $\begin{cases} x-2y=-13, \\ x+2y=-1 \end{cases} \Rightarrow \begin{cases} x_3=-7 \\ y_3=3 \end{cases}$

г) $\begin{cases} x-2y=-1, \\ x+2y=-13 \end{cases} \Rightarrow \begin{cases} x_4=-7 \\ y_4=-3 \end{cases}$

Javob: (7;-3), (7;3), (-7;3), (-7;-3).

Misol. Tenglamani butun sonlarda yeching:

a) $x^2-y^2=4$

(x - y)(x + y)=4

$\begin{cases} x-y=2 \\ x+y=2 \end{cases}$	$\begin{cases} x-y=1 \\ x+y=4 \end{cases}$	$\begin{cases} x-y=4 \\ x+y=1 \end{cases}$
2x = 4	2x = 5	2x = 5
x = 2	x = 5/2	x = 5/2
y = 0	butunmas	butunmas
$\begin{cases} x-y=-2 \\ x+y=-2 \end{cases}$	$\begin{cases} x-y=-1 \\ x+y=-4 \end{cases}$	$\begin{cases} x-y=-4 \\ x+y=-1 \end{cases}$
2x = -4	butunmas	butunmas
x = -2		
y = 0		

Javob: (-2;0), (2;0).

b) $x^2+xy=10$

Javob: (-10;9), (-5;3), (-2;-3), (-1;-9), (1;9), (2;3), (5;-3), (10;-9).

в) $(x+y)(y-1)=4$

Javob: (2;-3), (-1;-1), (-4;0), (2;2), (-1;3), (-4;5).

Mashqlar.

1) Butun sonlarda yeching.

а) $8x + 12y = 32$	$x = 1 + 3n,\ y = 2 - 2n,\ n \in Z$
б) $7x + 5y = 29$	$x = 2 + 5n,\ y = 3 - 7n,\ n \in Z$
в) $4x + 7y = 75$	$x = 3 + 7n,\ y = 9 - 4n,\ n \in Z$
г) $9x - 2y = 1$	$x = 1 - 2m,\ y = 4 + 9m,\ m \in Z$
д) $9x - 11y = 36$	$x = 4 + 11n,\ y = 9n,\ n \in Z$
е) $7x - 4y = 29$	$x = 3 + 4n,\ y = -2 + 7n,\ n \in Z$
ж) $19x - 5y = 119$	$x = 1 + 5p,\ y = -20 + 19p,\ p \in Z$
з) $28x - 40y = 60$	$x = 45 + 10t,\ y = 30 + 7t,\ t \in Z$

2) Tenglamaning nomanfiy butun yechimlarini toping:

а) $8x + 65y = 81$	$x = 2,\ y = 1$
б) $17x + 23y = 183$	$x = 4,\ y = 5$

3) Quyidagi shartlarni qanoatlantiruvchi barcha (x; y) juftliklarni toping

а) $x + y = xy$	(0;0), (2;2)
б) $x^2 - 3xy + 2y^2 = 3$	(1;2), (5;2), (-1;-1), (-5;-2)

Yechilishi:

$x^2 - 2xy + y^2 - xy + y^2 = 3$
$(x - y)^2 - y(x - y) = 3$
$(x - y)(x - 2y) = 3$

3 sonini ko'paytuvchilarga ajratsak,:

$3 = 1 \cdot 3 = 3 \cdot 1 = (-1) \cdot (-3) = (-3) \cdot (-1)$

а) $\begin{cases} x-y=1 \\ x-2y=3 \end{cases}$ $\begin{cases} y=-2 \\ x=-1 \end{cases}$	б) $\begin{cases} x-y=3 \\ x-2y=1 \end{cases}$ $\begin{cases} y=2 \\ x=5 \end{cases}$	в) $\begin{cases} x-y=-1 \\ x-2y=-3 \end{cases}$ $\begin{cases} y=2 \\ x=1 \end{cases}$	г) $\begin{cases} x-y=-3 \\ x-2y=-1 \end{cases}$ $\begin{cases} y=-2 \\ x=-5 \end{cases}$

Javob: (-1; -2), (5; 2), (1;2), (-5; -2).

в) $x^3+23=y^2$	(11;12), (-11;-12), (-11;12), (11;-12)
г) $x^2-48=y^2$	(24;23), (24;-23), (-24;-23), (-24;23)
д) $x(y^2+1)=48$	(48;0), (24;1), (24;-1)
е) $y=\frac{2}{3}x$	$x = 3m; y = 2m, m \in Z$
ж) $y = 2x - 1$	$x = m; y = 2m - 1, m \in Z$
з) $x^2=4y^2$	$x = 2m; y = m; x = 2m; y = -m, m \in Z$
и) $x^2=2y^2$	Yechimi yo'q

4) Tenglamani butun sonlarda yeching

$(x-y)(x-1)=4$	(-3;-2), (-1;1), (0;4), (2;-2), (3;1), (5;4)
$(x - 3)(xy + 5) = 5$	(-2;3), (2;-5), (4;0)
$(y + 1)(xy - 1)=3$	(0;-4), (1;-2), (1;2)
$x^2-2xy-3y^2=5$	(-4;-1), (-2;1), (2;-1), (4;1)
$x^2+23=y^2$	(-11;-12), (-11;12), (11;-12), (11;12)
$x^2-47=y^2$	(-24;23), (-24;23), (24;-23), (24;23)

5) Tenglamani butun sonlarda yeching.

a) $(x+1)^2+y^2=0$	(-1;0)
b) $x^2-10x+25+y^2=0$	(5;0)
c) $x^2-4x+y^2+2y+5=0$	(2;-1)
d) $x^2+5y^2+4xy+2y+1=0$	(2; -1)

Quyidagi tenglamalarni butun sonlar to‘plamida yeching.

1. $143x+169y=5$

2. $237x+44y=1$

3. $275x+145y=10$

4. $3x+8y=5$

5. $2x+5y=7$

6. $5x+28y=59$

7. $12x+7y=41$

8. $12x-7y=29$

9. $8x+3y=63$

10. $7x-19y=23$

11. $9x-22y=10$

12. $122x+129y=2$

13. $26x+34y=13$

14. $258x-172y=56$

15. $70x+33y=1$

16. $45x-37y=25$

17. $60x-91y=2$

18. 289. 440 kg donni tashish uchun 60kg va 80kg li qoplar mavjud. Shu donni tashish uchun har bir hil qopdan nechtadan olingan?

19. Kinoteatrga tushish uchun 14900 so‘mga 300 va 500 so‘mlik biletlardan sotib olindi. Har bir xil biletdan nechtadan sotib olingan?

Mustaqil yechish uchun tenglamalar

1. $y^2-xy-2x^2-13=0$

2. $y^2-x^2+3=0$

3. $x^2+xy-5=0$

4. $x^2-2xy-3y^2+11=0$

5. $4y^2-x^2+5=0$

6. $x^2 - xy - 2x^2 - 13 = 0$

7. $x^2 - 2x - y^2 - 2 = 0$

8. $y^2 - 2y - 4x^2 + 4 = 0$

9. $x^2 + 2x - y^2 - 2y + 3 = 0$

10. $x^2 + 2x + 2(x + 1)y - 3y^2 + 4 = 0$

11. $x^2 - xy - y - 4 = 0$

12. $x^2 - xy + 2x + 2 = 0$

13. $y^2 - xy + x + 1 = 0$

14. $xy - 2x^2 + 5x - 2y - 3 = 0$

15. $3xy - x^2 + 3x - 6y = 3$

16. $2x^2 - 3xy + 4x - 3y = 0$

17. $x^2 + xy - 5x + 2y + 5 = 0$

18. $2y^2 + 2xy - 3y - 2x = 0$

19. $x^2 + xy - x - y - 7 = 0$

20. $xy - 2y^2 + x - 2y - 5 = 0$

21. $x^2 + a^2 + 2x - 2a + 2 = 0$

22. $x^2 + a^2 - 4x + 2a + 5 = 0$

23. $2x^2 + 2a^2 + 12x + 8a + 26 = 0$

24. $2x^2 + 2a^2 - 2x + 6a + 5 = 0$

25. $4x^2 + 4a^2 - 8x - 4a + 5 = 0$

26. $x^2 + a^2 - 6x - 2a + 10 = 0$

27. $x^2 + a^2 - 10x + 4a + 29 = 0$

28. $3x^2 + 3a^2 - 12x - 12a + 24 = 0$

29. $4x^2 + 4a^2 - 8x + 32a + 68 = 0$

30. $2x^2 + 2a^2 - 12x - 12a + 36 = 0$

31. 440 kg donni tashish uchun 60 kg va 80 kg li qoplar mavjud. Shu donni tashish uchun har bir xil qopdan nechtadan olingan?

32. Kinoteatrga tushish uchun 14900 so'mga 300 va 500 so'mlik biletlardan sotib olindi. Har bir xil biletdan nechtadan sotib olingan?

4.Ratsional sonlar ustida amallarga oid misollar yechish.

1. $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\cdots\left(1-\frac{1}{n+1}\right)=$
2. $1\cdot 4+2\cdot 7+3\cdot 10+\cdots+n\cdot(3n+1)=$
3. $\left(1-\frac{1}{4}\right)\left(1-\frac{1}{9}\right)\left(1-\frac{1}{16}\right)\cdots\left(1-\frac{1}{(n+1)^2}\right)=$
4. $1\cdot 1!+2\cdot 2!+3\cdot 3!+\cdots n\cdot n!=$
5. $\frac{0}{1!}+\frac{1}{2!}+\frac{2}{3!}+\cdots+\frac{n-1}{n!}=$
6. $\frac{1^2}{1\cdot 3}+\frac{2^2}{3\cdot 5}+\cdots+\frac{n^2}{(2n-1)\cdot(2n+1)}=$
7. $\frac{1}{1\cdot 3\cdot 5}+\frac{2}{3\cdot 5\cdot 7}+\cdots+\frac{n}{(2n-1)\cdot(2n+1)\cdot(2n+3)}=$
8. $\frac{1}{1\cdot 2\cdot 3}+\frac{1}{2\cdot 3\cdot 4}+\cdots+\frac{1}{n(n+1)\cdot(n+2)}=$
9. $1\cdot 2\cdot 3+2\cdot 3\cdot 4+\cdots+n\cdot(n+1)(n+2)=$
10. $2\cdot 1^2+3\cdot 2^2+\cdots+(n+1)\cdot n^2=$
11. $\frac{1}{1\cdot 2\cdot 3\cdot 4}+\frac{1}{2\cdot 3\cdot 4\cdot 5}+\cdots+\frac{1}{n(n+1)(n+2)(n+3)}=$
12. $\left(1-\frac{4}{1}\right)\left(1-\frac{4}{9}\right)\left(1-\frac{4}{25}\right)\cdot\cdot\left(1-\frac{4}{(2n-1)^2}\right)=$
13. $\frac{2^3-1}{2^3+1}\cdot\frac{3^3-1}{3^3+1}\cdot\frac{4^3-1}{4^3+1}\cdot\ldots\cdot\frac{(n+1)^3-1}{(n+1)^3+1}$
14. $2^{20}-2^{19}-2^{18}-\cdots-2-1$
15. $\frac{1}{2!}+\frac{2}{3!}+\cdots+\frac{n}{(n+1)!}=$

Yig'indini hisoblang.

1. $S_n=\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\cdots+\frac{1}{(2n-1)(2n+1)}$
2. $S_n=\frac{1}{1\cdot 4}+\frac{1}{4\cdot 7}+\cdots+\frac{1}{(3n-1)(3n+1)}$
3. $S_n=\frac{1}{1\cdot 5}+\frac{1}{5\cdot 9}+\cdots+\frac{1}{(4n-3)(4n+1)}$
4. $S_n=\frac{1}{1\cdot 6}+\frac{1}{6\cdot 11}+\cdots+\frac{1}{(5n-4)(5n+1)}$
5. $S_n=1^2-2^2+3^2-4^2+\cdots+(-1)^{n-1}n^2.$

5. Irratsional sonlarga oid misollar yechish. Haqiqiy sonlar ustida amallar.

Tengliklarni isbotlang.

1. a) $\sqrt{9-4\sqrt{5}}+\sqrt{14-6\sqrt{5}}=1$

b) $\sqrt{11-4\sqrt{7}}+\sqrt{16-6\sqrt{7}}=1$

c) $\sqrt{19-8\sqrt{3}}-\sqrt{7-4\sqrt{3}}=2$

d) $\sqrt{18-8\sqrt{2}}-\sqrt{6-4\sqrt{2}}=2$

2. $\dfrac{\sqrt{\sqrt[4]{8}-\sqrt{\sqrt{2}-1}}}{\sqrt{\sqrt[4]{8}+\sqrt{\sqrt{2}-1}}-\sqrt{\sqrt[4]{8}-\sqrt{\sqrt{2}-1}}}=\dfrac{1}{\sqrt{2}}$

3. $\sqrt[3]{26+15\sqrt{3}}(2-\sqrt{3})=1$

4. $\dfrac{2\sqrt[3]{2}}{1+\sqrt{3}}=\dfrac{\sqrt[3]{20+12\sqrt{3}}}{2+\sqrt{3}}$.

5. $\dfrac{\sqrt{5-2\sqrt{6}}\cdot(5+2\sqrt{6})(49-20\sqrt{6})}{\sqrt{27}-3\sqrt{18}+3\sqrt{12}-\sqrt{8}}=1$

6. $\left(\dfrac{6+4\sqrt{2}}{\sqrt{2}+\sqrt{6+4\sqrt{2}}}+\dfrac{6-4\sqrt{2}}{\sqrt{2}-\sqrt{6-4\sqrt{2}}}\right)^2=8$

7. $\left(\dfrac{3}{\sqrt[3]{64}-\sqrt[3]{25}}+\dfrac{\sqrt[3]{40}}{\sqrt[3]{8}+\sqrt[3]{5}}-\dfrac{10}{\sqrt[3]{25}}\right)^{-1}\cdot(13-4\sqrt[3]{5}-2\sqrt[3]{25})+\sqrt[3]{25}=4$

8. $\sqrt[3]{6+\sqrt{\dfrac{847}{27}}}+\sqrt[3]{6-\sqrt{\dfrac{847}{27}}}=3$

9. $\sqrt[3]{5\sqrt{2}+7}-\sqrt[3]{5\sqrt{2}-7}=2$

10. Quyidagi sonlarning butun qismini toping:

a) $\left[\dfrac{8}{3}\right]$, c) $\left[-3\dfrac{1}{2}\right]$, e) $\left[\sqrt[3]{30}\right]$, j) $\left[\sqrt{175}+1\right]$,

b) $[2,8]$, d) $\left[\sqrt{13}\right]$, f) $\left[\sqrt[4]{200}\right]$, k) $\left[\dfrac{\sqrt{542}+2}{3}\right]$

g) $[2-\lg 2512]$.

11. Quyidagilarni toping.

a) $\{2,6\}$; c) $\{7\}$; d) $\{0,4\}$; j) $\{-4,8\}$;

b) $\left\{\dfrac{8}{3}\right\}$; g) $\{-4,5\}$ e) $\left\{-2\dfrac{1}{2}\right\}$; z) $\{-0,5\}$.

6.Algebraik va transtsendent sonlar. Taqribiy hisoblashlar va ularning tadbiqiga oid misollar yechish.

Transsedent sonlar xossalari

6. Agar *t* – transsendent son bo 'lsa, u holda –*t* va *1/t* lar ham transsendent sonlar bo 'ladi.
7. Agar *a* – algebraik son, *t* – transsendent son bo 'lsa u holda *a+t, a-t, at, a/t, t/a* sonlar ham transsendent son bo 'ladi .
8. Agar *t* – transsendent son, *n* – butun son bo 'lsa, u holda t^nva $\sqrt[n]{t}$ transsendent son bo 'ladi.

Masalan, c va d lar har xil nomanfiy butun sonlar bo 'lsin. $lg(2^c \cdot 5^d)$ irratsional son ekanligini isbotlaymiz.

Yechilishi: Irratsional son haqidagi mulohazalarga asosan isbotlaymiz. Shartga ko'ra $2^c \cdot 5^d$ ifoda 1 dan katta, shuning uchun ham $lg(2^c \cdot 5^d)$ ifoda 0 dan katta. Teskari faraz qilaylik, ya'ni $lg(2^c \cdot 5^d)$ ratsional son bo 'lsin, u holda

$lg(2^c \cdot 5^d) = \frac{a}{b}$, bu yerda a va b lar musbat butun sonlar. U holda

$2^c \cdot 5^d = 10^{\frac{a}{b}}$ bo 'ladi.

