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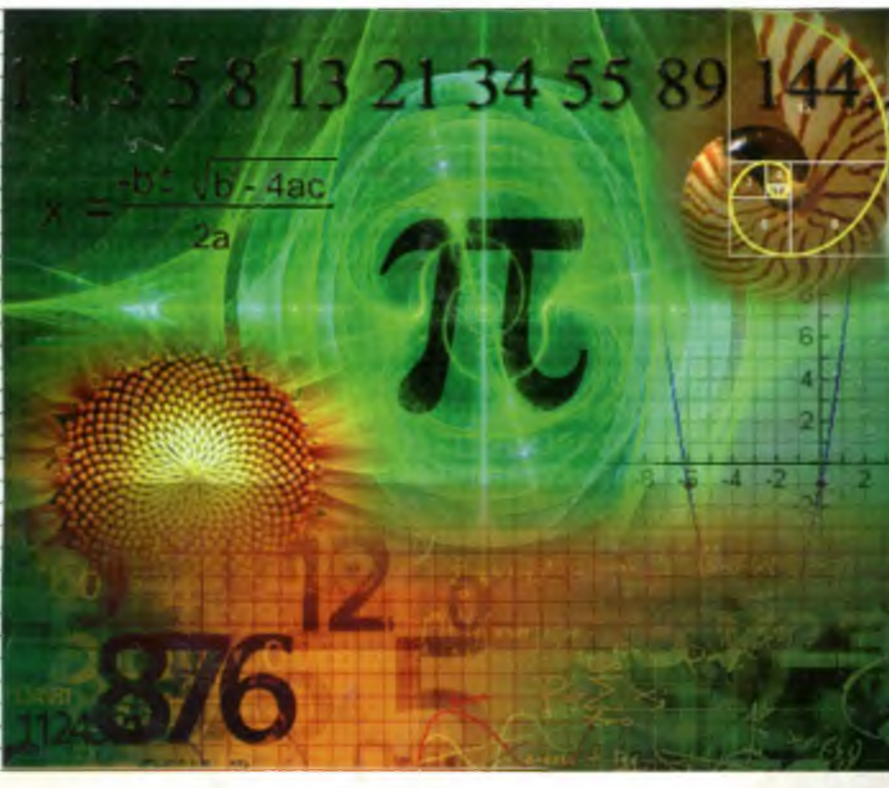
TENGLAMALAR YECHISH



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Ushbu qo'llanma sodda va murakkab algebraik tenglamalar, ko'phadlar va funksional tenglamalarni yechish bo'yicha dastur asosida akademik litsey va kasb-hunar kollejlari uchun tuzilgan. Unda mavzuga oid amaliy masalalar avvalo yechib ko'rsatilgan va mustaqil yechish uchun shunga monand misol, masalalar berilgan. Qo'llanmada DTM axborotnomalari savollaridan va mavzuga oid adabiyotlardan foydalanilgan.

Qo'llanma umumiy o'rta ta'lim maktablari, akademik litsey va kasb-hunar kollejlari o'quvchilari, pedagogika oliy o'quv yurtlarining matematika fakulteti talabalari, o'qituvchilar va oliy o'quv yurtlariga kiruvchilar uchun mo'ljallangan.

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KIRISH

Qo'lingizdagi o'quv-uslubiy qo'llanma akademik litsey va kasb-hunar kollejlari uchun mo'ljallangan bo'lib, bu qo'llanma algebraik tenglamalar, ko'phadlar va funksional tenglamalar mavzularini o'z ichiga qamrab olgan.

Mazkur qo'llanmaning mazmuni Davlat ta'lim standartlari va o'rta maxsus kasb-hunar ta'limining matematika dasturi asosida ishlab chiqilgan. Shu bilan birgalikda qo'llanmadan o'rin olgan mavzular umumta'lim maktablari, akademik litsey va kasb-hunar kollejlari o'quvchilari hamda oliy o'quv yurtlariga kiruvchilar uchun foydali bo'ladi.

Funksional tenglamalarga doir adabiyotlar kamligini nazarda tutgan holda qo'llanmada mazkur bo'limga alohida o'rin ajratilgan. Shu bilan birga, murakkab olimpiada masalalari ham turli usullarda yechib ko'rsatilgan bo'lib, bu o'quvchilarni matematika fanidan olimpiadaga tayyorlanishlari uchun muhim omil bo'ladi.

Mualliflar nazariy materialni ixcham shaklda, o'quvchilarga tushunarli tilda bayon etishga intilishdi. Darslarda asosiy turdagi masalalarni yechish bo'yicha bilim va malakalarini shakllantirish maqsadida izchil amaliy ish olib borilgan. Birinchi navbatda, misollar yechish malakalarini mashq qilishga e'tibor beriladi. O'quvchilar bilan tayanch (asosiy) masalalarni yechishga yondashgan holda ularning yechilish yo'llari muhokama etiladi. Bu yerda misollarning yechilish usullaridan namunalar keltirilgan bo'lib, natijada shu kabi misollarni o'quvchilar bemalol mustaqil yecha oladilar. Dars jarayonini jadallashtirish maqsadida qo'llanmada ta'limiy va o'z-o'zini nazorat etish ruhidagi turli misollar javoblari bilan keltirilgan. Ulardan foydalanish esa darsda ko'rib chiqiladigan material hajmini orttirishga imkon berib, uning yaxshiroq o'zlashtirilishiga yordam beradi.

Qo'llanmada DTM axborotnomalari misollaridan namunalar yechib ko'rsatilgan va mustaqil yechish uchun amaliy misollar berilgan.

Mazkur qo'llanma o'quvchilarning tafakkurini oshirish, diqqat-e'tibor, kuzatuvchanlik, o'rgatilayotgan bo'limni chuqur o'zlashtirishiga katta yordam berishini nazarda tutamiz.

Mualliflar

Ayniyat. Tenglama

Ta'rif. *Agar tenglik shu tenglikka kirgan harflarning qabul qilishi mumkin bo'lgan barcha qiymatlarida to'g'ri sonli tenglikka aylansa, bunday tenglik ayniyat deyiladi.*

Misol. 1) $(a-b)^2 = a^2 - 2ab + b^2$ 2) $(a+b)^2 = a^2 + 2ab + b^2$

3) $a+b = b+c$ 4) $(a+b)+c = a+(b+c)$ 5) $(ab)c = a(bc)$

Ayniyatlar quyidagi xossalarga ega:

1. Agar $A=B$ va $B=C$ bo'lsa, u holda $A=C$ bo'ladi.
2. Agar $A=B$ bo'lsa, u holda $A+C=B+C$ bo'ladi.
3. Agar $A=B$ bo'lsa, u holda $A\cdot C=B\cdot C$ bo'ladi.

Bu yerda A, B, C bir xil to'plamga tegishli.

Ikki algebraik ifoda yoki son tenglik ishorasi bilan birlashtirilsa, tenglik deb ataluvchi ifoda hosil bo'ladi.

Ta'rif. *Harf bilan belgilangan noma'lum son qatnashgan tenglik tenglama deyiladi.*

Tenglik belgisidan chap va o'ngda turgan ifodalar tenglamaning chap va o'ng qismlari deyiladi. Tenglamaning chap yoki o'ng qismidagi har bir qo'shiluvchi tenglamaning hadi deyiladi.

Noma'lumning tenglamani to'g'ri tenglikka (ayniyatga) aylantiradigan qiymatlari *tenglamaning ildizi* deyiladi.

Tenglamani yechish uning barcha ildizlarini topish yoki ildizlari yo'qligini isbotlash demakdir.

Demak, umumiy holda tenglamani $P(x)=0$ ko'rinishida yozish mumkin. $P(x)$ – bu x ga bog'liq ifoda.

Ta'rif. *x o'zgaruvchining $P(x)$ ifoda ma'noga ega bo'ladigan barcha qiymatlari to'plami tenglamaning aniqlanish sohasi deyiladi.*

Teng kuchli tenglamalar

Ta'rif. *Agar ikki tenglamadan birining ildizlari ikkinchisining ham ildizlari bo'lsa va aksincha, ikkinchi tenglamaning ildizlari birinchi tenglamaning ham ildizlari bo'lsa yoki ikkala tenglama ham ildizlarga ega bo'lmasa, bunday ikki tenglama teng kuchli yoki ekvivalent tenglamalar deyiladi.*

Ta'rif. *Agar ikki tenglamadan birining hamma ildizlari ikkinchisining ham ildizlari bo'lsa, ikkinchi tenglama birinchi tenglamaning natijasi deyiladi.*

Bu ta'rifdan ildizga ega bo'lgan ikki tenglama teng kuchli bo'lishi uchun ularning biri ikkinchisining natijasi bo'lishi kerakligi kelib chiqadi.

Misol. $5x+4=3x+20$ tenglama $5x-x=2x+16$ tenglama bilan teng kuchli, ikkala tenglama ham $x=8$ ildizga ega.

Tenglamalar ustida algebraik almashtirishlar bajarishda unga teng kuchli tenglamalar hosil bo'lishini quyidagi xossalarda ko'rish mumkin:

1. Agar berilgan tenglamaning har ikkala tomoniga bir xil o'zgarmas son (yoki ma'noga ega ifoda) qo'shilsa, natijada berilgan tenglamaga teng kuchli tenglama hosil bo'ladi.

2. Tenglamaning biror hadini qarama-qarshi ishora bilan uning bir tomonidan ikkinchi tomoniga o'tkazish mumkin.

Misol. $3x+4=7 \Leftrightarrow 3x+4-7=0 \Leftrightarrow 3x+4=7$

3. Berilgan tenglamaning ikkala tomoni nolga teng bo'lmagan songa yoki ma'noga ega bo'lgan ifodaga ko'paytirilsa (yoki bo'linsa), berilgan tenglamaga teng kuchli tenglama hosil bo'ladi.

Misol. $6-\frac{x-1}{2}=\frac{3-x}{2}+\frac{x-2}{3} \Leftrightarrow 36-3x+3=9-3x+2x-4$

tenglamalar teng kuchli, chunki ikkala tenglama ham $x=17$ ildizga ega.

4. Agar $f_1(x)=f_2(x)$ tenglamaning ikkala tomonini ham uning qiymatlar qabul qilish mumkin bo'lgan sohasini o'zgartirmaydigan qilib, ya'ni ayniy almashtirilsa, natijada berilgan tenglamaga teng kuchli bo'lgan tenglama hosil bo'ladi.

Misol. $(x+4)^2-x=(x-2)^2+1$ va $x^2+7x+16=x^2-4x+5$ tenglamalar teng kuchli, chunki ikkinchi tenglama $11x+11=0$ tenglamaga teng kuchli bo'lib, umumiy yechim $x=-1$ dan iborat.

1-§. CHIZIQLI TENGLAMALAR

Yig'indi, ayirma, ko'paytma va bo'linma hadlarining nomlanishini ko'rib o'taylik.

1. ***Yig'indi*: A + B = C**

A – *qo'shiluvchi*, **B** – *qo'shiluvchi*, **C** – *yig'indi*

5+7=12

5 – qo'shiluvchi, 7 – qo'shiluvchi, 12 – yig'indi.

