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ALGEBRA VA SONLAR NAZARIYASIDAN MODUL TEXNOLOGIYASI ASOSIDA TAYYORLANGAN MUSTAQIL ISHLAR TO'PLAMI

IV QISM

O`ZBEKISTON RESPUBLIKASI OLIY VA O`RTA MAXSUS TA'LIM VAZIRLIGI

NIZOMIY NOMIDAGI TOSHKENT DAVLAT PEDAGOGIKA UNIVERSITETI

«Matematika va uni o`qitish metodikasi» kafedrasi

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Algebra va sonlar nazariyasidan modul texnologiyasi asosida tayyorlangan mustaqil ishlar to`plami/ IV qism/. 56 б.

Ushbu o`quv-metodik qo`llanma uquv rejasiga «Algebra va sonlar nazariyasi» fani kiritilgan oliy o`quv yurtlar talabalari uchun mo`ljallangan bo`lib, talabalarining nazariy hamda amaliy bilimlarini, mustaqil ishlash malaka, ko`nikmalarini shakllantirish, rivojlvntirish, nazorat qilish va baholash uchun tuzilgan.

O`quv-metodik qo`llanma «Algebra va sonlar nazariyasi» fani davlat ta'lim standarti hamda dasturi asosida nashrdan chiqqan mustaqil ishlar to`plami 1,2,3-qismlarining uzviy davomi sifatida tayyorlangan. Tavsiya etilayotgan nazariy savollar va amaliy topshiriilar modullarda jamlangan bo`lib, ularda keltirilgan metodik tavsiyalar ushbu ishlanmadan foydalanuvchilarga keng imkoniyatlar yaratadi.

Taqrizchilar :
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O`quv-metodik qo`llanma Nizomiy nomidagi Toshkent davlat pedagogika universiteti uslubiy kengashi (2017- yil "20" apreldagi 9-sonli majlis)da ko`rib chiqilgan va nashrga tavsiya etilgan. Bu nashr takomillashtirilgan qayta nashr.

SO`Z BOSHI

«Algebra», «Algebra va sonlar nazariyasi», «Oliy matematika» fanlari o`quv rejasiga kiritilgan pedagogika va boshqa oliy ta'lim muassasalari talabalarining nazariy hamda amaliy bilim, malaka, ko`nikmalarini nazorat qilish va baholash tizimiga ko`ra talabalar oraliq, yakuniy nazoratlar topshiradilar.

Talabalarining «Algebra va sonlar nazariyasi» fani bo`yicha nazariy hamda amaliy bilimlarini tartiblash, nazorat qilish va baholash, mustaqil ta'limini tashkil etish maqsadida tuzilgan o`quv-metodik qo`llanma dasturga ko`ra modul texnologiyasiga asoslangan.

Talabalar bilimini nazorat qilish va baholash tizimiga ko`ra talabalar ON va YaNni yozma ish ko`rinishida topshiradilar va bu yozma ish variantlari nazariy hamda amaliy topshiriqlardan iborat bo`ladi. Ushbu o`quv-metodik qo`llanmada keltirilgan amaliy topshiriqlar, takrorlash uchun savollar fanning «Butun sonlar halqasida bo`linish nazariyasi», «Taqqoslamalar va ular ustida amallar» bo`limlari mazmunini qamrab olganligi sababli OB va YaB variantlariga asos bo`la oladi.

Talaba bajaradigan mustaqil ish variantini talabaning guruh jurnalidagi tartib raqami asosida belgilashni tavsiya etamiz.

Mazkur o`quv-metodik qo`llanma "Algebra", "Algebra va sonlar nazariyasi", "Oliy matematika", "Matematika" «Geometriya», «Matematik analiz», «Algebra va matematik analiz asoslari» fanlarini o`qitayotgan professor-o`qituvchilarga, akademik litsey, kasb-hunar kollejlari, umumiy o`rta maktab matematika o`qituvchilariga, matematikani o`rganayotgan talabalarga mo`ljallangan.

XV MODUL. BUTUN SONLAR HALQASIDA BO`LINISH NAZARIYASI

1. Butun sonlar halqasida bo`linish munosabati, xossalari.
2. Qoldiqli bo`lish haqidagi teorema.
3. Evklid algoritmi.
4. Tub va murakkab sonlar.
5. Butun sonni tub ko`paytuvchilarga yoyish.
6. Butun son bo`luvchilari.
7. Tub sonlar to`plamining cheksizligi.
8. Eratosfen g'alviri.
9. Tub sonlar taqsimoti.
10. Arifmetik progressiyada tub sonlar.
11. Sonli funksiyalar.
12. EKUB, xossalari.
13. O`zaro tub sonlar, xossalari.
14. EKUK, xossalari.
15. CHekli zanjir kasrlar.
16. Munosib kasrlar, xossalari.
17. Butun sistematik sonlar.
18. Sistematik sonlar ustida arifmetik amallar.
19. Bir sanoq sistemasidan ikkinchisiga o`tish.

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15- MUSTAQIL ISH

0-variant

1-misol. $\forall n \in N$ **uchun** $n(n+1)(2n+1)$ **ning 6 ga bo`linishini isbotlang.**

Yechish: 1-usul. Matematik induksiya metodi. $n=1$ bo`lsa, u holda $n(n+1)(2n+1)=6\vdots 6$. Faraz qilamiz $n=k$ uchun $k(k+1)(2k+1)\vdots 6$ bo`lsin, u holda, $n=k+1$ da $(k+1)(k+2)(2k+3)\vdots 6$. Haqiqatdan ham $(k+1)(k+2)(2k+3)=k(k+1)2k+1)+6(k+1)^2$ bo`lganligi va qo`shiluvchilarning har biri 6 ga birikganligi uchun $(k+1)(k+2)(2k+3)\vdots 6$.

2-usul. Natural sonlar qatoridan 2 ta ketma-ket kelgan sonlar $n(n+1)\vdots 2$ bo`linganligidan $n(n+1)(2n+1)\vdots 2$ va $6=2\cdot 3$ bo`lib, $(2,3)=1$ ekanligidan uchun $n(n+1)(2n+1)\vdots 3$ ekanligini ko`rsatish kifoya. Qoldiqli bo`lishi haqidagi teoremaga ko`ra har qanday natural sonni $n=3k$ yoki $n=3k+1$ yoki $n=3k+2$ ko`rinishida ifodalash mumkin. Bundan

1) Agar $n=3k$ bo`lsa, u holda $n(n+1)(2n+1)\vdots 3$;

2) Agar $n=3k+1$ ko`rinishida bo`lsa, u holda $2n+1=6k+3$ va $n(n+1)(2n+1)\vdots 3$;

3) Agar $n=3k+2$ ko`rinishida bo`lsa, u holda, $n+1=3k+3$ va $n(n+1)(2n+1)\vdots 3$;

Demak, $n(n+1)(2n+1)\vdots 6$.

3-usul. Agar $n(n+1)(2n+1)=n(n+1)[(n+1)+(n+2)]=$ $=(n+1)^n(n+1)+n(n+1)(n+2)$ shakl almashtirishdan foydalansa, u holda $n(n+1)(2n+1)$ ifodani 2 ta ketma-ket keluvchi 3 son ko`paytmasining yig'indisi ko`rinishiga keltirish mumkin. Ketma-ket kelgan 3 natural sonning 6 ga bo`linishidan $n(n+1)(2n+1)\vdots 6$ ekanligi kelib chiqadi.

2-misol. Berilgan 150 va 200 sonlar orasidagi barcha tub sonlarni aniqlang.

Yechish. 150 va 200 sonlar orasidagi barcha natural sonlarni tartib bilan yozib olamiz:

150 151 152 153 154 155 156 157 158 159
160 161 162 163 164 165 166 167 168 169
170 171 172 173 174 175 176 177 178 179
180 181 182 183 184 185 186 187 188 189
190 191 192 193 194 195 196 197 198 199
200

Tuzilgan qatorning birinchi soni 150 juft son. Demak, 2 ga bo`linadi. 150 dan boshlab qatorning har 2-sonini o`chirib chiqamiz:

~~150~~ 151 ~~152~~ 153 ~~154~~ 155 .... ~~200~~

Berilgan qatordan 2 ga bo`linuvchi sonlarni o`chirib chiqdik. Endi qolgan sonlar qatoridan raqamlarni yig'indisi 3 ga bo`linadigan birinchi sonni topamiz. Bu son qatordan o`chiramiz. Bunda o`chirilgan sonlar o`rni ham hisobga olinadi. Bu jarayonni $\sqrt{200} \approx 14$ dan katta bo`lmagan tub songa bo`linadigan sonlarni o`chirguncha davom ettiramiz. Berilgan qatorning o`chirilmay qolgan sonlari 150 dan 200 gacha bo`lgan tub sonlardir. Ular

~~150~~ 151 ~~152~~ ~~153~~ ~~154~~ ~~155~~ ~~156~~ 157 ~~158~~ ~~159~~
~~160~~ ~~161~~ ~~162~~ 163 ~~164~~ ~~165~~ ~~166~~ 167 ~~168~~ ~~169~~
~~170~~ 171 ~~172~~ ~~173~~ ~~174~~ ~~175~~ ~~176~~ ~~177~~ ~~178~~ ~~179~~
~~180~~ 181 ~~182~~ 183 ~~184~~ ~~185~~ ~~186~~ 187 ~~188~~ 189
190 191 192 193 194 195 196 197 198 199
200

Demak, 150 bilan 200 orasidagi tub sonlarni topish uchun 2.3.5.7.11.13 ga bo`linadigan sonlar qatordan o`chirildi va berilgan oraliqdagi tub sonlar Eratosfen g'alviri yordamida aniqlandi. Ular 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199.

3-misol. Berilgan 1321 sonning tub yoki murakkab ekanligini aniqlang.

Yechish. Berilgan a natural sonning tub yoki murakkab ekanligini aniqlash uchun $\sqrt{a}$ songacha bo`lgan tub sonlarga berilgan sonning bo`linishi yoki bo`linmasligi aniqlanadi. Agar berilgan $a$ son $\sqrt{a}$ gacha bo`lgan birorta ham tub songa bo`linmasa, u holda u tub son bo`ladi.

Demak, $\sqrt{1321} \approx 36$ ni topamiz. 36 gacha bo`lgan tub sonlar 2,3,5,7,11,13,17,19,23,29,31 ga berilgan 1321 sonni bo`linishini tekshiramiz.

2 ga bo`linmaydi, chunki 1321 toq son;

3 ga bo`linmaydi, chunki $1+3+2+1=7/3$;

5 ga bo`linmaydi, chunki 1321 ning oxirgi raqami 1;

$1321:7 \approx 188$

$1321:11 \approx 120$

$1321:13 \approx 101$

$1321:17 \approx 77$

$1321:19 \approx 69$

$1321:23 \approx 54$

$1321:29 \approx 45$

$1321:31 \approx 42$

Demak, 1321 36 gacha bo`lgan tu sonlarga bo`linmaydi. U tub son.

4-misol. Berilgan 123 va 321 sonlarning EKUB va EKUKlarini ikki usulda toping. EKUBni berilgan sonlar orqali chiziqli ifodalang.

Yechish. Berilgan natural sonlarning EKUB va EKUKlarini topish uchun ularni tub ko`paytiruvchilaridan yoki Uvklid algoritmidan foydalanish mumkin.

1-usul. Berilgan sonlarni tub ko`paytiruvchilarga kanonik yoyilmasini topamiz:

123	3	321	3
41	41	107	107
1		1	

$123 = 3\cdot 41 = 3^1\cdot 41^1\cdot 107^0$;

$321 = 3\cdot 107 = 3^1\cdot 41^0\cdot 107^1$

$n = P_1^{\alpha 1}....P_n^{\alpha n}$ va $m = p_1^{\beta 1}....p_n^{\beta n}$ sonlarning

EKUBi $(n;m)=P_1^{\min(\alpha_1,\beta_1)}\cdot P_2^{\min(\alpha_2,\beta_2)}\cdots P_n^{\min(\alpha_n,\beta_n)}$

EKUKi $[n;m]=P_1^{\max(\alpha_1,\beta_1)}\cdot P_2^{\max(\alpha_2,\beta_2)}\cdots P_n^{\max(\alpha_n,\beta_n)}$

Demak, $(123;321)=3$ va $[123;321]=3\cdot 41\cdot 107=13161$.

2-usul. Berilgan sonlar uchun qoldiqli bo`lish teoremasi yordamida Evklid algoritmini tuzamiz:

$321=123\cdot 2+75;$	$75=321-123\cdot 2;$
$123=75\cdot 1+48;$	$48=123-75\cdot 1;$
$75=48\cdot 1+27;$	$27=75-48\cdot 1;$
$48=27\cdot 1+21;$	$21=48-27\cdot 1;$
$27=21\cdot 1+6;$	$6=27-21\cdot 1;$
$21=6\cdot 3+3;$	$3=21-6\cdot 3$
$6=3\cdot 2+0$	

Demak,

$3=21-6\cdot 3=(48-27\cdot 1)-(27-21\cdot 1)\cdot 3=48-27\cdot 4+21\cdot 3=123-75\cdot 1-$

$-(75-48\cdot 1)\cdot 4+(48-27\cdot 1)\cdot 3=123-75\cdot 5+48\cdot 7-27\cdot 3=$

$=123-(321-123\cdot 2)\cdot 5+(123-75\cdot 1)\cdot 7-(75-48\cdot 1)\cdot 3=$

$=123\cdot 18-321\cdot 5-75\cdot 10+48\cdot 3=123\cdot 18-321\cdot 5-$

$-(321-123\cdot 2)\cdot 10+(123-75\cdot 1)\cdot 3=123\cdot 41-321\cdot 15-75\cdot 3=$

$=123\cdot 41-321\cdot 15-(321-123\cdot 2)\cdot 3=123\cdot 47-321\cdot 18=123\cdot 47+321\cdot(-18)$.

Bundan, $3=123\cdot 47+321\cdot(-18)$ kelib chiqadi.

Evklid algoritmidagi oxirgi noldan farqli qoldiq EKUB ni beradi. Demak,

$(321,123)=3$. Bundan $[321,123]=\dfrac{321\cdot 123}{(321,123)}=13161$.

Topilgan EKUB $(321,123)=3$ ning 123 va 321 lar yordamidagi chiziqli ifodasini topamiz. Tuzilgan Evklid algoritmidagi qoldiqlarni bo`linuvchi va bo`luvchilar yordamidagi ifodalarini topamiz.

5-misol. Berilgan $n=126$ soning natural bo`linuvchilari soni va yig'indisini, undan katta bo`lmagan va u bilan o`zaro tub sonlar sonini toping.

Yechish. Berilgan n sonining natural bo`luvchilari soni $\tau(n)$ va natural bo`luvchilari yig'indisini $\sigma(n)$, n dan katta bo`lmagan u bilan o`zaro tub sonlar soni $\varphi(n)$ larni aniqlash uchun n sonining tub ko`paytuvchilarga kanonik yoyilmasini topamiz. Agar $n=p_1^{\alpha_1}\dots p_n^{\alpha_n}$ bo`lsa, u holda

$$\tau(n)=(\alpha_1+1)(\alpha_2+1)\dots(\alpha_n+1);$$

$$\sigma(n)=\frac{p_1^{\alpha_1+1}-1}{p_1-1}\cdot\frac{p_2^{\alpha_2+1}-1}{p_2-1}\cdot\dots\cdot\frac{p_n^{\alpha_n+1}-1}{p_n-1};$$

$$\varphi(n)=n\left(1-\frac{1}{p_1}\right)\left(1-\frac{1}{p_2}\right)\dots\left(1-\frac{1}{p_n}\right) \text{ bo`ladi.}$$

$n=126$ ning tub bo`luvchilarga kanonik yoyilmasini topamiz:

126	2
63	3
21	3
7	7
1	

Bundan, $126=2^1\cdot3^2\cdot7^1$ ekan. U holda

a) $\tau(126)=(1+1)(2+1)(1+1)=2\cdot3\cdot2=12$. Demak, 126 ning natural bo`luvchilari 12 ta. Haqiqatdan ham ular: 1,2,3,6,7,9,14,18,21,42,63,126

v) $\sigma(126)=\frac{2^2-1}{2-1}\cdot\frac{3^3-1}{3-1}\cdot\frac{7^2-1}{7-1}=\frac{3}{1}\cdot\frac{26}{2}\cdot\frac{48}{6}=26\cdot12=312$

Haqiqatdan ham 1+2+3+6+7+9+14+18+21+42+63+126=312

s) $\varphi(126)=126\cdot\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{7}\right)=126\cdot\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{6}{7}=36$.