Bu tenglikning ikkala tomonini b darajaga ko'tarib

$2^{bc} \cdot 5^{bd} = 10^a = 2^a \cdot 5^a$ tenglikka ega bo 'lamiz.

Arifmetikaning asosiy teoremasiga asosan bu tenglik $bc = a$ va $bd = a$ bo'lgandagina to'g'ri bo'ladi ya'ni, $bc = bd$. Ammo c va d lar har xil sonlar edi, u holda bd va bc lar ham har xil sonlar bo 'lishi kerak edi. Demak, $lg(2^c \cdot 5^d)$ son irratsional son ekan.

Ammo hamma logarifmik ifodalar qatnashgan sonlar transsendent son bo'lavermaydi. Masalan, $lob_8 32, lob_9 27, ...$

Misollar.

a. $lg\frac{3}{4}$ sonning irratsional son ekanligini isbotlang.

b. $lg15$ sonning irratsional son ekanligini isbotlang.

c. $lg5 + lg3$ sonning irratsional son ekanligini isbotlang.

Haqiqiy sonlar: ratsional va irratsional bo 'ladi.

Irratsional sonlar: algebraik (masalan, $\sqrt{2}$ *va* $\sqrt[3]{5}$) va transsendent(masalan, $3^{\sqrt{3}}, lg2, \pi, e$)

Haqiqiy sonlar: algebraik(ratsional va irratsional) va transsendent(hammasi irratsional) sonlar bo'ladi.

Quyidagi sonlarning algebraik son ekanlini tekshiring:

1. $\frac{2018}{2019}$, 2. $3^{\frac{4}{5}}$, 3. $\left(\frac{2}{3}\right)^{\frac{3}{4}}$, 3. $\sqrt{3}$, 4. $\sqrt[3]{4}$, 5. $\sqrt{2}+\sqrt{3}$,

6. $\sqrt{2}+\sqrt{3}+\sqrt{5}$, 7. $\sqrt[3]{\frac{2}{3}}$, 8. $\sqrt{2+\sqrt{3}}$, 9. $\frac{\sqrt{2}}{\sqrt{2+\sqrt{3}}}$, 10. $\frac{\sqrt{3}}{\sqrt{5}}$

7.Bir o`zgaruvchili va bir jinsli ko`phad. Uning kanonik formulasi, ko'phadlar ustida amallar. Ko`phadning bo`linishi. Ko`phadlarni ko'paytuvchilarga ajratish.

Ko'phadni ko'paytuvchilarga ajratish - ko'phadni ikki yoki undan ortiq ko'phadlar ko'paytmasi shaklida ifodalashdir, masalan:

$$4x^2-9y^2=(2x+3y)(2x-3y).$$

Ko'phadni ko'paytuvchilarga ajratishda quyidagi usullardan foydalaniladi:

1) Umumiy ko'paytuvchini qavsdan tashqariga chiqarish, masalan:

$$3ax+6ay=3a(x+2y).$$

2) Guruhlashu suli, masalan:

$$a^3-2a^2-2a+4=(a^3-2a^2)-(2a-4)=a^2(a-2)-2(a-2)=(a-2)=(a-2)(a^2-2)$$

yoki

$$a^2-2a^2-2a+4=(a^3-2a)-(2a^2-4)=a(a^2-2)-2(a^2-2)=(a^2-2)(a-2).$$

3) Qisqa ko'paytirish formulalarini qo'llash, masalan:

1) $9x^2-\frac{1}{16}y^2=\left(3x-\frac{1}{4}y\right)\left(3x+\frac{1}{4}y\right)$

2) $27x^3+8y^6=(3x+2y^2)(9x^2-6xy^2+9y^4)$;

3) $z^2-14z+49=(z-7)^2$.

Bo'lishni bajaring

1. $(-2x+x^2-1+2x^3):(x+1)$
2. $(1-x^2-3x+6x^3):(2x-1)$
3. $(4x^2-x-x^3+2x^4+2):(x^2+1)$
4. $(3x^4+2+5x^2+2x+3x^3):(3x^2+2)$
5. $(4x-x^2-x^3-2):(1-x)$
6. $(2-x^2+2x-x^3):(2-x^2)$
7. $(2+3x^2+x^3+3x):(1+x+x^2)$
8. $(x^4+x^3-x-1):(x^2-1)$
9. $(x^4+x^2-4+2x^3):(x+2)$

10. $(x^3 - 2x^2 - 4x + 3) : (x - 1 + x^2)$

Ko'paytuvchilarga ajrating

1. $a^3 + 9a^2 + 27a + 19$
2. $a^4 + 6a^3 + 11a^2 + 6a$
3. $a^2(a^3 + 14) + 49$
4. $a^{12} - 2a^6 + 1$
5. $a^3 + a - 2$
6. $2a^3 - a^2 + 3$
7. $a^3 + 5a^2 + 3a - 9$
8. $a^3(a^2 - 7)^2 - 36a$
9. $(ab+ac+bc)(a+b+c)-abc$
10. $a^2b^2(b - a) + c^2b^2(c - b) + a^2c(a - c)$
11. $(a + b)^5 - (a^5 + b^5)$
12. $(a + b + c)^3 - (a^3 + b^3 + c^3)$
13. $(a^2 + a + 3)(a^2 + a + 4) - 12$
14. $a(a + 1)(a + 2)(a + 3) + 1$
15. $(a + 1)(a + 3)(a + 5)(a + 7) + 15$
16. $2(a^2 + 2a - 1)^2 + 5(a^2 + 2a - 1)(a^2 + 1) + 2(a^2 + 1)^2$
17. $(a - b)^3 + (b - c)^3 - (a - c)^3$
18. $(a^2 + b^2)^3 - (b^2 + c^2)^3 - (a^2 - c^2)^3$
19. $a^4 + b^4 + c^4 - 2a^2b^2 - 2a^2c^2 - 2c^2b^2$
20. $a^4 + 2a^3 + 3a^2 + 2a + 1$
21. $a^2b + ab^2 + a^2c + ac^2 + cb^2 + bc^2 + 3abc$
22. $a^{10} + a^5 + 1$
23. $a^5 + a^4 + a^3 + a^2 + a + 1$
24. $(a - b)c^3 - (a - c)b^3 + (b - c)a^3$

8.Ratsional ifodalarni ayniy almashtirishlarga oid misollar yechish.

1. $\left(\left(\frac{m}{n}-\frac{n}{m}\right):(m+n)+m\left(\frac{1}{n}-\frac{1}{m}\right)\right):\left(\frac{1+m}{mn}\right)$ *bu yerda* $m=190$ *va* $n=13$

2. $\left((a^2+b^2+ab)\left(b-\frac{b^2}{a+b}\right)\right):\left(\frac{a^3-b^3}{a^2-b^2}\right)$ *bu yerda* $a=15$ *va* $b=17$

3. $\left(\frac{(m+n)^2+2n^2}{m^3-n^3}-\frac{1}{m-n}+\frac{m+n}{m^2+mn+n^2}\right)\left(\frac{1}{n}-\frac{1}{m}\right)$; $m=16$ *va* $n=\frac{10}{176}$

4. $\left(\frac{4}{a^2+a}-\frac{2}{1-a^2}-\frac{1}{a^2-a}\right):\frac{2a-1}{a^2+a}$; $a=\frac{18}{35}$

5. $\left(\frac{a+1}{2a-2}+\frac{6}{2a^2-2}-\frac{a+3}{2a+2}\right)\left(\frac{4a^2-4}{5a}\right)$; $a=\frac{2}{121}$

6. $\left(\frac{ab}{a+b}+\frac{b^2}{a-b}+\frac{2ab^2}{a^2-b^2}\right)\left(\frac{1}{a+b}+\frac{b}{a^2-ab}-\frac{2b}{a^2-b^2}\right)$; $a=13$ *va* $b=78$

7. $\left(\frac{2m+1}{2m-1}-\frac{2m-1}{2m+1}\right):\left(\frac{4m}{10m-5}\right)$; $m=\frac{3}{14}$

8. $\left(\frac{a}{b-a}-\frac{a}{b+a}\right)\cdot\frac{c+2ab+a^2}{2a^2}$; $a=23$ *va* $b=33$

9. $\left(1+\frac{a}{b}+\frac{a^2}{b^2}\right)\left(1-\frac{a}{b}\right)\cdot\frac{ab^2}{a^3-b^3}$; $a=121$ *va* $b=11$

10. $\left(\frac{m^2}{n^2}+\frac{n}{m}\right):\left(\frac{m}{n^2}-\frac{1}{n}+\frac{1}{m}\right)$; $m=97$ *va* $n=41$

A , B, C larning qanday qiymatlarida quyidagiolar ayniyat bo'ladi?

1. $\frac{1}{x(x-1)(x+2)}=\frac{A}{x}+\frac{B}{x-1}+\frac{C}{x+2}$;
2. $\frac{x}{(x-1)(x+2)(x+3)}=\frac{A}{x-1}+\frac{B}{x+2}+\frac{C}{x+3}$;
3. $\frac{2x+1}{(x^2+1)(x+2)}=\frac{A}{x+2}+\frac{Bx+C}{x^2+1}$;
4. $\frac{x-2}{x(x+1)(x+2)}=\frac{A}{x}+\frac{B}{x+1}+\frac{C}{x+2}$;
5. $\frac{x}{(x-1)^2(x+1)}=\frac{A}{x+1}+\frac{B}{x-1}+\frac{C}{(x-1)^2}$;
6. $\frac{1}{(x^2+x+1)x}=\frac{A}{x}+\frac{Bx+C}{x^2+x+1}$;

7. $\frac{x^2+4}{(x-1)(x+1)(x-2)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x-2}$;

8. $\frac{x^2-1}{x^2(x+3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+3}$.

Ifodalarni soddalashtiring.

1. $\frac{\frac{1}{a}+\frac{1}{b+c}}{\frac{1}{a}-\frac{1}{b+c}} \cdot \left(1 + \frac{b^2+c^2-a^2}{2bc}\right)$;

2. $\frac{1}{(a-b)(a-c)} + \frac{1}{(b-c)(b-a)} + \frac{1}{(c-a)(c-b)}$;

3. $\frac{a+b}{(b-c)(c-a)} + \frac{b+c}{(c-a)(a-b)} + \frac{c+a}{(a-b)(b-c)}$;

4. $\frac{a-c}{a^2+ac+c^2} \cdot \frac{a^3-b^3}{a^2b-bc^2} \cdot \left(1 + \frac{c}{a-c} - \frac{1+c}{c}\right) : \frac{c(c+1)-a}{bc}$;

5. $\frac{\frac{a}{8b^3}+\frac{1}{4b^2}}{a^2+2ab+2b^2} - \frac{\frac{a}{8b^3}-\frac{1}{4b^2}}{a^2-2ab+2b^2} - \frac{1}{4b^2(a^2-2b^2)}$;

6. $\frac{a-b}{a+b} + \frac{b-c}{b+c} + \frac{c-a}{c+a} + \frac{(a-b)(b-c)(c-a)}{(a+b)(b+c)(c+a)}$;

7. $\frac{a^3b-ab^3+b^3c-bc^3+c^3a-ca^3}{a^2b-ab^2+b^2c-bc^2+c^2a-ca^2}$;

8. $\frac{(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3}{(a-b)^3+(b-c)^3+(c-a)^3}$;

9. Ayniyatni isbotlang.

$$\frac{b-c}{(a-b)(a-c)} + \frac{c-a}{(b-c)(b-a)} + \frac{a-b}{(c-a)(c-b)} = \frac{2}{a-b} + \frac{2}{b-c} + \frac{2}{c-a}$$

10. Ayniyatni isbotlang.

$$a^2\frac{(d-b)(d-c)}{(a-b)(a-c)} + b^2\frac{(d-c)(d-a)}{(b-c)(b-a)} + c^2\frac{(d-a)(d-b)}{(c-a)(c-b)} = d^2$$

9.Qaytma va yuqori darajali tenglamalarga oid misollar yechish.

1. $x^4 - 2x^3 - x^2 - 2x + 1 = 0$
2. $7\left(x + \frac{1}{x}\right) - 2\left(x^2 + \frac{1}{x^2}\right) = 9$
3. $2x^4 + 3x^3 - x^2 + 3x + 2 = 0$
4. $x^2 + \frac{4}{x^2} - 8\left(x - \frac{2}{x}\right) = 4$
5. $\frac{x}{x^2-x+1} - \frac{2x}{x^2+x+1} = -\frac{1}{12}$
6. $\frac{3x^2-1}{X} + \frac{5X}{3x^2-X-1} = 7$
7. $\frac{x^2-x+2}{x} + \frac{x^4+x^2+4}{x^2} = 8$
8. $\frac{x}{x^2-x+1} - \frac{3x}{x^2+x+1} = 2$
9. $\frac{3x^2-1}{X} + \frac{3X}{3x^2-X-1} = -3$
10. $\frac{3x}{x^2-4x+1} - \frac{2x}{x^2+x+1} = \frac{3}{8}$
11. $(x-3)(x-4)(x-7)(x-8) = 60$
12. $(x+6)(x+7)(x+9)(x+10) = 10$
13. $(x+3)(x+6)(x+9)(x+12) = 45{,}5625$
14. $(x-1)(x-2)(x-3)(x-4) = \frac{9}{16}$
15. $x(x+1)(x-1)(x-2) = -\frac{7}{16}$
16. $(12x-1)(6x-1)(4x-1)(3x-1) = 5$
17. $x(x+1)(x+2)(x+3) = 48$
18. $x(x+1)(x-1)(x+2) = 3$
19. $(x+2)(x+5)(x+15)(x+18) = -360$
20. $\left(x - \frac{1}{2}\right)\left(x - \frac{1}{3}\right)\left(x + \frac{3}{2}\right)\left(x - \frac{3}{2}\right) = -\frac{1}{3}$
21. . $6x^4 - 13x^3 - 27x^2 + 40x - 12 = 0$.
22. $9x^2 + 4x^3 = 1 + 12x^4$.
23. $x^4 + x^3 + x^2 + x + 1 = 0$.
24. $x^5 + x^3 + x = 0$.

25. $x^5-6x^4+9x^3-6x^2+8x=0$.

26. $3x^7+x^6+3x^4+x^3+15x+5=0$.

27. $8x^7-6x^6-4x^4+3x^3+8x-6=0$.

28. $x^7+2x^5+4x^4-36x^3+32x^2-72x+48=0$.

29. $(x^3+x^2+1)^2+(x^3-x^2+1)^2=2x^4$.

30. $(x-1)^3+(2x+3)^3=27x^3+8$.

10.Kasr-ratsional tenglamalarga oid misollar yechish.