2. ***Ayirma*: A – B = C**

A – *kamayuvchi*, **B** – *ayriluvchi*, **C** – *ayirma*

24 – 11=13

24 – kamayuvchi, 11 – ayriluvchi, 13 – ayirma.

3. ***Ko'paytma*: A · B = C**

A – *ko'paytuvchi* (*kop'ayuvchi*), **B** – *ko'paytuvchi* (*ko'paytiruvchi*), **C** – *ko'paytma*

7·9=63

7 – ko'paytuvchi (kop'ayuvchi), 9 – ko'paytuvchi (ko'paytiruvchi), 63 – ko'paytma.

4. ***Bo'linma*: A : B = C**

A – *bo'linuvchi*, **B** – *bo'luvchi*, **C** – *bo'linma*

48:6=8

48 – bo'linuvchi, 6 – bo'luvchi, 8 – bo'linma.

1. Sodda tenglamalar

Qoida. *Noma'lum qo'shiluvchini topish uchun, yig'indidan ma'lum qo'shiluvchini ayirish kerak*:

$$X + A = B \qquad A + X = B$$

$$X = B - A \qquad X = B - A$$

1-misol. Tenglamalarni yeching: 1) $x + 9 = 28$

2) $13 + x = 49$.

Yechish: 1) $x+9=28 \Rightarrow x=28-9 \Rightarrow x=19$.

Tekshirish: $19+9=28$; Javob: $x=19$.

2) $13+x=49 \Rightarrow x=49-13 \Rightarrow x=36$.

Tekshirish: $13+36=49$; Javob: $x=36$.

Qoida. *Noma'lum kamayuvchini topish uchun ayirmaga ayriluvchini qo'shish kerak*:

$$X-A=B$$
$$X=B+A$$

2-misol. $y-6=15$ tenglamani yeching.

Yechish: $y-6=15 \Rightarrow y=15+6 \Rightarrow y=21$.

Tekshirish: $21-6=15$ Javob: $y=21$.

Qoida. *Noma'lum ayriluvchini topish uchun kamayuvchidan ayirmani ayirish kerak*:

$$A-X=B$$
$$X=A-B$$

3-misol. $75-z=25$ tenglamani yeching.

Yechish: $75-z=25 \Rightarrow z=75-25 \Rightarrow z=50$.

Tekshirish: $75-50=25$; Javob: $z=50$.

Qoida. *Noma'lum ko'paytuvchini topish uchun ko'paytmani ma'lum ko'paytuvchiga bo'lish kerak*:

$$A\cdot X=B \qquad X\cdot A=B$$
$$X=B:A \qquad X=B:A$$

4-misol. 1) $4\cdot a=48$ 2) $z\cdot 7=98$ tenglamalarni yeching.

Yechish: 1) $4\cdot a=48 \Rightarrow a=48:4 \Rightarrow a=12$.

Tekshirish: $4\cdot 12=48$; Javob: $a=12$

2) $z\cdot 7=98 \Rightarrow z=98:7 \Rightarrow z=14$.

Tekshirish: $14\cdot 7=98$; Javob: $z=14$.

Qoida. *Noma'lum bo'linuvchini topish uchun bo'linmaga bo'luvchini ko'paytirish kerak*:

$$X:A=B$$
$$X=B\cdot A$$

5-misol. $s:8=18$ tenglamani yeching.

Yechish: $s:8=18 \Rightarrow s=18\cdot 8 \Rightarrow s=144$.

Tekshirish : $144:8=18$; Javob : $s=144$.

Qoida. *No'malum bo'luvchini topish uchun bo'linuvchini bo'linmaga bo'lish kerak*:

$$A:X=B$$
$$X=A:B$$

6-misol. $68:x=17$ tenglamani yeching.

Yechish: $68:x=17 \Rightarrow x=68:17 \Rightarrow x=4$.

Tekshirish : $68:4=17$; Javob : $x=4$.

7-misol. $(2520:(480-3000:x)+48)\cdot 8=576$ tenglamani yeching.

$A)$ 4 $B)$ 8 $C)$ 12 $D)$ 16

Yechish:

$$(2520:(480-3000:x)+48)\cdot 8=576$$
$$2520:(480-3000:x)+48=576:8$$
$$2520:(480-3000:x)=72-48$$
$$480-3000:x=2520:24$$
$$3000:x=480-105$$
$$x=3000:375$$
$$x=8.$$

Javob: $B)$ 8.

8-misol. x ni toping: $420:(160-1000:x)=12$.

$A)$ 8 $B)$ $\frac{1}{8}$ $C)$ 35 $D)$ 36 $E)$ -8

Yechish:

$$420:(160-1000:x)=12$$
$$160-1000:x=420:12$$
$$160-1000:x=35$$
$$1000:x=160-35$$
$$1000:x=125$$
$$x=1000:125$$
$$x=8$$

Javob: $A)$ 8.

9-misol. $((8600-325\cdot(576:x))\cdot 42):165-220=480$ tenglamani yeching.

$A)\ 16$ $B)\ 8$ $C)\ 64$ $D)\ 32$

Yechish: $((8600-325\cdot(576:x))\cdot 42):165-220=480$

$$((8600-325\cdot(576:x))\cdot 42):165=480+220$$
$$(8600-325\cdot(576:x))\cdot 42=700\cdot 165$$
$$8600-325\cdot(576:x)=115500:42$$
$$325\cdot(576:x)=8600-2750$$
$$576:x=5850:325$$
$$x=576:18$$
$$x=32.$$

Javob: $D)\ 32$.

Mustaqil yechish uchun misollar

1.1-misol. Quyidagi tenglamalarni yeching.

1. $z:291+5197=6490$ **2.** $10000-3105:y=9931$

3. $4515-x\cdot 228=2007$ **4.** $900000-7\cdot a=857643$

5. $(47843-3\cdot a):4=6824$ **6.** $(64-10x):4+11=22$

7. $(12+34x)\cdot 56-789=18923$ **8.** $24960:\left(3360-\frac{300\cdot(200-6x)}{115}\right)=8$

9. $4520:\left(225-4209520:\frac{1000795+(250+x)\cdot 50}{27}\right)=40$.

1.2-misol. Test topshiriqlarini bajaring.

1. x ni toping: $(360+x)\cdot 1002=731460$.

$A)\ 370$ $B)\ 270$ $C)\ 470$ $D)\ 730$ $E)\ 1090$

2. $(10000-3333x)\cdot 10000-9999=1$ tenglamani yeching.

$A)\ 2$ $B)\ 4$ $C)\ 3$ $D)\ 1$

3. $((56\cdot(666+x)+12600):40-700)\cdot 24=21000$ tenglamani yeching.

$A)\ 134$ $B)\ 234$ $C)\ 184$ $D)\ 334$

2. Chiziqli tenglamalar

Ta'rif. $ax = b\ (a \neq 0)$ *ko'rinishidagi tenglamalar chiziqli tenglamalar deyiladi.*

Chiziqli tenglamalarni yechishning uchta holi mavjud:

1-hol. $a \neq 0$ *bo'lsa, tenglama yagona yechimga ega:* $x = \frac{b}{a}$.

2-hol. $a = 0$, $b = 0$ *bo'lsa, tenglama* $0 \cdot x = 0$ *ko'rinishda bo'lib, cheksiz ko'p yechimga ega bo'ladi.*

3-hol. $a = 0$, $b \neq 0$ *bo'lsa, tenglama* $0 \cdot x = b, b \neq 0$ *ko'rinishda bo'lib, tenglama ildizga ega emas.*

1-misol. $4{,}5 - 1{,}6 \cdot (5x - 3) = 1{,}2 \cdot (4x - 1) - 15{,}1$ tenglamani yeching.

$A)$ 20 $B)$ 2 $C)$ 0,2 $D)$ 0,5

$E)$ To'g'ri javob keltirilmagan

Yechish: Qavslarni ochib, o'xshash hadlarni ixchamlaymiz:

$$4{,}5 - 8x + 4{,}8 = 4{,}8x - 1{,}2 - 15{,}1$$

$$9{,}3 - 8x = 4{,}8x - 16{,}3$$

$$-8x - 4{,}8x = -16{,}3 - 9{,}3$$

$$-12{,}8x = -25{,}6$$

$$x = \frac{25{,}6}{12{,}8} = 2$$

Javob: $B)\ 2$.

2-misol. Tenglamani yeching: $\frac{3x - 11}{4} - \frac{3 - 5x}{8} = \frac{x + 6}{2}$.

$A)\ 5$ $B)\ -4{,}5$ $C)\ 6{,}5$ $D)\ 7$ $E)\ 8$

Yechish: Tenglamaning ikkala tomonini unda qatnashgan kasrlarning umumiy maxrajiga ko'paytiramiz:

$$8\cdot\frac{3x-11}{4}-8\cdot\frac{3-5x}{8}=8\cdot\frac{x+6}{2}$$

$$2\cdot(3x-11)-(3-5x)=4\cdot(x+6)$$

$$6x-22-3+5x=4x+24$$

$$11x-25=4x+24$$

$$11x-4x=24+25$$

$$7x=49$$

$$x=7$$

Javob: $D)\ 7$.

3-misol. Tenglamani yeching: $\frac{x-3}{6}+x=\frac{2x-1}{3}-\frac{4-x}{2}$.

$A)\ 3$ $B)\ 2$ $C)\ -2$ $D)\ -4$ $E)\ \varnothing$

Yechish: Tenglamaning ikkala tomonini undagi kasrlarning umumiy maxraji 6 ga koʻpaytiramiz:

$$6\cdot\frac{x-3}{6}+6x=6\cdot\frac{2x-1}{3}-6\cdot\frac{4-x}{2}$$

$$x-3+6x=2(2x-1)-3(4-x)$$

Qavslarni ochamiz va oʻzgaruvchilar qatnashgan hadlarni tenglamaning chap tomoniga, sonli hadlarni oʻng tomoniga oʻtkazamiz:

$$7x-3=4x-2-12+3x$$

$$7x-3=7x-14$$

$$7x-7x=-14+3$$

$$0\cdot x=-11 \text{ yoki } 0=-11$$

Demak, notoʻgʻri tenglik hosil boʻldi. Bu tenglama yuqoridagi 3-holga keladi. Bu esa berilgan tenglama ildizga ega emasligini bildiradi.

Javob: E) $\varnothing$.

4-misol. $\frac{2x+3}{2}+\frac{2-3x}{3}=2{,}1(6)$ tenglamani yeching.

$A)\ \varnothing$ $B)\ 2$ $C)\ -2$ $D)\ -\frac{1}{2}$

E) cheksiz koʻp yechimga ega

Yechish: Davriy kasrni oddiy kasrga aylantiramiz:

$$2,1(6)=2\frac{16-1}{90}=2\frac{15}{90}=2\frac{1}{6}=\frac{13}{6}.$$

$\frac{2x+3}{2}+\frac{2-3x}{3}=\frac{13}{6}$ tenglamaning ikkala tomonini umumiy maxrajiga ko'paytiramiz:

$$6\cdot\frac{2x+3}{2}+6\cdot\frac{2-3x}{3}=6\cdot\frac{13}{6}$$

$$3(2x+3)+2(2-3x)=13$$

$$6x+9+4-6x=13$$

$$6x-6x=13-13$$

$$0\cdot x=0 \quad \text{yoki} \quad 0=0$$

To'g'ri tenglik hosil bo'ldi. Yuqoridagi 2-holga asosan tenglama cheksiz ko'p yechimga ega.