Demak, 126 dan katta bo`lmagan, u bilan o`zaro tub sonlar soni 36 ta.

6-misol. 23! **ni tub ko`paytiruvchilarga kanonik yozilmasini toping.**

Yechish. Berilgan $n!$ sonning tub ko`paytuvchilarga yoyilmasini topish uchun, $n$ dan katta bo`lmagan tub sonlar qanday daraja bilan kanonik yoyilmada qatnashishini topamiz.

23 dan katta bo`lmagan tub sonlar 2,3,5,7,11,13,17,19,23

2 ning 23! ning kononik yoyilmasidagi darajasini topamiz. Buning uchun 23 ni 2 ga bo`lamiz. Bo`linma 2 dan kichik son bo`lguncha bu jarayonni davom ettiramiz:

$23 = 2 \cdot 11 + 1$

$11 = 2 \cdot 5 + 1$

$5 = 2 \cdot 2 + 1$

$2 = 2 \cdot 1 + 0$

Demak, 2 ning kanonik yoyilmadan darajasi 11+5+2+1=19.

3 ning darajasini topamiz: $23 = 3 \cdot 7 + 2$
$7 = 3 \cdot 2 + 1$

3 ning darajasi 7+2=9.

5 ning darajasini topamiz:

$$23 = 5 \cdot 4 + 3$$

5 ning darajasi 4.

$$23 = 7 \cdot 3 + 2$$

7 ning darajasi 3.

$$23 = 11 \cdot 2 + 1$$

11 ning darajasi 2.

13 ning darajasi 1, chunki $23 = 13 \cdot 1 + 10$.

Huddi shunday 17,19,23 larning ham yoyilmadagi darajalari 1 ga teng.

Demak, $23! = 2^{19} \cdot 3^9 \cdot 5^4 \cdot 7^3 \cdot 11^2 \cdot 13 \cdot 17 \cdot 19 \cdot 23$.

7-misol. $\begin{cases} a \cdot b = 768 \\ (a,b) = 8 \end{cases}$ **sistemani qanoatlantiruvchi a va b sonlarni toping.**

Yechish Berilgan a va b sonlarning eng katta umumiy bo`luvchisi 8 ekanligidan, bu sonlarni $a = 8k$ va $b = 8l$ ko`rinishda yozib olamiz. Bu erda $(l,k) = 1$. Bundan $a \cdot b = 8k \cdot 8l = 64 \cdot k \cdot l = 768$ ni, bundan esa $k \cdot l = 12$ ni hosil qilamiz. Demak, 12 o`zaro tub $k$ va $l$ sonlarning ko`paytmasi ko`rinishida ifodalanadi. Quyidagi holatlar bo`lishi mumkin:

k	l	$k\cdot l$
1	12	12
3	4	12
4	3	12
12	1	12

Bundan,

a	b	$a\cdot b$
8	96	768
24	32	768
32	24	768
96	8	768

Demak, $(a,b):(8;96);\ (24;32);\ (32;24),\ (96;8)$

8-misol. Berilgan $\frac{104}{23}$ kasrni chekli zanjir kasr ko`rinishida ifodalang va uning munosib kasrlarini toping.

Yechish. $\frac{104}{23}$ kasrni chekli zanjir kasr ko`rinishida ifodalash uchun 104 va 53 sonlari uchun Evklid algoritmini tuzamiz.

$$104 = 23\cdot 4 + 12;$$
$$23 = 12\cdot 1 + 11;$$
$$12 = 11\cdot 1 + 1;$$
$$11 = 1\cdot 11 + 0.$$

Evklid algoritmidagi tengliklarning har ikkala tomonini bo`luvchilarga bo`lamiz:

$$\frac{104}{23}=4+\frac{12}{23};$$
$$\frac{23}{12}=1+\frac{11}{12};$$
$$\frac{12}{11}=1+\frac{11}{11};$$
$$\frac{11}{1}=11$$

Hosil bo`lgan tengliklarning o`ng tomonidagi kasr sonni uning teskarisi bilan almashtirish natijasida

$$\frac{104}{23}=4+\frac{12}{23}=4+\frac{1}{\frac{23}{12}}=4+\frac{1}{1+\frac{11}{12}}=4+\frac{1}{1+\frac{1}{\frac{12}{11}}}=4+\frac{1}{1+\frac{1}{1+\frac{1}{11}}}$$

chekli zanjirni hosil qilamiz. Uni qisqacha $\frac{104}{23}=[4;1,1,11]$ ko`rinishida ifodalaymiz.

Agar berilgan kasr manfiy bo`lsa, birinchi qoldiqni musbat qilib olamiz. Masalan, $-\frac{23}{13}=-2+\frac{3}{13}$ va kasr qismi chekli zanjir ko`rinishida ifodalanadi.

$$-\frac{23}{13}=-2+\frac{3}{13}=-2+\frac{1}{\frac{13}{3}}=-2+\frac{1}{4+\frac{1}{3}}=[-2;4,3]$$

Berilgan $\frac{104}{23}=[4;1,1,11]$ ning munosib kasrlarini topish uchun quyidagi jadvalni tuzamiz:

k	-1	0	1	2	3
q_k	–	4	1	1	11
P_k	1	4	5	9	104
Q_k	0	1	1	2	23

Demak, $\frac{P_o}{Q_o}=4;\ \frac{P_1}{Q_1}=5;\ \frac{P_2}{Q_2}=\frac{9}{2};\ \frac{P_3}{Q_3}=\frac{104}{23}.$

9-misol. Berilgan $\sqrt{14}$ sonni zanjir kasr ko`rinishida ifodalang.

Yechish.

$$\sqrt{14}=3+\frac{1}{\alpha_1};$$

$$\alpha_1=\frac{1}{\sqrt{14}-3}=\frac{\sqrt{14}+3}{5}=1+\frac{1}{\alpha_2};$$

$$\alpha_2=\frac{1}{\frac{\sqrt{14}+3}{5}-1}=\frac{5}{\sqrt{14}-1}=\frac{\sqrt{14}+2}{2}=2+\frac{1}{\alpha_3};$$

$$\alpha_3=\frac{1}{\frac{\sqrt{14}+2}{2}-2}=\frac{2}{\sqrt{14}-2}=\frac{\sqrt{14}+2}{5}=1+\frac{1}{\alpha_4};$$

$$\alpha_4=\frac{1}{\frac{\sqrt{14}+2}{5}-1}=\frac{5}{\sqrt{14}-3}=\sqrt{14}+3=6+\frac{1}{\alpha_5};$$

$$\alpha_5=\frac{1}{\sqrt{14}+3-6}=\frac{1}{\sqrt{14}-3}.$$

$\alpha_5=\alpha_1$ bo`lganligi uchun, yana yuqoridagi jarayon hosil bo`ladi. Demak,

$\sqrt{14}=[3;(1,2,1,6)]$.

10-misol. $-117x+343y=119$ tenglamani butun sonlar to`plamida yeching.

Yechish. Tenlamani $117(-x)+343y=119$ ko`rinishida yozib olamiz va $ax+by+c$ tenglama agar $(a,b)=1$ bo`lsa

$$x=(-1)^{n-1}\cdot c\cdot Q_{n-1}+bt$$

$$y=(-1)^{n}\cdot c\cdot P_{n-1}-at,\quad t\in z$$

formulalar orqali topiladigan butun yechimlarga ega. Buning uchun $\frac{a}{b}$ kasrning munosib kasrlari topiladi.

$\frac{a}{b}=\frac{117}{343}$ uchun chekli zanjir kasrni topamiz.

$117 = 0 \cdot 343 + 117;$

$343 = 117 \cdot 2 + 109;$

$117 = 109 \cdot 1 + 8;$

$109 = 8 \cdot 13 + 5;$

$8 = 5 \cdot 1 + 3;$

$5 = 3 \cdot 1 + 2;$

$3 = 2 \cdot 1 + 1;$

$2 = 1 \cdot 2 + 0.$

Demak, $\frac{117}{343} = [0;2,1,13,1,1,1,2]$. Munosib kasrlar jadvalini tuzamiz:

k	-1	0	1	2	3	4	5	6	7
q_k	–	0	2	1	13	1	1	1	2
P_k	1	0	1	1	14	15	29	44	117
Q_k	0	1	2	3	41	44	85	129	343

$P_6 = 44,\ Q_6 = 129$ lardan foydalanamiz.

Xususiy yechim: $\begin{cases} -x_o = (-1)^6 \cdot 119 \cdot 129 = 15351; \\ y_o = (-1) \cdot 119 \cdot 44 = -5236 \end{cases}$

Umumiy yechim:

$$\begin{cases} -x = 15351 + 343t \\ y = -5236 - 117t, \quad t \in z \end{cases} \quad \text{yoki} \quad \begin{cases} x = -15351 - 343t \\ y = -5236 - 117t, \quad t \in z \end{cases}$$

Berilgan misolni Yechishda $-\frac{117}{343}$ uchun zanjir kasmi tuzish ham mumkin. U holda $-\frac{117}{343} = [-1;1,1,1,13,1,1,1,2]$ bo`lib, $k = 8,\ a = -117, b = 343,$ $c = 119,\ P_{n-1} = P_7 = -44,\ Q_{n-1} = Q_7 = 129$ bo`ladi.

Undan $\begin{cases} x = -15351 + 343t \\ y = 5236 + 117t, \quad t \in z \end{cases}$ yechimlar hosil bo`ladi.

11-misol. Hisoblang:

$$(202332_4 + 22201_4) + (220111_4 - 32303_4) - 23230301_4 : 113_4$$

Yechish. 4 lik sanoq sistemasida berilgan amallarni bajarish uchun qo`shish va ko`paytirish amallari jadvallarini tuzib olamiz:

+	0	1	2	3
0	0	1	2	3
1	1	2	3	10
2	2	3	10	11
3	3	10	11	12

x	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	10	12
3	0	3	12	21

Berilgan misoldagi amallarni bajaramiz

1)

$+\ 202332_4$

222201_4

231133_4

Tekshirish:

$-\ 231133_4$

202332_4

222201_4

2)

$-\ 22011$

1_4

32303

$_4$

12120

2_4

Tekshirish:

$+\ 121202_4$

32303_4

220111_4

3)

Tekshirish:

$+\ 231133_4$

121202_4

1013001_4

$-\ 1013001_4$

231133_4

121202_4

4)

$$
\begin{array}{l|l}
-\,23230301_4 & 113_4 \\
\underline{232} & \overline{200203_4} \\
-\,303 & \\
\underline{232} & \\
-\,1101 & \\
\underline{1011} & \\
30_4 &
\end{array}
$$

Tekshirish:

$$
\begin{array}{r}
\times\ 200203_4 \\
113_4 \\
\hline
+\ 1201221 \\
200203 \\
200203 \\
\hline
23230211_4
\end{array}
$$

$23230211_4+30_4=23233014_4$

5)

$$
\begin{array}{r}
-\ 1013001_4 \\
200203_4 \\
\hline
1213210_4
\end{array}
$$

Demak, javob: 1213210_4

12-misol. n asosda berilgan a sonni m va k asoslarga o`tkazing:

$$a=211,\quad n=3,\quad m=2,\quad k=4$$

Yechish. Berilgan a sonni 3 lik sanoq sistemasida uni 2 lik sanoq sistemasiga o`tkazish uchun berilgan sonni hosil bo`ladigan bo`linmalarni 2 ga bo`lamiz:

$$
\begin{array}{l|l}
-\,211_3 & 2_3 \\
\underline{2} & \overline{102_3} \\
-\,11 & \\
\underline{11} & \\
0 &
\end{array}
\qquad
\begin{array}{l|l}
102_3 & 2_3 \\
\underline{2} & \overline{12_3} \\
-\,12 & \\
\underline{11} & \\
1 &
\end{array}
\qquad
\begin{array}{l|l}
-\,12_3 & 2_3 \\
\underline{11} & \overline{2_3} \\
1 &
\end{array}
\qquad
\begin{array}{l|l}
-\,2_3 & 2_3 \\
\underline{2_3} & \overline{1_3} \\
0 &
\end{array}
\qquad
\begin{array}{l|l}
-\,1_3 & 2_3 \\
\underline{0} & \overline{0_3} \\
1 &
\end{array}
$$

Bu jarayonni bo`linmada 0 hosil bo`lguncha davom ettiramiz. Oxirgi qoldiqdan boshlab barcha qoldiqlar yordamida berilgan sonning 2 lik sanoq sistemasidagi ifodasini topamiz: $211_3=10110_2$

Tekshirish ikki usulda bajariladi:

1-usul. 211_3 va 10110_2 sonlarni o`nlik asosga o`tkazilib solishtiriladi.

2-usul. 10110_2 uchlik asosga o`tkaziladi.

$$211_3 = 2\cdot 3^2 + 1\cdot 3^1 + 1\cdot 3^0 = 18 + 3 + 1 = 22_{10}$$

$$10110_2 = 1\cdot 2^4 + 0\cdot 2^3 + 1\cdot 2^2 + 1\cdot 2^1 + 0\cdot 2^0 = 16 + 4 + 2 = 22_{10}$$

Demak, 211_3 ni ikkilik asosda to`g'ri ifodalangan. $211_3$ ni to`rtlik asosdagi ifodasini topamiz. Buning uchun 211_3 ning o`nlik asosdagi ifodasini topib, hosil bo`lgan sonni to`rtlik asosga o`tkazamiz:

$211_3 = 22_{10}$

$$-\begin{array}{r|l} 22_{10} & 4_{10} \\ \cline{2-2} 20 & 5_{10} \\ \hline 2 & \end{array} \qquad -\begin{array}{r|l} 5_{10} & 4_{10} \\ \cline{2-2} 4 & 1_{10} \\ \hline 1 & \end{array} \qquad -\begin{array}{r|l} 1_{10} & 4_{10} \\ \cline{2-2} 0 & 0_{10} \\ \hline 1 & \end{array}$$

Demak, $22_{10} = 112_4$ bundan $211_3 = 112_4$. Tekshirish yuqoridagi usullarda bajariladi.

15-MUSTAQIL ISH TOPSHIRIQLARI

1-misol. Isbotlang:

1.1. $(n^4 + 6n^3 + 11n^2 + 6n) \vdots 24$.

1.2. $(n^5 - 5n^3 + 4n) \vdots 120$.

1.3. $(n^5 - n) \vdots 30$.

1.4. $(n^7 - n) \vdots 42$.

1.5. $(2^{4n} - 6') \vdots 10$.

1.6. $(4^{2n} - 3^{2n} - 7) \vdots 84$.

1.7. $(6^{2n-1} + 1) \vdots 7$.

1.8. $(n+1)(n+2) \ldots (n+n) \vdots 2^n$.

1.9. $(11^{n+2} - 12^{2n+1}) \vdots 133$.

1.10. $(10^n + 18n - 1) \vdots 27$.

1.11. $(3^{2n+3} + 40n - 27) \vdots 64$.