1. $\frac{x^2+2x+2}{x^2+2x+3}-\frac{1}{x^2+2x+2}=\frac{1}{6}$

2. $\frac{2x^2+5x+15}{2x^2+5x+3}-\frac{2x^2+5x+13}{2x^2+5x+5}=1$

3. $\frac{8x^2+4x+4}{4x^2+2x+3}-\frac{1}{2x^2+x+1}=\frac{1}{3}$

4. $6x^2+3x+1=-\frac{2x^2+x-5}{2x^2+x}$

5. $\frac{4x^2+4x-3}{x^2+x-1}-\frac{5x^2+5x}{3x^2+3x+4}=4$

6. $\frac{4x^2+10x+9}{2x^2+5x+4}+\frac{2x^2+5x+1}{6x^2+15x+8}=2$

7. $\frac{1}{5x^2+8x+5}+\frac{35x^2+56x+25}{20x^2+32x+28}=\frac{3}{4}$

8. $\frac{4x^2+7x+7}{8x^2+14x+10}+\frac{4x^2+7x+1}{12x^2+21x+13}=\frac{1}{2}$

9. $-6x^2+9x-\frac{2x^2-3x-4}{2x^2-3x+1}=4$

10. $\frac{4x^2+2x+3}{2x^2+x+1}+\frac{2x^2+x-2}{6x^2+3x-1}=2$

11. $\frac{19-2x}{x^2+5x+4}-\frac{2x+9}{x^2+3x+2}=\frac{4x}{x^2+6x+8}$

12. $\frac{2x}{x^2+x-2}+\frac{2}{3(x^2-4x+3)}=\frac{5}{3(x^2-x-6)}$

13. $\frac{1-9x}{x^2+2x-3}+\frac{3x-1}{x-1}=\frac{2x}{x+3}$

14. $\frac{3(2x^2-x-1)}{x^2+x-6}=1+\frac{4x}{x+3}$

15. $\frac{2(4x+13)}{x^2+8x+7}+\frac{2x+9}{x+7}=3$

16. $\frac{2x}{x+2}+\frac{2(11x+6)}{x^2-4x-12}=\frac{3x-1}{x-6}$

17. $\frac{3x-1}{x+3}-\frac{x^2-27x-10}{x^2-2x-15}=\frac{x+1}{x-5}$

18. $\frac{2x^2+15x+27}{2x^2+7x+3}+\frac{3x-1}{2x+1}=2$

19. $5-\frac{x^2-14x-51}{x^2-x-12}=\frac{3x}{x-4}$

20. $\frac{x^2+2x+2}{x+1}+\frac{x^2+8x+20}{x+4}=\frac{x^2+4x+6}{x+2}+\frac{x^2+6x+12}{x+3}$

21. $\frac{x+4}{2x^3-8x+6}-\frac{x-3}{8-2x^2}=\frac{x+6}{x^3+3x^2-x+3}$

22. $\frac{2x+5}{3x^2-3x-6}+\frac{3x}{8-2x^2}=\frac{5x+7}{x^3+x^2-4x-4}$

23. $\frac{x+5}{2x^2-6x-8}+\frac{x-7}{64-4x^2}+\frac{9}{x^3-x^2-16x+16}=0$

24. $\frac{242}{48-10x-2x^2}+\frac{x^2+8x}{x^2-3x}+\frac{x+2}{x+8}=1$

25. $\frac{14}{20-6x-2x^2}+\frac{x^2+4x}{x^2+5x}-\frac{x+3}{2-x}+3=0$

26. $\frac{40}{x^2+10x+21}-\frac{3-x}{7+x}+\frac{6+x}{x-4}-2=0$

27. $\frac{22}{x^2+7x-18}+1=\frac{x^2+8x}{x^2+9x}+\frac{7-x}{x-2}$

11.Modul qatnashgan tenglamalarga oid misollar yechish

1. $|3x+2|=|2x-3|$;
2. $|6x+5|=|1-x|$;
3. $|x^2+x-1|=2x-1$;
4. $|x^2-x-3|=-x-1$;
5. $2|x^2+2x-5|=x-1$;
6. $2|x^2-x|=x^2+1$;

7. $x^2 + 3|x| + 2 = 0$

8. $2x^2 - |x| - 15 = 0$

9. $(x+1)^2 - 2|x+1| + 1 = 0$

10. $x^2 + 2x - 3|x+1| + 3 = 0$

11. $|x| + |x+1| = 1$

12. $|x+1| + |x+2| = 2$

13. $|x-1| - |x-2| = 1$

14. $|x-2| + |4-x| = 3$

15. $|x-1| + |x-2| = 1$

16. $2|x+3| - |x-4| = 4$

17. $|x-2| + |x-3| + |2x-8| = 9$

18. $|x+1| - |x-2| + |3x+6| = 5$

19. $|2x+1| - |3-x| = |x-4|$

20. $|x-1| + |1-2x| = 2|x|$

21. $|x| - 2|x+1| + 3|x+2| = 0$

22. $|x+1| - |x| + 3|x-1| = 2$

23. $|x| - 2|x+1| + 3|2x-4| = 1$

24. $|x| + 2|x+1| - 3|x-3| = 0$

25. $|x^2 - 9| + |x+2| = 5$

26. $|x^2 - 1| + |x+1| = 0$

27. $|x^2 - 4| - |9 - x^2| = 5$

28. $|x^2 - 9| + |x^2 - 4| = 5$

29. $|x - x^2 - 1| = |2x - 3 - x^2|$

30. $|x^2 + 2x| - |2 - x| = |x^2 - x|$

31. $\left||3 - 2x| - 1\right| = 2|x|$

32. $\left||x+4| - 2x\right| = 3x - 1$

33. $|2|x-1| + 3x - 4| = x - 2$

34. $\left|3x - |2x-5|\right| = 1 - 5x$

35. $|-5x - 3|2x-3| + 2| = 11 + x$

36. $\frac{|x^2-4x|+4}{x^2+|x-5|}=1$

37. $\frac{|x^2-x|+1}{|x+1|-x^2}=1$

38. $\frac{|x^2-4x+3|+x}{|x^2-x|+1}=1$

39. $\frac{|x^2+4x+4|-x}{x-|2x+x^2|}=1$

Quyidagi tenglamalarni grafik usulda yeching.

40. $|x-2|=3,$

41. $|x|=x+2,$

42. $|x|=2x+1,$

43. $|-x+2|=2x+1,$

44. $|3x-4|=-x+4,$

45. $\frac{7x+4}{5}-x=\frac{|3x-5|}{2},$

46. $|x-1|+|x-2|=1.$

Quyidagi tenglamalarni yeching.

47. $|x-2|+|x-3|+|2x-8|=9,$

48. $|4x-1|-|2x-3|+|x-2|=0,$

49. $|x-1|+|x+2|-|x-3|=4,$

50. $|x-1|-|x+2|-|2x-5|+|3-x|=-3,$

51. $|||x-2|-1|-2|=2,$

52. $|2-|-1|x||=1.$

Tenglamalarni yeching.

53. $|x^4-4|=x^2-4,$

54. $|-x^2+1|=-x^2+1,$

55. $|x^2-3x+2|=3x-x^2-2,$

56. $|2x-x^2-1|=2x-x^2-1,$

57. $|5x-x^2-6|=x^2-5x+6.$

58. $\left|x^2-5x+6\right|=5x-x^2-6.$

59. $|x-1|=-|x|+1.$

60. $\left|\frac{1}{2}x2x+\frac{3}{2}\right|+\left|\frac{1}{2}x^2-3x+4\right|=\frac{3}{4}.$

12. Tenglamalar sistemasiga oid misollar yechish.

1. $\begin{cases} x+y=-8 \\ x^2+y^2+6x+2y=0 \end{cases}$

2. $\begin{cases} x^2(x+y)=80 \\ x^2(2x-3y)=80 \end{cases}$

3. $\begin{cases} 2x-y=1 \\ 2x^2-y^2+x+y=-11 \end{cases}$

4. $\begin{cases} x-y=2 \\ x^3-y^3=8 \end{cases}$

5. $\begin{cases} x-y=1 \\ x^2+y^2=41 \end{cases}$

6. $\begin{cases} x+y=-1 \\ 16x^2-y^4=0 \end{cases}$

7. $\begin{cases} 3x+5y=2 \\ 3x^2+10xy-25y^2=0 \end{cases}$

8. $\begin{cases} x+2y+3z=-1 \\ 2x+3y+4z=-1 \\ 3x+4y+6z=-1 \end{cases}$

9. $\begin{cases} 2x^2-3y=23 \\ 3y^2-8x=59 \end{cases}$

10. $\begin{cases} x+y+z=3 \\ x+2y-z=2 \\ x+yz+zx=3 \end{cases}$

11. $\begin{cases} x^2+2y=-5 \\ 2x^2+3y^2=29 \end{cases}$

12. $\begin{cases} x^2+3y^2-xz=6 \\ 2x-y+3z=11 \\ x+2y-2z=1 \end{cases}$

13. $\begin{cases} 2x-3y-xy=4 \\ 3x+y+3xy=3 \end{cases}$

14. $\begin{cases} xy=2 \\ 9x^2+y^2=13 \end{cases}$

15. $\begin{cases} 5x^2+14y=19 \\ 7y^2+10x=17 \end{cases}$

16. $\begin{cases} 3xy+3x^2-3y^2-2x-y=-7 \\ x^2+xy-y^2+x-2y=-4 \end{cases}$

17. $\begin{cases} x^2+y^2-2x+3y-9=0 \\ 2x^2+2y^2+x-5y-1=0 \end{cases}$

18. $\begin{cases} x-y=\frac{1}{4}xy \\ x^2+y^2=\frac{5}{2}xy \end{cases}$

19. $\begin{cases} x+yz=2 \\ y+xz=2 \\ z+yx=2 \end{cases}$

20. $\begin{cases} \frac{4}{x+y}+\frac{4}{x-y}=3 \\ (x+y)^2+(x-y)^2=20 \end{cases}$

21. $\begin{cases} \dfrac{1}{x+y} + \dfrac{1}{x-y} = 2 \\ \dfrac{3}{x+y} + \dfrac{4}{x-y} = 7 \end{cases}$

22 $\begin{cases} \dfrac{x-y}{x+y} + \dfrac{x+y}{x-y} = \dfrac{5}{2} \\ x^2 + y^2 = 20 \end{cases}$

23. $\begin{cases} (x+y)^2 + 2x = 35 - 2y \\ (x-y)^2 - 2y = 3 - 2x \end{cases}$

24. $\begin{cases} 12(x+y)^2 + x = 2{,}5 - y \\ 6(x-y)^2 + x = 0{,}125 + y \end{cases}$

25. $\begin{cases} y^2(x^2-3) + xy + 1 = 0 \\ y^2(3x^2-6) + xy + 2 = 0 \end{cases}$

26. $\begin{cases} x^2 - yz = 3 \\ y^2 - zx = 5 \\ z^2 - xy = -1 \end{cases}$

27. $\begin{cases} x^2 + xy + y^2 = 7 \\ y^2 + yz + z^2 = 3 \\ z^2 + zx + x^2 = 1 \end{cases}$

28. $\begin{cases} 2x^2 + y^2 + z^2 = 9 + yz \\ x^2 + 2y^2 + z^2 = 6 + zx \\ x^2 + y^2 + 2z^2 = 3 + xy \end{cases}$

29. $\begin{cases} x^3 - y^3 = 19(x-y) \\ x^3 + y^3 = 7(x+y) \end{cases}$

30. $\begin{cases} x^2 + xy + y^2 = 19(x-y)^2 \\ x^2 - xy + y^2 = 7(x+y)^2 \end{cases}$

31. $\begin{cases} x^3 + y^3 = 19 \\ (xy+8)(x+y) = 2 \end{cases}$

32. $\begin{cases} x^2 - yz = 3 \\ y^2 - xz = 5 \\ z^2 - xy = -1 \end{cases}$

33. $\begin{cases} x^2 + xy + y^2 = 7 \\ z^2 + zy + y^2 = 3 \\ x^2 + xz + z^2 = 1 \end{cases}$

34. $\begin{cases} x^2 + y^2 = 34 \\ x + y + xy = 23 \end{cases}$

35. $\begin{cases} x + y + x^2 + y^2 = 18 \\ xy + x^2 + y^2 = 19 \end{cases}$

13.Teng kuchli tenglamalar. Tenglamalar sitemasini yechishning elementar usullariga oid misollar yechish.

1. $x^2 + 1 = \sqrt{x}$ *va* $x^2 + 1 + \sqrt{1-x} = \sqrt{x} + \sqrt{1-x}$

2. $x^2 - 1 = \sqrt{x}$ *va* $x^2 - 1 + \sqrt{1-x} = \sqrt{x} + \sqrt{1-x}$

3. $x^3 + x = 0$ *va* $\frac{x^3+x}{x} = 0$

4. $x^2 + 1 = 0$ *va* $\frac{x^2+1}{x} = 0$

5. $\frac{2x^2+2x+3}{x+3} = \frac{3x^2+2x-1}{x+3}$ va $2x^2 + 2x + 3 = 3x^2 + 2x - 1$

6. $\frac{2x^2+2x+3}{x+2} = \frac{3x^2+2x-1}{x+2}$ va $2x^2 + 2x + 3 = 3x^2 + 2x - 1$

7. $\sqrt{x} + 2 = \sqrt{2x} + 1$ va $(\sqrt{x} + 2)^2 = (\sqrt{2x} + 1)^2$

8. $(\sqrt{x} - 2)^2 = (\sqrt{2x} + 1)^2$ va $x - 4\sqrt{x} + 4 = 2x + 2\sqrt{2x} + 1$

9. $2\sqrt{x} - 7x^2 = 2x + 2\sqrt{x}$ va $-7x^2 = 2x$

10. $(x-4)(x+3) = 0$ va $x - 4 = 0;\ \ x + 3 = 0$

11. $(x-4)(x+\frac{1}{x+3}) = 0$ va $x - 4 = 0;\ \ x + \frac{1}{x+3} = 0$

12. $\sqrt{x-2}\sqrt{x+3} = 0$ va $\sqrt{x-2} = 0;\ \ \sqrt{x+3} = 0$

13. $\sqrt{2-x}\sqrt{x+3} = 0$ va $\sqrt{2-x} = 0;\ \ \sqrt{x+3} = 0$

14. $(x-3)\ ln(2-x) = 0$ va $x - 3 = 0;\ \ ln(2-x) = 0$

15. $(2-x)\ ln(x-3) = 0$ va $2 - x = 0;\ \ ln(x-3) = 0$

16. $\frac{x^2-5x+6}{x^2-6x+8}\left(2^{\frac{x+4}{x^2-9}} - 1\right) = 0$ va $x^2 - 5x + 6 = 0;\ 2^{\frac{x+4}{x^2-9}} - 1 = 0$

14.Tengsizliklarni isbotlashga oid misollar yechish. Birinchi va ikkinchi darajali tengsizliklarni yechishga oid misollar.

o'rta arifmetik qiymat $A(a)=A_n=\frac{a_1+a_2+...+a_n}{n}$,

o'rta geometrik qiymat $G(a)=G_n=\sqrt[n]{a_1a_2...a_n}$,

o'rta kvadratik qiymat $K(a)=K_n=\sqrt{\frac{a_1^2+a_2^2+...+a_n^2}{n}}$ va

o'rta garmonik qiymat $N(a)=N_n=\frac{n}{a_1^{-1}+a_2^{-1}+...+a_n^{-1}}$ larni aniqlaymiz.

Xususan x, y musbat sonlar uchun bu o'rta qiymatlar quyidagicha aniqlanadi:

$$A_2=\frac{x+y}{2};\quad G_2=\sqrt{xy};\quad K_2=\sqrt{\frac{x^2+y^2}{2}};\quad N_2=\frac{2xy}{x+y}.$$

2.Tengsizliklarni isbotlashning usullari haqida.

1–misol. Istalgan a, b va C sonlari uchun $2a^2+b^2+c^2 \geq 2a(b+c)$ ekanligini isbotlang.

Yechilishi. Istalgan a, b va C sonlari uchun $(2a^2+b^2+c^2)-2a(b+c)$ ayirmani manfiy emasligini ko'rsatamiz:

$$(2a^2+b^2+c^2)-2a(b+c)=(a^2-2ab+b^2)+(a^2-2ac+c^2)=$$
$$=(a-b)^2+(a-c)^2.$$

Istalgan sonning kvadrati nomanfiy son bo'lgani uchun $(a-b)^2 \geq 0$ va $(a-c)^2 \geq 0$. Demak, $(2a^2+b^2+c^2)-2a(b+c)$ istalgan a, b va C sonlari uchun manfiy emas. Shuning uchun berilgan tengsizlik istalgan a, b va C sonlari uchun o'rinli. Jumladan, tenglik belgisi $a=b=c$ bo'lgandagina bajariladi. Δ

Tengsizlikning to'g'riligini ko'rsatish uchun uning har ikkala qismining ayirmasini musbat yoki manfiyligini aniqlash, ya'ni yuqoradagi misoldagidek bevosita ta'rifdan foydalanib isbotlashga harakat qilish ayrim hollarda qiyinchiliklarni tug'diradi. Shuning uchun tengsizliklarni isbotlashda tengsizliklarning xossalaridan foydalanish tavsiya etiladi.

2-misol. Musbat a, b va C sonlari uchun $\frac{b+c}{a}+\frac{c+a}{b}+\frac{a+b}{c}\geq 6$ tengsizlikni isbotlang.

Yechilishi: Tengsizlikning chap qismida shakl almashtirish bajarib, uni quyidagi ko'rinishda yozamiz:

$$\left(\frac{a}{b}+\frac{b}{a}\right)+\left(\frac{a}{c}+\frac{c}{a}\right)+\left(\frac{b}{c}+\frac{c}{b}\right)\geq 6. \qquad (1)$$

Ikkita musbat son uchun o'rta arifmetik va o'rta geometrik qiymatlar [illegible] Koshi tengsizligidan foydalanamiz:

$$\frac{a}{b}+\frac{b}{a}\geq 2\sqrt{\frac{a}{b}\cdot\frac{b}{a}}=2, \quad \frac{a}{c}+\frac{c}{a}\geq 2, \quad \frac{b}{c}+\frac{c}{b}\geq 2.$$

Bu tengsizliklarni hadma-had qo'shib, (1) tengsizlikni hosil qilamiz.

1-misol. $x, y>0$ bo'lsa, $x^2+y^2+1\geq xy+x+y$ tengsizlikni isbotlang.