Javob: E) cheksiz ko'p yechimga ega.

5-misol. Tenglamani yeching: $\left(x+2\frac{22}{25}\right):7\frac{1}{3}=3$.

$A)\ 20\frac{22}{25}$ $B)\ 19\frac{22}{25}$ $C)\ 19\frac{3}{25}$ $D)\ 18\frac{3}{25}$ $E)\ 18\frac{28}{75}$

Yechish: $\left(x+2\frac{22}{25}\right):7\frac{1}{3}=3\Rightarrow x+2\frac{22}{25}=3\cdot7\frac{1}{3}\Rightarrow x+2\frac{22}{25}=3\cdot\frac{22}{3}\Rightarrow$

$\Rightarrow x+2\frac{22}{25}=22\Rightarrow x=22-2\frac{22}{25}=19\frac{3}{25}.$

Javob: $C)\ 19\frac{3}{25}$.

6-misol. Tenglamani yeching: $\frac{0,(3)+0,1(6)}{0,(319)+1,(680)}\cdot x=8^{0,(6)}$

$A)\ 4$ $B)\ 32$ $C)\ 2$ $D)\ 1$ $E)\ 16$

Yechish: Tenglamadagi davriy kasrlarni oddiy kasrga aylantiramiz va kasrni sodda holga keltiramiz:

$0,(3)=\frac{3}{9}=\frac{1}{3}$; $0,1(6)=\frac{16-1}{90}=\frac{15}{90}=\frac{1}{6}$; $0,(319)=\frac{319}{999}$;

$1,(680)=1\frac{680}{999}$; $0,(6)=\frac{6}{9}=\frac{2}{3}$; $8^{0,(6)}=8^{\frac{2}{3}}=\sqrt[3]{8^2}=\sqrt[3]{64}=4$;

1) $\frac{1}{3}+\frac{1}{6}=\frac{2+1}{6}=\frac{3}{6}=\frac{1}{2}$ 2) $\frac{319}{999}+1\frac{680}{999}=1+\frac{319+680}{999}=2$

3) $\frac{1}{2}:2=\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{4}$.

Bu almashtirishlardan keyin tenglama sodda holga keladi.

$$\frac{1}{4}\cdot x=4\Rightarrow x=4:\frac{1}{4}=4\cdot 4=16.$$

Javob: $E)16$.

7-misol. Tenglamani yeching: $\dfrac{(x-12):\frac{3}{8}}{0{,}3\cdot 3\frac{1}{3}+7}=1$.

$A)$ 25 $B)$ 14 $C)$ 15 $D)$ 16 $E)$ 18

Yechish: Tenglamadagi kasrlarni soddalashtiramiz:

$$0{,}3\cdot 3\frac{1}{3}+7=\frac{3}{10}\cdot\frac{10}{3}+7=1+7=8.$$

Tenglama quyidagi ko'rinishga keladi:

$$\frac{(x-12):\frac{3}{8}}{8}=1\Rightarrow(x-12):\frac{3}{8}=8\Rightarrow x-12=8\cdot\frac{3}{8}\Rightarrow x-12=3\Rightarrow x=15.$$

Javob: $C)15$.

8-misol. Tenglamani yeching: $\left(4\frac{3}{8}x+5\frac{1}{16}\right)\cdot\frac{4}{15}=\frac{5}{12}x+2\frac{2}{5}$.

$A)\ \frac{1}{15}$ $B)\ 1\frac{2}{5}$ $C)\ \frac{3}{185}$ $D)\ 2\frac{1}{5}$ $E)\ \frac{7}{15}$

Yechish:

$$\left(4\frac{3}{8}x+5\frac{1}{16}\right)\cdot\frac{4}{15}=\frac{5}{12}x+2\frac{2}{5}$$

$$\frac{35}{8}x\cdot\frac{4}{15}+\frac{81}{16}\cdot\frac{4}{15}=\frac{5}{12}x+2\frac{2}{5}$$

$$\frac{7}{6}x+\frac{27}{20}=\frac{5}{12}x+2\frac{2}{5}$$

$$\left(\frac{7}{6}-\frac{5}{12}\right)x=2\frac{2}{5}-1\frac{7}{20}$$

$$\frac{3}{4}x=1\frac{1}{20}$$

$$x=1\frac{1}{20}:\frac{3}{4}=\frac{21}{20}\cdot\frac{4}{3}=\frac{7}{5}=1\frac{2}{5}$$

Javob: $B)\,1\frac{2}{5}$.

9-misol. $\left(1-\frac{1}{5^2}\right)\left(1-\frac{1}{6^2}\right)\cdots\left(1-\frac{1}{14^2}\right)\cdot(x-1)=\frac{3}{7}$ tenglamani yeching.

$A)\ 2$ $B)\ 1$ $C)\ 1{,}5$ $D)\ 0{,}5$

Yechish:

$$\left(1-\frac{1}{5^2}\right)\left(1-\frac{1}{6^2}\right)\cdots\left(1-\frac{1}{14^2}\right)=\left(1-\frac{1}{5}\right)\left(1+\frac{1}{5}\right)\cdot$$

$$\cdot\left(1-\frac{1}{6}\right)\left(1+\frac{1}{6}\right)\cdots\left(1-\frac{1}{13}\right)\left(1+\frac{1}{13}\right)\cdot\left(1-\frac{1}{14}\right)\left(1+\frac{1}{14}\right)=$$

$$=\frac{4}{5}\cdot\frac{6}{5}\cdot\frac{5}{6}\cdot\frac{7}{6}\cdot\ldots\cdot\frac{12}{13}\cdot\frac{14}{13}\cdot\frac{13}{14}\cdot\frac{15}{14}=\frac{4}{5}\cdot\frac{15}{14}=\frac{6}{7}.$$

$$\frac{6}{7}(x-1)=\frac{3}{7}\Rightarrow x-1=\frac{3}{7}\cdot\frac{7}{6}\Rightarrow x-1=\frac{1}{2}\Rightarrow x=1{,}5.$$

Javob: $C)\,1{,}5$.

10-misol. $0{,}35(0{,}35x-1)-0{,}45(0{,}45x-2)=0{,}55(0{,}55x-3)-$ $-0{,}65(0{,}65x-4)$ tenglamani yeching.

$A)\ 10$ $B)\ 100$ $C)\ 110$ $D)\ 1000$

Yechish: Tenglamaning ikkala tomonini 10000 ga ko'paytiramiz:

$$35(35x-100)-45(45x-200)=55(55x-300)-65(65x-400)$$

$$1225x-3500-2025x+9000=3025x-16500-4225x+26000$$

$$-800x+5500=-1200x+9500$$

$$400x=4000$$

$$x=10$$

Javob: $A)\,10$.

11-misol. $\frac{x-a}{bc}+\frac{x-b}{ac}+\frac{x-c}{ab}=2\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ tenglamani yeching: ($abc\neq 0$).

$A)\ abc$ $B)\ a-b+c$ $C)\ a-b-c$ $D)\ a+b+c$

Yechish:

$$\frac{x-a}{bc}+\frac{x-b}{ac}+\frac{x-c}{ab}=2\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$

$$\frac{x}{bc}-\frac{a}{bc}+\frac{x}{ac}-\frac{b}{ac}+\frac{x}{ab}-\frac{c}{ab}=2\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$

$$\left(\frac{1}{bc}+\frac{1}{ac}+\frac{1}{ab}\right)x=2\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)+\frac{a}{bc}+\frac{b}{ac}+\frac{c}{ab}$$

$$\frac{a+b+c}{abc}x=\frac{2ab+2ac+2bc+a^2+b^2+c^2}{abc}$$

$$\frac{a+b+c}{abc}x=\frac{(a+b+c)^2}{abc}$$

$$x=\frac{(a+b+c)^2}{abc}\cdot\frac{abc}{a+b+c}=a+b+c$$

Javob: *D*) $a+b+c$.

Mustaqil yechish uchun misollar

2.1-misol. Quyidagi tenglamalarni yeching.

1. $5(5x-1)-2{,}7x+0{,}2x=6{,}5-0{,}5x$

2. $4(3x+2)-7(x+1)=3(x-1)$

3. $5-3(x-2(x-2(x-2)))=2$

4. $6(1{,}2x-0{,}5)-1{,}3x=5{,}9x-3$

5. $2x+3-6(x-1)=4(1-x)+5$

6. $8(1{,}3x+0{,}25)-6{,}6x=3{,}8x+2$

7. $5(1{,}14+1{,}44-0{,}171x)=0{,}05(319{,}77+69x)$

8. $4x+(5x-(6x-(7x-(8x-9))))=10$

9. $\frac{1}{9}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}(x+2)+4\right)+6\right)+8\right)=1$

10. $11\frac{2}{3}x-5\frac{1}{6}=3\frac{3}{4}+2\frac{3}{4}x$

11. $12\frac{3}{4}+\frac{3}{7}y=\frac{y}{2}-10\frac{1}{28}$

12. $\frac{3x-7}{4}-\frac{9x+11}{8}=\frac{3-x}{2}$

13. $\frac{9x-5}{2}-\frac{3+5x}{3}-\frac{8x-2}{4}=2$

14. $\frac{x-1}{3}+\frac{5x+2}{12}=\frac{5+3x}{4}$

15. $\frac{2x+1}{3}-\frac{7x+5}{15}=\frac{x-2}{5}$

16. $\frac{x}{2}+\frac{x}{6}+\frac{x}{12}+\frac{x}{20}+\frac{x}{30}+\frac{x}{42}=-6$.

2.2-misol. Test topshiriqlari.

1. Tenglamani yeching: $6{,}4\cdot(2-3x)=6\cdot(0{,}8x-1)+6{,}8$.

A) 5 *B*) −0,5 *C*) 0,5 *D*) −2 *E*) 2,5

2. Tenglamani yeching: $0{,}9(4x-2)=0{,}5(3x-4)+4{,}4$.

A) 1,2 *B*) 2,5 *C*) −3 *D*) 2 *E*) 0,2

3. Tenglamani yeching: $0{,}2\cdot(5y-2)=0{,}3\cdot(2y-1)-0{,}9$.

A) 2 *B*) 0,2 *C*) −2 *D*) −1,2 *E*) $2\frac{1}{2}$

4. Tenglamani yeching: $2{,}8x-3(2x-1)=2{,}8-3{,}19x$.

A) −20 *B*) 20 *C*) −2 *D*) 200 *E*) 0,2

5. Tenglamani yeching: $6-\frac{x-1}{2}=\frac{3-x}{2}+\frac{x-2}{3}$.

A) 4,5 *B*) 8 *C*) 17 *D*) 11 *E*) 14

6. $\frac{x}{3}-\frac{x+8}{6}=\frac{3x+2}{9}-\frac{x+11}{6}$ tenglamani yeching.