1.12. $(4^{n} + 6n - 1) \vdots 9$.

1.13. $(10^{n+1} - 9n - 10) \vdots 81$.

1.14. $(9^{n+1} - 8n - 9) \vdots 16$.

1.15. $(5^{2n} - 1) \vdots 24$.

1.16. $(3^{2n+3} - 24n + 37) \vdots 64$.

1.17. $(6^{2n} + 3^{n+2} + 3^{n}) \vdots 11$.

1.18. $(n^{3} + 3n^{2} - n - 3) \vdots 48$.

1.19. $(3^{2n+3} - 24n + 37) \vdots 32$.

1.20. $(3^{2n+3} - 24n + 37) \vdots 16$.

1.21. $(n^{4} + 6n^{3} + 11n^{2} + 6n) \vdots 12$.

1.22. $(n^{5} - 5n^{3} + 4n) \vdots 12$.

1.23. $(9^{n+1} - 8n - 9) \vdots 4$.

1.24. $(3^{2n+3} - 24n + 37) \vdots 8$.

1.25. $(n^{4} + 6n^{3} + 11n^{2} + 6n) \vdots 12$.

2-misol. Eratosfen g'alviri yordamida berilgan sonlar orasidagi barcha tub sonlarni aniqlang:

2.1. 1050 va 1150 ;

2.2. 1060 va 1160 ;

2.3. 1070 va 1170 ;

2.4. 1100 va 1200 ;

2.5. 1250 va 1350 ;

2.6. 1435 va 1545 ;

2.7. 1675 va 1780 ;

2.8. 1880 va 2000 ;

2.9. 5555 va 5750 ;

2.10. 5890 va 6000 ;

2.11. 6437 va 6540 ;

2.14. 2100 va 2200 ;

2.15. 2300 va 2400 ;

2.16. 2350 va 2450 ;

2.17. 2550 va 2650 ;

2.18. 2745 va 2900 ;

2.19. 2900 va 3100 ;

2.20. 3390 va 3450 ;

2.21. 4550 va 4670 ;

2.22. 4660 va 4770 ;

2.23. 6100 va 6250 ;

2.24. 2355 va 2455 ;

2.12. 4422 va 4525 ; 2.25. 1122 va 1222 .

2.13. 3333 va 3444 ;

3-misol. Berilgan natural sonning tub yoki murakkab ekanligini aniqlang:

3.1. n = 1559 ; 3.10. n = 1627 ; 3.18. n = 1783 ;

3.2. n = 3061 ; 3.11. n = 3709 ; 3.19. n = 4057 ;

3.3. n = 1987 ; 3.12 . n = 2339 ; 3.20. n = 2671 ;

3.4. n = 3343 ; 3.13. n = 3659 ; 3.21. n = 4007 ;

3.5. n = 1051 ; 3.14. n = 1423 ; 3.22. n = 3623 ;

3.6. n = 3989 ; 3.15. n = 4027 ; 3.23. n = 3739 ;

3.7. n = 3083 ; 3.16. n = 1699 ; 3.24. n = 2803;

3.8. n = 3001 ; 3.17. n = 3229 ; 3.25. n = 1459 .

3.9. n = 1181;

4-misol. Ikki usulda berilgan sonlarning EKUB va EKUK larini toping:

4.1. a = 1786 ; b = 705 . 4.14. a = 4373; b = 3281 .

4.2. a = -826 ; b = 822 . 4.15. a = 1068 ; b = 899 .

4.3. a = 3655 ; b = 1023 . 4.16. a = 31605 ; b = 498 .

4.4. a = 3059 ; b = 1352 . 4.17. a = 1518 ; b = 731 .

4.5. a = 2737 ; b = 1627 . 4.18. a = 2516 ; b = 3360 .

4.6. a = 1488 ; b = 1126 . 4.19. a = 9163 ; b = 22083 .

4.7. a = 9234 ; b = 6574 . 4.20. a = 294 ; b = 2048 .

4.8. a = 3928 ; b = 2937 . 4.21. a = 5473 ; b = 2739 .

4.9. a = 7362 ; b = 632 . 4.22. a = 3726 ; b = 27364 .

4.10. a = 37261 ; b = 372 . 4.23. a = 8372 ; b = 3726 .

4.11. a = 7261 ; b = 1372 . 4.24. a = 372 ; b = 726 .

4.12. a = 2261 ; b = 272 . 4.25. a = 5312 ; b = 1326 .

4.13. a = 3243 ; b = 145 .

5-misol. **Berilgan n natural sonning natural bo`luvchilari soni va yig'indisini; n dan katta bo`lmagan va n bilan o`zaro tub sonlar sonini toping:**

5.1. n = 360 ;
5.2. n = 542 ;
5.3. n = 635 ;
5.4. n = 784 ;
5.5. n = 234 ;
5.6. n = 654 ;
5.7. n = 765 ;
5.8. n = 558 ;
5.9. n = 912 ;
5.10. n = 430 ;
5.11. n = 894 ;
5.12. n = 324 ;
5.13. n = 895 ;
5.14. n = 324 ;
5.15. n = 865 ;
5.16. n = 779 .
5.17. n = 410 ;
5.18. n = 345 ;
5.19. n = 895 ;
5.20. n = 890 ;
5.21. n = 334 ;
5.22. n =534 ;
5.23. n = 990 ;
5.24 n = 745 ;
5.25. n = 525 .

6-misol . n ! ni tub ko`paytuvchilarga kanonik yoyilmasini toping:

6.1. n = 55 ;
6.2. n = 92 ;
6.3. n = 87 ;
6.4. n = 63 ;
6.5. n = 34 ;
6.6. n = 66 ;
6.7. n = 87 ;
6.8. n = 57 ;
6.9. n = 67 .
6.10 n = 53 ;
6.11. n = 45 ;
6.12. n = 50 ;
6.13. n = 38 ;
6.14. n = 90 ;
6.15. n = 96 ;
6.16. n = 37 ;
6.16. n = 79 ;
6.18. n = 64 ;
6.19. n = 67 ;
6.20. n = 52 ;
6.21. n = 65 ;
6.22. n = 35 ;
6.23. n = 68 ;
6.24. n = 99 ;
6.25. n = 94 ;

7-misol. x va y natural sonlarni toping :

7.1. $\begin{cases} x+y=150, \\ (x,y)=30; \end{cases}$;

7.2. $\begin{cases} x\cdot y=8400, \\ (x,y)=20; \end{cases}$;

7.3. $\begin{cases} (x,y)=4, \\ [x,y]=12; \end{cases}$;

7.10. $\begin{cases} x+y=144, \\ (x,y)=24; \end{cases}$;

7.11. $\begin{cases} x\cdot y=720, \\ (x,y)=4; \end{cases}$;

7.12. $\begin{cases} (x,y)=24, \\ [x,y]=2496; \end{cases}$;

7.18. $\begin{cases} x\cdot y=20, \\ [x,y]=10; \end{cases}$;

7.19. $\begin{cases} (x,y)=4, \\ [x,y]=24; \end{cases}$;

7.20. $\begin{cases} x+y=667, \\ [x,y]=120\cdot (a,b); \end{cases}$;

7.4. $\begin{cases} x\cdot y=168, \\ (x,y)=14; \end{cases}$; 7.13. $\begin{cases} \frac{x}{y}=\frac{11}{7}, \\ (x,y)=45; \end{cases}$; 7.21. $\begin{cases} \frac{x}{y}=\frac{5}{9}, \\ (x,y)=28; \end{cases}$;

7.5. $\begin{cases} \frac{x}{(x,y)}+\frac{y}{(x,y)}=18, \\ [x,y]=975; \end{cases}$; 7.14. $\begin{cases} \frac{x}{y}=\frac{4}{3}, \\ (x,y)=25; \end{cases}$; 7.22. $\begin{cases} x+y=180, \\ (x,y)=30; \end{cases}$;

7.6. $\begin{cases} x+y=168, \\ (x,y)=24; \end{cases}$; 7.15. $\begin{cases} (x,y)=12, \\ [x,y]=72; \end{cases}$; 7.23. $\begin{cases} x+y=60, \\ [x,y]=72; \end{cases}$;

7.7. $\begin{cases} (x,y)=5, \\ [x,y]=495; \end{cases}$; 7.16. $\begin{cases} x+y=100, \\ [x,y]=495; \end{cases}$; 7.24. $\begin{cases} x+y=40, \\ (x,y)=4; \end{cases}$;

7.8. $\begin{cases} x+y=70, \\ (x,y)=7; \end{cases}$; 7.16. $\begin{cases} x+y=100, \\ (x,y)=10; \end{cases}$; 7.25. $\begin{cases} x+y=100, \\ [x,y]=90; \end{cases}$.

7.9. $\begin{cases} x+y=49, \\ [x,y]=70; \end{cases}$;

8-misol. Berilgan kasrni chekli zanjir kasr ko\`rinishida ifodalang va uning munosib kasrlarini toping:

8.1. $\frac{707}{500}$; 8.8. $\frac{157}{225}$; 8.14. $\frac{167}{153}$; 8.20. $\frac{3107}{2341}$;

8.2. $-\frac{602}{367}$; 8.9. $-\frac{117}{343}$; 8.15. $-\frac{99}{170}$; 8.21. $-\frac{83}{217}$;

8.3. $\frac{521}{143}$; 8.10. $-\frac{602}{367}$; 8.16. $-\frac{149}{330}$; 8.22. $\frac{105}{38}$;

8.4. $\frac{245}{83}$; 8.11. $\frac{64}{25}$; 8.17. $\frac{73}{43}$; 8.23. $\frac{99}{464}$;

8.5. $-1\frac{11}{50}$; 8.12. $-2\frac{11}{39}$; 8.18. $-4\frac{25}{41}$; 8.24. $\frac{2633}{1810}$;

8.6. $\frac{121}{35}$; 8.13. $-2\frac{25}{64}$; 8.19. $-4\frac{5}{11}$; 8.25. $\frac{2432}{1713}$.

8.7. $\frac{2367}{1313}$;

9-misol. Berilgan irrasional sonlarni zanjir kasr orqali ifodalang:

9.1. $\frac{\sqrt{37}-3}{4}$;

9.2. $\frac{\sqrt{13}-13}{3}$;

9.3. $\frac{5\sqrt{2}}{2}$;

9.4. $\frac{29+\sqrt{21}}{10}$;

9.5. $\frac{4\sqrt{95}-18}{13}$;

9.6. $1-\sqrt{31}$;

9.7. $\frac{7-\sqrt{5}}{3}$;

9.8. $\frac{5-\sqrt{23}}{13}$;

9.9. $\frac{6-\sqrt{22}}{7}$;

9.10. $\frac{\sqrt{37}-1}{3}$;

9.11. $\frac{\sqrt{101}-1}{4}$;

9.12. $\frac{2(\sqrt{14}+2)}{5}$;

9.13. $\frac{138-\sqrt{5}}{79}$;

9.14. $\frac{2+\sqrt{5}}{3}$;

9.15. $\frac{1+\sqrt{31}}{2}$;

9.16. $\frac{76+\sqrt{285}}{94}$;

9.17. $\frac{4+\sqrt{37}}{32}$;

9.18. $\frac{\sqrt{7925}-69}{14}$;

9.19. $\frac{\sqrt{37}+3}{4}$;

9.20. $\frac{25-\sqrt{61}}{4}$;

9.21. $\frac{18+\sqrt{506}-3}{28}$;

9.22. $\frac{2+\sqrt{7}}{2}$;

9.23. $\frac{3-\sqrt{7}}{3}$;

9.24. $\frac{23-\sqrt{17}}{3}$;

9.25. $\frac{24-\sqrt{41}}{5}$.

10-misol Berilgan tenglamalarni butun sonlar to`plamida yeching:

10.1. $38x+117y=209$;

10.2. $119x-68y=34$;

10.3. $41x+114y=5$;

10.4. $49x+9y=400$;

10.5. $12x+31y=170$;

10.6. $37x+23y=15$;

10.7. $53x+17y=25$;

10.8. $64x-39y=15$;

10.9. $3827x+3293y=1869$;

10.10. $571x+359y=-10$;

10.14. $23x-42y=72$;

10.15. $15x+28y=185$;

10.16. $90x-5y=5$;

10.17. $10x-11y=15$;

10.18. $31x-47y=23$;

10.19. $101x+39y=89$;

10.20. $-26x+174y=2$;

10.21. $-6x+11y=29$;

10.22. $-10x+23y=17$;

10.23. $903x+5y=43$;

10.11. $51x+39y=-10$;

10.12. $71x+59y=210$;

10.13. $38x+35y=30$;

10.24. $93x+5y=123$;

10.25. $43x+34y=23$.

11-misol. Amallarni bajaring:

11.1. $((351_6 \cdot 14_6 - 1153_6 : 31_6 - 150_6) : 205_6) : 25_6$;

11.2. $((215_8 + 532_8) \cdot 16_8 - (11031_8 - 527_8) : 32_8) : 14775_8$;

11.3. $(3333_4 + 2222_4) \cdot 12_4 - (231020_4 + 3333333_4) : 23_4$;

11.4. $(4123_8 - 4221_8) \cdot 11_8 + (1222_8 + 773_8) : 3_8$;

11.5. $3215_7 \cdot 24_7 - 11461_7 : 25_7 + 1532_7 - 115044_7$;

11.6. $(6325_7 + 456_7 - 150335_7 : 23_7 - 551_7) \cdot 5623_7$;

11.7. $120111_3 : 102_3 + (201_3 \cdot 12_3 - 11220_3) \cdot 20110_3$;

11.8. $(563_8 + 217_8) \cdot 15_8 + (2365_8 - 636_8) : 17_8 - 15122_8$;

11.9. $232011_5 : 104_5 + 1234_5 \cdot 322_5 - 122334_5$;

11.10. $23213_5 : 32_5 + 113_5 \cdot 34_5 - 15643_5$;

11.11. $20671_8 : 131_8 - 23765_8 + 53241_8 \cdot 453_8$;

11.12. $(425_6 \cdot 54_6 - 531_6 \cdot 43_6) : 245_6 + 321453_6$;

11.13. $150335_7 : 23_7 + 2341152_7 \cdot 321_7 - 23142_7$;

11.14. $11111101_2 : 10111_2 + 1100101_2 \cdot 1011_2 - 1010101_2$;

11.15. $33162_8 : 457_8 - 3422_8 + 1232145_8 \cdot 3452_8$;

11.16. $111100011_2 : 10101_2 + 1011001_2 \cdot 101_2 - 100101_2$;

11.17. $1141043_5 : 23_5 + 23411_5 \cdot 32_5 - 34231_5$;

11.18. $471222_8 : 27_8 + 432564_8 \cdot 23134_8 - 345214_8$;

11.19. $51(10)3406_{11} : 548_{11} + 98(10)12_{11} \cdot 1232_{11} - 234219_{11}$;

11.20. $(2032_4 : 22_4 + 33211_4 \cdot 3221_4 - 321121_4) \cdot 21_4$;

11.21. $21452_5 + 1141043_5 : 23_5 - 23411_5 \cdot 132_5$;

11.22. $11221_4 - 3121_4 \cdot 223_4 + 2032_4 : 22_4$;

11.23. $21120_3 + 20112_3 \cdot 221_3 - 120111_3 : 102_3$;

11.24. $3452_6 \cdot 4354_6 + 1153_6 : 31_6 - 52341_6$;

11.25. $57623_8 \cdot 5634_8 - 3527_8 + 20671_8 : 131_8$.

12-misol. a natural sonni n asosdan m va k asosga o`tkazing:

12.1. a = 124352 ; n = 6 ; m = 7 ; k = 12 .

12.2. a = 675438 ; n = 9 ; m = 5 ; k = 11 .