Yechilishi:

$$x^2+y^2+1\geq xy+x+y \Leftrightarrow \frac{x^2}{2}+\frac{x^2}{2}+\frac{y^2}{2}+\frac{y^2}{2}+\frac{1}{2}+\frac{1}{2}\geq xy+x+y$$

$$+\begin{cases}\frac{x^2}{2}+\frac{y^2}{2}\geq xy,\\ \frac{y^2}{2}+\frac{1}{2}\geq y, \\ \frac{x^2}{2}+\frac{1}{2}\geq x.\end{cases} \Rightarrow x^2+y^2+1\geq xy+x+y.$$

2-misol. $x>0$ bo'lsa, $2^{\sqrt[12]{x}}+2^{\sqrt[4]{x}}\geq 2\cdot 2^{\sqrt[6]{x}}$ tengsizlikni isbotlang.

Yechilishi. $2^{\sqrt[12]{x}}+2^{\sqrt[4]{x}}\geq 2\cdot\sqrt{2^{\sqrt[12]{x}}\cdot 2^{\sqrt[4]{x}}}=2\cdot\sqrt{2^{[illegible]}}=2\cdot 2^{[illegible]}=2\cdot 2^{[illegible]}$

Misollar:

1. Agar $x, y>0$ bo'lsa, $x^4+y^4+8\geq 8xy$ ni isbotlang.
2. $x_1, x_2, x_3, x_4, x_5>0$ bo'lsa, quyidagini isbotlang:

$$x_1^2+x_2^2+x_3^2+x_4^2+x_5^2 \ge x_1\left(x_2+x_3+x_4+x_5\right).$$

3. $x, y, z>0$ bo'lsa, $x^2+y^2+z^2 \ge xy+yz+xz$ ni isbotlang.

4. $a, b, c>0$ bo'lsa, $\frac{a}{b}+\frac{b}{c}+\frac{c}{a} \ge 3$ ni isbotlang.

5. $a, b, c>0$ bo'lsa, $(a+1)(b+1)(c+a)(b+c) \ge 16abc$ ni isbotlang.

1-misol. Agar $a, b, c>0$ bo'lsa, $\frac{3}{1/a+1/b+1/c} \le \frac{a+b+c}{3}$ tengsizlikni isbotlang.

Yechilishi: $9 \le (a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$,

$$\begin{cases} a+b+c \ge 3\sqrt[3]{abc}, \\ \frac{1}{a}+\frac{1}{b}+\frac{1}{c} \ge 3\cdot\frac{1}{\sqrt[3]{abc}}. \end{cases} \Rightarrow (a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \ge \frac{9\sqrt[3]{abc}}{\sqrt[3]{abc}}=9.$$

2-misol. Agar $a, b, c>0$, $ab^2c^3=1$ bo'lsa, $\frac{1}{a}+\frac{1}{b}+\frac{1}{c} \ge 6$ ni isbotlang.

Yechilishi: $\frac{1}{a}+\frac{2}{b}+\frac{3}{c}=\frac{1}{a}+\frac{1}{b}+\frac{1}{b}+\frac{1}{c}+\frac{1}{c}+\frac{1}{c} \ge 6\frac{1}{\sqrt[6]{ab^2c^3}}=6$.

2-masala. Tengsizliklarni isbotlang:

$$x_n=\underbrace{\sqrt{2+\sqrt{2+\ldots+\sqrt{2}}}}_{n+1 \text{ ta ildiz}}<\sqrt{2}+1, \quad n \in N; \qquad (5)$$

$$x_n=\underbrace{\sqrt{4+\sqrt{4+\ldots+\sqrt{4}}}}_{n \text{ ta ildiz}}<3, \quad n \in N.$$

\6-masala. $x^2+y^2+z^2 \ge xy+xz+yz$ tengsizlikni isbotlang, bu yerda x, y, z - musbat sonlar.

7-masala. $x^4+y^4+z^4 \ge xyz(x+y+z)$ tengsizlikni isbotlang, bu yerda x, y, z - musbat sonlar.

Yechilishi. 6-masalaga ko'ra:

$x^4+y^4+z^4=(x^2)^2+(y^2)^2+(z^2)^2 \geq x^2y^2+y^2z^2+x^2z^2$ ga egamiz. Bu yerdan esa $x^2y^2+y^2z^2+x^2z^2 \geq xyyz+yzzx+zxxy=xyz(x+y+z)$ ni olamiz.

8-masala. $x^4+y^4+z^4+u^4 \geq 4xyzu$ tengsizlikni isbotlang, bu yerda x, y, z, u. musbat sonlar.

Yechilishi. $x^4+y^4 \geq 2x^2y^2$, $z^4+u^4 \geq 2z^2u^2$ ga egamiz. Demak $x^4+y^4+z^4+u^4 \geq 2x^2y^2+2z^2u^2$. Bundan tashqari $x^2y^2+z^2u^2 \geq 2xyzu$. Demak $x^4+y^4+z^4+u^4 \geq 4xyzu$.

Misollar.

1. $2a^2+b^2+c^2 \geq 2a(b+c)$
2. $a+b+c \geq \sqrt{ab}+\sqrt{ac}+\sqrt{bc}$
3. $ab+bc+ac \geq \sqrt{abc}(\sqrt{a}+\sqrt{b}+\sqrt{c})$

15.Kasr-ratsional va yuqori darajali tengsizliklarga oid misollar yechish.

1. $x(x-1)^2 > 0$
2. $(2-x)(3x+1)(2x-3) > 0$
3. $(3x-2)(x-3)^3(x+1)^3(x+2)^4 < 0$
4. $x^3-64x > 0$
5. $x^2-10 \leq 7x$
6. $x^2-7x < 3$
7. $-x^2-16+8x \geq 0$
8. $x^2+5x+8 > 0$
9. $x^4+8x^3+12x^2 \geq 0$
10. $(x-1)(x^2-3x+8) < 0$
11. $(x-1)(x^2-1)(x^3-1)(x^4-1) \leq 0$
12. $\frac{(x-1)(3x-2)}{5-2x} > 0$
13. $\frac{(x+1)(x+2)(x+3)}{(2x-1)(x+4)(3-x)} > 0$

14. $(16-x^2)(x^2+4)(x^2+x+1)(x^2-x-3) \le 0$

15. $(x^2-4)(x^2-4x+4)(x^2-6x+8)(x^2+4x+4) < 0$

16. $(2x^2-x-5)(x^2-9)(x^2-3x) \le 0$

17. $\frac{x^2-5x+6}{x^2-12x+35} > 0$

18. $\frac{x^2-4x-2}{9-x^2} < 0$

19. $\frac{x^3+x^2+x}{9x^2-25} \ge 0$

20. $\frac{x^4+x^2+1}{x^2-4x-5} < 0$

21. $\frac{x^3-x^2+x-1}{x+8} \le 0$

22. $\frac{7x-4}{x+2} \ge 1$

23. $\frac{1}{x} \le \frac{1}{3}$

24. $\frac{1}{x+1} + \frac{2}{x+3} > \frac{3}{x+2}$

25. $\frac{1}{3x-2-x^2} > \frac{3}{7x-4-3x^2}$

26. $\frac{3}{6x^2-x-12} < \frac{25x-47}{10x-15} - \frac{3}{3x+4}$

27. $\frac{2-x}{x^3+x^2} \ge \frac{1-2x}{x^3-3x^2}$

28. $\frac{1}{x+1} - \frac{2}{x^2-x+1} \le \frac{1-2x}{x^3+1}$

[illegible]

$$\sqrt[n]{a^m} = (\sqrt[n]{a})^m$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

$$\sqrt[n]{a}\sqrt[n]{b} = \sqrt[n]{ab}$$

$$\sqrt[n]{a^m} = \sqrt[nk]{a^{mk}}$$

$$\sqrt[n]{\sqrt[m]{a}} = \sqrt[nm]{a}$$

[illegible]

[illegible]

[illegible]

[illegible]

Kasrning maxrajini irratsionallikdan qutqaring:

1. $\frac{2\sqrt{30}}{\sqrt{5}+\sqrt{6}+\sqrt{7}}$

2. $\frac{1}{\sqrt[3]{9}+\sqrt[3]{6}+\sqrt[3]{27}+3}$

3. $\frac{1}{\sqrt[3]{13}-\sqrt[3]{7}}$

4. $\frac{1}{\sqrt[3]{4}+\sqrt[3]{6}+\sqrt[3]{9}}$

Tengliklar to'g'riligini isbotlang:

5. $\sqrt{\frac{\sqrt{5}+2}{\sqrt{5}}}+\sqrt{\frac{\sqrt{5}-2}{\sqrt{5}}}=\frac{1}{\sqrt[4]{5}}$

6. $\frac{2\sqrt{6}+\sqrt{65}}{\sqrt{10}-\sqrt{3}}=\frac{\sqrt{10}+\sqrt{3}}{2\sqrt{6}-\sqrt{65}}$

7. $\frac{1+\sqrt{3}}{2\sqrt{2}}=\frac{2+\sqrt{3}}{\sqrt{20+12\sqrt{3}}}$

8. $\sqrt[3]{38+17\sqrt{5}}=\sqrt{9+4\sqrt{5}}$

9. $\frac{2+\sqrt{3}}{\sqrt{2}+\sqrt{2+\sqrt{3}}}+\frac{2-\sqrt{3}}{\sqrt{2}-\sqrt{2-\sqrt{3}}}=\sqrt{2}$

Ifodalarni soddalashtiring:

10. $\frac{a^2+2a-3+(a+1)\sqrt{a^2-9}}{a^2-2a-3+(a-1)\sqrt{a^2-9}}$

11. $\sqrt{\sqrt{a}+\sqrt{\frac{a^2-4}{a}}}+\sqrt{\sqrt{a}-\sqrt{\frac{a^2-4}{a}}}$

12. $\left(\frac{\sqrt{x^3}-2}{\sqrt{x}-2x}+\sqrt{x}\right)\cdot\left(\frac{\sqrt{x^3}+2}{\sqrt{x}+2x}-\sqrt{x}\right):\frac{1+\frac{1}{4}x^2}{x-\frac{1}{4}}$.

13. $\left(\sqrt{m(1-m)}+\frac{\sqrt{m^3}}{\sqrt{1-m}}\right):\left(\frac{1}{1+\sqrt{m}}+\frac{\sqrt{m}}{1-m}\right);\ 0<m<1.$

14. $\left(\frac{ab^3}{\sqrt{(a+b)^5}}-\frac{2ab^2}{\sqrt{(a+b)^3}}+\frac{a}{\sqrt{a+b}}\right):\frac{a^2}{\sqrt{(a+b)^5}}-\frac{a^2b}{\sqrt{(a+b)^7}}$

15. $\left(\frac{\sqrt[3]{a+b}}{\sqrt[3]{a-b}}+\frac{\sqrt[3]{a-b}}{\sqrt[3]{a+b}}-2\right):\left(\frac{1}{\sqrt[3]{a-b}}-\frac{1}{\sqrt[3]{a+b}}\right);\ a>b.$

16. $\left(\frac{\sqrt{a-b}}{\sqrt{a+b}+\sqrt{a-b}}+\frac{a-b}{\sqrt{a^2-b^2}-a+b}\right):\sqrt{\frac{a^2}{b^2}-1};\ a>b.$

17. $\left(\frac{1+x+x^2}{2x+x^2}+2-\frac{1-x+x^2}{2x-x^2}\right)^{-1}\left(5-2x^2\right);\ x=\sqrt{3,92}$

18. $\frac{\left(x^2-y^2\right)\sqrt[3]{x}+\sqrt[3]{y}}{\sqrt[3]{x^5}+\sqrt[3]{x^2y^3}-\sqrt[3]{x^3y^2}-\sqrt[3]{y^5}}-\left(\sqrt[3]{xy}+\sqrt[3]{y^2}\right);\ x=64;y=\frac{31}{78}$

19. $\frac{a^3-a-2b-\frac{b^2}{a}}{\left(1-\sqrt{\frac{1}{a}+\frac{b^2}{a}}\right)\cdot\left(a+\sqrt{a+b}\right)}:\left(\frac{a^3+a^2+ab+a^2b}{a^2-b^2}+\frac{b}{a-b}\right);\ a=23;b=22$

20. $\sqrt{\frac{\sqrt{2}}{a}+\frac{a}{\sqrt{2}}+2}-\frac{a^2\sqrt[4]{2}-2\sqrt{a}}{a\sqrt{2a}-\sqrt[4]{8a^4}}$

21. $\left(\frac{\sqrt[4]{a^3}-1}{\sqrt[4]{a}-1}+\sqrt[4]{a}\right)^{\frac{1}{2}}\left(\frac{\sqrt[4]{a^3}+1}{\sqrt[4]{a}+1}-\sqrt{a}\right)\left(a-\sqrt{a^3}\right)^{-1};\ a>0, a\neq 1$

22. $\left(\frac{t\sqrt{t+2}}{\sqrt{t-2}}-\frac{2\sqrt{t-2}}{\sqrt{t+2}}-\frac{4t}{\sqrt{t^2-4}}\right)^{\frac{1}{2}}:\sqrt[4]{t^2-4};\ |t|>2$

23. $\frac{8-n}{2+\sqrt[3]{n}}:\left(2+\frac{\sqrt[3]{n^2}}{2+\sqrt[3]{n}}\right)-\left(\sqrt[3]{n}+\frac{2\sqrt[3]{n}}{\sqrt[3]{n}-2}\right)\times\frac{4-\sqrt[3]{n^2}}{\sqrt[3]{n^2}+2\sqrt[3]{n}};\ n\neq\pm 8$

24. $\frac{\sqrt{(2p+1)^3}+\sqrt{(2p-1)^3}}{\sqrt{4p+2\sqrt{4p^2-1}}},\qquad p\geq\frac{1}{2}$

25. $\dfrac{\sqrt[4]{x^5}+\sqrt[4]{xy^4}-\sqrt[4]{x^4y}-\sqrt[4]{x^5}}{\sqrt{x}-\sqrt{y}}\cdot\left(\sqrt[4]{x}+\sqrt[4]{y}\right)$

26. $\left(\dfrac{a\sqrt{a}+b\sqrt{b}}{\sqrt{a}+\sqrt{b}}-\sqrt{ab}\right)\cdot\left(\dfrac{\sqrt{a}+\sqrt{b}}{a-b}\right)^2$

27. $\dfrac{\dfrac{x+y}{\sqrt{x}-\sqrt{y}}-\dfrac{x-y}{\sqrt{x}+\sqrt{y}}}{\dfrac{\sqrt{x}-\sqrt{y}}{x+y}-\dfrac{\sqrt{x}+\sqrt{y}}{x-y}}\cdot\dfrac{y-\sqrt{xy}+x}{2\sqrt{xy}}$

28. $\dfrac{\sqrt{4x+4+\dfrac{1}{x}}}{\sqrt{x}\left|2x^2-x-1\right|}$.

29. $\dfrac{x^4+x^2+x\sqrt{2}+2}{x^2-x\sqrt{2}+2}-x\sqrt{2}$

30. $\dfrac{\left(\sqrt{x}+\sqrt{2}\right)^2-\sqrt{2x}}{x^2+x-\sqrt{2x}+2}$, $\quad x>0$

31. $\sqrt{\dfrac{\left(x^2-3\right)^2+12x^2}{x^2}}+\sqrt{(x+2)^2-8x}$

32. $\dfrac{(x-1)\sqrt{(x-1)^2+4x}}{x^2+1+2|x|}$

33. $\sqrt{\left(\dfrac{x^2-4}{2x}\right)^2+4}+\sqrt{1+\dfrac{4}{x^2}+\dfrac{4}{x}}$

IRRATSIONAL TENGLAMALAR TIPLARI

I Tip. Bir xil o'zgaruvchili tenglamalar

1) $\sqrt{x^2+x-3}=\sqrt{1-2x}$

2) $\sqrt{5x-1}-\sqrt{3x+19}=0$

3) $\sqrt{8-5x}=\sqrt{x^2-16}$

Yechish usuli:

Ikki tomonini ham bir xil darajaga ko'tarish.

II Tip. Tenglamaning chap tomonini asoslari yoki [illegible] tomonini esa biror o'zgaruvchi yoki musbat son orqali ifodalash.