A) −5 *B*) 5 *C*) ∅ *D*) −4 *E*) cheksiz ko‘p ildizga ega

7. $\frac{3x-2}{4}+\frac{2x+3}{2}-2{,}5x+2=0$ tenglamani yeching.

A) ∅ *B*) 4 *C*) 10 *D*) −10 *E*) yechimlari cheksiz ko‘p

8. Tenglamani yeching: $\left(18\frac{1}{3}+x\right):3\frac{1}{7}=7$.

$A)\ 4\frac{1}{3}$ $B)\ 3\frac{2}{3}$ $C)\ 3\frac{1}{3}$ $D)\ 5\frac{2}{3}$ $E)\ 4\frac{2}{3}$

9. Tenglamani yeching: $\left(x+3\frac{2}{9}\right):4\frac{1}{6}=6.$

$A)\ 22\frac{2}{9}$ $B)\ 21\frac{7}{9}$ $C)\ 22\frac{1}{3}$ $D)\ 20\frac{4}{9}$ $E)\ 21\frac{5}{6}$

10. $\frac{0,1(6)+0,(6)}{0,(3)+1,1(6)}(x+1)=0,3(8)x$ tenglamani yeching.

$A)\ 2,(6)$ $B)\ -2,(6)$ $C)\ 3,(6)$ $D)\ -3,(6)$ $E)\ -3,(3)$

11. Tenglamani yeching: $\frac{\left(2x+6\frac{6}{13}\right)}{3}=4\frac{1}{3}.$

$A)\ 3\frac{3}{13}$ $B)\ 3\frac{19}{13}$ $C)\ 3\frac{7}{26}$ $D)\ 4\frac{3}{13}$ $E)\ 4\frac{7}{26}$

12. $12\cdot\left(1\frac{3}{4}x+\frac{5}{8}\right)=-6\frac{1}{2}$ tenglamani yeching.

$A)\ -\frac{1}{3}$ $B)\ -\frac{2}{3}$ $C)\ \frac{2}{3}$ $D)\ -\frac{13}{21}$ $E)\ \frac{3}{4}$

13. $0,2(x-1)+0,5(3x-9)=0,(3)x-2$ tenglamani yeching.

$A)\ \frac{1}{3}$ $B)\ \frac{81}{41}$ $C)\ \frac{85}{41}$ $D)\ \frac{80}{41}$

14. $\frac{9}{19}-\frac{n}{11}=\frac{6}{11}-\frac{10}{19}$ tenglamadan n ni toping.

$A)\ 15$ $B)\ -5$ $C)\ 27$ $D)\ 5$

15. $\frac{8-a}{6}+\frac{5-4a}{3}=\frac{a+6}{2};\ a=?$

$A)\ 2$ $B)\ -1$ $C)\ 0$ $D)\ 3$

16. $(x-a-b)ab+(x-b-c)bc+(x-c-a)ac=3abc$ tenglamani yeching.

$A)\ a-b+c$ $B)\ a+b+c$

$C)\ b+c-a$ $D)\ a+b$

3. Proporsiyaga doir tenglamalar

Ta'rif. *Ikki nisbatning tengligiga proporsiya deyiladi:*

$$\frac{a}{b}=\frac{c}{d} \text{ yoki } a:b=c:d$$

bunda $b\neq 0$, $d\neq 0$.

Bu yerda *a* va *d* lar proporsiyaning *chetki hadlari*, *b* va *c* lar proporsiyaning *o'rta hadlari* deyiladi.

1-xossa(asosiy xossa). Proporsiyaning chetki hadlari ko'paytmasi o'rta hadlari ko'paytmasiga teng: $a\cdot d=b\cdot c$.

2-xossa. $\frac{a}{b}=\frac{c}{d}\Rightarrow\left[\frac{a}{c}=\frac{b}{d};\ \frac{d}{c}=\frac{b}{a};\ \frac{d}{b}=\frac{c}{a}.\right.$

3-xossa. $\frac{a}{b}=\frac{c}{d}\Rightarrow\left[\frac{am}{c}=\frac{bm}{d};\ \frac{a}{cn}=\frac{b}{dn};\right.\quad m,n\neq 0.$

$\frac{a}{b}=\frac{c}{d}$ proporsiyadan quyidagi hosilaviy proporsiyalarni hosil qilamiz:

1. $\frac{a+b}{b}=\frac{c+d}{d}$ 2. $\frac{a-b}{b}=\frac{c-d}{d}$ 3. $\frac{a}{a+b}=\frac{c}{c+d}$

4. $\frac{a}{a-b}=\frac{c}{c-d}$ 5. $\frac{a+b}{a-b}=\frac{c+d}{c-d}$

1-misol. Tenglamani yeching: $6{,}9:4{,}6=x:5{,}4$.

A) 7,1 *B*) 7,7 *C*) 8,1 *D*) 8,4 *E*) 9,2

Yechish: Proporsiyaning 1-xossasiga asosan $4{,}6\cdot x=5{,}4\cdot 6{,}9$ bo'ladi.

Bundan

$$x=\frac{5{,}4\cdot 6{,}9}{4{,}6}=\frac{5{,}4\cdot 3\cdot 2{,}3}{2\cdot 2{,}3}=2{,}7\cdot 3=8{,}1.$$

Javob: *C*)8,1.

2-misol. $12\frac{1}{2}:2\frac{1}{2}=16\frac{2}{3}:y$ tenglamani yeching.

A) $3\frac{1}{3}$ *B*) $3\frac{2}{3}$ *C*) $3\frac{1}{6}$ *D*) $3\frac{5}{6}$ *E*) $3\frac{1}{9}$

Yechish: Asosiy xossaga binoan $12\frac{1}{2}\cdot y=16\frac{2}{3}\cdot 2\frac{1}{2}$ ni hosil qilamiz. Natijada

$$y=16\frac{2}{3}\cdot 2\frac{1}{2}:12\frac{1}{2}=\frac{50}{3}\cdot\frac{5}{2}\cdot\frac{2}{25}=\frac{10}{3}=3\frac{1}{3}.$$

Javob: A) $3\frac{1}{3}$.

3-misol. Tenglamani yeching: $\left(\frac{1}{3}+x\right):7=\left(\frac{3}{4}+x\right):9$.

A) $1\frac{3}{8}$ *B*) $1\frac{1}{8}$ *C*) $1\frac{5}{8}$ *D*) $1\frac{7}{8}$ *E*) $1\frac{1}{4}$

Yechish: Asosiy xossadan foydalanamiz va chiziqli tenglama hosil qilib, uni yechamiz:

$$9\cdot\left(\frac{1}{3}+x\right)=7\cdot\left(\frac{3}{4}+x\right)\Rightarrow 9\cdot\frac{1}{3}+9x=7\cdot\frac{3}{4}+7x\Rightarrow 3+9x=5\frac{1}{4}+7x\Rightarrow$$

$$\Rightarrow 9x-7x=5\frac{1}{4}-3\Rightarrow 2x=2\frac{1}{4}\Rightarrow x=2\frac{1}{4}:2=\frac{9}{4}\cdot\frac{1}{2}=\frac{9}{8}=1\frac{1}{8}.$$

Javob: *B*) $1\frac{1}{8}$.

4-misol. Agar $a-2b;4;a+3b;24$ sonlar proporsiyaning ketma-ket hadlari bo‘lsa, $\frac{a^2-b^2}{2ab}$ ifodaning qiymatini toping.

A) $\frac{4}{3}$ *B*) 2 *C*) 3 *D*) $\frac{8}{3}$ *E*) $\frac{7}{2}$

Yechish: Masala shartidan $(a-2b):4=(a+3b):24$ tenglikni hosil qilamiz va proporsiyaning asosiy xossasidan foydalanamiz.

$$24\cdot(a-2b)=4\cdot(a+3b)\Rightarrow 6(a-2b)=a+3b\Rightarrow 6a-12b=$$
$$=a+3b\Rightarrow 4a=12b\Rightarrow a=3b.$$

a ning qiymatini yuqoridagi ifodaga qo‘yamiz:

$$\frac{a^2-b^2}{2ab}=\frac{(3b)^2-b^2}{2\cdot 3b\cdot b}=\frac{9b^2-b^2}{6b^2}=\frac{8b^2}{6b^2}=\frac{4}{3}.$$

Javob: *A*) $\frac{4}{3}$.

5-misol. Proporsiyadan x ni toping: $13\frac{1}{3}:1\frac{1}{3}=26:(0{,}2x)$.

Yechish:

$$13\frac{1}{3}\cdot 0{,}2x=26\cdot 1\frac{1}{3}\Rightarrow x=26\cdot 1\frac{1}{3}:\left(13\frac{1}{3}\cdot 0{,}2\right)=26\cdot\frac{4}{3}\cdot\frac{3}{40}\cdot 5=13.$$

Javob: 13.

6-masala. Kumush va misdan iborat ikkita qotishma berilgan. Birinchi qotishmada kumush va misning miqdori 1:2 nisbatda, ikkinchi qotishmada 2:3 kabi. Agar yangi hosil qilingan qotishmada kumush va misning miqdori 17:27 bo'lsa, birinchi va ikkinchi qotishmalardan qanday nisbatda olish kerak?

Yechish: x orqali yangi qotishmadagi birinchi qotishmaning miqdorini, y orqali ikkinchi qotishmaning miqdorini belgilaymiz. U holda yangi qotishmada $\frac{x}{3}+\frac{2y}{5}$ miqdor kumush, $\frac{2x}{3}+\frac{3y}{5}$ miqdor mis bo'ladi. Bundan quyidagi tenglikni hosil qilamiz: $\dfrac{\frac{x}{3}+\frac{2y}{5}}{\frac{2x}{3}+\frac{3y}{5}}=\dfrac{17}{27}$

Soddalashtirishlardan keyin

$$\frac{5x+6y}{10x+9y}=\frac{17}{27}$$

$$27\cdot(5x+6y)=17\cdot(10x+9y)$$

$$135x+162y=170x+153y$$

$$35x=9y$$

$$\frac{x}{y}=\frac{9}{35}$$

Javob: Birinchi qotishmadan 9 miqdor, ikkinchi qotishmadan 35 miqdor olish kerak (9:35).

Mustaqil yechish uchun misollar

3.1-misol. Proporsiyaga oid tenglamalarni yeching.