12.3. a = 8709546 ; n = 11 ; m = 3 ; k = 13 .

12.4. a = 6738(10)4 ; n = 12 ; m = 2 ; k = 14 .

12.5. a = 5643432 ; n = 7 ; m = 4 ; k = 8 .

12.6. a = 87854632 ; n = 9 ; m = 5 ; k = 10 .

12.7. a = 3421342 ; n = 5 ; m = 3 ; k = 7 .

12.8. a = 234123564 ; n = 7 ; m = 5 ; k = 8 .

12.9. a = 7564352 ; n = 8 ; m = 9 ; k = 4 .

12.10. a = 1221221 ; n = 3 ; m = 2 ; k = 4 .

12.11. a = 657332 ; n = 8 ; m = 6 ; k = 9 .

12.12. a = 7756435; n = 8 ; m = 3 ; k = 10 .

12.13. a = 23433213 ; n = 6 ; m = 4 ; k = 11.

12.14. a = 34554365 ; n = 8 ; m = 5 ; k = 12 .

12.15. a = 445434 ; n = 7 ; m = 4 ; k = 8 .

12.16. a = 6554543 ; n = 9 ; m = 5 ; k = 13 .

12.17. a = 2245436 ; n = 8 ; m = 6 ; k = 11 .

12.18. a = 343454 ; n = 7 ; m = 4 ; k = 9 .

12.19. a = 567767 ; n = 9 ; m = 7 ; k = 12 .

12.20. a = 765654 ; n = 8 ; m = 4 ; k = 9 .

12.21. a = 54775 ; n = 8 ; m = 3 ; k = 11 .

12.22. a = 42112 ; n = 7 ; m = 4 ; k = 9 .

12.23. a = 153422 ; n = 6 ; m = 3 ; k = 7 .

12.24. a = 7(11)761 ; n = 12 ; m = 9 ; k = 13 .

12.25. a = 10(10)89 ; n = 11 ; m = 8 ; k =129 .

Takrorlash uchun savollar:

1. Butun sonlar halqasida bo`linish munosabati, xossalari.
2. Qoldiqli bo`lish haqidagi teorema.Evklid algoritmi.
3. Tub va murakkab sonlar.Butun sonni tub ko`paytuvchilarga yoyish.
4. Tub sonlar to`plamining cheksizligi.Eratosfen g'alviri.
5. Tub sonlar taqsimoti.Arifmetik progressiyada tub sonlar.
6. Sonli funksiyalar.EKUB, xossalari.
7. O`zaro tub sonlar, xossalari.EKUK, xossalari.
8. CHekli zanjir kasrlar.Munosib kasrlar, xossalari.
9. Butun sistematik sonlar.Sistematik sonlar ustida arifmetik amallar.

XVI MODUL.

TAQQOSLAMALAR VA ULAR USTIDA AMALLAR

1. Modul bo`yicha taqqoslamalar.
2. Taqqoslamalar xossalari.
3. m modul bo`yicha chegirmalar halqasi.
4. Chegirmalar to`liq sistemasi, xossalari.
5. Chegirmalar keltirilgan sistemasi.
6. Chegirmalar mul'tiplikativ gruppasi.
7. Eyler funksiyasi.
8. Eyler va Ferma teoremalari.
9. Taqqoslama darajasi va uning yechimi.
10. Teng kuchli taqqoslamalar.
11. Vil'son teoremasi.
12. Birinchi darajali taqqoslamalarni Yechish usullari.
13. Ikki o`zgaruvchili chiziqli tenglamalar.
14. Taqqoslamalar sistemasi.
15. Lejandr, Yakobi simvollari.
16. Chegirmalar sinfi tartibi.
17. Tub modul bo`yicha boshlang'ich ildizlar.
18. Indekslar, xossalari.
19. Indekslar jadvali.
20. Indekslarning tadbiqi.
21. Taqqoslamalar nazariyasining arifmetikaga tadbiqlari.

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16-MUSTAQIL ISH

0-variant

1-misol. a) a=2511 sonini b=123 ga bo`lgandagi qoldiqni toping.

Yechish. Qoldiqli bo`lishi xaqidagi teorimadan foydalanib a=bq+r ,

$0 \le r < b$ ifodani topamiz: 2511=123·20+51

Demak, a=2511 ni b=123ga bo`lganda r=51 qoldiq qoladi.

b) a=25^{112} ni b=16 ga bo`lgandagi qoldiqni toping.

Yechish. a=25^{112} sonini 16ga bo`lish uchun taqqoslamaning xossalaridan foydalanamiz. 25=16·1+9 ekanligidan 25≡9(mod16) qilib chiqadi. Bundan $25^{112} \equiv 9^{112} \equiv (9^2)^{56} \equiv 81^{56}$. 81=16·5+1 ekanligini e'tiborga olsak, u xolda $25^{112} \equiv 81^{56} \equiv 1^{56} \equiv 1$(mod16).

Demak, 25^{112} ni 16ga bo`lganda 1 qoldiq qoladi.

2-misol. Agar 100a+100b+s≡0(mod 21) bo`lsa ,u xolda a-2b+4s≡0(mod 21) ekanligini isbotlang.

Isbot. Taqqoslamaning ikkala tomonini modul bilan o`zaro tub 4 songa ko`paytiramiz : 400a+40b+4c≡0 (mod 21).

400=21·19+1 , 40=21·2+(-2) , 4=21·0+4 lardan foydalanib quyidagi taqqoslamalarni yozamiz :

400a ≡ a (mod21), chunki 400a-a = 399a ⋮ 21;

40b ≡ -2b (mod 21), chunki 40b-(-2b) = 42b ⋮ 21;

4c ≡ 4c (mod 21), chunki 4s - 4s = 0 ⋮ 21 ;

Birilgan taqqoslamadan yuqoridagi taqqoslamalarni e'tiborga olib 400a+40b+4s ≡ a-2b+4s (mod 21) taqqoslamani hosil qilamiz.

Demak, $400a + 40b + 4c \equiv 0(mod21)$ shartdan $a - 2b + 4c \equiv 0(\mathrm{mod}\,21)$ kelib chiqadi .

3.1-misol. $7 \cdot x \equiv 10(\mathrm{mod}\,4)$ **taqqoslamaning yechimlarini taqqoslama xossalaridan foydalanib toping.**

Yechish. (7,4)=1 ekanligidan taqqoslama yagona yechimga ekanligi kelib chiqadi.7 va 11 sonlari 4 dan katta bo`lganligi uchun $7\cdot x\equiv 3x(\bmod 4)$ va $10\equiv 2(\bmod 4)$ lardan foydalanib $3x\equiv 2(\bmod 4)$ ni hosil qilamiz. Bundan $3x\equiv -x(\bmod 4)$ etiborga olib $-x\equiv 2(\bmod 4)$ ni, va nihoyat $x\equiv -2(\bmod 4)$ ni hosil qilamiz.

Agar $-2\equiv 2(\bmod 4)$ ni qo`llasak ,u holda $x\equiv 2(\bmod 4)$ kelib chiqadi.

Tekshirish :

$7\cdot 2\equiv 10(\bmod 4)$

$14\equiv 10(\bmod 4)\Rightarrow(14-10)=4\vdots 4$ kelib chiqadi

3.2-misol. $27x\equiv 47\ (\bmod 38)$ **taqqoslamani taqqoslama xossalaridan foydalanib yechimlarini toping.**

Yechish. $47\equiv 9\ (\bmod 38)$ dan $27x\equiv 9\ (\bmod 38)$ hosil bo`ladi. $(27,38)=1$ bo`lgani uchun taqqoslama yagona yechimga ega. $(9,38)=1$ bo`lgani uchun taqqoslamani ikkala tomonini 9 ga bo`lamiz: $3x\equiv 1\ (\bmod 38)$.

Taqqoslamaning o`ng tomoniga 38 ni qo`shamiz: $3x\equiv 39\ (\bmod 38)$. Hosil bo`lgan taqqoslamani ikkala tomonini $(3,38)=1$ bo`lgani uchun 3 ga bo`lamiz: $x\equiv 13\ (\bmod 38)$.

Tekshirish. $27\cdot 13-47=304=(38\cdot 8)\vdots 38$

4.1-misol. Berilgan $7\cdot x\equiv 10(\bmod 4)$ **taqqoslamani tanlash usuli bilan yeching.**

Yechish. $ax\equiv b(\bmod m)$ taqqoslamani tanlash usuli bilan yechimlarini topish uchun avval yechimlar sonini aniqlaymiz. So`ngra m modul bo`yicha chegirmalar to`la sistemasidagi har bir sinfning yechimi bo`lish bo`lmasligini tekshiramiz.

$7\cdot x\equiv 10(\bmod 4)$ taqqoslamada (7,4) =1.

Demak, yagona yechim mavjud. 4 modul bo`yicha chegirmalari to`la sistemasi 0,1,2,3 x noma'lum o`rniga birma-bir qo`yib tekshiriladi. Qaysidir chegirmalar sinfi yechim bo`lishi ma'lum bo`lsa tekshirish jarayonini to`xtamiz:

x=0da $7\cdot 0 \equiv (\mathrm{mod}\,4)$ o`rinli emas , chunki $(0-10) \not\vdots\, 4$;

x=1da $7\cdot 1 \equiv (\mathrm{mod}\,4)$ o`rinli emas, chunki $7-10=3 \not\vdots\, 4$;

x=2 da $7\cdot 2 \equiv (\mathrm{mod}\,4)$ o`rinli, chunki $14-10=4 \vdots 4$.

$x \equiv 2(\mathrm{mod}\,4)$ yechim bo`ladi. Qolgan sinflar berilgan taqqoslamaning birgina yechimi mavjud bo`lganligi sababli, tekshirilmaydi.

Tekshirish. $7\cdot 2-10=14-10=4\vdots 4$.

4.2-misol. $2x \equiv 5 \;(\mathrm{mod}\,9)$ **taqqoslamani tanlash usuli yordamida yechimlarini toping.**

Yechish. 9 modul bo`yicha $0, \pm 1, \pm 2, \pm 3, \pm 4$ chegirmalar sinflaridan $(2;9)=1$ bo`lganligi uchun berilgan taqqoslamaning yagona yechimini topamiz.

$$2\cdot 0 = 0 \not\equiv 5 \quad (mod\ 9);$$
$$2\cdot 1 = 2 \not\equiv 5 \quad (mod\ 9);$$
$$2\cdot(-1) = -2 \not\equiv 5 \quad (mod\ 9);$$
$$2\cdot 2 = 4 \not\equiv 5 \quad (mod\ 9);$$
$$2\cdot(-2) = -4 \equiv 5 \quad (mod\ 9).$$

Demak, $x \equiv -2 \;(\mathrm{mod}\,9)$, ya'ni $x \equiv 7 \;(\mathrm{mod}\,9)$ berilgan taqqoslamaning yechimi.

Tekshirish. $2\cdot 7-5=14-5=9\vdots 9$

5.1-misol. $7\cdot x \equiv 10(\mathrm{mod}\,4)$ **taqqoslamani Eyler teoremasi yordamida yeching .**

Yechish.Agar $a\cdot x \equiv b(\mathrm{mod}\,m)$ taqqoslama (a,m)=1 bo`lsa ,u holda uning yechimi $x = b\cdot a^{\varphi(m)-1}(\mathrm{mod}\,m)$ formula yordamida topiladi .Haqiqatdan ham Eyler teoremasiga ko`ra $a^{\varphi(m)} \equiv 1(\mathrm{mod}\,m)$ Bundan

$a^{\varphi(m)}b \equiv b(\bmod m)$ va $a \cdot a^{\varphi(m)-1}b \equiv b(\bmod m)$ larni hosil qilsak $x \equiv ba^{\varphi(m)-1}(\bmod m)$ kelib chiqadi.

$7 \cdot x \equiv 10(\bmod 4)$ dan a=7,b=10, m=4 yechim $x \equiv 10 \cdot 7^{\varphi(4)-1}(\bmod 4)$ ni topish uchun $\varphi(4)$ ni aniqlaymiz. $4 = 2^2$ ekanligidan $\varphi(4) = 4 \cdot (1 - \frac{1}{2}) = 2$ kelib chiqadi .

Demak. $x = 10 \cdot 7^{2-1}(\bmod 4)$. Agar $10 \equiv 2(\bmod 4)$, $7 \equiv 3(\bmod 4)$ va $6 \equiv 2(\bmod 4)$ taqqoslamalardan foydalansak ,$x \equiv 10 \cdot 7^{2-1} \equiv 2 \cdot 3 \equiv 6 \equiv 2(\bmod 4)$, ya'ni $x \equiv 2(\bmod 4)$ yechimni hosil qilamiz.

Tekshirish: $2 \cdot 10 - 10 = 14 - 10 = 4 \vdots 4$.

5.2-misol. $27x \equiv 24 \ (\bmod 102)$ **taqqoslamani Eyler metodidan foydalanib yechimlarini toping.**

Yechish. $(27,102) = 3$ va $24 = 3 \cdot 8$. Demak, taqqoslama 3 ta yechimga ega. Berilgan taqqoslamaning ikkala qismi va modulni 3 ga bo`lamiz: $9x \equiv 8 \ (\bmod 34)$.

Bunda $a = 9, m = 34, b = 8$ bo`lgani uchun $x \equiv b \cdot a^{\varphi(m)-1}$ (mod m) dan $x \equiv 8 \cdot 9^{\varphi(34)-1} \ (\bmod 34)$ ga ega bo`lamiz. $\varphi(34) = 2 \cdot 17\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{17}\right) = 16$ ekanligini e'tiborga olamiz:

$$x \equiv 8 \cdot 9^{15} \equiv 8 \cdot 9 \cdot 9^{14} \equiv 4 \cdot (9^2)^7 \equiv 4 \cdot 13^7 \equiv 4 \cdot 13^7 \equiv 4 \cdot 13 \cdot (13^2)^3 \equiv$$

$$\equiv 18 \cdot 33^3 \equiv 18 \cdot 33 \cdot (33)^2 \equiv 16 \cdot 1^2 \equiv 16 \ (mod\ 34)$$

Bundan $x \equiv 16 \ (\bmod 34)$ ga ega bo`lamiz.

Tekshirish. $9 \cdot 16 - 8 = 136 \vdots 34$. U holda $27x \equiv 24 \ (\bmod 102)$ taqqoslama

$x \equiv 16 \ (\bmod 102)$

$x \equiv 16 + 34 \ (\bmod 102)$

$x \equiv 16 + 34 \cdot 2 \ (\bmod 102)$ yechimlarga ya'ni,

$x \equiv 16 \pmod{102}$

$x \equiv 50 \pmod{102}$

$x \equiv 84 \pmod{102}$ yechimlarga ega.

Tekshirish.

$27 \cdot 16 - 24 = 408 \vdots 102;$

$27 \cdot 50 - 24 = 3126 \vdots 102;$

$27 \cdot 84 - 24 = 2244 \vdots 102.$

6.1-misol. $7x \equiv 10 \pmod 4$ **taqqoslamani munosib kasrlar yordamida yeching.**

Yechish. Agar $ax \equiv b$ (mod m) taqqoslamada $(a,m)=1$ va P_{n-1} son $\frac{m}{a}$ ning oxiridan oldingi munosib kasr surati bo`lsa, u holda $x \equiv b \cdot (-1)^{n-1} P_{n-1} \pmod m$ berilgan taqqoslamaning yechimi bo`ladi.

Berilgan taqqoslamada $m=4$, $a=7$ bo`lganidan $\frac{4}{7}$ ning munosib kasrlarini

topamiz:
$$4 = 7 \cdot 0 + 4;$$
$$7 = 4 \cdot 1 + 3;$$
$$4 = 3 \cdot 1 + 1;$$
$$3 = 1 \cdot 3 + 0.$$

Bundan $\frac{4}{7} = [0;1,1,3]$ ko`rinishda bo`ladi.