1) $\sqrt{x-1}\cdot\sqrt{x+4}=\sqrt{6}$

2) $\sqrt{x-1}\cdot\sqrt{2x+6}=x+3$

3) $\sqrt{x+2}\cdot\sqrt{5-x}=2$

Yechish usuli:

Tenglamaning o'ng tomoni musbat ekanligiga asosan kvadratga ko'tarish [illegible]

III Tip. Tenglamaning ikki tomoni bir xil ko'paytmaga ega bo'lsa

1) $(x-3)\sqrt{x^2-5x+4}=2x-6$

2) $(x+1)\sqrt{x^2+x-2}=2x+2$

3) $(x+2)\sqrt{16x+33}=(x+2)(8x-15)$

Yechish usuli:

Umumiy ko'paytuvchini qavsdan tashqariga chiqarib va ko'paytuvchilarni tenglashtirish yo'li bilan va albatta aniqlanish sohasini e'tiborga olinadi.

IV Tip.

1) $x^2+\sqrt{x^2+2x+8}=12-2x$

2) $\sqrt{3x^2-2x+15}+\sqrt{3x^2-2x+8}=7$

3) $3x^2+\sqrt{x^2+5+3x}=\sqrt{5}-9x$

Yechish usuli:

Yangi o'zgaruvchi kiritish orqali.

V Tip.

1) $\sqrt{x+2+2\sqrt{x+1}}+\sqrt{x+2-2\sqrt{x+1}}=2$

2) $\sqrt{x+5-4\sqrt{x+1}}+\sqrt{x+2-2\sqrt{x+1}}=1$

Yechish usuli:

Ildiz ostidagi ifodani to'la kvadratga ajratish

VI Tip.

$$\sqrt{2x+3}+\sqrt{4x+1}=4$$

Yechish usuli:

Aniqlanish sohasinini e`tiborga olib kvadratga ko'tarish.

VII Tip.

$$\sqrt{1+4x-x^2}=x-1$$

Yechish usuli:

O'ng tomon nomanfiy ekanligidan foydalanib kvadratga oshirish.

VIII Tip.

1) $\sqrt[3]{x-7}+\sqrt[3]{x+1}=2$

2) $\sqrt[3]{15+2x}+\sqrt[3]{13-2x}=4$

Yechish usuli:

Qisqa ko'paytirish qoidalariga ko'ra ishlash yoki yangi o'zgaruvchi kiritish.

IX Tip.

$$(\sqrt{x+1}+1)\cdot(\sqrt{x+10}-4)=x$$

Yechish usuli:

Tenglamani yechish uchun nolga teng bo'lmagan ifodaga ko'paytirish.

X Tip.

$$10x^2-2x-1-3x\sqrt{2x+1}=0$$

Yechish usuli:

Tenglamani $x^2 \neq 0$ va x=0 tenglab, x Tenglamaning yechimi bo'lmasin. Keyin yangi o'zgartiruvchi kiritiladi.

XI Tip.

$$\sqrt{2x^2+x-1}+\sqrt{x^2-x-2}=\sqrt{x^2-3x-4}$$

Yechish usuli:

Ildiz ostidagi ifodani umumiy ko'paytuvchiga ajratish

XII Tip.

$$\sqrt{3x^2+6x+7}+\sqrt{5x^2+10x+14}=4-2x-x^2$$

Yechish usuli:

Baholash metodi.

Tenglamani yeching 1. $\sqrt{x-10}+\sqrt{3-x}=2.$

Tenglamani yeching 2. $\sqrt{x+3}+\sqrt{x+8}=5$

Namuna 1.

$$\sqrt{6-4x-x^2}=x+4$$

$$\begin{cases}6-4x-x^2=x+4\\ x+4\geq 0\end{cases}$$

$$\begin{cases}x^2+6x+5=0\\ x\geq -4\end{cases}$$

$$\begin{cases}x=-5\\ x=-1\\ x\geq -4\end{cases}$$

Javob: $\{-1\}$

Namuna 2. $\sqrt{1+\sqrt{x^2-24}}=x+1$

$$\begin{cases}1+x\sqrt{x^2-24}=x^2-2x+1\\ x-1\geq 0\end{cases} \qquad \begin{cases}x(\sqrt{x^2-24}-x+2)=0\\ x\geq 1\end{cases}$$

$$\begin{cases}x=0\\ x=7\\ x\geq 1\end{cases} \qquad x=7$$

Javob: $\{7\}$

Quyidagi tenglamalarni ratsional sistemaga yoki modul qatnashgan tenglamaga keltirish usuli bilan yeching.

1. $\sqrt{(x-2)^2}+\sqrt{(x+1)^2}=\sqrt{(x+2)^2}$
2. $\sqrt{x^2-4x+4}-\sqrt{x^2-6x+9}=\sqrt{x^2-2x+1}$
3. $\sqrt{x+5-4\sqrt{x+1}}+\sqrt{x+2-2\sqrt{x+1}}=1$
4. $\sqrt{5+x+4\sqrt{x+1}}=2+\sqrt{x+1}$
5. $\sqrt{x+2+2\sqrt{x+1}}+\sqrt{x+2-2\sqrt{x+1}}=2$
6. $\sqrt{x^2+9}-\sqrt{x^2-7}=2$
7. $\sqrt{10-x^2}+\sqrt{x^2+3}=5$
8. $\sqrt{4x+2}+\sqrt{4x-2}=4$
9. $\sqrt{2-x}+\sqrt{9-x}=5$

10. $\sqrt[3]{x-5} - \sqrt[3]{x-8} = 2$

11. $\sqrt[3]{[illegible] - \sqrt{x-1}} - \sqrt[3]{[illegible] - \sqrt{x-1}} = 4$

Mustaqil yeching.

1. $2x^2 + 3x + \sqrt{2x^2 + 3x + 9} = 33$

2. $2x^2 + (2x + 1)\sqrt{x^2 - x + 1} = 1$

3. $(x + 3)\sqrt{x - 1} = 3\sqrt{x^2 - 1}$

4. $\sqrt{2x^2 - 1} + \sqrt{x^2 - 3x - 2} = \sqrt{2x^2 + 2x + 3} + \sqrt{x^2 - x + 2} = 0$

5. $\sqrt{x + 3 - 4\sqrt{x - 1}} + \sqrt{x + 8 - 6\sqrt{x - 1}} = 1$

6. $\sqrt{4x^2 + 8x + 8} + \sqrt{3x^2 + 6x + 12} = 4 - 2x - x^2$

7. $x^2 + 2 + \frac{4}{x^2 - 2x + 2} = 2x + \sqrt{12 - x^2 + 4x}$

$x \in R$ *uchun*

$\sqrt{12 - x^2 + 4x} = \sqrt{16 - (x-2)^2} \le \sqrt{16} = 4,$

$x^2 - 2x + 2 = (x-1)^2 + 1 > 0.$

Koshi tengsizligiga asosan:

$x^2 - 2x + 2 + \frac{4}{x^2 - 2x + 2} \ge 2\sqrt{(x^2 - 2x + 2)\cdot\frac{4}{x^2 - 2x + 2}} = 4,$

$x = 0$ *va* $x = 2.$

$x = 2$ *-tenglamaning yechimi.*

8. $\sqrt[3]{2 - x} = 1 - \sqrt{x - 1}$

9. $\sqrt[3]{54 + \sqrt{x}} + \sqrt[3]{54 - \sqrt{x}} = \sqrt[3]{18}$

10. $\sqrt[4]{12 - x} + \sqrt{14 + x} = 2$

11. $\sqrt[4]{12 - x} + \sqrt[4]{14 + x} = 2$

12. $\sqrt[3]{(8 - x)^2} + 2\sqrt[3]{(x + 27)^2} = 3\sqrt[3]{(8 - x)(x + 27)}$

Ko'rsatma: $u=\sqrt[3]{8-x}, v=\sqrt[3]{x+27}$.

Tenglamani yozsak.

$u^2+2v^2=3uv$

$\left(\frac{u}{v}\right)^2-3\left(\frac{u}{v}\right)+2=0,$ *bundan :* $\frac{u}{v}=1, \frac{u}{v}=2$

13. $x+\sqrt[8]{x^5}-12\sqrt[4]{x}=0$

14. $\sqrt{2x^3+2x^2-3x+3}=x+1$

15. $\sqrt{\frac{1+2x\sqrt{1-x^2}}{2}}+2x^2=1$

16. $\sqrt{x-1}+\sqrt{x}=\frac{1}{\sqrt[3]{x^2}}$

17. $\sqrt{18+3x}-\sqrt{9-x^2}=\sqrt{3x}$

18. $\sqrt{\frac{x+5}{5}}-\sqrt{\frac{x-5}{x}}=\sqrt{2\left(1-\frac{5}{x}\right)}$

19. $\sqrt{\frac{1}{x}-1}+\sqrt{x+1}=\sqrt{\frac{2}{x}}$

20. $\sqrt{x}+\sqrt{x+7}+2\sqrt{x^2+7x}=35-2x$

21. $x^2+2x\sqrt{x}+2x+\sqrt{x}=30$

22. $\sqrt{3x+5}+\sqrt{5x-4}=\sqrt{3x+8}+\sqrt{5x-7}$

23. $\frac{\sqrt{x}+\sqrt{3x}-\sqrt{6}}{\sqrt{2x+6}}=\sqrt{2}+\sqrt{6}-\sqrt{x}$

24. $\sqrt[3]{1+\sqrt{x}}=\sqrt{1+\sqrt[3]{x}}$

25. $x+\sqrt{17-x^2}+x\sqrt{17-x^2}=9$

26. $\sqrt{x}+\sqrt{2-x}+\sqrt{2x-x^2}=\sqrt{2}$

27. $4\left(\sqrt{1+x}-1\right)\left(\sqrt{1-x}+1\right)=x$

28. $\sqrt[4]{77+x}+\sqrt[4]{20-x}=5$

29. $\sqrt{6-x}+\sqrt{x-2}+2\sqrt[4]{(6-x)(x-2)}=2$

30. $\sqrt[5]{(x-2)(x-32)} - \sqrt[5]{(x-1)(x-33)} = 1$

18.Irratsional tengsizliklar va tengsizliklar sistemasiga oid misollar yechish.

18.1. $\sqrt{x^2+3x+4} + \sqrt{x} + 1 > 1{,}4$

18.2. $\sqrt{x+1} + 1 < 4x^2 + \sqrt{3x}$

18.3. $\sqrt{2-x} > \sqrt{7-x} - \sqrt{-3-2x}$

18.4. $\sqrt{-x} - \sqrt{x+1} > \frac{1}{\sqrt{3}}$

18.5. $\sqrt{x+6} > \sqrt{x+1} + \sqrt{2x-5}$

18.6. $\sqrt{x+2} - \sqrt{x-5} \leq \sqrt{5-x}$

18.7. $\sqrt{x-2} - \sqrt{x-3} > -\sqrt{x-5}$

18.8. $\sqrt{7x-13} - \sqrt{3x-19} > \sqrt{5x-27}$

18.9. $\sqrt{x+2} < \sqrt{x+12} - \sqrt{2x-10}$

18.10. $\sqrt{x^2+3x+2} - \sqrt{x^2-x+1} < 1$

18.11. $\sqrt{\frac{2x-1}{3x-2}} \leq 3$

18.12. $\sqrt{\frac{x+3}{4-x}} \geq 2$

18.13. $\sqrt{(x-3)(x+1)} > 3(x+1)$

18.14. $\sqrt{(x+2)(x-5)} < 8-x$

18.15. $\sqrt{x^2-4x} > x-3$

18.16. $\sqrt{x^2-5x+6} \leq x+4$

18.17. $x^2 + \sqrt{x^2+11} < 31$

18.18. $\frac{x-4}{\sqrt{x}+2} < x-8$

18.19. $x^2+5x+4 < 5\sqrt{x^2+5x+28}$

18.20. $\sqrt{2x+\sqrt{6x^2+1}} < x+1$

16. Ko'rsatkichli va logarifmik ifodalarni ayniy almashtirishga oid misollar yechish

$a^x = b \Leftrightarrow x = \log_a b \quad (a > 0,\ a \neq 1,\ b > 0)$,

Masalan:

$\log_3 x = 4 \Leftrightarrow x = 3^4 \Leftrightarrow x = 81$; $\log_{a} x = -\frac{1}{3}$ [illegible]

$$\lg b = \log_{10} b,$$

Misol

$\lg 10 = 1$;

$\lg 100 = 2$, $\quad 10^2 = 100$;

$\lg x = 3 \Leftrightarrow x = 10^3 \Leftrightarrow x = 1000$,

$$\ln b = \log_e b,$$

Misol

$\ln e = 1$;

$\ln e^2 = 2$,

$$\log_a a = 1 \quad (a > 0;\ a \neq 1) \qquad 1 = \log_a a \quad (a > 0;\ a \neq 1),$$

Xossalari

1. $\log_a 1 = 0$
2. $\log_a a = 1$
3. $\log_a(xy) = \log_a x + \log_a y$ yoki $\log_a x + \log_a y = \log_a(xy)$,
4. $\log_a \frac{x}{y} = \log_a x - \log_a y$ yoki $\log_a x - \log_a y = \log_a \frac{x}{y}$,
5. $\log_a x^p = p \cdot \log_a x$ yoki $p \cdot \log_a x = \log_a x^p$,
6. $\log_a x = \frac{\log_b x}{\log_b a}$ natijasi $\log_a b = \frac{1}{\log_b a}$ yoki $\frac{1}{\log_b a} = \log_a b$,

Misollar

1. $log_8 log_4 log_2 16$

2. $lglg\sqrt{\sqrt[5]{10}}$

3. $\left(\frac{16}{25}\right)^{log_{\frac{125}{64}}3}$

4. $36^{log_6 5} + 10^{1-lg2} - 3^{log_9 36}$

5. $3^{log_6 5} - 5^{log_6 3}$

6. $36^{\sqrt{log_{36}5}} - 5^{\sqrt{log_5 36}}$

7. $lg\left(7 - log_2 log_3\sqrt{\sqrt[4]{3}}\right)$ ni hisoblang.

8. $3^{\sqrt{\log_3 2}} - 2^{\sqrt{\log_2 3}}$ ni hisoblang.

9. $\log_2 5 = a$ ga ko'ra $\log_{100} 40$ ni hisoblang.

10. $\log_{12} 27 = a$ ga ko'ra $\log_6 16$ ni hisoblang.

11. $\log_3 20 = a$ va $\log_3 15 = b$ ga ko'ra $\log_2 360$ ni hisoblang.

12. $\log_{12} 5 = a$ va $\log_{12} 11 = b$ ga ko'ra $\log_{275} 60$ ni hisoblang.

13. $\lg_{ab} k = \dfrac{\log_a k \log_b k}{\log_a k + \log_b k}$ ni isbotlang.

14. $\lg_{ab} ak = \dfrac{\log_b a + \log_b k}{1 + \log_b k}$ ni isbotlang.

15. $\dfrac{1}{(\log_{a_1} b)^{-1} + (\log_{a_2} b)^{-1} + \ldots + (\log_{a_n} b)^{-1}} = \log_{a_1 a_2 \ldots a_n} b$ ni isbotlang.

16. Agar $a^2 + 4b^2 = 12ab$ bo'lsa, $\lg\dfrac{a+2b}{4} = \dfrac{1}{2}(\lg a + \lg b)$ ni isbotlang.

17. $(\log_a b + \log_b a + 2)\cdot(\log_a b - \log_{ab} b)\log_b a - 1$ ni soddalashtiring.

18. $0.2(2a^{\log_2 b} + 3b^{\log_{\sqrt{2}}\sqrt{a}})$ ni soddalashtiring.

19. $\sqrt{1 + 2^{\frac{\lg a}{\lg\sqrt{2}}} - a^{1+\frac{1}{\log_4 a^2}}} - 1$ ni soddalashtiring.

20. $2\log_a^{\frac{1}{2}} b\left[\left(\log_a \sqrt[4]{ab} + \log_b \sqrt[4]{ab}\right)^{\frac{1}{2}} - \left(\log_a \sqrt[4]{\frac{b}{a}} + \log_b \sqrt[4]{\frac{a}{b}}\right)^{\frac{1}{2}}\right]$, (a>1) ni soddalashtiring.

21. Agar $m^2 = a^2 - b^2$ ekanligi ma'lum bo'lsa, $\log_{a+b} m + \log_{a-b} m - 2\log_{a+b} m \cdot \log_{a-b} m$ ifodani soddalashtiring.

22. $\beta = 10^{\frac{1}{1-\lg\alpha}}$ va $\gamma = 10^{\frac{1}{1-\lg\beta}}$ ekanligi ma'lum bo'lsa, α *ning* γ ga bog'lanishini toping.

23. Agar b^2=ac bo'lsa, $\frac{\log_a x - \log_b x}{\log_b x - \log_c x} = \frac{\log_a x}{\log_c x}$ ni isbotlang.

24. $(\log_a b + \log_b a + 1)(\log_a b - \log_{ab} b)\log_b a - 1$.