1. Tenglamani yeching: $3{,}5 : x = 0{,}8 : 2{,}4$.

A) 10,5 *B*) 9,2 *C*) 13,5 *D*) 7,8 *E*) 11,5

2. Tenglamani yeching: $5{,}4 : 2{,}4 = x : 1{,}6$.

A) 3,6 *B*) 4 *C*) 2,8 *D*) 4,6 *E*) 3,9

3. Tenglamani yeching: $0{,}25 : 1{,}4 = 0{,}75 : x$.

A) 3,6 *B*) 2,4 *C*) 4,2 *D*) 5,2 *E*) 3,4

4. Proporsiyaning noma'lum hadini toping: $2\frac{4}{5} : x = 1\frac{2}{3} : 2\frac{6}{7}$.

A) $\frac{1}{2}$ *B*) $\frac{2}{3}$ *C*) $4\frac{4}{5}$ *D*) $\frac{3}{5}$ *E*) $2\frac{1}{5}$

5. Proporsiyaning noma'lum hadini toping: $3\frac{3}{5} : 2\frac{7}{10} = 3\frac{3}{4} : x$.

A) $2\frac{13}{16}$ *B*) $2\frac{3}{10}$ *C*) $3\frac{1}{3}$ *D*) $1\frac{15}{16}$ *E*) $1\frac{13}{18}$

6. Proporsiyaning noma'lum hadini toping: $5\frac{5}{8} : 7\frac{1}{2} = x : 6\frac{2}{5}$.

A) $4\frac{4}{5}$ *B*) $3\frac{2}{5}$ *C*) $5\frac{1}{8}$ *D*) $4\frac{1}{5}$ *E*) $3\frac{3}{8}$

7. $x : 0{,}75 = 2\frac{3}{8} : 3\frac{9}{16}$ proporsiyaning noma'lum hadini toping.

A) 4 *B*) 2 *C*) 0,25 *D*) 0,5

8. Tenglamani yeching: $\frac{37{,}02}{x+3} = \frac{1{,}234}{10{,}1}$.

A) 303 *B*) 30,3 *C*) 300 *D*) 30

9. Tenglamani yeching: $x : 2{,}0(6) = 0{,}(27) : 0{,}4(09)$.

A) 1,3 *B*) 1,37 *C*) 1,(37) *D*) 1,(32) *E*) 1,3(7)

10. Tenglamani yeching: $(12{,}5 - x) : 5 = (3{,}6 + x) : 6$.

A) $5\frac{2}{11}$ *B*) $5\frac{3}{11}$ *C*) $5\frac{4}{11}$ *D*) $5\frac{1}{11}$ *E*) $5\frac{5}{11}$

11. Proporsiyaning dastlabki uchta hadi yig'indisi 28 ga teng. Uning ikkinchi hadi birinchi hadining $\frac{1}{2}$ qismini, uchinchi hadi esa $\frac{2}{3}$ qismini tashkil etadi. Proporsiyaning oxirgi hadini toping.

A) $4\frac{1}{13}$ *B*) $4\frac{2}{13}$ *C*) $4\frac{3}{13}$ *D*) $4\frac{4}{13}$ *E*) $4\frac{5}{13}$

12. Qotishma kumush va oltindan iborat bo'lib, 3:5 nisbatda. Agar qotishmada 0,45 kilogramm oltin bo'lsa, qotishmaning og'irligini toping.

A) 0,72 *B*) 0,21 *C*) 1,21 *D*) 0,8 *E*) 0,9

13. Ikkita qotishma ikkita metalldan iborat. Birinchi qotishmada metallar miqdori 1:2, ikkinchi qotishmada esa 3:2 nisbatda. Metallar miqdori 8:7 nisbatda bo'lgan yangi qotishma hosil qilish uchun yuqoridagi qotishmalardan qanday nisbatda olish kerak?

A) 2:3 *B*) 1:3 *C*) 3:1 *D*) 3:2 *E*) 2:5

2-§. KVADRAT TENGLAMALAR

1. $x^2 = d$ tenglama

Biz $x^2 = d$ tenglamani ko'rib o'taylik. Bu tenglamani yechish uchun uchta holni ko'ramiz.

1-hol. $x^2 = d, d > 0$ *bo'lsa, ikkita haqiqiy ildizga ega*:

$$x = \pm\sqrt{d} \Rightarrow x_1 = -\sqrt{d};\ x_2 = \sqrt{d}.$$

Isbot. d ni tenglamaning chap qismiga olib o'tamiz: $x^2 = d$. $d > 0$ bo'lganligi uchun arifmetik kvadrat ildizning ta'rifiga ko'ra $d = (\sqrt{d})^2$. Shuning uchun tenglamani quyidagicha yozish mumkin:

$$x^2 - (\sqrt{d})^2 = 0.$$

Bu tenglamaning chap qismini ko'paytuvchilarga ajratib, quyidagilarni hosil qilamiz:

$$(x - \sqrt{d})(x + \sqrt{d}) = 0,$$

bundan, $x_1 = -\sqrt{d}; x_2 = \sqrt{d}$.

1-misol. $x^2 - 4 = 0$ tenglamani yeching.

Yechish: $x^2 - 4 = 0 \Rightarrow x^2 = 4 > 0 \Rightarrow x = \pm\sqrt{4} = \pm 2 \Rightarrow x_1 = -2; x_2 = 2$.

Javob: $\{-2; 2\}$.

2-misol. $(x+5)^2 - 36 = 0$ tenglamani yeching.

Yechish: $(x+5)^2 = 36 > 0 \Rightarrow x + 5 = \pm 6$.

$a)\ x + 5 = -6 \Rightarrow x_1 = -11 \quad b)\ x + 5 = 6 \Rightarrow x_2 = 1$.

Javob: $\{-11; 1\}$.

2-hol. $x^2 = d, d = 0$ *bo'lsa, yagona ildizga ega.*

$$x^2 = 0 \Rightarrow x = 0.$$

3-misol. $3x^2 - 5 = -5$ tenglamani yeching.

Yechish: $3x^2 = -5 + 5 \Rightarrow 3x^2 = 0 \Rightarrow x^2 = 0 \Rightarrow x = 0$.

Javob: $\{0\}$.

3-hol. $x^2 = d, d < 0$ *bo'lsa, haqiqiy ildizga ega emas (Bu holda tenglama ikkita mavhum (kompleks) ildizga ega bo'ladi*).

4-misol. $x^2 + 16 = 0$ tenglamani yeching.

Yechish: $x^2 = -16 < 0 \Rightarrow x \in \varnothing$.

Javob: $\varnothing$.

Mustaqil yechish uchun misollar

1.1-misol. Quyidagi tenglamalarning haqiqiy ildizlarini toping.

1. $x^2 - 49 = 0$	**2.** $x^2 - 121 = 0$	**3.** $x^2 - 8 = 0$
4. $x^2 + 25 = 0$	**5.** $y^2 + 81 = 0$	**6.** $x^2 = \frac{9}{16}$
7. $x^2 = 2\frac{1}{4}$	**8.** $x^2 = 41$	**9.** $\frac{x^2}{12} = 0$
10. $(a-4)^2 - 64 = 0$	**11.** $(x+8)^2 = 225$	**12.** $\left(x - \frac{1}{2}\right)^2 - \frac{9}{16} = 0$.

2. Kvadrat tenglamalar

Ta'rif. $ax^2 + bx + c = 0$ $(a \neq 0)$ *ko'rinishidagi tenglamalar kvadrat tenglama deyiladi. Bu yerda* a , b, c – *berilgan sonlar,* x *esa noma'lum.*

Odatda a – *birinchi yoki bosh koeffitsiyent*, b – *ikkinchi koeffitsiyent*, c – *ozod had* deb ataladi.

Kvadrat tenglamani to'la kvadratga ajratish orqali ildizlarini topish formulasini keltirib chiqaramiz. Umumiy ko'rinishdagi $ax^2 + bx + c = 0$ kvadrat tenglamani qaraylik, bunda $a \neq 0$.

Tenglamaning ikkala qismini a ga bo'lamiz: $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$.

Bu tenglamaning shaklini shunday almashtiramizki, uning chap qismida ikkihadning to‘la kvadrati hosil bo‘lsin:

$$x^2+\frac{b}{a}x=-\frac{c}{a}$$

$$x^2+2\cdot\frac{b}{2a}\cdot x+\left(\frac{b}{2a}\right)^2=-\frac{c}{a}+\left(\frac{b}{2a}\right)^2$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$$

Agar $b^2-4ac\geq 0$ bo‘lsa, u holda

$$\left(x+\frac{b}{2a}\right)^2=\left(\frac{\sqrt{b^2-4ac}}{2a}\right)^2.$$

Bundan

$$x+\frac{b}{2a}=\pm\frac{\sqrt{b^2-4ac}}{2a}\Rightarrow x=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}\Rightarrow x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Kvadrat tenglamaning umumiy formulasida **diskriminant** $D=b^2-4ac$ deb belgilash kiritiladi va

$$x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-b\pm\sqrt{D}}{2a}$$

formulani hosil qilamiz. Bunda uchta hol bo‘lishi mumkin.

1-hol. Agar $D>0$ bo‘lsa, kvadrat tenglama ikkita haqiqiy ildizga ega:

$$x_{1,2}=\frac{-b\pm\sqrt{D}}{2a}.$$

2-hol. Agar $D=0$ bo‘lsa, kvadrat tenglama yagona ildizga ega:

$$x_1=x_2=-\frac{b}{2a}.$$

3-hol. Agar $D<0$ bo‘lsa, kvadrat tenglama haqiqiy ildizga ega emas, bu holda ikkita kompleks ildizga ega bo‘ladi.

1-misol. $2x^2+5x+2=0$ tenglamani yeching.

Yechish: $a=2;\ b=5;\ c=2$ va $D=b^2-4ac=5^2-4\cdot 2\cdot 2=$ $=25-16=9>0$.

Demak, tenglama ikkita haqiqiy ildizga ega.

$$x_{1,2}=\frac{-b\pm\sqrt{D}}{2a}=\frac{-5\pm\sqrt{9}}{2\cdot 2}=\frac{-5\pm 3}{4}\Rightarrow x_1=\frac{-5-3}{4}=-2;$$

$$x_2=\frac{-5+3}{4}=-\frac{1}{2}=-0,5.$$

Javob: $\{-2;-0,5\}$.

2-misol. $-2x^2+x+1=0$ tenglamani yeching.

Yechish: $a=-2;\ b=1;\ c=1,\ D=1^2-4\cdot(-2)\cdot 1=1+8=9>0.$

$$x_{1,2}=\frac{-1\pm\sqrt{9}}{2\cdot(-2)}=\frac{-1\pm 3}{-4}\Rightarrow x_1=\frac{-1-3}{-4}=1;\ x_2=\frac{-1+3}{-4}=-0,5.$$

Javob: $\{-0,5;1\}$.

3-misol. $36x^2+12x+1=0$ tenglamani yeching.

Yechish: $a=36;\ b=12;\ c=1,\ D=12^2-4\cdot 1\cdot 36=144-144=0$ yagona ildizga ega.

$$x_1=x_2=-\frac{b}{2a}=-\frac{12}{2\cdot 36}=-\frac{1}{6}.$$

Yuqoridagi tenglamani ikkihad yig'indisining kvadrati ko'rinishiga keltirib, sodda holda yechish ham mumkin:

$$36x^2+12x+1=(6x)^2+2\cdot 6x\cdot 1+1^2=(6x+1)^2=0\Rightarrow 6x+1=0\Rightarrow x=-\frac{1}{6}$$

Javob: $x_1=x_2=-\frac{1}{6}$.

Bu misolni kvadrat tenglamaning umumiy formulasi yordamida yechsak ham shu natija hosil bo'ladi.

4-misol. $x^2-5x+6=0$ tenglamani yeching.