Munosib kasrlar jadvalini tuzamiz:

k	-1	0	1	2	3
q_k	–	0	1	1	3
P_k	1	0	1	1	4
Q_k	0	1	1	2	7

Demak, $P_{n-1} = P_2 = 1$ va $x \equiv b \cdot (-1)^{n-1} P_{n-1} \equiv 10 \cdot (-1)^{3-1} \cdot 1 \equiv 10 \equiv 2 \pmod 4$.

Berilgan taqqoslamaning $x \equiv 2 \pmod 4$ yechimi mavjud ekan.

Tekshirish. $7 \cdot 2 - 10 = 14 - 10 = 4 \vdots 4$.

6.2-misol. $220x \equiv 28 \pmod{348}$ **taqqoslamani munosib kasrlar yordamida yechimlarini toping.**

Yechish. $(220,348)=4$ va $28\vdots 4$ dan berilgan taqqoslama 4 ta yechimga ega ekanligi kelib chiqadi. Taqqoslamani ikkala tomoni va modulni 4 ga bo`lamiz:

$$55x \equiv 7 \pmod{87}.$$

$\frac{87}{55}$ kasrni chekli zanjir kasr ko`rinishiga keltirib, munosib kasrlar jadvalini tuzamiz:

$\frac{87}{55}=[1;1,1,1,2,1,1,4]$. Bundan,

k	1	0	1	2	3	4	5	6
q_k	–	1	1	1	2	1	1	4
P_k	1	1	2	3	8	11	19	87

va $n=6$, $P_{n-1}=P_5=19$, $b=7$, $m=87$ larni $x \equiv (-1)^n P_{n-1} b \pmod{m}$ formulaga qo`ysak, $x \equiv (-1)^6 \cdot 19 \cdot 7 \equiv 133 \equiv 46 \pmod{87}$ kelib chiqadi.

Demak, $55x \equiv 7 \pmod{87}$ ning yechimi $x \equiv 46 \pmod{87}$ va $220x \equiv 28 \pmod{348}$ ning yechimlari $x \equiv 46; 133; 220; 307 \pmod{348}$.

Tekshirish.

$220 \cdot 46 - 28 = 10092 \vdots 348;$

$220 \cdot 133 - 28 = 29232 \vdots 348;$

$220 \cdot 220 - 28 = 48372 \vdots 348;$

$220 \cdot 307 - 28 = 67512 \vdots 348.$

7. 1-misol. $7x \equiv 10 \pmod{4}$ **taqqoslamani 7 ga 4 modul bo`yicha teskari sinfi orqali yeching.**

Yechish. $ax \equiv b \pmod{m}$ taqqoslamada $(a,m)=1$ bo`lsa, u holda 1 ning $a$ va $m$ sonlarga chiziqli yoyilmasini topamiz: $1 = au + mv$ yoyilmadagi $u$ soni $a$ soniga $m$ modul bo`yicha teskari son bo`ladi.

Evklid algoritmi yordamida berilgan $\frac{7}{4}$ sonlarning eng katta umumiy bo`luvchisining chiziqli ifodasini topamiz:

$7 = 4 \cdot 1 + 3;$ $\qquad 3 = 7 - 4 \cdot 1;$

$4 = 3 \cdot 1 + 1;$ $\qquad 1 = 4 - 3 \cdot 1$

$3 = 1 \cdot 3 + 0.$

Bundan $1 = 4 - 3 \cdot 1 = 4 - (7 - 4 \cdot 1) = 4 \cdot 2 - 7 = 4 \cdot 2 + 7(-1)$. Demak, $1 = 4 \cdot 2 + 7(-1)$. 7 soniga 4 modul bo`yicha teskari son $-1$ yoki $-1 \equiv 3 \pmod{4}$ ekanligidan 3 soni bo`ladi.

$7x \equiv 10 \pmod{4}$ taqqoslamaning ikkala tomonini 7 ga 4 modul bo`yicha teskari 3 soniga ko`paytiramiz $((3,4)=1)$:

$7 \cdot 3x \equiv 10 \cdot 3 \pmod{4}$

$21x \equiv 30 \pmod{4}$

$21x \equiv x \pmod{4}$

$30x \equiv 2 \pmod{4}$

lardan $x \equiv 2 \pmod{4}$ yechimini topamiz.

Tekshirish. $7 \cdot 2 - 10 = 14 - 10 = 4 \vdots 4$.

7.2-misol. $37x \equiv 25 \pmod{107}$ **taqqoslamani teskari sinf yordamida yeching.**

Yechish. $(37,107)=1$ dan berilgan taqqoslamaning yagona yechimi mavjudligi kelib chiqadi. 37 ga 107 modulda teskari sonni topamiz:

$107 = 37 \cdot 2 + 33;$

$37 = 33 \cdot 1 + 4;$

$33 = 4 \cdot 8 + 1;$

$4 = 1 \cdot 4 + 0.$

$1 = 33 - 4 \cdot 8 = 33 - (37 - 33 \cdot 1) \cdot 8 = 33 \cdot 9 + 37(-8) = (107 - 37 \cdot 2) \cdot 9 + 37(-8) =$
$107 \cdot 9 + 37(-26).$

Bundan $1=107\cdot 9+37(-26)$, ya'ni 107 modulda 37 ga teskari sinf –26. musbat son bilan almashtiramiz: $-26+107=81$. Hosil bo`lgan 81 ga berilgan taqqoslamaning ikkala qismini ko`paytiramiz va $37\cdot 81x\equiv 28\cdot 81 \pmod{107}$ dan $x\equiv 2025 \pmod{107}$ ya'ni $x\equiv 99 \pmod{107}$ yechimni topamiz.

Tekshirish. $37\cdot 99-25=3638\vdots 107$.

8-misol. $27x+38y=47$ **tenglamani taqqoslamalar yordamida yeching.**

Yechish. Tenlamaning butun yechimlarini taqqoslamalardan foydalanib topish uchun $27x\equiv 47 \pmod{38}$ bir o`zgaruvchili taqqoslamani tuzib olamiz. $(27,38)=1$ ekanligidan taqqoslamaning bitta yechimi mavjud. $47\equiv 9 \pmod{38}$ dan $27x\equiv 9 \pmod{38}$ni hosil qilamiz. Bundan $3x\equiv 1 \pmod{38}$ va $x\equiv 13 \pmod{38}$ kelib chiqadi.

$x\equiv 13 \pmod{38}$ berilgan $27x\equiv 9 \pmod{38}$ taqqoslamaning yechimi. U holda $\left\{13, \frac{47-27\cdot 13}{38}\right\}=\{13,-8\}$ berilgan tenglamaning yechimlaridan biri bo`ladi.

$ax+by=c$ tenglamaning barcha yechimlari $x'=x_o+\frac{m}{d}t$, $y'=y_o+\frac{a}{d}t$ ko`rinishda bo`lib, bu erda $x_o=13$, $y_o=-8$, $m=18$, $a=27$, $d=1$. Demak,

$$\begin{cases} x'=13+38t, \\ y'=-8-27t, \quad t\in z \end{cases}$$

Tekshirish. $27(13+38t)+38(-8-27t)=47;$

$$351+1026t-304-1026t=47$$

$$47=47$$

9-misol. $\begin{cases} 3x\equiv 11 \pmod{17} \\ 15x\equiv 35 \pmod{13} \\ 21x\equiv 33 \pmod{30} \end{cases}$ **taqqoslamalar sistemasini yeching.**

Yechish. Berilgan taqqoslamalar sistemasidagi har bir taqqoslama yechimlari yuqoridagi misollarda keltirilgan usullardan biri yordamida topiladi.

$$\begin{cases} x \equiv 15 \pmod{17} \\ x \equiv 11 \pmod{13} \\ x \equiv 3 \pmod{10} \end{cases}$$

Hosil qilingan taqqoslamalar sistemasidagi taqqoslamalar modullari o`zaro tub bo`lganligi uchun ularning eng kichik umumiy karralisi M bo`yicha sistema yechimini topamiz:

$M = 17 \cdot 13 \cdot 10 = 2210;$

$M_1 = \frac{2210}{17} = 130;$

$M_2 = \frac{2210}{13} = 170;$

$M_3 = \frac{2210}{10} = 221.$

Quyidagi taqqoslamalarni tuzib yechimini topamiz:

1) $130y_1 \equiv 1 \pmod{17}$
$y_1 = 14;$

2) $170y_2 \equiv 1 \pmod{13}$
$y_2 = 1;$

3) $221y_3 \equiv 1 \pmod{10}$
$y_3 = 1.$

Bundan berilgan taqqoslamalar sistemasining yechimi

$x = x_o = 130 \cdot 14 \cdot 15 + 170 \cdot 1 + 11 + 211 \cdot 1 \cdot 3 = 29833 \equiv 1103 \pmod{2210}$

ya'ni, $x \equiv 1103 \pmod{2210}$ kelib chiqadi.

Agar berilgan taqqoslamalar sistemasidagi uchinchi taqqoslamaning 3 ta yechimi borligini e'tiborga olsak, u holda taqqoslamalar sistemasining 3 ta yechimini topish mumkin:

$$\begin{cases} x \equiv 15 \pmod{17} \\ x \equiv 11 \pmod{13} \\ x \equiv 3 \pmod{30} \end{cases} \quad \begin{cases} x \equiv 15 \pmod{17} \\ x \equiv 11 \pmod{13} \\ x \equiv 13 \pmod{30} \end{cases} \quad \begin{cases} x \equiv 15 \pmod{17} \\ x \equiv 11 \pmod{13} \\ x \equiv 23 \pmod{30} \end{cases}$$

$x \equiv 5523 \pmod{6630}$; $x \equiv 3313 \pmod{6630}$ $x \equiv 1103 \pmod{6630}$.

yechimlar hosil qilinadi.

10-misol. $\begin{cases} x \equiv 2 \pmod{15} \\ x \equiv 7 \pmod{20} \\ x \equiv 12 \pmod{35} \end{cases}$ **taqqoslamalar sistemasini yeching.**

Yechish. Taqqoslama ta`rifiga ko`ra birinchi taqqoslamadan $x = 2 + 15t$, $t \in z$ ifodani hosil qilamiz. Bu qiymatni ikkinchi taqqoslamaga qo`yamiz:

$$2 + 15t \equiv 7 \pmod{20}$$

Bundan, $15t \equiv 5 \pmod{20}$ yoki $t \equiv 3 \pmod 4$ ni olamiz. YAna taqqoslama ta'rifini qo`llab $z = 3 + 4k$, $k \in z$ ifodani olamiz. Bu ifodadan $x = 2 + 15t = 2 + 15(3 + 4k) = 47 + 60k$ kelib chiqadi. Hosil qilingan $x$ ning ifodasini uchinchi taqqoslamaga qo`yamiz: $47 + 60k \equiv 12 \pmod{35}$ taqqoslamani yechib $k \equiv 0 \pmod 7$ yechimni topamiz.

Bundan $k = 7l$, $l \in z$ kelib chiqadi. Hosil bo`lgan ifodani $x$ ning ifodasiga qo`llaymiz: $x = 47 + 60k = 47 + 60 \cdot 7l = 47 + 420l$.

Demak, $x \equiv 47 \pmod{420}$ berilgan taqqoslamalar sistemasining yechimi.

Tekshirish. $\begin{cases} 47 - 2 = 45 \vdots 15; \\ 47 - 7 = 40 \vdots 20; \\ 47 - 12 = 35 \vdots 35. \end{cases}$

11-misol. $251x^{54} + 63x^{25} - 7x^{11} + 4x^{3} + 2 \equiv 0 \pmod 5$ **taqqoslamani soddalashtiring.**

Yechish. Berilgan taqqoslamani soddalashtirish uchun taqqoslamalar xossalaridan va Eyler teoremasidan foydalanamiz:

$251 \equiv 1 \pmod 5$;

$63 \equiv 3 \pmod 5$;

$7 \equiv 2 \pmod 5$;

$4 \equiv 4 \pmod 5$;

$2 \equiv 2 \pmod 5$.

$$x^{54} \equiv (x^4)^{13} \cdot x^2 \equiv x^2 \pmod 5;$$
$$\varphi(5) = 4 \text{ dan } \quad x^{25} \equiv (x^4)^6 \cdot x \equiv x \pmod 5;$$
$$x^{11} \equiv (x^4)^2 \cdot x^3 \equiv x^3 \pmod 5$$

Keltirilgan taqqoslamalar yordamida berilgan taqqoslamani soddalashtiramiz:

$$251x^{54} + 63x^{25} - 7x^{11} + 4x^3 + 2 = x^2 + 3x - 2x^3 + 4x^3 + 2 = 2x^3 + x^2 + 3x + 2 = 0 \pmod 5)$$

12-misol. $\frac{219}{383}$ **ning Lejandr simvolini toping.**

Yechish: Lejandr simvoli deb $\frac{a}{p}$ kasr songa 1, -1 ni quyidagicha mos qo`yish tushuniladi:

$$\left(\frac{a}{p}\right) = \begin{cases} 1 & agar\ a\ soni\ p\ modul\ bo'yicha\ kvadrat\ chegirma\ bo'lsa; \\ -1 & agar\ a\ soni\ p\ modul\ bo'yicha\ kvadrat\ chegirma\ bo'lmasa; \end{cases}$$

Berilgan kasr sonning maxraji tub soni bo`lsa, uning Lejandr simvoli topiladi. Buning uchun quyidagi xossalardan foydalanamiz:

1. Agar $a = b \pmod p$ bo`lsa, u xolda $\left(\frac{a}{p}\right) = \left(\frac{b}{p}\right)$;

2. $\left(\frac{a^2}{p}\right) = 1$;

3. $\left(\frac{1}{p}\right) = 1$;

4. $\left(\frac{-1}{p}\right) = (1)^{\frac{p-1}{2}}$;

5. $\left(\frac{ab..c}{p}\right) = \left(\frac{a}{p}\right)\left(\frac{b}{p}\right)..\left(\frac{c}{p}\right)$;

6. $\left(\frac{ab^2}{p}\right) = \left(\frac{a}{p}\right)$;

7. $\left(\frac{a^n}{p}\right)=\left(\frac{a}{p}\right)^n$;

8. $\left(\frac{2}{p}\right)=(1)^{\frac{p^2-1}{8}}$;

9. Agar $(p,q)=1$, u xolda $\left(\frac{q}{p}\right)=\left(\frac{p}{q}\right)\cdot(-1)^{\frac{p-1}{2}\cdot\frac{q-1}{2}}$.

Berilgan $\frac{219}{383}$ kasrning Lejandr simvolini topamiz:

1-usul

$$\left(\frac{219}{383}\right)=\left(\frac{3\cdot 73}{383}\right)=|5-xossaga\ ko'ra|=\left(\frac{3}{383}\right)\cdot\left(\frac{73}{383}\right)=|9-xossaga\ ko'ra|=$$

$$=\left(\frac{383}{3}\right)(-1)^{\frac{383-1}{2}\cdot\frac{3-1}{2}}\cdot\left(\frac{383}{73}\right)(-1)^{\frac{383-1}{2}\cdot\frac{73-1}{2}}=\left(\frac{2}{3}\right)(-1)^{191\cdot1}\left(\frac{18}{73}\right)(-1)^{191\cdot36}=-\left(\frac{2}{3}\right)\cdot\left(\frac{2\cdot3^2}{73}\right)=$$

$$=|5,6\text{-}\ xossaga\ ko'ra|=-\left(\frac{2}{3}\right)\cdot\left(\frac{2}{73}\right)=|8-xossaga\ ko'ra|=-(-1)^{\frac{3^2-1}{8}}\cdot(-1)^{\frac{73^2-1}{8}}=$$

$$=-(-1)(-1)^{666}=1.$$

Demak, $\left(\frac{219}{383}\right)=1$. Bundan $x^2\equiv 219 \pmod{383}$ taqqoslama uchun 219 kvadrat chegirma bo`ladi, ya'ni hosil qilingan taqqoslama kamida bitta yechimga ega.