25. $(\sqrt[\lg a]{b^{\log_{100} a}}\sqrt[\lg b]{a^{\log_{100} b}})2\log_{ab}(a+b)$.

26. $[(\log_b a + \log_a^4 b + 2)]^{\frac{1}{2}} - \log_b a - \log_a b$.

27. $\log_2 2x_2 + \log_2 x \cdot x^{\log_x(\log_2 x+1)} + \frac{1}{2}\log_4^2 x^4 + 2^{3\log_{\frac{1}{2}}\log_2 x}$.

28. $\frac{\log_a b - \log_{b-3\sqrt{a}} \sqrt{b}}{\log_{\frac{a}{b^4}} b - \log_{\frac{a}{b^8}} b} \cdot \log_{a^2} b^{-12}$.

29. $[6(\log_b a \cdot \log_{a^2} b + 1) + \log_a b^{-6} + \log_a^2 b]^{\frac{1}{2}} - \log_a b;\ a > 1$.

30. $\sqrt{\log_n p + \log_p n + 2}(\log_n p - \log_{np} p)\sqrt{\log_n p}$.

31. $(\frac{\log_a^2 b + 1}{2\log_a b} - 1)^{\frac{1}{2}}(\frac{\log_a^2 b + 1}{2\log_a b} + 1)^{\frac{1}{2}} \cdot \sqrt{2}\log_a^{\frac{1}{2}} b;\quad a > 1$.

20.Ko`rsatkichli va logarifmik tenglamalar va tenglamalar sistemasiga oid misollar yechish.

Ko'rsatkichli tenglamalar ko'rinishlari va yechish usullari.

1-ko'rinish $a^{f(x)} = 1$

a=1 da cheksiz ko'p yechimga ega bo'ladi. Boshqa hollarda $f(x) = 0$

Misol. $2^{x^2} \cdot 3^{x^2} = 1$.

2-ko'rinish $q(x)^{f(x)} = 1$

Misol. $(x^2 - 4)^{2x+3} = 1$ $\qquad |x-2|^{x^2 - 6x + 8} = 1$.

3-ko'rinish $a^{f(x)} = b^{f(x)}$

Misol. $2^{4-3x} = 3^{4-3x}$, $6^{2x+1} = 3^{3x+1}2^{x+1}$

4-ko‘rinish $a^{f(x)} = a^{q(x)}$

Misol. $6^{0,5x^2-x} = \sqrt[3]{6^{-5}}$, $\left(\frac{1}{3}\right)^{x-1} = 2^{x^2-1}$.

5-ko‘rinish $a_0m^{nx+c_1} + a_1m^{nx+c_2} + \dots + a_nm^{nx+c_n} = F$

Misollar. $3^{2x+1} - 5^{2x-1} = 2\cdot 3^{2x} + 4\cdot 5^{2(x-1)}$, $4^{x^2+x-1} - 4^{x^2-x-1} = 1,5\cdot 4^{x^2-1}$

6-ko‘rinish $ma^{2f(x)} + na^{f(x)} + p = 0$

Misollar. $8^{x^2} + 2^{2x^2-x} - 3\cdot 2^{(x-1)^2} = 0$.

7- ko‘rinish Ba’zi ko‘rsatkichli tenglamalarni yechishda o‘rniga qo‘yish usulining tatbiqlari (Misolning berilishiga qarab o‘rniga qo‘yishlar turlicha bo‘ladi. $3(3^x + 3^{-x}) + 9(9^x + 9^{-x}) = 92$, $3^x + 3^{-x} = t$, va h.k.) $2^x + \sqrt{1-2^x} + 2^x\sqrt{1-2^x} = 1$. da $2^x + \sqrt{1-2^x} = t$, masalan, $\frac{4^x+9^x}{2^x+3^x} = \sqrt{6^x}$. da $\frac{4^x+9^x}{2^x+3^x} = \sqrt{6^x}$.

8- ko‘rinish $ma^{2f(x)} + na^{f(x)}b^{f(x)} + b^{2f(x)} = 0$ bunda m va n noldan farqli. Berilgan tenglamani $a^{2f(x)}$ ga yoki $b^{2f(x)}$ bo‘lib, $m\left(\frac{a}{b}\right)^{2f(x)} + n\left(\frac{a}{b}\right)^{f(x)} + 1 = 0$ ga olib kelamiz. Endi $\left(\frac{a}{b}\right)^{f(x)} = t$ o‘rniga qo‘yishni bajarib, so‘ngra kvadrat tenglamani hosil qilish mumkin.

9- ko‘rinish Tenglamani yechishning nostandart usuli.

$\left(\sqrt{3+2\sqrt{2}}\right)^x + \left(\sqrt{3-2\sqrt{2}}\right)^x = 6$ tenglamani yechishni ko‘raylik.

Yechilishi. Bilamizki, $\left(\sqrt{3+2\sqrt{2}}\right)^x \cdot \left(\sqrt{3-2\sqrt{2}}\right)^x = \left(\sqrt{(3+2\sqrt{2})(3-2\sqrt{2})}\right)^x = \left(\sqrt{9-8}\right)^x = 1$, u holda $\left(\sqrt{3-2\sqrt{2}}\right)^x = \frac{1}{\left(\sqrt{3+2\sqrt{2}}\right)^x}$

Bulardan $\left(\sqrt{3+2\sqrt{2}}\right)^x = t$ belgilashni kiritib, tenglamani yechish mumkin bo‘ladi.

Yana bir nechta misol ko'raylik.

1. $\left(2+\sqrt{3}\right)^{(x+1)^2} + \left(2-\sqrt{3}\right)^{(x-1)^2} = \left(2+\sqrt{3}\right)^{2x^2+2} + 1.$ misolda ko'rinishini o'zgartirib,

$$\left(2+\sqrt{3}\right)^{x^2+1}\left(2+\sqrt{3}\right)^{2x} + \frac{\left(2+\sqrt{3}\right)^{2x}}{\left(2+\sqrt{3}\right)^{x^2+1}} = \left(2+\sqrt{3}\right)^{2x^2+2} + 1.$$

ni hosil qilamiz.

$\left(2+\sqrt{3}\right)^{x^2+1} = a;\ \left(2+\sqrt{3}\right)^{2x} = b,$ belgilashlarni kiritib, tenglama yechimiga kelish mumkin.

2. $6\cdot 2^{|x-2|+|x-4|} + 8\cdot 3^{|x-1|+|x-3|-1} = 48.$ misolda tenglamaning ikkala tomonini 24ga bo'lib, $2^{|x-2|+|x-4|-2} + 3^{|x-1|+|x-3|-2} = 2.$ tenglamaga kelamiz. $|a-b| \le |a|+|b|$ tengsizlikdan foydalanib, $|x-2|+|x-4| \ge |x-2-(x-4)| = 2$ ni va $|x-1|+|x-3| = |x-1-(x-3)| = 2$ ni yozish mumkin. Bulardan $2^{|x-2|+|x-4|-2} \ge 1$ va $3^{|x-1|+|x-3|-2} \ge 1$ larni hosil qilamiz. Tenglik belgisi quyidagi sistema yechimga ega bo'lgandagina bajarilishi ma'lum, $\begin{cases}|x-2|+|x-4|=2,\\ |x-1|+|x-3|=2\end{cases} \Leftrightarrow \begin{cases}x\in[2;\ 4],\\ x\in[1;\ 3]\end{cases} \Leftrightarrow x\in[2;\ 3].$

Demak, javob: $[2;\ 3]$.

10- ko'rinish. Ko'rsatkichli tenglamalar sistemasini yechish.

Ko'rsatkichli tenglamalar sistemasini yechishda ham algebraik tenglamalarni yechish kabi usullardan foydalaniladi. Masalan, tenglamalarni qo'shish, o'rniga qo'yish va hakozo. Ba'zan, misolning berilishiga qarab ba'zi sun'iy usullardan foydalanishga to'g'ri keladi. Masalan, $\begin{cases}y^{\frac{1}{x}}=3,\\ y^x=81\end{cases}$, $\begin{cases}x^{2y}=3x^y-2,\\ 3y^{2x}=2y^x+1\end{cases}$, $\begin{cases}x^{x+y}=y^{12}\\ y^{x+y}=x^3\end{cases}$, $\begin{cases}x^y=y^x\\ 2^x=3^y\end{cases}$, $\begin{cases}2^x\cdot 3^y=24\\ 2^y\cdot 3^x=54\end{cases}$,

$$\begin{cases}4^{x+1}+2^{x+y+2}-2^{y+1}-1=0,\\ 4^x=\left(\frac{1}{3}\right)^y-2^y+\frac{1}{4}.\end{cases}$$

Logarifm

1. Logarifm ta'rifidan foydalanish

$\log_a b = c$ dan $a^c = b$ ga o'tish. Bulardan $a^{\log_a b} = b\ (a>0,\ b>0,\ a\ne 1)$ kelib chiqadi.

Misol, $x^2+2\cdot 3^{\log_3 x}-3=0$, $x^{\log_{\sqrt{x}}(x-2)^2}=81$ va hakozo.

2. Logarifm haqidagi teorema.

$$\log_c(ab) = \log_c a + \log_c b \ (a > 0,\ b > 0,\ c > 0,\ c \neq 1). \quad (1)$$

$$\log_c\left(\frac{a}{b}\right) = \log_c a - \log_c b \ (a > 0,\ b > 0,\ c > 0\ c \neq 1). \quad (2)$$

$$\log_c a^k = k \cdot \log_c a \ (a > 0,\ c > 0,\ c \neq 1). \quad (3)$$

Misollar. $\lg x = \frac{1}{2}\lg 16 - \lg 5 + \lg 3$, $\log_3 x = \frac{1}{3}(2\log_3 2 - 3\log_3 6)$;

3. Logarifmda bir asosdan boshqa asosga o'tish formulasi

$$\log_a b = \frac{\log_c b}{\log_c a}$$

Misollar. $\log_2 3 = a$, $\log_3 5 = b$; bo'lsa, $\log_6 15$ ni toping va hakozo.

4. Zaruriy logarifmik ifodalar

4.1. $a^{\log_c b} = b^{\log_c a}$ (isboti mustaqil).

Misollar. 1. $2^{\log_3 5} \cdot 3^{\log_2 5} > 25$ isbotlang. 2. $x^{\lg 2} + 2^{\lg x} = 4$ tenglamani yeching.

4.2. $a^{\log_a^2 b} = b^{\log_a b}$ isboti mustaqil.

Misollar. 1. $2^{\log_2^2 3} \cdot \left(\frac{2}{3}\right)^{\log_2 3}$ hisoblang.

4.3. $a^{\sqrt{\log_a b}} = b^{\sqrt{\log_b a}}$ isboti mustaqil.

Misol. 1. $4^{\sqrt{\log_2 5}} = 25^{\sqrt{\log_5 2}}$ ni isbotlang. 2. $x^{\sqrt{\log_x 2}} + 2^{\sqrt{\log_2 x}} = 4$ tenglamani yeching.

3. $a^{\frac{1}{\log_{ab} a - 1}} = b^{\frac{1}{1 - \log_{ab} b}}$ ni isbotlang.

5. Noma'lum(o'zgaruvchisi) logarifm belgisi ostida yoki logarifm asosida qatnashgan tenglamalar logarifmik tenglamalar deyiladi.

1-ko'rinish $\log_a f(x) = b$

2-ko'rinish $\log_{\varphi(x)} f(x) = b$ tenglama quyidagi sistemaga teng kuchli (mustaqil yozing)

3-ko'rinish $a^{\log_a f(x)} = b$ ga teng kuchli $f(x) = b$

$(\varphi(x))^{\log_{\varphi(x)} f(x)} = b$ *tenglamani yechishda* $\varphi(x) > 0, \varphi(x) \neq 1, f(x) > 0$ *larni tekshirib*

$f(x) = b > 0$ *tenglama yechiladi.*

Misollar.

1. $\log_2(x^2 - 5x + 6) = 1$

2. $\log_3\log_2\log_2 x = 0.$

3. $\log_{2-x}(x^2 + 3x - 6) = 1$

4. $(x^2 + x)^{\log_{x^2+x}(x^2-2x)} = 3$

6. Tenglamalarni logarimflab yechish

Bunday tenglamalarga misollar keltiraylik.

Misollar. 1. $x^{\log_2 x-1} = 8x$

2. $2^{x^2} \cdot 5^x = 10$ 3. $2^{x-2} = 3^{x^2-5x+6}$

7. Potensirlab logarifmik tenglamani yechish.

1. $\log_3(\sin x + \cos x + x^2) = \log_3(\sin x + \cos x + 1)$

2. $x - x\lg 5 = \lg(2^x + x - 3).$

3. $2x - 1 - \log_3(2 \cdot 3^x - 9) = \log_3(3^x - 6).$

8. $f(\log_a \varphi(x)) = 0$ ko'rinishdagi tenglamalar. Bunda *f(x)* - qandaydir funksiya.

1. $3\lg x^2 - \lg^2(-x) = 5$

2. $\log_2^2(4x) + \log_2\frac{x^2}{8} = 8$

9. Logarifmda bir asosdan boshqa asosga o'tish formulasi orqali logarifmik tenglamani yechish.

1. $\log_a b = \dfrac{\log_c b}{\log_c a}$ $(a > 0,\ b > 0,\ c > 0,\ a \neq 1,\ c \neq 1)$

2. $\log_a b = \dfrac{1}{\log_b a}$

3. $\log_{a^k} b^n = \dfrac{n}{k}\log_a b.$

Misollar.

1. [illegible]

2. [illegible]

10. [illegible] logarifmning [illegible] tenglamalar.

10.1. $(f(x))^{g(x)} = 1$ ko'rinishdagi tenglama, [illegible]

bo'ladi. $\begin{cases} f(x) = 1; \\ g(x) = 0; \\ f(x) > 0. \end{cases}$

Misollar.

1. $(x^2 - 2x - 3)^{\lg(x^2+3x-3)} = 1$ $\quad \left[\begin{cases} x^2 - 2x - 3 = 1, \\ x^2 + 3x - 3 > 0; \end{cases} \quad \begin{cases} \lg(x^2 + 3x - 3) = 0, \\ x^2 - 2x - 3 > 0. \end{cases}\right.$

10.2. $(f(x))^{q_1(x)} = (f(x))^{q_2(x)}$ ko'rinishdagi tenglamalar. Bular $\begin{cases} f(x) = 1 \\ q_1(x) = q_2(x) \\ f(x) > 0 \end{cases}$

Sistema bilan almashtiriladi.

Misollar.

$|x-2|^{x^2-3x} = \dfrac{1}{(x-2)^2}$ $\qquad (x+1)^{\lg x+1} = (x+1)^{\lg^2 x-1}$.

10.3. Logarifm daraja ko'rsatkichida qatnashganda.

Bunday tenglamalar logarifmlash usulida yechiladi. Logarifmlashdan keyin hosil bo'lgan tenglamaga qarab yechish usuli tanlanadi.

Misollar. 1. $x^{\lg x+1} = 10^6$, $\quad$ 2. $x \cdot 2^{\log_x 2} = 4$

10.4. $(f(x))^{p(x)} = (q(x))^{p(x)}$ ko'rinishdagi tenglamalar.

Misollar.

1. $\left(\dfrac{x}{16}\right)^{-\log_2(x-7)} - (x-6)^{\log_2(x-7)} = 0$.

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

10. [illegible]

11. $\log_{13}\log_{0}\log_{7x+16}\left((7x+16)^{9}+x^{2}-x-56\right)=0$ [illegible]

12. [illegible] $\log^2(2x-1)$ [illegible] $4\log(2x-1)=0$

13. $2\lg^2 x - 5\lg x - 7 = 0$ [illegible]

14. [illegible] $2 = 3\log_x 3$.

15. [illegible] $(4-x)^2 + 2\log(2x-1) = 4\log 3$.

16. $(x^2-7x-18)\log(x+4)=0$ [illegible]

17. [illegible] $(8x^2-x)\cdot\log(8x-1)=2$.

18. $\lg^2(4-x)+\lg(4-x)\cdot\lg\left(x+\frac{1}{2}\right)=2\lg^2\left(x+\frac{1}{2}\right)$

19. [illegible] $=\frac{5}{x}$.

20. $\log_3(x^2-6x+12)=\cos(2\pi x)$.

$x^2-6x+12=(x-3)^2+3\geq 3$ $\log_3(x^2-6x+12)\geq 1$,

$\cos(2\pi x)\leq 1$, $\begin{cases}\log_3(x^2-6x+12)=1,\\ \cos(2\pi x)=1.\end{cases}$ $x^2-6x+12=3\Leftrightarrow(x-3)^2=0\Leftrightarrow x=3$,

Tenglamalarni yeching.