Yechish: $a=1;\ b=-5;\ c=6,\ D=(-5)^2-4\cdot 1\cdot 6=25-24=1$

$$x_{1,2}=\frac{-(-5)\pm\sqrt{1}}{2\cdot 1}=\frac{5\pm 1}{2}\Rightarrow x_1=\frac{5-1}{2}=2;\ x_2=\frac{5+1}{2}=3.$$

Javob: $\{2;3\}$.

5-misol. $x^2-2x+10=0$ tenglamani yeching.

Yechish: $a=1; b=-2; c=10,\ D=(-2)^2-4\cdot 1\cdot 10=4-40=-36<0$ bo'lib, tenglama haqiqiy ildizlarga ega emas.

Javob: $\varnothing$.

6-misol. Tenglamaning nechta ildizi bor?

$$x+6=-\frac{3}{x}.$$

$A)$ 1 $B)$ 2 $C)$ 3 $D)$ ildizi yo'q $E)$ cheksiz ko'p

Yechish: $x=0$ tenglamaning ildizi emasligi ravshan. Tenglamaning ikkala tomonini x ga ko'paytiramiz: $x+6=-\frac{3}{x}$ $/\times x$ va $x^2+6x+3=0$ kvadrat tenglamani hosil qilamiz. Bu kvadrat tenglamaning diskriminantini hisoblaymiz: $D=6^2-4\cdot1\cdot3=24>0$. Bundan kvadrat tenglamaning ikkita haqiqiy ildizga ega ekanligi kelib chiqadi.

Javob: $B)$ 2.

7-misol. Agar $(4x+1)\cdot\left(x-\frac{1}{4}\right)=0$ bo'lsa, $4x+1$ qanday qiymatlar qabul qilishi mumkin?

$A)$ faqat $-\frac{1}{4}$ $B)$ faqat $\frac{1}{4}$

$C)$ faqat 0 $D)$ 0 yoki 2 $E)$ $-\frac{1}{4}$ yoki $\frac{1}{4}$

Yechish: $A\cdot B=0 \Leftrightarrow A=0$ yoki $B=0$ o'rinli. Bundan:

1) $4x+1$ ifodamiz 0 qiymat qabul qiladi. Chunki birinchi qavs yuqoridagi shart bajarilsa 0 ga teng.

2) Ikkinchi qavsdagi ifoda $x-\frac{1}{4}=0$ dan $x=\frac{1}{4}$ qiymatni qabul qiladi. Bu qiymatda $4x+1$ ifoda $4x+1=4\cdot\frac{1}{4}+1=2$ qiymatni qabul qiladi.

Javob: D) 0 yoki 2.

8-misol. $2013x^2-2015x+2=0$ tenglamani yeching.

$A)$ $1; -\frac{2}{2013}$ $B)$ $1; \frac{2015}{2013}$ $C)$ $1; \frac{2}{2013}$ $D)$ $-1; \frac{2}{2013}$

Yechish: Berilgan tenglamani quyidagi ko'rinishda yozib olamiz:

$$x^2-\frac{2015}{2013}x+\frac{2}{2013}=0\Rightarrow x^2-\left(1+\frac{2}{2013}\right)x+\frac{2}{2013}=0\Rightarrow$$

$$\Rightarrow x^2-x-\frac{2}{2013}x+\frac{2}{2013}=0\Rightarrow x(x-1)-\frac{2}{2013}(x-1)=0\Rightarrow$$

$$\Rightarrow(x-1)\left(x-\frac{2}{2013}\right)=0\Rightarrow x_1=1;\ x_2=\frac{2}{2013}.$$

Bu tenglamani Viyet teoremasi yordamida ham yechsa bo‘ladi.

$2013x^2-2015x+2=0$ kvadrat tenglama uchun

$$\begin{cases} x_1+x_2=\dfrac{2015}{2013}=1+\dfrac{2}{2013} \\ x_1+x_2=\dfrac{2}{2013}=1\cdot\dfrac{2}{2013} \end{cases}\Rightarrow x_1=1;\ x_2=\frac{2}{2013}.$$

Javob: $C)\ 1;\ \frac{2}{2013}$.

Xossa. Agar $ax^2+bx+c=0\,(a\neq 0)$ kvadrat tenglamada $a+b+c=0$ bo‘lsa, $x_1=1,\ x_2=\frac{c}{a}$ o‘rinli bo‘ladi.

Bu xossaning isboti sodda. Yuqoridagi 8-misolga xossani qo‘llasak, $x_1=1,\ x_2=\frac{2}{2013}$ larni topamiz.

Mustaqil yechish uchun misollar

2.1-misol. Kvadrat tenglamalarning haqiqiy ildizlarini toping.

1. $2x^2+3x+1=0$ **2.** $2x^2-7x+3=0$

3. $4x^2-11x+6=0$ **4.** $2x^2-7x-4=0$

5. $3x^2+2x-1=0$ **6.** $6x^2-5x-1=0$

7. $x^2+4x+3=0$ **8.** $x^2-8x+7=0$

9. $16x^2+8x+1=0$ **10.** $25x^2-30x+9=0$

11. $x^2+10x+25=0$ **12.** $9x^2-6x+1=0$

13. $x^2+x+\frac{1}{4}=0$ **14.** $3x^2-5x+4=0$

15. $7x^2-6x+2=0$

16. $3x^2-x+2=0$

17. $x^2-6x+13=0$

18. $4x^2-8x-1=0$

19. $9x^2-3x-4=0$

20. $3x^2+4x-1=0$

2.2-misol. Tenglamani yeching.

1. $x(x-1)=72$

2. $x(x+1)=56$

3. $2x(x+2)=8x+3$

4. $5x^2+1=6x$

5. $\dfrac{2x^2+x}{3}-\dfrac{2-3x}{4}=\dfrac{x^2-6}{6}$

6. $\dfrac{x^2-3x}{7}+x=11$

7. $\dfrac{x^2+x}{4}-\dfrac{3-7x}{20}=0{,}3$

2.3-misol. Test topshiriqlari.

1. Tenglamaning nechta ildizi bor? $3-x=-\dfrac{4}{x}$.

A) 1 *B*) 2 *C*) 3 *D*) ildizi yo‘q *E*) cheksiz ko‘p

2. Tenglamaning nechta ildizi bor? $\dfrac{2}{x}=x+2$.

A) 3 *B*) 2 *C*) 1 *D*) ildizi yo‘q *E*) cheksiz ko‘p

3. Agar $(3x-1)\cdot(x-2)=0$ bo‘lsa, $3x-1$ qanday qiymatlar qabul qilishi mumkin?

A) faqat $\dfrac{1}{3}$ *B*) faqat 0 *C*) $\dfrac{1}{3}$ yoki 0 *D*) $\dfrac{1}{3}$ yoki 2 *E*) 0 yoki 5

4. Agar $(x-5)\cdot\left(\dfrac{1}{5}x+4\right)=0$ bo‘lsa, $\dfrac{1}{5}x+4$ qanday qiymatlar qabul qiladi?

A) faqat 0 *B*) faqat -20

C) 0 yoki 5 *D*) 0 yoki 8 *E*) -20 yoki 0

5. Tenglamani yeching: $1998x^2-2000x+2=0$.

A) $1;\ \dfrac{2}{1998}$ *B*) $-1;\ \dfrac{3}{1998}$ *C*) $1;\ -\dfrac{2}{1998}$ *D*) $-1;\ -\dfrac{2}{1998}$ *E*) $1;\ -1$

6. Tenglama ildizlarining o‘rta arifmetigi ularning ko‘paytmasidan qancha kam?

$$\frac{x^2+16}{x}=10$$

$A)\ 13$ $B)\ 12$ $C)\ 14$ $D)\ 11$ $E)\ 10$

7. $(x+1)^2-(x+2)^2=(x+3)^2-(x+5)^2$ tenglamani yeching.

$A)\ 6{,}5$ $B)\ 4$ $C)\ -6{,}5$ $D)\ 1{,}5$

8. $x^2-3ax+2a^2-ab-b^2=0$ tenglamani yeching.

$A)\ a+b;\ 2a+b$ $B)\ -a-b;\ 2a-b$

$C)\ a-b;\ 2a-b$ $D)\ a-b;\ 2a+b$

3. Chala kvadrat tenglamalar

Ta'rif. $ax^2+bx+c=0$ *kvadrat tenglamada* b *yoki* c *koeffitsiyentlardan aqalli bittasi nolga teng bo'lsa, bunday kvadrat tenglamalar chala kvadrat tenglamalar deyiladi.*

Demak, chala kvadrat tenglama quyidagi tenglamalardan biri ko'rinishida bo'ladi:

1) $ax^2=0$ 2) $ax^2+c=0,\ c\neq 0$ 3) $ax^2+bx=0,\ b\neq 0$.

Bu chala kvadrat tenglamalarda a *koeffitsiyent nolga teng emas.*

Yuqoridagi chala kvadrat tenglamalarni uchta holga ajratib yechishni ko'rib o'tamiz:

1-hol. $b=0$ va $c=0$ bo'lsa $ax^2=0$ ko'rinishga ega bo'ladi.

Bundan $ax^2=0\Rightarrow x^2=0\Rightarrow x=0$ bo'lib, tenglama yagona $x=0$ ildiziga ega bo'ladi.

1-misol. $7x^2=0$ tenglamani yeching.

Yechish: $7x^2=0$ tenglamaning ikkala qismini 7 ga bo'lib, $x^2=0$ ni hosil qilamiz, bundan $x=0$.

2-hol. $b=0$ va $c\neq 0$ bo'lsa, $ax^2+c=0$ ko'rinishga ega bo'ladi. Bu hol yuqoridagi $x^2=d$ tenglamani yechishga keladi.

2-misol. $3x^2-27=0$ tenglamani yeching.

Yechish: $3x^2-27=0 \Rightarrow x^2=9 \Rightarrow x_{1,2}=\pm\sqrt{9}=\pm 3 \Rightarrow x_1=-3;\ x_2=3.$

Javob: $x_1=-3;\ x_2=3.$

3-hol. $b\neq 0$ va $c=0$ bo'lsa, $ax^2+bx=0$ ko'rinishga ega bo'ladi. Bunda tenglamaning chap qismini ko'paytuvchilarga ajratib, tenglamani yechamiz:

$$ax^2+bx=0 \Rightarrow x(ax+b)=0.$$

Har bir ko'paytuvchini nolga tenglaymiz va tenglamaning ildizlarini topamiz:

$$a)\ x_1=0;$$

b) Qavs ichidagi ifodani nolga tenglasak, $ax+b=0$ chiziqli tenglama hosil bo'ladi. Bundan ikkinchi ildiz $x_2=-\frac{b}{a}$ ni hosil qilamiz. Demak, bu holda tenglama ikkita ildizga ega bo'lar ekan.

3-misol. $x^2-3x=0$ tenglamani yeching.

Yechish: $x^2-3x=0 \Rightarrow x(x-3)=0 \Rightarrow x_1=0;\ x_2=3.$

Javob: $\{0; 3\}$.

4-misol. $5x^2-7x=0$ tenglamani yeching.