2-usul. $\left(\frac{219}{383}\right)=\left(\frac{3}{383}\right)\cdot\left(\frac{73}{383}\right)$ tenglikdan foydalanib, ko`paytuvchilarni alohida-alohida topish mumkin:

a) $\left(\frac{3}{383}\right)=\left(\frac{383}{3}\right)\cdot(-1)^{\frac{383-1}{2}\cdot\frac{3-1}{2}}=-\left(\frac{383}{3}\right)=-\left(\frac{2}{3}\right)=-(-1)^{\frac{3^2-1}{8}}=-(-1)=1.$

v) $\left(\frac{73}{383}\right)=\left(\frac{383}{73}\right)\cdot(-1)^{\frac{383-1}{2}\cdot\frac{73-1}{2}}=-\left(\frac{383}{73}\right)=-\left(\frac{18}{73}\right)=\left(\frac{2\cdot3^2}{73}\right)=$

$=\left(\frac{2}{73}\right)=-(-1)^{\frac{73^2-1}{2}}=1.$

Bunda $\left(\frac{219}{383}\right)=\left(\frac{3}{383}\right)\cdot\left(\frac{73}{383}\right)=1\cdot1=1$ kelib chiqadi.

3-usul. Berilgan $\frac{219}{383}$ kasrning maxraji suratidan katta bo`lgani uchun 9-xossani qo`llash mumkin:

$$\left(\frac{219}{383}\right)=\left(\frac{383}{219}\right)\cdot(-1)^{\frac{383-1}{2}\cdot\frac{219-1}{2}}=-\left(\frac{383}{219}\right)=\left(\frac{164}{219}\right)=\left(\frac{41\cdot 2^2}{219}\right)=$$

$$=-\left(\frac{41}{219}\right)=-\left(\frac{219}{41}\right)\cdot(-1)^{\frac{219-1}{2}\cdot\frac{41-1}{2}}=-\left(\frac{219}{41}\right)=\left(\frac{14}{41}\right)=-\left(\frac{2}{41}\right)\left(\frac{7}{41}\right)=(-1)^{\frac{41^2-1}{8}}\left(\frac{7}{41}\right)=$$

$$=-\left(\frac{7}{41}\right)=-\left(\frac{41}{7}\right)\cdot(-1)^{\frac{41-1}{2}\cdot\frac{7-1}{2}}=-\left(\frac{41}{7}\right)=-\left(\frac{-1}{7}\right)=(-1)^{\frac{7-1}{2}}=1$$

Demak, $\left(\frac{219}{383}\right)=1$.

13-misol. $\frac{383}{219}$ **ning Yakobi simvolini aniqlang.**

Yechish: Yakobi simvolining Lejandr simvolidan farqi Yakobi simvoli o`zaro tub bo`lgan a ва m $(m>1)$sonlardan tuzilgan $\frac{a}{m}$ uchun aniqlanadi $\left(\frac{a}{m}\right)$ belgilash "a ning m modul bo`yicha Yakobi simvoli" deb o`qiladi. Yuqoridagi 12-misoldagi Lejandr simvolining xossalari va $\left(\frac{a}{m}\right)=\left(\frac{a}{p_1\cdots p_n}\right)=\left(\frac{a}{p_1}\right)\cdots\left(\frac{a}{p_n}\right)$ xossadan:

$$\left(\frac{383}{219}\right)=\left(\frac{383}{3\cdot 73}\right)=\left(\frac{383}{3}\right)\left(\frac{383}{73}\right)=\left(\frac{2}{3}\right)\cdot\left(\frac{18}{73}\right)=\left(\frac{2}{3}\right)\left(\frac{23^2}{73}\right)=$$

$$\left(\frac{2}{3}\right)\left(\frac{2}{73}\right)=(-1)^{\frac{3^2-1}{8}}(-1)^{\frac{73^2-1}{8}}=(-1)\cdot 1=-1$$

Demak, $\left(\frac{383}{219}\right)=-1$, ya'ni $x^2\equiv 383(\mathrm{mod}\ 219)$ taqqoslama uchun 383 kvadrat chegirma emas.

14-misol. $p=17$ **modul bo`yicha** $g=6$ **boshlang'ich ildizning indekslar jadvalini tuzing.**

Yechish. p tub modul bo`yicha boshlang'ich ildiz bu shunday g chegirmalar sinfini, uning uchun $g^{p-1} \equiv 1 \pmod p$ bo`lib, $p-1$dan kichik natural darajalarda p modulda 1 bilan taqqoslanmaydi.

$g-6$ning mod 17 da boshlang'ich ildiz bo`lishini tekshiramiz. Buning uchun - $p-1$ning $n$ bo`luvchilarida $6^n \equiv 1 \pmod p$ shartni tekshiramiz:

$p=17$, $p-1=16$, 16ning natural bo`luvchilari $n=1, 2, 4, 8, 16$. Bundan:

$6^1 \equiv 6 \pmod{17}$

$6^2 \equiv 2 \pmod{17}$

$6^4 \equiv 4 \pmod{17}$

$6^8 \equiv 16 \pmod{17}$

$6^{16} \equiv 1 \pmod{17}$

Demak, 17 modulda 6 boshlang'ich ildiz bo`ladi. $6^0, 6^1, 6^2 \ldots 6^{15}$ lardan 17 modul bo`yicha taqqoslamalar tuzamiz:

$6^0 \equiv 1 \pmod{17}$;

$6^1 \equiv 6 \pmod{17}$;

$6^2 \equiv 2 \pmod{17}$;

$6^3 \equiv 12 \pmod{17}$;

$6^4 \equiv 4 \pmod{17}$;

$6^5 \equiv 7 \pmod{17}$;

$6^6 \equiv 8 \pmod{17}$;

$6^7 \equiv 14 \pmod{17}$;

$6^8 \equiv 16 \pmod{17}$;

$6^9 \equiv 11 \pmod{17}$;

$6^{10} \equiv 15 \pmod{17}$;

$6^{11} \equiv 5 \pmod{17}$;

$6^{12} \equiv 13 \pmod{17}$;

$6^{13} \equiv 10 \pmod{17}$;

$6^{14} \equiv 9 \pmod{17}$;

$6^{15} \equiv 3 \pmod{17}$.

Tuzilgan taqqoslamalar yordamida quyidagi jadvallarni tuzamiz:

1-jadval

N	0	1	2	3	4	5	6	7	8	9
0		0	2	15	4	11	1	5	6	14
1	13	9	3	12	7	10	8			

1-jadval uchun taqqoslamalarning ikkinchi tomonidagi songa mos daraja topiladi.

2-jadval

I	0	1	2	3	4	5	6	7	8	9
0	1	6	2	12	4	7	8	14	16	11
1	15	5	13	10	9	3				

2-jadval uchun taqqoslamalarning birinchi tomonidagi darajaga mos qoldiq topiladi.

15-misol. $15x^{19} \equiv 28 \pmod{17}$ **taqqoslamani yeching.**

Yechish. $15x^{19} \equiv 28 \pmod{17}$taqqoslamani taqqoslama xossalari yordamida soddalashtiramiz: $15x^3 \equiv 11 \pmod{17}$. Hosil bo`lgan taqqoslamani indekslar xossalariga ko`ra: $ind15 + 3indx \equiv ind11 \pmod{16}$ taqqoslamani hosil qilamiz.

14-misolda tuzilgan jadvaldan $ind15 = 10$, $ind11 = 9$larni topib,

$10 + 3indx \equiv 9 \pmod{16}$

$3indx = -1 \pmod{16}$

(3,16)=1 ekanligidan taqqoslamaning yagona yechimi bor. Taqqoslama xossalaridan

$$3indx \equiv 15 \pmod{16};$$
$$indx \equiv 5 \pmod{16}$$

larni va 2-jadval yordamida $x \equiv 7 \pmod{17}$yechimni hosil qilamiz.

Tekshirish:

$$15 \cdot 7^{19} - 28 = -2(7^2)^9 7 - 11 \equiv -2(49)^9 \cdot 7 - 11 \equiv -2(-2)^9 \cdot 7 - 11 \equiv$$
$$-2(-2)^5 \cdot (-2)^4 7 - 11 \equiv -2(-32) \cdot 16 \cdot 7 - 11 \equiv -2 \cdot 2 \cdot (-1) \cdot 7 - 11 \equiv$$
$$\equiv 28 - 11 \equiv 17 \equiv 0 \pmod{17}$$

Demak, $15 \cdot 7^{19} - 28 \equiv 0$.

16-MUSTAQIL ISH TOPSHIRIQLARI

1-misol. a sonni b songa bo`lgandagi qoldiqni toping:

1.1. $a = 34562$, $b = 234$; $a = 245^{837}$, $b = 23$.

1.2. $a = 74653$, $b = 657$; $a = 854^{132}$, $b = 94$.

1.3. $a = 23415$, $b = 534$; $a = 9584^{245}$, $b = 75$.

1.4. $a = 23147$, $b = 126$; $a = 657^{34}$, $b = 89$.

1.5. $a = 74645$, $b = 324$; $a = 4536^{26}$, $b = 53$.

1.6. $a = 76354$, $b = 123$; $a = 654^{768}$, $b = 356$.

1.7. $a = 74856$, $b = 64$; $a = 2635^{12}$, $b = 36$.

1.8. $a = 96847$, $b = 238$; $a = 172^{172}$, $b = 72$.

1.9. $a = 24352$, $b = 342$; $a = 857^{123}$, $b = 85$.

1.10. $a = 12485$, $b = 342$; $a = 357^{423}$, $b = 75$

1.11. $a = 20394$, $b = 21$; $a = 905^{456}$, $b = 74$.

1.12. $a = 12903$, $b = 372$; $a = 732^{45}$, $b = 34$.

1.13. $a = 28045$, $b = 2834$; $a = 433^{564}$, $b = 35$.

1.14. $a = 18847$, $b = 3823$; $a = 863^{6433}$, $b = 53$.

1.15. $a = 27421$, $b = 283$; $a = 7423^{32}$, $b = 23$.

1.16. $a = 84054$, $b = 3743$; $a = 313^{542}$, $b = 12$.

1.17. $a = 37950$, $b = 129$; $a = 632^{542}$, $b = 64$.

1.18. $a = 28406$, $b = 2632$; $a = 986^{65}$, $b = 33$.

1.19. $a = 36412$, $b = 430$; $a = 632^{544}$, $b = 74$.

1.20. $a = 27363$, $b = 6473$; $a = 832^{63}$, $b = 78$.

1.21. $a = 73263$, $b = 4173$; $a = 832^{233}$, $b = 58$.

1.22. $a = 6363$, $b = 473$; $a = 632^{123}$, $b = 38$.

1.23. $a = 56463$, $b = 4473$; $a = 542^{63}$, $b = 45$.

1.24. $a = 54263$, $b = 3413$; $a = 322^{431}$, $b = 51$.

1.25. $a = 84133$, $b = 976$; $a = 232^{143}$, $b = 87$.

2-misol. Isbotlang:

2.1. Agar $(a + b - c) \vdots 2$ bo`lsa, u holda $(a - b - c) \vdots 2$.

2.2. Agar $(11a + 2b) \vdots 19$ bo`lsa, u holda $(18a + 5b) \vdots 19$.

2.3. Agar $(a - 5b) \vdots 17$ bo`lsa, u holda $(2a + 7b) \vdots 17$.

2.4. Agar $(12a - 7b) \vdots 16$ bo`lsa, u holda $(4a + 23b) \vdots 16$.

2.5. Agar $(a - 5b) \vdots 19$ bo`lsa, u holda $(10a + 7b) \vdots 19$.

2.6. Agar $(16a - 11b + c) \vdots 21$ bo`lsa, u holda $(11a - b + 2c) \vdots 21$.

2.7. Agar $(6a - 11b) \vdots 31$ bo`lsa, u holda $(a - 7b) \vdots 31$.

2.8. Agar $(50a + 8b + c) \vdots 21$ bo`lsa, u holda $(a + b + 8c) \vdots 21$.

2.9. Agar $(15a + 3b) \vdots 17$ bo`lsa, u holda $(5a + b) \vdots 17$.

2.10. Agar $(50a - b + 60c) \vdots 388$ bo`lsa, u holda $(a - 4b + 41c) \vdots 194$.

2.11. Agar $(a + b - 2c) \vdots 13$ bo`lsa, u holda $(7a - 6b - 5c) \vdots 13$.

2.12. Agar $(a + b - 2c) \vdots 15$ bo`lsa, u holda $(-5a + 10b - 5c) \vdots 3$.

2.13. Agar $(2a + 3b - c) \vdots 11$ bo`lsa, u holda $(a + 7b - 6c) \vdots 11$.

2.14. Agar $(2a + 3b - c) \vdots 14$ bo`lsa, u holda $(2a - 4b - 8c) \vdots 7$.

2.15. Agar $(7a - 5b + 2c) \vdots 18$ bo`lsa, u holda $(-2a - 4b + 2c) \vdots 9$.

2.16. Agar $(7a - 5b + 2c) \vdots 18$ bo`lsa, u holda $(-2a - 4b + 2c) \vdots 18$.

2.17. Agar $(7a - 5b + 2c) \vdots 18$ bo`lsa, u holda $(-2a - 4b + 2c) \vdots 3$.

2.18. Agar $(16a - 11b + c) \vdots 21$ bo`lsa, u holda $(11a - b + 2c) \vdots 7$.

2.19. Agar $(16a - 11b + c) \vdots 21$ bo`lsa, u holda $(11a - b + 2c) \vdots 3$.

2.20. Agar $(12a - 7b) \vdots 16$ bo`lsa, u holda $(4a + 23b) \vdots 8$.

2.21. Agar $(12a - 7b) \vdots 16$ bo`lsa, u holda $(4a + 23b) \vdots 4$.

2.22. Agar $(50a + 8b + c) \vdots 21$ bo`lsa, u holda $(a + b + 8c) \vdots 7$.

2.23. Agar $(50a + 8b + c) \vdots 21$ bo`lsa, u holda $(a + b + 8c) \vdots 3$.

2.24. Agar $(9a + 5b - 3c) \vdots 24$ bo`lsa, u holda $(12a + 20b - 12c) \vdots 8$.

2.25. Agar $(9a + 5b - 3c) \vdots 24$ bo`lsa, u holda $(12a + 20b - 12c) \vdots 6$.

3-misol. Berilgan taqqoslamalarni xossalar yordamida yeching:

3.1. $7x \equiv 8 \pmod{13}$;
3.2. $6x \equiv 11 \pmod{14}$;
3.3. $8x \equiv 10 \pmod{14}$;
3.4. $11x \equiv -32 \pmod{27}$;
3.5. $16x \equiv 50 \pmod{23}$;
3.6. $25x \equiv 1 \pmod{37}$;
3.7. $17x \equiv 23 \pmod{41}$;
3.8. $32x \equiv 43 \pmod{51}$;
3.9. $27x \equiv 38 \pmod{17}$;
3.10. $-7x \equiv 5 \pmod{3}$;
3.11. $23x \equiv 8 \pmod{11}$;
3.12. $29x \equiv 13 \pmod{19}$;
3.13. $39x \equiv 25 \pmod{13}$;
3.14. $4x \equiv 3 \pmod{16}$;
3.15. $12x \equiv 7 \pmod{21}$;
3.16. $24x \equiv 3 \pmod{13}$;
3.17. $32x \equiv 5 \pmod{19}$;
3.18. $24x \equiv 3 \pmod{11}$;
3.19. $5x \equiv 8 \pmod{6}$;
3.20. $41x \equiv 32 \pmod{17}$;
3.21. $54x \equiv 32 \pmod{15}$;
3.22. $45x \equiv 23 \pmod{13}$;
3.23. $-4x \equiv 7 \pmod{11}$;
3.24. $52x \equiv 31 \pmod{13}$;
3.25. $15x \equiv 64 \pmod{9}$.