1. $5^{x-1}=0{,}2$. 1. $\log_{\frac{1}{5}}(4x-1)=-1$.

7. $4^x - 3^{x-\frac{1}{2}} = 3^{x+\frac{1}{2}} - 2^{2x-1}$;

8. $3*4^{-\frac{1}{x}} + 2*9^{-\frac{1}{x}} = 5*6^{-\frac{1}{x}}$

9. $2^{10x^2-8x-23} + 2^{5x^2-4x-12} - 3 = 0$.

10. $5^{2(\log_3 x)^2} - 6*5^{(\log_3 x)} + 5 = 0$.

11. $\left(\sqrt[3]{3+\sqrt{8}}\right)^x + \left(\sqrt[3]{3-\sqrt{8}}\right)^x = 6$.

12. $4^x + 6*9^x = 5*6^x$.

13. $4^x + 5^x = 41$.

14. $5*x^{\log_3 2} + 2^{\log_3 x} = 24$.

15. $4^x + 5^x = 9^x$.

16. $2^{x^2+1} + |x| = 2$.

17. $4^{\log_4(2x+1)} = x^2 + 3x - 5$

$$4^{\log_4(2x+1)} = x^2 + 3x - 5 \Leftrightarrow \begin{cases} 2x+1 > 0, \\ 2x+1 = x^2+3x-5 \end{cases} \Leftrightarrow \begin{cases} x > -\frac{1}{2}, \\ x^2 + x - 6 = 0 \end{cases} \Leftrightarrow \begin{cases} x > -\frac{1}{2}, \\ \left[\begin{array}{l} x = -3, \\ x = 2 \end{array}\right. \end{cases} \Leftrightarrow x = 2.$$

Logarifmik tenglamalar

1. $\log_5\left(x^2 - 7x - 35\right) = 2$.

2. $\log_{16} x^5 - \log_4 x^3 + \log_2 x = -3$.

3. $\log^2{}_5 x + 5 = \log_4 x \log_3 x + 7\log^2{}_2 x = 0$

a>0, $a \neq 1$, x, y >0

1. $\log_a \frac{x}{y} = \log_a x - \log_a y$, 2. $\log_a(xy) = \log_a x + \log_a y$, 3. $\log_a x^k = k\log_a x$,

4. $\log_{a^k} x = \frac{1}{k}\log_a x$, 5. $\log_{a^n} x^k = \frac{k}{n}\log_a x$, $n \neq 0$ 6. $\log_a b = \frac{\log_c b}{\log_c a}, c > 0, c \neq 1$.

7. $a^{\log_a b} = b, b > 0$

4. $\log_x 5 + \log_5 (5x - 2) = \log_5 5$

5. $(x-2)^{x^2-x} = 15$ [illegible]

6. $(x-2)^{[illegible]} = 6$

7. $x^{\lg x} = 400$ $(10^{x})^{2}$ [illegible]

8. $\lg_2(10 - x^2) = \log_2(1 + x) + 1$

9. $\log_2(x^2 - 12) + 0{,}5\log_{\frac{1}{2}} x^2 = 0$

10. $\log_{\sin x}(2x^2 + x^2 - x - 48) = \log_{\sin x}(2x^2 + 4x - 4)$ [illegible]

11. $\log_3 \log_2 \log_{7x+6}\left((7x+6)^2 + x^2 - x - 56\right) = 0$

12. $\log^2_2(2x - 1) - 4\log_2(2x - 1) = 0.$

13. $2\lg^2 x - 5\lg x - 7 = 0$

14. $\log_3 x - 2 = 3\log_x 3.$

15. $\log_a(4 - x)^2 + 2\log_a(2x - 1) = 4\log_a 3.$ [illegible]

16. $(x^2 - 7x - 18)\log_{x+1}(x + 4) = 0$

17. $\log_x(8x^2 - x) \cdot \log_{x}(8x - 1) = 2.$ [illegible]

18. $\lg^2(4 - x) + \lg(4 - x) \cdot \lg\left(x + \frac{1}{2}\right) = 2\lg^2\left(x + \frac{1}{2}\right)$

19. $\log_x x = \frac{9}{x}.$ [illegible]

20. $\log_3(x^2 - 6x + 12) = \sin(2\pi x).$

$x^2 - 6x + 12 = (x - 3)^2 + 3 \geq 3$ $\quad \log_3(x^2 - 6x + 12) \geq 1,$

$\sin(2\pi x) \leq 1,$ $\begin{cases} \log_3(x^2 - 6x + 12) = 1, \\ \sin(2\pi x) = 1. \end{cases}$ $x^2 - 6x + 12 = 3 \Leftrightarrow (x - 3)^2 = 0 \Leftrightarrow x = 3,$

Tenglamalarni yeching:

1. $3^{x-1} = 0{,}7$ [illegible] $\quad$ [illegible] $\log_{\frac{1}{4}}(4x - 1) = -1.$

2. $\left(\frac{1}{2}\right)^{[illegible]} = 8\sqrt{2}$.

3. $\left(\frac{2}{3}\right)^{[illegible]} = 1,5^{[illegible]}$.

4. $2^{x} \cdot \left(\frac{3}{2}\right)^{x} = \frac{1}{9}$.

5. $\left(\frac{1}{3}\right)^{[illegible]} + \left(\frac{1}{3}\right)^{[illegible]} = \frac{4}{9}$.

6. $\frac{9^{x+20}}{11} = \frac{9}{11^{x+20}}$.

7. $25^{x} + 24 * 5^{x} - 1 = 0$.

8. $3^{2x-1} - \frac{8}{3^{2x-1}-1} = -1$.

9. $5 * \left(\frac{5}{6}\right)^{x-1} - 9 * \left(\frac{6}{5}\right)^{x} + 3 = 0$.

10.. $\left(2-\sqrt{3}\right)^{x} + \left(2+\sqrt{3}\right)^{x} - 2 = 0$.

11. $2 * 5^{2x} + 10^{x} = 15 * 4^{x}$.

12. $64 * 9^{\frac{1}{x}} + 12 * 12^{\frac{1}{x}} - 27 * 16^{\frac{1}{x}} = 0$.

13. $x^{\log_2 x+4} = 32$.

14. $\frac{4^{1-2x}}{8} = 0,5 * 2^{1,2+2x}$.

15. $\frac{2^{x+1} + 2 * 3^{x}}{2^{x}} = \frac{3 * 2^{x} + 3^{x+1}}{3^{x}}$.

2. $\log_{[illegible]}\left(1 + \log_{[illegible]} x\right) = 1$.

3. $\log_{4}(2x+3) + \log_{4}(x-1) = \log_{4} 3 [illegible]$

4. $(\log_{0,5} x)^2 + \log_{0,5} x - 6 = 0$.

5. $\frac{\log^2_{3} x - \log_{3} x - 2}{\log_{3} x + 1} = 1$.

6. $(x^2-1)^{\log_2(x^2-1)} = 2$.

7. $\log_{x^2-4}(2x^2 - 5x - 10) = 1$.

8. $(x^2-1)^{\log_{x^2-1} 2} = 2$.

9. $(x+2)(x-5)\log_{6-x}(x+7) = 0$.

10. $\frac{\log^2_{x+1}(x-1) + \log^2_{5}(2x-5)}{\log^2_{x+1}(x-1) + \log^2_{5}(x-2)} = 1$.

11. $x^2 - 6x - \log_3(1-x) = 7 - \log_3(1-x)$.

12. $\log_{x^2} 13 = \log_{4-3x} 13$.

13. $\log_5 \frac{3x-1}{x} * \log_{\frac{x}{5}}(3x-1) = -1$

14. $(x^2 - 5x + 3) * \lg(1 - \frac{x}{3}) = \lg\left(\frac{3}{3-x}\right)$

15. $\log_x \frac{x+3}{x-1} = 1$.

Ko`rsatkichki tenglamalarni yeching:

1) $3 \cdot 9^{x} = 91$

2) $2 \cdot 4^{x} = 64$

2) $3^{x-\frac{1}{2}}\cdot 3^{x+2}=[illegible]$

3) $4\cdot 5^{x-1}\cdot 0,5^{1-2x}=2$

4) $0,6^{x+3}=0,6^{2x-5}$;

5) $6^{3x-1}=6^{1-2x}$.

6) $3^{2x-1}+3^{2x}=108$

7) $2^{[illegible]}-2^{[illegible]}=30$

8) $2^{x+2}+2^{x+1}+2^{x}=28$

9) $3^{x+1}-3^{x}+3^{x-1}=63$

10) $5^{x}=8^{x}$

12) $(\frac{1}{2})^{x}=(\frac{1}{3})^{x}$

13) $3^{x}=5^{2x}$

14) $4^{x}=3^{\frac{x}{2}}$

15) $9^{x}-4\cdot 3^{x}+3=0$

16) $16^{x}-17*4^{x}+16=0$

21.Ko'rsatkichli va logarifmik tengsizliklar. Tengsizliklar sistemasi.

Ko'rsatkichli tengsizliklarni yeching:

1) [illegible]

2) [illegible]

3) [illegible]

4) [illegible]

5) $0,64<(0,8^{[illegible]})[illegible]<1$

6) [illegible]

7) [illegible]

8) $64^x - 8^x - 56 < 0$

9) $8 \cdot 4^x - 6 \cdot 2^x + 1 > 0$

10) $\left(\frac{1}{4}\right)^2 + \left(\frac{1}{2}\right)^x - 6 > 0$

11) $13^{2x+1} - 13^x - 12 < 0$

12) $3^{2x+1}10 \cdot 3^x + 3 \le 0$

13) $2^{3x} + 8 \cdot 2^x - 6 \cdot 2^{2x} \le 0$

14) $5^{3x+1} + 34 \cdot 5^{2x} - 7 \cdot 5^x > 0$

Tenglama va tengsizliklarni yeching.

1) $\log_2^2 x - 4\log_2 x + 3 = 0$

2) $1 + \log_x(5 - x) = \log_7 4 \cdot \log_x 7$

3) $(\log_9(7 - x) + 1)\log_{3-x} 3 \ge 1$

4) $\log_5(4^x + 144) - 4\log_5 2 = 1 + \log_5(2^{x-2} + 1)$

5) $x^{\log_4 x - 2} \le 2^{3(\log_4 x - 1)}$

6) $(3,(3) - 1,(1))^{\log_{4x} x} = 2,(2)^{\log_x 4x}$

7) $3^{\log_3 x + \log_3 x^2 + \log_3 x^3 + \dots + \log_3 x^8} \le 27x^{30}$

8) $x^{\frac{\lg x - 5}{3}} \ge 10^{5 + \lg x}$

22.Parametr qatnashgan tenglama va tengsizliklarga oid misollar yechish.

Parametr qatnashgan tenglama va tengsizliklar sistemasiga oid misollar yechish.

1. $(a^2 - 2a + 1)x = a^2 + 2a - 3$
2. $(a^3 - a^2 - 4a + 4)x = a - 1$
3. $\frac{x}{a} + \frac{a}{x} + \frac{x+a}{a+3} = 1$
4. $\frac{x+a}{a+1} = \frac{x-a}{a+2}$

5. $\frac{3x-2}{a^2-2a}+\frac{x-1}{a-2}+\frac{2}{a}=0$

6. $x^2-4ax+3a^2=0$

7. $ax^2-(1-2a)x+a-2=0$

8. $(2a-1)x^2-(3a+1)x+a-1=0$

9. $(a^2+a-2)x^2+(2a^2+a+3)x+a^2-1=0$

10. $(4a-15)x^2+2a|x|+4=0$

11. $144^{|x|}-2\cdot 12^{|x|}+a=0$

12. $3\cdot 4^{x-2}+27=a+a\cdot 4^{x-2}$

13. $lg2x+\lg(2-x)=lglga$

14. $log_a x+log_{\sqrt{a}}x+log_{\sqrt[3]{a^2}}x=27$

15. $x+\sqrt{x^2-x}=a$

16. $\frac{1}{\sqrt{x+a}}+\frac{1}{\sqrt{x-a}}=\frac{1}{\sqrt{x^2-a}}$

17. $\begin{cases}(3+a)x+2y=3\\ ax-y=3\end{cases}$

18. $\begin{cases}(7-a)x+ay=5\\ (1-a)x+3y=5\end{cases}$

19. $\begin{cases}x+ay=1\\ ax+y=a^2\end{cases}$

20. $a^2+ax<1-x$

21. $2x+3(ax-8)+\frac{x}{3}<4\left(x+\frac{1}{2}\right)-5$

22. $(2{,}5a+1)x^2+(a+2)x+a\le 0$

23. a ning qanday qiymatlarida $x^2-6ax+(2-2a+9a^2)=0$ tenglamaning ikkala ildizi ham 3 dan katta bo'ladi.

24. a ning qanday qiymatlarida $x^2+ax+2=0$ tenglamaning ikkala ildizi ham [0;3] da yotadi.

25. a ning qanday qiymatlarida $4^x-a\cdot 2^x-a+3\le 0$ tengsizlik kamida bitta yechimga ega bo'ladi.

23.Aylana doira va ularda metrik munosabatlarga oid masalalar yechish.

1. Ikkita aylana A nuqtada tashqi urinadi. BC ularning umumiy tashqi urinmasi. $\angle BAC = 90^0$ ekanligini isbotlang.

2. Ikkita aylana A va B nuqtalarda kesishadi. A va B nuqtalar aylanalarni C, D, E va M nuqtalarda kesib o'tuvchi l to'g'ri chiziqdan turli tomonlarda yotadi. DBE va CAM burchaklar yig'indisi 180^0 ekanligini isbotlang.

3. Ikkita aylana A va B nuqtalarda kesishadi. l_1 va l_2 parallel to'g'ri chiziqlar shunday o'tkazilganki, l_1 A nuqtadan o'tib aylanalarni E va K nuqtalarda, l_2 to'g'ri chiziq B nuqtadan o'tib aylanalarni M va P nuqtalarda kesib o'tadi. EKMP to'rtburchakning parallelogramm ekanligini isbotlang.

4. M nuqtadan markazi O bo'lgan aylanaga MA va MB urinmalar o'tkazilgan. l to'g'ri chiziq aylanaga C nuqtada urinib, MA va MB lar bilan mos ravishda D va E nuqtalarda kesishadi. a) MDE uchburchakning perimetri C nuqtaning joylashuviga bog'liq emasligini isbotlang. b) DOE burchak kattaligi C nuqtaning tanlanishiga bog'liq emasligini isbotlang.

5. A, B, C va D nuqtalar aylanani 1:3:5:6 nisbatda bo'ladi. A, B, C va D nuqtalardan aylanaga o'tkazilgan urinmalar orasidagi burchaklarni toping.

6. Ikkita o'zaro teng aylanalar bir-biriga va radiusi 8 sm bo'lgan uchinchi aylanaga urinadi. Ikkita teng aylanalarning uchinchi aylana bilan urinish nuqtalarini tutashtiruvchi kesma uzunligi 12 sm. Teng aylanalarning radiusini toping.

7. Ikkita kesishuvchi aylanalarning umumiy a vatari shu aylanalardan biriga ichki chizilgan oltiburchak tomoni, ikkinchisiga ichki chizilgan kvadrat tomoni bo'ladi. Shu aylanalarning markazlari orasidagi masofani toping.

8. Radiuslari r va R bo'lgan ikkita aylana tashqi urinadi. Ularning umumiy tashqi urinmasini toping.

9. Radiuslari r va R bo'lgan ikkita aylana tashqi urinadi. l to'g'ri chiziq bu aylanalarni AB=BC=CD shartni qanoatlantiradigan A, B, C va D nuqtalarda aylanalarni kesib o'tadi. AD kesmaning uzunligini toping.