Yechish: Tenglamaning chap qismini ko'paytuvchilarga ajratamiz va har bir ko'paytuvchini nolga tenglaymiz:

$$5x^2-7x=0 \Rightarrow x(5x-7)=0$$

$$a)\ x_1=0 \quad b)\ 5x-7=0 \Rightarrow x_2=\frac{7}{5}=1{,}4.$$

Javob: $\{0; 1{,}4\}$.

Mustaqil yechish uchun misollar

3.1-misol. Tenglamalarning haqiqiy ildizlarini toping.

1. $6x^2=0$	**2.** $4x^2-64=0$	**3.** $3x^2-81=0$
4. $6x^2+96=0$	**5.** $4-x^2=0$	**6.** $25-16x^2=0$
7. $-x^2+64=0$	**8.** $-16x^2+49=0$	**9.** $0{,}01x^2=4$
10. $3x^2=5\frac{1}{3}$		

3.2-misol. Chala kvadrat tenglamalarni yeching.

1. $x^2-7x=0$ 2. $x^2+10x=0$ 3. $4x^2=9x$
4. $9x^2-x=0$ 5. $4x^2-3x=0$ 6. $25x-16x^2=0$
7. $10x^2-100x=0$ 8. $x^2-0{,}01x=0$

4. Viyet teoremasi

Teorema. $ax^2+bx+c=0\ (a\neq 0)$ tenglamaning x_1 va x_2 ildizlari, koeffitsiyentlari uchun quyidagi munosabatlar o'rinli:

$$\begin{cases} x_1+x_2=-\dfrac{b}{a} \\ x_1\cdot x_2=\dfrac{c}{a} \end{cases}$$

Bu teorema *Viyet teoremasi* deb nomlanadi.

Isbot. Kvadrat tenglamaning x_1 va x_2 ildizlari uchun $x_1=\dfrac{-b-\sqrt{D}}{2a}$ va $x_2=\dfrac{-b+\sqrt{D}}{2a}$ formulalar o'rinli. Bundan

$$x_1+x_2=\frac{-b-\sqrt{D}}{2a}+\frac{-b+\sqrt{D}}{2a}=-\frac{b}{a}$$

$$x_1\cdot x_2=\frac{-b-\sqrt{D}}{2a}\cdot\frac{-b+\sqrt{D}}{2a}=\frac{b^2-D}{4a^2}=\frac{b^2-(b^2-4ac)}{4a^2}=\frac{4ac}{4a^2}=\frac{c}{a}.$$

Isbot tugadi.

1-misol. x_1 va x_2 sonlari $ax^2+bx+c=0$ kvadrat tenglamaning ildizlari bo'lib, bu yerda $c\neq 0$ bo'lsin. Quyidagi ifodalarni a, b, c koeffitsiyentlar orqali ifodalang:

1) x_1-x_2 2) $x_1^2-x_2^2$ 3) $x_1^3-x_2^3$ 4) $x_1^4-x_2^4$.

Yechish: Viyet teoremasiga asosan $x_1+x_2=-\dfrac{b}{a}$; $x_1\cdot x_2=\dfrac{c}{a}$ o'rinli.

1) $|x_1-x_2|=\sqrt{(x_1-x_2)^2}=\sqrt{(x_1-x_2)^2-4x_1x_2}=\sqrt{\dfrac{b^2}{a^2}-\dfrac{4c}{a}}=$

$$=\sqrt{\frac{b^2-4ac}{a^2}}=\frac{\sqrt{b^2-4ac}}{|a|}$$

Javob: $\frac{\sqrt{b^2-4ac}}{|a|}$, agar $x_1>x_2$ bo'lsa; $-\frac{\sqrt{b^2-4ac}}{|a|}$, agar $x_1<x_2$ bo'lsa.

2) $x_1^2-x_2^2=(x_1-x_2)(x_1+x_2)=-\frac{b}{a}(x_1-x_2)$. x_1-x_2 ning yuqoridagi misoldagi qiymatlarini qo'yamiz:

Javob: $-\frac{b\sqrt{b^2-4ac}}{a|a|}$, agar $x_1>x_2$ bo'lsa; $\frac{b\sqrt{b^2-4ac}}{a|a|}$, agar $x_1<x_2$ bo'lsa.

3) $x_1^3-x_2^3=(x_1-x_2)(x_1^2+x_1x_2+x_2^2)=(x_1-x_2)[(x_1+x_2)^2-$
$-x_1x_2]=(x_1-x_2)\left[\frac{b^2}{a^2}-\frac{c}{a}\right]=(x_1-x_2)\cdot\frac{b^2-ac}{a^2}$

Javob: $\frac{(b^2-ac)\sqrt{b^2-4ac}}{a^2|a|}$, agar $x_1>x_2$ bo'lsa;

$\frac{(ac-b^2)\sqrt{b^2-4ac}}{a^2|a|}$, agar $x_1<x_2$ bo'lsa.

4) $x_1^4-x_2^4=(x_1^2-x_2^2)(x_1^2+x_2^2)=(x_1-x_2)(x_1+x_2)(x_1^2+x_2^2)=$
$=(x_1-x_2)\cdot\left(-\frac{b}{a}\right)\cdot\left(\frac{b^2}{a^2}-\frac{2c}{a}\right)=(x_1-x_2)\cdot\frac{b(2ac-b^2)}{a^3}$

Javob: $\frac{b(2ac-b^2)\sqrt{b^2-4ac}}{a^3|a|}$, agar $x_1>x_2$ bo'lsa;

$\frac{b(b^2-2ac)\sqrt{b^2-4ac}}{a^3|a|}$, agar $x_1<x_2$ bo'lsa.

2-misol. a va b sonlari $3x^2-2x-6=0$ tenglamaning ildizlari bo'lsa, a^2+b^2 ni hisoblang.

$A)\ 6$ $B)\ 8$ $C)\ 4\frac{4}{9}$ $D)\ 4\frac{2}{9}$ $E)\ 3\frac{5}{9}$

Yechish: Viyet teoremasiga asosan $x_1+x_2=-\frac{-2}{3}=\frac{2}{3}$ va $x_1\cdot x_2=\frac{-6}{3}=-2$ bo'ladi. Masala shartidan $x_1=a$ va $x_2=b$ bo'lib, $a^2+b^2=x_1^2+x_2^2=(x_1+x_2)^2-2x_1x_2=\left(\frac{2}{3}\right)^2-2\cdot(-2)=\frac{4}{9}+4=4\frac{4}{9}$.

Javob: $C)4\frac{4}{9}$.

3-misol. x_1 va x_2 lar $3x^2-8x-15=0$ tenglamaning ildizlari bo'lsa, $\frac{x_1}{x_2}+\frac{x_2}{x_1}$ ning qiymatini hisoblang.

$A)-3\frac{19}{45}$ $B)-3\frac{1}{45}$ $C)\,5$ $D)-\frac{8}{3}$ $E)-1\frac{11}{13}$

Yechish: Viyet teoremasiga asosan $x_1+x_2=-\frac{b}{a}=$ $=-\frac{-8}{3}=\frac{8}{3}$ va $x_1\cdot x_2=\frac{c}{a}=\frac{-15}{3}=-5$ o'rinli. Masala shartidan

$$\frac{x_1}{x_2}+\frac{x_2}{x_1}=\frac{x_1^2+x_2^2}{x_1x_2}=\frac{(x_1+x_2)^2-2x_1x_2}{x_1x_2}=\frac{(x_1+x_2)^2}{x_1x_2}-2=$$
$$=-\frac{64}{9}\cdot\frac{1}{5}-2=-1\frac{19}{45}-2=-3\frac{19}{45}.$$

Javob: $A)-3\frac{19}{45}$.

4-misol. Tenglama ildizlari kublarining yig'indisini toping:

$$2x^2-5x+1=0.$$

$A)\,11\frac{7}{8}$ $B)\,12$ $C)\,12\frac{8}{9}$ $D)\,12\frac{7}{8}$ $E)\,13$

Yechish: Masala sharti bo'yicha $x_1^3+x_2^3$ ni topishimiz kerak. Viyet teoremasidan $x_1+x_2=-\frac{b}{a}=\frac{5}{2}$ va $x_1\cdot x_2=\frac{c}{a}=\frac{1}{2}$ ga ega bo'lamiz. $(a+b)^3=a^3+b^3+3ab(a+b)$ qisqa ko'paytirish formulasini qo'llaymiz:

$$x_1^3+x_2^3=(x_1+x_2)^3-3x_1x_2(x_1+x_2)=\left(\frac{5}{2}\right)^3-3\cdot\frac{1}{2}\cdot\frac{5}{2}=\frac{125}{8}-\frac{15}{4}=\frac{95}{8}=11\frac{7}{8}.$$

Javob: $A)\ 11\frac{7}{8}$.

Mustaqil yechish uchun misollar

4.1-misol. Agar x_1 va x_2 sonlari $2x^2-11x+13=0$ kvadrat tenglamaning ildizlari bo'lsa, quyidagilarni toping:

$$1)\ \frac{x_1}{x_2}+\frac{x_2}{x_1}\quad 2)\ x_1^2+x_2^2\quad 3)\ x_1^3+x_2^3\quad 4)\ x_1^4+x_2^4$$

$$5)\ x_1^2-x_2^2\quad 6)\ x_1^3-x_2^3\quad 7)\ x_1^4-x_2^4$$

4.2-misol. Test topshiriqlari.

1. Tenglamaning ildizlari yig'indisi va ko'paytmasining yig'indisini hisoblang: $2x^2-5x+2=0$.

$A)$ 2,5 $B)$ 7 $C)$ 2,8 $D)$ 3,5 $E)$ 3,2

2. Agar x_1 va x_2 lar $9x^2+3x-1=0$ tenglamaning ildizlari bo'lsa, $\frac{3x_1\cdot x_2}{x_1+x_2}$ ning qiymatini toping.

$A)\ -1$ $B)\ 1$ $C)\ 2$ $D)\ \frac{1}{3}$ $E)\ 3$

3. Agar x_1 va x_2 sonlari $2x^2+3x-4=0$ tenglamaning ildizlari bo'lsa, $\frac{x_1^3-x_2^3}{x_1-x_2}$ ning qiymatini toping.

$A)$ 0,25 $B)$ $-0{,}25$ $C)$ 4,25 $D)$ $-4{,}25$ $E)$ 3,25

4. $ax^2+bx+c=0$ tenglamaning koeffitsiyentlari $b=a+c$ tenglikni qanoatlantiradi. Agar x_1 va x_2 berilgan kvadrat tenglamaning ildizlari bo'lsa, $\frac{x_2}{x_1}+\frac{x_1}{x_2}-2$ ning qiymatini toping.

$A)\ \frac{1}{a}+\frac{1}{c}$ $B)\ \frac{a}{c}+\frac{c}{a}$ $C)\ \frac{2(a+c)}{ac}$ $D)\ \frac{(a-c)^2}{ac}$

5. Keltirilgan kvadrat tenglama

Ta'rif. *Ushbu* $x^2 + px + q = 0$ *ko'rinishidagi kvadrat tenglama keltirilgan kvadrat tenglama deyiladi.*

Bu tenglamaning bosh koeffitsiyenti birga teng. Har qanday $ax^2 + bx + c = 0$ kvadrat tenglamani uning ikkala qismini $a \neq 0$ ga bo'lib, keltirilgan kvadrat tenglama ko'rinishiga keltirish mumkin.