4-misol. Berilgan taqqoslamalarni tanlash usuli bilan yeching:

4.1. $4x \equiv 7 \pmod{3}$;
4.2. $13x \equiv 11 \pmod{4}$;
4.3. $-8x \equiv 10 \pmod{6}$;
4.4. $11x \equiv -32 \pmod{7}$;
4.5. $16x \equiv 50 \pmod{3}$;
4.6. $25x \equiv 1 \pmod{6}$;
4.7. $17x \equiv 23 \pmod{9}$;
4.8. $3x \equiv -4 \pmod{5}$;
4.9. $3x \equiv 7 \pmod{5}$;
4.10. $23x \equiv 5 \pmod{3}$;
4.11. $22x \equiv -3 \pmod{7}$;
4.12. $24x \equiv 17 \pmod{5}$;
4.13. $14x \equiv 5 \pmod{7}$;
4.14. $5x \equiv 13 \pmod{7}$;
4.15. $12x \equiv 7 \pmod{2}$;
4.16. $24x \equiv 3 \pmod{5}$;
4.17. $32x \equiv 5 \pmod{9}$;
4.18. $45x \equiv 3 \pmod{11}$;
4.19. $5x \equiv 18 \pmod{6}$;
4.20. $4x \equiv 32 \pmod{13}$;
4.21. $5x \equiv 3 \pmod{13}$;
4.22. $4x \equiv 23 \pmod{13}$;
4.23. $14x \equiv 5 \pmod{11}$;
4.24. $35x \equiv 13 \pmod{7}$;
4.25. $34x \equiv 7 \pmod{9}$.

5-misol. Berilgan taqqoslamalarni Eyler teoremasi yordamida yeching:

5.1. $10x \equiv 3 \pmod 7$;
5.2. $13x \equiv 5 \pmod{17}$;
5.3. $4x \equiv 23 \pmod 9$;
5.4. $14x \equiv 24 \pmod{16}$;
5.5. $21x \equiv -32 \pmod 7$;
5.6. $3x \equiv 12 \pmod 7$;
5.7. $33x \equiv 7 \pmod 8$;
5.8. $26x \equiv 32 \pmod{15}$;
5.9. $11x \equiv 2 \pmod{24}$;
5.10. $52x \equiv 22 \pmod{18}$;
5.11. $16x \equiv 50 \pmod{13}$;
5.12. $25x \equiv 1 \pmod{16}$;
5.13. $17x \equiv 23 \pmod{19}$;
5.14. $2x \equiv 5 \pmod 9$;
5.15. $8x \equiv 15 \pmod{19}$;
5.16. $34x \equiv 15 \pmod{29}$;
5.17. $45x \equiv 32 \pmod{29}$;
5.18. $22x \equiv 5 \pmod{19}$;
5.19. $24x \equiv 15 \pmod 9$;
5.20. $41x \equiv 25 \pmod{19}$;
5.21. $27x \equiv 25 \pmod{29}$;
5.22. $56x \equiv 11 \pmod{19}$;
5.23. $58x \equiv 3 \pmod{15}$;
5.24. $45x \equiv 3 \pmod{31}$;
5.25. $5x \equiv 18 \pmod{26}$.

6-misol. Berilgan taqoslamalarni chekli zanjir kasrlar yordamida yeching:

6.1. $47x \equiv 8 \pmod{133}$;
6.2. $56x \equiv 11 \pmod{144}$;
6.3. $38x \equiv 10 \pmod{149}$;
6.4. $121x \equiv 32 \pmod{247}$;
6.5. $46x \equiv 50 \pmod{273}$;
6.6. $25x \equiv 1 \pmod{367}$;
6.7. $117x \equiv 23 \pmod{451}$;
6.8. $32x \equiv 43 \pmod{501}$;
6.9. $247x \equiv 38 \pmod{817}$
6.10. $37x \equiv 5 \pmod{243}$;
6.11. $13x \equiv 32 \pmod{19}$;
6.12. $13x \equiv 7 \pmod{58}$;
6.13. $26x \equiv 32 \pmod{115}$;
6.14. $74x \equiv 3 \pmod{156}$;
6.15. $62x \equiv 7 \pmod{421}$;
6.16. $24x \equiv 3 \pmod{153}$;
6.17. $32x \equiv 5 \pmod{219}$;
6.18. $24x \equiv 3 \pmod{101}$;
6.19. $15x \equiv 8 \pmod{756}$;
6.20. $41x \equiv 32 \pmod{717}$;
6.21. $54x \equiv 32 \pmod{185}$;
6.22. $45x \equiv 23 \pmod{193}$;
6.23. $4x \equiv 7 \pmod{111}$;
6.24. $24x \equiv 15 \pmod{69}$;
6.25. $43x \equiv 25 \pmod{119}$;

7-misol. Berilgan $ax \equiv b(\mathrm{mod}\ m)$ taqoslamalarni a ga teskari sinf orqali yeching:

7.1. $57x \equiv 8(\mathrm{mod}\ 33)$; 7.14. $14x \equiv 3(\mathrm{mod}\ 56)$;

7.2. $67x \equiv 11(\mathrm{mod}\ 44)$; 7.15. $42x \equiv 7(\mathrm{mod}\ 21)$;

7.3. $28x \equiv 10(\mathrm{mod}\ 49)$; 7.16. $14x \equiv 3(\mathrm{mod}\ 53)$;

7.4. $21x \equiv 32(\mathrm{mod}\ 47)$ 7.17. $32x \equiv 5(\mathrm{mod}\ 19)$;

7.5. $86x \equiv 50(\mathrm{mod}\ 73)$; 7.18. $54x \equiv 3(\mathrm{mod}\ 81)$;

7.6. $35x \equiv 1(\mathrm{mod}\ 67)$; 7.19. $35x \equiv 8(\mathrm{mod}\ 56)$;

7.7. $17x \equiv 23(\mathrm{mod}\ 51)$; 7.20. $61x \equiv 32(\mathrm{mod}\ 17)$;

7.8. $2x \equiv 43(\mathrm{mod}\ 51)$; 7.21. $84x \equiv 32(\mathrm{mod}\ 85)$;

7.9. $47x \equiv 38(\mathrm{mod}\ 17)$; 7.22. $35x \equiv 23(\mathrm{mod}\ 93)$;

7.10. $37x \equiv 5(\mathrm{mod}\ 43)$; 7.23. $42x \equiv 7(\mathrm{mod}\ 11)$;

7.11. $7x \equiv 138(\mathrm{mod}\ 27)$; 7.24. $5x \equiv 23(\mathrm{mod}\ 13)$;

7.12. $12x \equiv 17(\mathrm{mod}\ 19)$; 7.25. $15x \equiv 23(\mathrm{mod}\ 33)$.

7.13. $14x \equiv 35(\mathrm{mod}\ 18)$;

8-misol. Berilgan tenglamalarni taqqoslamalar yordamida yeching:

8.1. $16x + 5y = 23$; 8.14. $24x + 13y = 11$;

8.2. $25x + 11y = 37$; 8.15. $5x + 8y = 6$;

8.3. $17x + 23y = 41$; 8.16. $41x + 32y = 17$;

8.4. $32x + 43y = 51$; 8.17. $54x + 32y = 15$;

8.5. $27x + 38y = 17$; 8.18. $45x + 23y = 13$;

8.6. $-7x + 5y = 13$; 8.19. $-4x + 7y = 11$;

8.7. $47x + 8y = 133$; 8.20. $74x + 3y = 156$;

8.8. $56x + 11y = 44$; 8.21. $62x + 7y = 21$;

8.9. $38x + 10y = 48$; 8.22. $24x + 23y = 53$;

8.10. $21x + 32y = 24$; 8.23. $32x + 5y = 19$;

8.11. $47x + 38y = 17$; 8.24. $35x + 23y = 19$;

8.12. $7x + 3y = 17$; 8.25. $5x + 23y = 29$.

8.13. $4x + 13y = 27$;

9-misol. Taqqoslamalar sistemasini yeching:

9.1. $\begin{cases} 3x \equiv 5(\mathrm{mod}\,7), \\ 2x \equiv 1(\mathrm{mod}\,5), \\ 4x \equiv 7(\mathrm{mod}\,11); \end{cases}$

9.2. $\begin{cases} 13x \equiv 5(\mathrm{mod}\,27), \\ 22x \equiv 31(\mathrm{mod}\,5), \\ 14x \equiv 27(\mathrm{mod}\,11); \end{cases}$

9.3. $\begin{cases} -3x \equiv 5(\mathrm{mod}\,37), \\ 12x \equiv 31(\mathrm{mod}\,25), \\ 14x \equiv 37(\mathrm{mod}\,9); \end{cases}$

9.4. $\begin{cases} 7x \equiv 5(\mathrm{mod}\,13), \\ 22x \equiv 11(\mathrm{mod}\,5), \\ 34x \equiv 57(\mathrm{mod}\,31); \end{cases}$

9.5. $\begin{cases} 7x \equiv 85(\mathrm{mod}\,37), \\ 23x \equiv 11(\mathrm{mod}\,25), \\ 24x \equiv 47(\mathrm{mod}\,11); \end{cases}$

9.6. $\begin{cases} x \equiv 35(\mathrm{mod}\,27), \\ -2x \equiv 21(\mathrm{mod}\,5), \\ 4x \equiv -7(\mathrm{mod}\,15); \end{cases}$

9.7. $\begin{cases} 14x \equiv 5(\mathrm{mod}\,7), \\ -11x \equiv -3(\mathrm{mod}\,15), \\ 35x \equiv -7(\mathrm{mod}\,11); \end{cases}$

9.8. $\begin{cases} 3x \equiv -5(\mathrm{mod}\,7), \\ 2x \equiv 3(\mathrm{mod}\,5), \\ 4x \equiv 7(\mathrm{mod}\,19); \end{cases}$

9.9. $\begin{cases} -3x \equiv 5(\mathrm{mod}\,37), \\ 12x \equiv 31(\mathrm{mod}\,25), \\ 14x \equiv 37(\mathrm{mod}\,9); \end{cases}$

9.10. $\begin{cases} 2x \equiv 15(\mathrm{mod}\,17), \\ 2x \equiv 11(\mathrm{mod}\,5), \\ 14x \equiv 7(\mathrm{mod}\,21); \end{cases}$

9.11. $\begin{cases} 11x \equiv 5(\mathrm{mod}\,17), \\ 21x \equiv 11(\mathrm{mod}\,15), \\ 34x \equiv 27(\mathrm{mod}\,21); \end{cases}$

9.12. $\begin{cases} 7x \equiv 4(\mathrm{mod}\,7), \\ 9x \equiv 27(\mathrm{mod}\,15), \\ 4x \equiv 37(\mathrm{mod}\,21); \end{cases}$

9.13. $\begin{cases} 43x \equiv 15(\mathrm{mod}\,57), \\ 52x \equiv 11(\mathrm{mod}\,35), \\ 8x \equiv 47(\mathrm{mod}\,21); \end{cases}$

9.14. $\begin{cases} 45x \equiv 49(\mathrm{mod}\,17), \\ 52x \equiv 35(\mathrm{mod}\,25), \\ 8x \equiv 72(\mathrm{mod}\,23); \end{cases}$

9.15. $\begin{cases} 5x \equiv -4(\mathrm{mod}\,17), \\ -12x \equiv 11(\mathrm{mod}\,15), \\ 14x \equiv 7(\mathrm{mod}\,21); \end{cases}$

9.16. $\begin{cases} 36x \equiv 75(\mathrm{mod}\,17), \\ 42x \equiv 101(\mathrm{mod}\,5), \\ 19x \equiv 47(\mathrm{mod}\,21); \end{cases}$

9.17. $\begin{cases} 17x \equiv 4(\mathrm{mod}\,27), \\ 3x \equiv 27(\mathrm{mod}\,15), \\ 14x \equiv 37(\mathrm{mod}\,21); \end{cases}$

9.18. $\begin{cases} 5x \equiv 9(\mathrm{mod}\,17), \\ 21x \equiv 4(\mathrm{mod}\,15), \\ 4x \equiv 7(\mathrm{mod}\,9); \end{cases}$

9.19. $\begin{cases} 6x \equiv 19(\mathrm{mod}\,17), \\ 3x \equiv 54(\mathrm{mod}\,15), \\ 41x \equiv 7(\mathrm{mod}\,29); \end{cases}$

9.20. $\begin{cases} 43x \equiv 9(\mathrm{mod}\,17), \\ 23x \equiv 4(\mathrm{mod}\,15), \\ 26x \equiv 7(\mathrm{mod}\,9); \end{cases}$

9.21. $\begin{cases} 33x \equiv 19(\mathrm{mod}\,17), \\ 21x \equiv 34(\mathrm{mod}\,15), \\ 24x \equiv 27(\mathrm{mod}\,9); \end{cases}$

9.22. $\begin{cases} 32x \equiv 9(\mathrm{mod}\,17), \\ 18x \equiv 24(\mathrm{mod}\,15), \\ 29x \equiv 37(\mathrm{mod}\,9); \end{cases}$

9.23. $\begin{cases} 32x \equiv 69(\mathrm{mod}\,17), \\ 26x \equiv 24(\mathrm{mod}\,15), \\ 15x \equiv 17(\mathrm{mod}\,9); \end{cases}$

9.24. $\begin{cases} 27x \equiv 4(\mathrm{mod}\,17), \\ 9x \equiv 2(\mathrm{mod}\,15), \\ 4x \equiv 7(\mathrm{mod}\,11); \end{cases}$

9.25. $\begin{cases} 33x \equiv 9(\mathrm{mod}\,17), \\ 3x \equiv 14(\mathrm{mod}\,15), \\ 6x \equiv 17(\mathrm{mod}\,9); \end{cases}$

10-misol. Taqqoslamalar sistemasini yeching:

10.1. $\begin{cases} x \equiv 5(\mathrm{mod}\,7), \\ x \equiv 1(\mathrm{mod}\,5), \\ x \equiv 7(\mathrm{mod}\,11); \end{cases}$

10.2. $\begin{cases} x \equiv 5(\mathrm{mod}\,27), \\ x \equiv 31(\mathrm{mod}\,5), \\ x \equiv 27(\mathrm{mod}\,11); \end{cases}$

10.3. $\begin{cases} x \equiv 5(\mathrm{mod}\,37), \\ x \equiv 31(\mathrm{mod}\,25), \\ x \equiv 37(\mathrm{mod}\,9); \end{cases}$

10.4. $\begin{cases} x \equiv 5(\mathrm{mod}\,13), \\ x \equiv 11(\mathrm{mod}\,5), \\ x \equiv 57(\mathrm{mod}\,31); \end{cases}$

10.5. $\begin{cases} x \equiv 85(\mathrm{mod}\,37), \\ x \equiv 11(\mathrm{mod}\,25), \\ x \equiv 47(\mathrm{mod}\,11); \end{cases}$

10.6. $\begin{cases} x \equiv 35(\mathrm{mod}\,27), \\ x \equiv 21(\mathrm{mod}\,5), \\ x \equiv -7(\mathrm{mod}\,15); \end{cases}$

10.7. $\begin{cases} x \equiv 15(\mathrm{mod}\,27), \\ x \equiv 2(\mathrm{mod}\,5), \\ x \equiv -7(\mathrm{mod}\,13); \end{cases}$