10. Radiuslari 1:3 nisbatda bo'lgan ikkita aylana tashqi urinadi. Ularning umumiy urinmasi $6\sqrt{3}$ sm. Tashqi urinmalar va aylanalarning tashqi yoylari bilan hosil bo'ladigan figuraning yuzini toping.
11. Aylanaga tashqaridagi nuqtadan uzunligi 48 sm bo'lgan kesuvchi va uzunligi kesuvchining ichkaridagi qismining 2/3 qismiga teng urinma o'tkazilgan. Agar kesuvchi aylana markazidan 24 sm uzoqlikda joylashgan bo'lsa, aylana radiusini toping.
12. Ikkita tashqi urinuvchi aylanalarning umumiy tashqi urinmalari yotgan to'g'ri chiziqlar α burchak tashkil qilsa, aylanalarning radiuslari nisbatini toping.
13. Markazi O bo'lgan aylanaga tashqaridagi A nuqtadan ikkita ABC va AMK kesuvchilar o'tkazilgan. Agar $AC = a$, $\angle CAO = \alpha, \angle COK = \beta$ bo'lib, AMK kesuvchi aylana markazidan o'tgan bo'lsa, BC kesma uzunligini toping.
14. Ikkita aylana A va B nuqtalarda kesishadi. A nuqtadan o'tkazilgan AC va AD kesmalar bitta aylana uchun vatarlar, ikkinchi aylana uchun urinma bo'lsin. $AC^2 \cdot BD = AD^2 \cdot BC$ tenglikni isbotlang.
15. AB va CD R radiusli aylananing kesishuvchi perpendikulyar vatarlari bo'lsin. $AC^2+BD^2 = 4R^2$ tenglikni isbotlang.
16. Ikkita aylana C nuqtada tashqi urinadi. AB ularning umumiy urinmasi. Agar $AC = 8$ $sm, BC = 6$ sm bo'lsa, aylanalarning radiusilarini toping.
17. Aylana ichida kesishuvchi to'g'ri chiziqlar orasidagi burchak va hosil bo'ladigan kesmalar uzunliklari orasidagi bog'lanishlarni aniqlash formulasini keltirib chiqaring
18. R radiusli doiraga o'zaro tashqi urinuvchi uchta teng aylana ichki chizilgan. Bu aylanalarning radiusini toping.
19. Radiuslari R va r bo'lgan ikki aylananing tashqi urinmasi ichki urinmasidan ikki marta uzun. Shu aylanalar markazlari orasidagi masofani toping.
20. r radiusli aylanagda o'tkazilgan vatar uzunligi bilan markazdan vatargacha bo'lgan masofalar yig'indisi a ga teng. Vatar uzunligini toping.
21. Radiuslari r va R bo'lgan aylanalar o'zaro ichki urinadi. Bu aylanalarga va ularning markazlari chizig'iga urinuvchi uchinchi aylananing radiusini toping.
22. Aylanaga ikkita parallel urinma o'tkazilgan. Aylanaga o'tkazilgan uchinchi urinmaning parallel urinmalar orasida qolgan kesmasi aylana markazidan 90^0 ko'rinishini isbotlang.
23. A nuqtada tashqi urinuvchi ikki O va O_1 aylanalarga (BC) umumiy urinma o'tkazilgan. B va C lar urinish nuqtalari bo'lsa, $\angle BAC$ ni toping.
24. Ikki aylana A va B nuqtalarda kesishadi. A nuqtadan (MAN) va B nuqtadan (PBQ) kesuvchilar o'tkazilgan. (M, P va N, Q lar alohida aylanalarda yotadi). MP va NQ kesmalar parallel ekanligini isbotlang.

24. Tashqi urinuvchi ikki aylanaga (radiuslari R va r) umumiy tashqi urinma o'tkazilgan va urinish nuqtalari orasidagi kesmalar diametr qilib aylana chizilgan. Shu aylananing ikki aylana markazlari orqali o'tuvchi chiziqqa urinishini isbotlang hamda radiusini toping.

25. Radiuslari R va r bo'lgan ikki aylananing tashqi urinmasi ichki urinmasidan ikki marta uzun. Shu aylanalar markazlari orasidagi masofani toping.

26. R va r radiusli aylanalar ichki urinadi. Bu aylanalarga va ularning markazlar chizig'iga urinuvchi uchinchi aylananing radiusini toping.

27. Teng yonli uchburchakka ichki chizilgan aylana radiusining tashqi chizilgan aylanasi radiusiga nisbati k ga teng. Uchburchakning asosidagi burchagini kosinusi topilsin.

28. R radiusli aylanaga diagonallari E nuqtada AE:EC=2:3 kabi nisbatda bo'linuvchi ABCD to'rtburchak ichki chizilgan. Agar ABC teng tomonli uchburchak bo'lsa, CD tomon topilsin.

29. O'tkir burchakli uchburchakning balandliklarining asoslari yangi uchburchak tashkil etadi. Berilgan uchburchakning balandliklari yangi uchburchak uchun bissektrisa bo'lishi isbotlansin.

30. Agar to'g'ri burchakli uchburchakka tashqi va ichki chizilgan aylanalar radiuslarini nisbati $\sqrt{3}+1$ ga teng bo'lsa, uning o'tkir burchaklarini toping.

31. Ikkita aylana A nuqtada tashqi ravishda urinadi. Agar A nuqta bilan umumiy tashqi urinmalardan birining urinish nuqtalarini tutashtiruvchi vatarlar 6 sm va 8 sm ga teng bo'lsa, bu aylanalarning radiuslarini toping.

24.Uchburchakda metrik munosabatlar.

1.To 'g'ri burchakli uchburchakning katetlari $2\sqrt{21}$ va $4\sqrt{7}$ ga teng.
To 'g'ri burchak uchidan tushirilgan balandlik gipotenuzani qanday kesmalarga ajratadi.

2. To 'g'ri burchakli uchburchakda to 'g'ri burchagidan balandlik tushirilgan. Agar gipotenuza 17 ga, unga tushirilgan balandlik 4 ga teng bo'lsa, gipotenuza bo 'laklari uzunliklarini toping.

3.Uchburchakning balandligi $\sqrt{8}$ ga teng. Asosiga parallel to 'g'ri chiziq berilgan uchburchak yuzining yarmiga teng kichik uchburchak ajratadi. Kichik uchburchakning balandligini toping.

4.ABC uchburchakda AB=3, AC=5, BC=6. C uchidan AC tomonga B uchdan tushirilgan balandlikkacha masofani toping.

5.Uchburchakning asosi $\sqrt{98}$ ga teng. Asosiga parallel va uchburchakning yuzini teng ikkiga bo 'luvchi kesma uzunligini toping.

6. Teng tomonli uchburchakka kvadrat shunday ichki chizilganki, uning bir tomoni uchburchak asosida yotadi. Agar kvadratning tomoni $(2-\sqrt{3})\sqrt{3}$ ga teng bo'lsa, uchburchakning tomonini toping.

7. Uchburchakning bir uchidan o'tkazilgan balandlik va mediana shu uchga yopishgan burchakni teng uch bo'lakka bo'ladi. Uchburchakning burchaklarini hisoblang.

8. Teng yonli uchburchakning teng B va C burchaklarining bissektrisalari E nuqtada kesishib, davomida uchburchakka tashqi chizilgan aylana bilan D va F nuqtalarda kesishadi. ADEF to'rtburchak romb ekanligini isbotlang.

9. ABC uchburchakning AC va AB tomonlari uzunliklari b va c ga AA medianasining uzunligi $\sqrt{bc}$ ga teng bo'lsa, A burchakning kattaligini toping.

10. ABC uchburchak tekisligida ixtiyoriy O nuqta berilgan. AOB, BOC va COA uchburchaklarning og'irlik markazlari mos ravishda P, Q va R bo'lsa, ABC va PQR uchburchaklarning og'irlik markazlari N, K va O nuqtalar bir to'g'ri chiziqda yotishini isbotlang.

11. Teng yonli uchburchakning yon tomoni 20 sm, asosi 24 sm ga teng uchburchakning medianalari kesishgan nuqtadan bissektrisalar kesishgan nuqtagacha bo'lgan masofani toping.

12. Teng yonli ABC uchburchakning teng AV va VS tomonlarida AE vaCF teng kesmalar olingan. CE=AF ekanini va bular kesishgan nuqta BD bissektrisada yotishini isbotlang.

13. $\angle XOY=60^{\circ}$ li burchakdan tashqarida M nuqta olinib, burchak tomonlariga MA=n_1, MB=n_2 va burchak bissektrisasiga MS tik chiziqlar tushirilgan bo'lsa, OS ni toping.

14. ABC uchburchakning tomonlarida P, Q, R nuqtalar shunday olinganki, AP, BQ va CR to'g'ri chiziqlar bir nuqtada kesishadi. $AR \cdot BP \cdot CQ = RB \cdot PC \cdot QA$ munosabatni tekshiring.

15. Uchburakning ikki medianasi o'zaro tik. Uchburchakning bu medianalar o'tgan tomonlari a va b ga teng. Shu uchburchakning tomonlari orasidagi bog'lanishni toping.

16. Agar uchburchakning ikki medianasi o'zaro teng bo'lsa, u holda bu uchburchak teng yonli bo'lishini va aksincha, agar uchburchak teng yonli bo'lsa, u holda uning ikkita medianasi teng bo'lishini isbotlang.

17. Berilgan M nuqtani uchburchakning uchlaridan uzoqligi m, n, p ga teng. Agar uchburchakning tomonlari a, b, c ga teng bo'lsa, berilgan nuqtaning shu uchburchak og'irlik markazidan uzoqligini toping.

18. ABC uchburchakda bissektrisalar kesishgan nuqtadan BC tomonga parallel to'g'ri chiziq utkazilgan, u AB tomonni B_1 nuqtada va AC tomonni C_1 nuqtada kesadi. $B_1C_1=BB_1+CC_1$ bo'lishini isbotlang.
19. Uchburchakning ikkita tomonlarining uzunliklarining nisbati uchga ular orasidagi burchak esa α ga teng. shu burchakning bissektrisasi bilan unga qarshi yotgan tomon orasidagi burchakni toping.
20. ABC uchburchakda $\angle A=30^0$, $\angle B=50^0$. Uchburchakning tomonlari uchun $c^2-b^2=ab$ munosabat o'rinli ekanligini isbotlang.
21. Agar $AC+CD=m$ va $AB-BD=n$ lar ma'lum bo'lsa, ABC uchburchakning AD bissektrisasini toping.
22. To'g'ri burchakli uchburchakda to'g'ri burchakning bissektrisasi mediana va balandlik tashkil etgan burchakni teng ikkiga bo'lishini isbotlang.

25. Uchburchakka ichki va tashqi chizilgan aylanalarga oid masalalar yechish.

1. ABC uchburchakda BAC burchak 60^0, unga ichki chizilgan O markazli aylana radiusi $\sqrt[4]{3}$ ga teng. OBC uchburchak yuzini toping.
2. ABC uchburchakda BAC va ABC burchaklarininig o 'lchovlari mos ravishda 30^0 va 45^0. Agar O nuqta uchbuchakka tashqi chizilgan radiusi $\sqrt{2-\sqrt{3}}$ bo 'lgan aylana markazi bo 'lsa, AOBC to'rtburchak yuzini toping.
3. Agar BAC burchak 60^0 bo 'lib, tashqi chizilgan aylanba markazidan BC tomongacha masofa 1,3 ga teng bo 'lsa, ABC uchburchakka tashqi chizilgan aylana radiusini toping.
4. ABC uchburchakning BAC va ABC burchaklari mos ravishda 30^0 va 45^0. O nuqta ABC uchburchakka tashqi chizilgan radiusi $3-\sqrt{2}$ bo'lgan aylana markazi bo 'lsa, AOBC to 'rtburchakning perimetrini toping.
5. ABC uchburchakning BC tomoni uzunligi unga tashqi chizilgan aylana radiusiga teng bo 'lsa, BAC burchakning kattaligini toping.
6. ABC uchburchakning ACB burchagi 1200. Agar tashqi chizilgan aylana radiusi $\sqrt{75}$ bo 'lsa, AB tomon uzunligini toping.
7. ABC uchburchakda BC tomon $2\sqrt{2}$ ga teng. BAC burchak esa 45^0 ga teng. Shu uchburchakka tashqi chizilgan aylana radiusini toping.
8. ABC uchburchakning BAC va ABC burchaklari mos ravishda 15^0 va 45^0. Agar O nuqta shu uchburchakka tashqi chizilgan aylana markazi bo 'lsa, AOB burchak kosinusini toping.

9. BAC burchagi o'tkir, hamda 45^0 ga teng bo'lgan ABC uchburchakka tashqi chizilgan aylana markazi O nuqta berilgan. Agar OBC uchburchak yuzi 18 ga teng bo 'lsa, aylana radiusini toping.
10. ABC uchburchakning ABC burchak 450 ga teng. Agar shu uchburchakka tashqi chizilgan aylana radiusi $\sqrt{8}$ bo 'lsa, AC tomon uzunligini toping.

26.Styuart, Ptolemey teoremalariga oid masalalar yechish.

1. Yon tomoni 4 sm bo'lgan teng yonli uchburchakda yon tomoniga mediana o'tkazilgan. Agar shu mediana 3 sm bo'lsa, uchburchakning asosini toping.

2. To'g'ri burchakli uchburchakning katetlariga tushurilgan medianalari $\sqrt{52}$ va $\sqrt{73}$ bo'lsa, uning gipotenuzasini toping.

3. To'g'ri burchakli uchburchakka tashqi chizilgan aylana radiusi 15m, ichki chizilgan aylana radiusi 6m bo'lsa uchburchakning tomonlarini toping.

4. Teng yonli uchburchakda asosi va yon tomoni mos ravishda 5m va 20m. asosidagi burchagining bissektrisasini toping.

5. Uchburchakning asosi 20sm, yon tomonlariga tushgan medianalari 18 va 24sm. Uchburchakning yuzini toping.

6. Uchburchakning ikki tomoni mos ravishda 6sm va 8 sm. Bu tomonlarga o'tkazilgan medianalar perpendikulyar bo'lsa, uchburchakning uchinchi tomonini toping.

7. Uchburchakning medianalari kvadratlarining yig'indisi uning tomonlari kvadratlari yig'indisining to'rtdan uch qismiga teng ekanligini isbotlang.

8. ABC uchburchakda AB tomon 21 ga teng, BD bissektrisasi $8\sqrt{3}$ ga, DC kesma uzunligi 8ga teng. ABC uchburchakning perimetrini toping.

9. KPM uchburchakda KP tomon 5ga, PM tomon $\sqrt{13}$ ga, PO mediana $3\sqrt{2}$ga teng. KPM uchburchakning yuzini toping.

10. ABC uchburchakda AB tomon 3 ga, BC=2AC, E – CD bissektrisa davomining berilgan uchburchakka tashqi chizilgan kesishish nuqtasi va DE ning uzunligi 1 ga teng. AC tomon uzunligini toping.

11. Bissektrisasi $24\sqrt{2}$ ga teng to'g'ri burchakli uchburchakning katetlari 3:4 nisbatda bo'lsa, uning perimetrini toping.

12. ABC uchburchakda AB=18sm, AC=15sm, AE bissektrisasi $4\sqrt{15}$sm. ABC uchburchak perimetrini toping.

13. ABC uchburchak yuzi $20\sqrt{3}$ ga teng. AB=8 va AC=14. ABC

uchburchakning BM medianasini toping.

14. ABC uchburchakka ichki chizilgan aylanani AL1 bissektrisa E va T nuqtalarda kesadi. AE va TL1 kesmalardan qaysi biri katta?

15. Agar ABC uchburchakda bissektrisalar O nuqtada bir nisbatda bo'linsa, ABC teng tomonli uchburchak ekanligini isbotlang.

16. ABC uchburchakda BC tomon AB va AC tomonlarning o'rta arifmetigi bo'lsin. MO to'g'ri chiziqning BC tomonga parallel ekanligini isbotlang(Bunda M – medianalar kesishish nuqtasi, O – bissektrisalar kesishish nuqtasi).

17. Uchburchakning medianasi tomonlari yig'indisi yarmidan kichik, shu yarim yig'indi bilan uchinchi tomon yarmi ayirmasidan katta ekanligini isbotlang.

18. Teng yonli uchburchakda uchidan tushurilgan balandligi 12sm. Asosining yon tomonga nisbati 4:3 nisbatda bo'lsa, uchburchakka ichki chizilgan aylana radiusini toping.

19. ABC uchburchakda AM mediana AC=b, AB=c tomonlarning o'rta proporsionali ya'ni $AM = \sqrt{bc}$. $CosA = \frac{4bc-b^2-c^2}{2bc}$ tenglikni isbotlang.

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Mundarija

Toxirova Malika

MATEMATIKADAN MISOL VA MASALALAR YECHISH METODIKASIDAN MASALALAR TO'PLAMI

(Metodik qo'llanma)

Босишга рухсат этилди. 11.10.2019 й.
Қогоз бичими 60х84 1/16. Times New Roman
гарнитурасида терилди.
Офсет усулбида оқ когозда чоп этилди.
Нашриёт ҳисоб табоғи 5.5, Адади 100. Буюртма № 34
Баҳоси келишув асосида

"BROK CLASS SERVIS" МЧЖ босмахонасида чоп этилди.
Манзил: Тошкент шаҳар Заргарлик кўчаси, Сегизбаева 10а.