10.8. $\begin{cases} x \equiv 5(\mathrm{mod}\,7), \\ x \equiv -3(\mathrm{mod}\,15), \\ x \equiv -7(\mathrm{mod}\,11); \end{cases}$

10.9. $\begin{cases} x \equiv 11(\mathrm{mod}\,17), \\ x \equiv 6(\mathrm{mod}\,15), \\ x \equiv 8(\mathrm{mod}\,9); \end{cases}$

10.10. $\begin{cases} x \equiv 15(\mathrm{mod}\,17), \\ x \equiv 11(\mathrm{mod}\,5), \\ x \equiv 7(\mathrm{mod}\,21); \end{cases}$

10.11. $\begin{cases} x \equiv 5(\mathrm{mod}\,17), \\ x \equiv 11(\mathrm{mod}\,15), \\ x \equiv 27(\mathrm{mod}\,21); \end{cases}$

10.12. $\begin{cases} x \equiv 4(\mathrm{mod}\,7), \\ x \equiv 27(\mathrm{mod}\,15), \\ x \equiv 37(\mathrm{mod}\,21); \end{cases}$

10.13. $\begin{cases} x \equiv 15(\mathrm{mod}\,57), \\ x \equiv 11(\mathrm{mod}\,35), \\ x \equiv 47(\mathrm{mod}\,21); \end{cases}$

10.14. $\begin{cases} x \equiv 49(\mathrm{mod}\,17), \\ x \equiv 35(\mathrm{mod}\,25), \\ x \equiv 72(\mathrm{mod}\,23); \end{cases}$

10.15. $\begin{cases} x \equiv -4(\mathrm{mod}\,17), \\ x \equiv 11(\mathrm{mod}\,15), \\ x \equiv 7(\mathrm{mod}\,21); \end{cases}$

10.16. $\begin{cases} x \equiv 4(\mathrm{mod}\,7), \\ x \equiv 5(\mathrm{mod}\,13), \\ x \equiv -7(\mathrm{mod}\,21); \end{cases}$

10.17. $\begin{cases} x \equiv 75(\mathrm{mod}\,17), \\ x \equiv 101(\mathrm{mod}\,5), \\ x \equiv 47(\mathrm{mod}\,21); \end{cases}$

10.18. $\begin{cases} x \equiv 9(\mathrm{mod}\,17), \\ x \equiv 4(\mathrm{mod}\,15), \\ x \equiv 7(\mathrm{mod}\,9); \end{cases}$

10.19. $\begin{cases} x \equiv 19(\mathrm{mod}\,17), \\ x \equiv 54(\mathrm{mod}\,15), \\ x \equiv 7(\mathrm{mod}\,29); \end{cases}$

10.20. $\begin{cases} x \equiv 9(\mathrm{mod}\,17), \\ x \equiv 4(\mathrm{mod}\,15), \\ x \equiv 7(\mathrm{mod}\,9); \end{cases}$

10.21. $\begin{cases} x \equiv 19(\mathrm{mod}\,17), \\ x \equiv 34(\mathrm{mod}\,15), \\ x \equiv 27(\mathrm{mod}\,9); \end{cases}$

10.22. $\begin{cases} x \equiv 9(\mathrm{mod}\,17), \\ x \equiv 24(\mathrm{mod}\,15), \\ x \equiv 37(\mathrm{mod}\,9); \end{cases}$

10.23. $\begin{cases} x \equiv 69(\mathrm{mod}\,17), \\ x \equiv 24(\mathrm{mod}\,15), \\ x \equiv 17(\mathrm{mod}\,9); \end{cases}$

10.24. $\begin{cases} x \equiv 9(\mathrm{mod}\,17), \\ x \equiv 4(\mathrm{mod}\,15), \\ x \equiv 7(\mathrm{mod}\,9); \end{cases}$

10.25. $\begin{cases} x \equiv 29(\mathrm{mod}\,17), \\ x \equiv 14(\mathrm{mod}\,15), \\ x \equiv 17(\mathrm{mod}\,9); \end{cases}$

11-misol. Berilgan taqqoslamalarni soddalashtiring:

11.1. $x^{233} + 345x^{132} - 567x^{109} + 346x^{98} + 45x^{56} - 463x^{37} - 24x^{15} + x^{9} + 467 \equiv 0 \pmod{5}$;

11.2. $245x^{274} + 345x^{123} - 507x^{119} + 346x^{98} + 45x^{54} - 463x^{27} - 24x^{15} + x - 67 \equiv 0 \pmod{7}$;

11.3. $x^{233} + 345x^{132} - 567x^{109} + 346x^{98} + 45x^{56} - 463x^{37} - 24x^{15} + x^{9} + 467 \equiv 0 \pmod{13}$;

11.4. $245x^{274} + 345x^{123} - 507x^{119} + 346x^{98} + 45x^{54} - 463x^{27} - 24x^{15} - x + 67 \equiv 0 \pmod{11}$;

11.5. $5^{435} + 325x^{203} - 57x^{100} + 634x^{98} + 45x^{52} - 63x^{25} - 24x^{15} + x + 167 \equiv 0 \pmod{3}$;

11.6. $45x^{294} + 545x^{223} - 677x^{97} + 334x^{90} - 465x^{57} + 43x^{28} - 264x^{11} + 244x + 674 \equiv 0 \pmod{3}$;

11.7. $245x^{274} + 345x^{123} - 507x^{119} + 346x^{98} + 45x^{54} - 463x^{27} - 24x^{15} + x + 67 \equiv 0 \pmod{5}$;

11.8. $245x^{274} + 345x^{123} + 507x^{119} + 346x^{98} + 45x^{54} - 463x^{27} - 24x^{15} + x + 67 \equiv 0 \pmod{13}$;

11.9. $x^{233} + 345x^{132} - 567x^{109} + 346x^{98} + 45x^{56} - 463x^{37} + 24x^{15} + x^{9} + 467 \equiv 0 \pmod{3}$;

11.10. $x^{233} + 345x^{132} - 567x^{109} + 346x^{98} - 45x^{56} - 463x^{37} - 24x^{15} + x^{9} + 467 \equiv 0 \pmod{11}$;

11.11. $x^{233} + 345x^{132} - 567x^{109} - 346x^{98} + 45x^{56} - 463x^{37} - 24x^{15} + x^{9} + 467 \equiv 0 \pmod{7}$;

11.12. $x^{233} + 345x^{132} - 567x^{109} + 346x^{98} + 45x^{56} - 463x^{37} - 24x^{15} + x^{9} + 467 \equiv 0 \pmod{13}$;

11.13. $745x^{394} + 545x^{223} - 677x^{97} + 334x^{90} - 465x^{57} - 43x^{28} - 264x^{11} + 244x - 674 \equiv 0 \pmod{13}$;

11.14. $-45x^{294} + 545x^{223} - 677x^{97} + 334x^{90} + 465x^{57} - 43x^{28} - 264x^{11}$-

$-244x+674 \equiv 0 \pmod 5$;

11.15. $453^{594} - 545x^{223} - 677x^{97} + 334x^{90} + 465x^{57} - 43x^{28}-$

$-264x^{11}-244x-674 \equiv 0 \pmod 7$;

11.16. $-45x^{294}+545x^{223}+677x^{97}+334x^{90}+465x^{57}-43x^{28}-$

$-264x^{11}+244x-674 \equiv 0 \pmod{11}$;

11.17. $145x^{245} - 55x^{123} - 77x^{99}+34x^{95}-165x^{50}-473x^{23}-64x^{12}+$

$+44x^{9}-124 \equiv 0 \pmod 3$;

11.18. $345x^{245}+55x^{123}+77x^{99}+34x^{95}+165x^{50}-473x^{23}+$

$+64x^{12}-44x^{9}+124 \equiv 0 \pmod{13}$;

11.19. $-145x^{345}+55x^{123}-77x^{99}+34x^{95}-165x^{50}-473x^{23}-64x^{12}+$

$+44x^{9}-124 \equiv 0 \pmod 5$;

11.20. $245x^{445}+55x^{123}-77x^{99}+34x^{95}+165x^{50}-473x^{23}+64x^{12}+$

$+44x^{9}+124 \equiv 0 \pmod 7$;

11.21. $145x^{145} + 76x^{103} -47x^{90}+32x^{65}+15x^{5}-43x^{2}+$

$+6x+24 \equiv 0 \pmod{17}$;

11.22. $761x^{345} + 415x^{233}-247x^{100}+337x^{84}+195x^{74}-$

$-73x^{23}+64x^{12}+44x^{9}+194 \equiv 0 \pmod 5$;

11.23. $975x^{285}+735x^{214}-767x^{119}+394x^{105}+465x^{70}-$

$-173x^{63}+344x^{62}+124x^{91} \equiv 0 \pmod 7$;

11.24. $763x^{534}+732x^{323}-237x^{129}+354x^{125}+865x^{110}-$

$-473x^{93}+604x^{72}+234x^{11} \equiv 0 \pmod 5$;

11.25. $934x^{333}+143x^{203}-232x^{119}+549x^{94}+765x^{53}-$

$-73x^{26}+984x^{22}+49x^{11}+9 \equiv 0 \pmod 3$.

12-misol. Lejandr simvolini aniqlang:

12.1. $\frac{4563}{197}$; 12.8. $\frac{673}{251}$; 12.14. $\frac{5467}{349}$; 12.20. $\frac{9876}{617}$;

12.2. $\frac{5798}{659}$; 12.9. $\frac{5876}{941}$; 12.15. $\frac{4566}{1021}$; 12.21. $\frac{5435}{1091}$;

12.3. $\frac{2435}{419}$; 12.10. $\frac{10234}{1511}$; 12.16. $\frac{14634}{1811}$; 12.22. $\frac{2545}{1777}$;

12.4. $\frac{3545}{1723}$; 12.11. $\frac{54376}{2011}$; 12.17. $\frac{5433}{2063}$; 12.23. $\frac{24354}{2371}$;

12.5. $\frac{54567}{2693}$; 12.12. $\frac{3543}{2699}$; 12.18. $\frac{43254}{2999}$; 12.24. $\frac{23543}{3323}$;

12.6. $\frac{4567}{693}$; 12.13. $\frac{13543}{2699}$; 12.19. $\frac{53254}{2999}$; 12.25. $\frac{73543}{3323}$.

12.7. $\frac{45543}{3699}$.

13-misol. Yakobi simvolini aniqlang:

13.1. $\frac{235}{414}$; 13.8. $\frac{1234}{1514}$; 13.14. $\frac{1434}{1812}$; 13.20. $\frac{255}{178}$;

13.2. $\frac{435}{455}$; 13.9. $\frac{234}{111}$; 13.15. $\frac{634}{411}$; 13.21. $\frac{545}{77}$;

13.3. $\frac{567}{693}$; 13.10. $\frac{243}{699}$; 13.16. $\frac{254}{99}$; 13.22. $\frac{543}{332}$;

13.4. $\frac{547}{264}$; 13.11. $\frac{353}{299}$; 13.17. $\frac{434}{272}$; 13.23. $\frac{233}{338}$;

13.5. $\frac{367}{125}$; 13.12. $\frac{4355}{1020}$; 13.18. $\frac{4344}{1032}$; 13.24. $\frac{3543}{323}$;

13.6. $\frac{567}{325}$; 13.13. $\frac{1355}{1120}$; 13.19. $\frac{1344}{1132}$; 13.25. $\frac{2543}{425}$;

13.7. $\frac{6543}{4120}$;

14-misol. r modul bo`yicha g boshlang'ich ildizning indekslar jadvalini tuzing:

14.1.	$p = 73,\ g=5;$	14.14.	$p=71,\ g=7;$
14.2.	$p = 61,\ g= 2;$	14.15.	$p=59,\ g = 2;$
14.3.	$p = 47,\ g = 5;$	14.16.	$p = 43,\ g = 3$
14.4.	$p = 37,\ g = 2;$	14.17.	$p = 31,\ g = 3$
14.5.	$p = 23,\ g = 5;$	14.18.	$p = 19,\ g = 2;$
14.6.	$p = 13,\ g = 2;$	14.19.	$p = 11,\ g = 2$
14.7.	$p = 23,\ g = 7;$	14.20.	$p = 19,\ g = 3;$
14.8.	$p = 7,\ g = 5;$	14.21.	$p = 19,\ g = 10$
14.9.	$p = 29,\ g = 3;$	14.22.	$p=67,\ g= 2;$
14.10.	$p= 53,\ g = 2;$	14.23.	$p = 17,\ g = 6;$
14.11.	$p = 41,\ g = 6;$	14.24.	$p = 29,\ g = 2;$
14.12.	$p = 17,\ g = 3;$	14.25.	$p = 17,\ g = 5;$
14.13.	$p = 7,\ g = 3.$		

15-misol. Berilgan taqqoslamalarni yeching:

15.1.	$5x^{23} \equiv 8 \pmod{31};$	15.14.	$14x^{9} \equiv 3 \pmod{59};$
15.2.	$17x^{34} \equiv 11 \pmod{47};$	15.15.	$12x^{32} \equiv 7 \pmod{41};$
15.3.	$8x^{14} \equiv 10 \pmod{43};$	15.16.	$14x^{52} \equiv 3 \pmod{7};$
15.4.	$2x^{45} \equiv 32 \pmod{37};$	15.17.	$32x^{3} \equiv 5 \pmod{19};$
15.5.	$6x^{54} \equiv 50 \pmod{7};$	15.18.	$54x^{14} \equiv 3 \pmod{13};$
15.6.	$35x^{24} \equiv 1 \pmod{5};$	15.19.	$35x^{32} \equiv 8 \pmod{11};$
15.7.	$17x^{21} \equiv 23 \pmod{13};$	15.20.	$61x^{35} \equiv 32 \pmod{17};$
15.8.	$24x^{34} \equiv 43 \pmod{3};$	15.21.	$4x^{8} \equiv 32 \pmod{73};$
15.9.	$47x^{27} \equiv 38 \pmod{17};$	15.22.	$35x^{34} \equiv 23 \pmod{71};$
15.10.	$37x^{5} \equiv 5 \pmod{19};$	15.23.	$2x^{45} \equiv 7 \pmod{11};$
15.11.	$123x^{12} \equiv 25 \pmod{13};$	15.24.	$45x^{62} \equiv 3 \pmod{13};$
15.12.	$29x^{52} \equiv 53 \pmod{17};$	15.25.	$53x^{45} \equiv 74 \pmod{17};$
15.13.	$57x^{5} \equiv 14 \pmod{11}.$		

Takrorlash uchun savollar:

1. Modul bo`yicha taqqoslamalar.
2. Taqqoslamalar xossalari.
3. m modul bo`yicha chegirmalar halqasi.
4. Chegirmalar to`liq sistemasi.
5. Chegirmalar keltirilgan sistemasi.
6. Chegirmalar mul'tiplikativ gruppasi.
7. Eyler funksiyasi.
8. Eyler va Ferma teoremalari.
9. Taqqoslama darajasi va uning yechimi.
10. Teng kuchli taqqoslamalar.
11. Birinchi darajali taqqoslamalarni Yechish usullari.
12. Ikki o`zgaruvchili chiziqli tenglamalar.
13. Taqqoslamalar sistemasi.
14. Lejandr, Yakobi simvollari.
15. Chegirmalar sinfi tartibi.
16. Tub modul bo`yicha boshlang'ich ildizlar.
17. Indekslar, xossalari. Indekslar jadvali.
18. Taqqoslamalar nazariyasining arifmetikaga tadbiqlari.

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MUNDARIJA

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ALGEBRA VA SONLAR NAZARIYASIDAN MODUL TEXNOLOGIYASI ASOSIDA TAYYORLANGAN MUSTAQIL ISHLAR TO`PLAMI

IV QISM

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