Keltirilgan kvadrat tenglamaning ildizlarini $ax^2 + bx + c = 0$ kvadrat tenglamaning

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{D}}{2a}$$

formulasidan foydalanib topamiz. Bunda $a = 1$, $b = p$, $c = q$ larni e'tiborga olamiz va

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-p \pm \sqrt{p^2 - 4q}}{2} = -\frac{p}{2} \pm \frac{\sqrt{p^2 - 4q}}{2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

ni hosil qilamiz.

Keltirilgan kvadrat tenglamaning ildizlari quyidagi formula yordamida topiladi:

$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

Bu formuladan p juft son bo'lganda foydalanish qulayroqdir.

1-misol. $x^2 - 14x - 15 = 0$ tenglamani yeching.

Yechish: $p = -14;\ q = -15$.

$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q} = 7 \pm \sqrt{49 + 15} = 7 \pm 8 \Rightarrow x_1 = -1;\ x_2 = 15.$$

Javob: $x_1 = -1;\ x_2 = 15$.

2-misol. $x^2 + 6x - 7 = 0$ tenglamani yeching.

Yechish: $p = 6;\ q = -7$.

$$x_{1,2}=-\frac{p}{2}\pm\sqrt{\left(\frac{p}{2}\right)^2-q}=-3\pm\sqrt{9+7}=-3\pm4\Rightarrow x_1=-7;\ x_2=1.$$

Javob: $x_1=-7;\ x_2=1$.

Mustaqil yechish uchun misollar

5.1-misol. Keltirilgan kvadrat tenglamalarning haqiqiy ildizlarini toping.

1. $x^2+4x-5=0$ **2.** $x^2-6x-7=0$ **3.** $x^2+6x-40=0$

4. $x^2-8x-9=0$ **5.** $x^2-18x+81=0$ **6.** $x^2+22x+121=0$

7. $x^2-4x+5=0$ **8.** $x^2-2x+10=0$

5.2-misol. Keltirilgan kvadrat tenglamani yeching.

1. $x^2-2\sqrt{3}x-1=0$ **2.** $x^2-2\sqrt{5}x+1=0$ **3.** $x^2+\sqrt{2}x-4=0$

4. $x^2-4\sqrt{7}x+4=0$ **5.** $x^2-(2+2\sqrt{2})x+2\sqrt{2}=0$

6. Keltirilgan kvadrat tenglama uchun Viyet teoremasi

Teorema. *Agar x_1 va x_2 lar $x^2+px+q=0$ tenglamaning ildizlari bo'lsa, u holda*

$$\begin{cases} x_1+x_2=-p \\ x_1\cdot x_2=q \end{cases}$$

formulalar o'rinli, ya'ni keltirilgan kvadrat tenglama ildizlarining yig'indisi qarama-qarshi ishora bilan olingan ikkinchi koeffitsiyentga, ildizlarining ko'paytmasi esa ozod hadga teng.

Isbot. Keltirilgan kvadrat tenglama ildizlarini topish formulasiga asosan:

$$x_1=-\frac{p}{2}-\sqrt{\left(\frac{p}{2}\right)^2-q}\ \text{ va }\ x_2=-\frac{p}{2}+\sqrt{\left(\frac{p}{2}\right)^2-q}.$$

Tengliklarni hadma-had qo'shamiz:

$$x_1 + x_2 = -\frac{p}{2} - \sqrt{\left(\frac{p}{2}\right)^2 - q} - \frac{p}{2} + \sqrt{\left(\frac{p}{2}\right)^2 - q} = -p$$

Bu tengliklarni ko'paytirib, kvadratlar ayirmasi formulasini qo'llab quyidagini hosil qilamiz:

$$x_1 \cdot x_2 = \left(-\frac{p}{2} - \sqrt{\left(\frac{p}{2}\right)^2 - q}\right)\left(-\frac{p}{2} + \sqrt{\left(\frac{p}{2}\right)^2 - q}\right) =$$

$$= \left(-\frac{p}{2}\right)^2 - \left(\sqrt{\left(\frac{p}{2}\right)^2 - q}\right)^2 = \left(\frac{p}{2}\right)^2 - \left(\frac{p}{2}\right)^2 + q = q$$

Isbot tugadi.

Masalan, $x^2 - 13x + 30 = 0$ tenglama $x_1 = 3$ va $x_2 = 10$ ildizlarga ega: uning ildizlari yig'indisi $x_1 + x_2 = 3 + 10 = 13$, ko'paytmasi esa $x_1 \cdot x_2 = 3 \cdot 10 = 30$.

Viyet teoremasi kvadrat tenglama ikkita teng $x_1 = x_2 = -\frac{p}{2}$ ildizlarga ega bo'lganda ham to'g'ri bo'ladi.

Masalan, $x^2 - 8x + 16 = 0$ tenglama ikkita teng $x_1 = x_2 = 4$ ildizlarga ega: ularning yig'indisi $x_1 + x_2 = 8$, ko'paytmasi $x_1 \cdot x_2 = 16$.

Ko'pgina masalalarni yechishda *Viyet teoremasiga teskari bo'lgan* quyidagi teorema ham qo'llaniladi.

Teorema. *Agar p, q, x_1, x_2 sonlari uchun $x_1 + x_2 = -p$ va $x_1 \cdot x_2 = q$ munosabatlar bajarilsa, u holda x_1 va x_2 sonlar $x^2 + px + q = 0$ tenglamaning ildizlari bo'ladi.*

Isbot. Tenglamaning chap qismidagi $x^2 + px + q$ kvadrat uchhaddagi p ning o'rniga $-(x_1 + x_2)$ ni, q ning o'rniga esa $x_1 \cdot x_2$ ko'paytmani qo'yamiz. Natijada quyidagi ifoda hosil bo'ladi:

$$x^2 + px + q = x^2 - (x_1 + x_2)x + x_1x_2 = x^2 - x_1x - x_2x + x_1x_2 =$$
$$= x(x - x_1) - x_2(x - x_1) = (x - x_1)(x - x_2)$$

Shunday qilib, agar p, q, x_1, x_2 sonlari $x_1 + x_2 = -p$ va $x_1 \cdot x_2 = q$ munosabatlar bilan bog'langan bo'lsa, u holda x *ning har qanday qiymatida*

$$x^2 + px + q = (x - x_1)(x - x_2)$$

tenglik bajariladi, bundan x_1 va x_2 lar $x^2 + px + q = 0$ tenglamaning ildizlari ekanligi kelib chiqadi. Isbot tugadi.

Keltirilgan kvadrat tenglama $x^2 + px + q = 0$ ning x_1 va x_2 ildizlari uchun Viyet teoremasining shartlari $x_1 + x_2 = -p$ va $x_1 \cdot x_2 = q$ bajarilsa, quyidagi xulosalarni hosil qilishimiz mumkin:

$$x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1x_2 = p^2 - 2q$$

$$(x_1 - x_2)^2 = (x_1 + x_2)^2 - 4x_1x_2 = p^2 - 4q$$

$$x_1^3 + x_2^3 = (x_1 + x_2)^3 - 3x_1x_2(x_1 + x_2) = 3pq - p^3$$

$$x_1^4 + x_2^4 = (x_1^2 + x_2^2)^2 - 2(x_1x_2)^2 = (p^2 - 2q)^2 - 2q^2 = p^4 - 4p^2q + 2q^2$$

$$x_1^5 + x_2^5 = (x_1^2 + x_2^2)(x_1^3 + x_2^3) - (x_1x_2)^2(x_1 + x_2) = -p^5 + 5p^3q - 5pq^2$$

Bu formulalar yordamida ko'pgina masalalarni hal qilish mumkin. Yuqoridagi formulalarni $ax^2 + bx + c = 0$ tenglamaga ham qo'llash mumkin. Chunki kvadrat tenglamani har doim keltirilgan kvadrat tenglama ko'rinishiga keltirsa bo'ladi.

Viyet teoremasiga teskari teoremadan foydalanib, kvadrat tenglama ildizlarini *tanlash usuli* bilan topish mumkin.

1-misol. $x^2 - 5x + 6 = 0$ tenglamaning ildizlarini toping.

Yechish: Bu yerda $p = -5$, $q = 6$. Ikkita x_1 va x_2 sonlarni $x_1 + x_2 = -p = -(-5) = 5$ va $x_1x_2 = q = 6$ bo'ladigan qilib tanlaymiz.

$2 \cdot 3 = 6$ va $2 + 3 = 5$ ekanligini e'tiborga olib, Viyet teoremasiga teskari teorema bo'yicha $x_1 = 2$, $x_2 = 3$ ga, ya'ni $x^2 - 5x + 6 = 0$ tenglamaning ildizlariga ega bo'lamiz.

Javob: $x_1 = 2$, $x_2 = 3$.

2-misol. $x^2 - x - 6 = 0$ tenglama ildizlarini toping.

Yechish: $p = -1$, $q = -6$,

$$1) = 1 \Rightarrow x_1 = -2\,;\ x_2 = 3\,;\ \textit{Tekshirish:}\ \begin{cases} -2 + 3 = 1 \\ -2 \cdot 3 = -6 \end{cases}.$$

ι. $x^2 + x - 6 = 0$ tenglama ildizlarini toping.

h: $p = 1,\ q = -6$,

$$\begin{matrix} = -1 \\ = -6 \end{matrix} \Rightarrow x_1 = 2\,;\ x_2 = -3\,;\ \textit{Tekshirish:}\ \begin{cases} 2 + (-3) = -1 \\ 2 \cdot (-3) = -6 \end{cases}.$$

isol. $x^2 + 8x + 7 = 0$ tenglama ildizlarini toping.

hish: $p = 8,\ q = 7$,

$$\begin{matrix} {}_2 = -8 \\ {}_2 = 7 \end{matrix} \Rightarrow x_1 = -1\,;\ x_2 = -7\,;\ \textit{Tekshirish:}\ \begin{cases} -1 + (-7) = -8 \\ -1 \cdot (-7) = 7 \end{cases}.$$

-misol. Ushbu $2x^2 - 26x + 72 = 0$ tenglama ildizlarining ›roporsionalini toping.

$A)\,4 \quad B)\,5 \quad C)\,7 \quad D)\,6 \quad E)\,8$

Yechish: $2x^2 - 26x + 72 = 0$ tenglamaning ikkala tomoniga bo'lamiz va u $x^2 - 13x + 36 = 0$ keltirilgan kvadrat tengıaga keladi. Tanlash usuli yordamida tenglama ildizlarini ›amiz:

$$\begin{cases} x_1 + x_2 = 13 \\ x_1 \cdot x_2 = 36 \end{cases} \Rightarrow x_1 = 4\,;\ x_2 = 9$$

Bu tenglama ildizlarining o'rta proporsionali $\sqrt{x_1 \cdot x_2} = \sqrt{4 \cdot 9} = 6$ ga teng.

Javob: $D)6$.

6-misol. $x^2 + 5x - 6 = 0$ tenglama kichik ildizining katta ildiziga nisbatini toping.

$A)\,6 \quad B)\,-6 \quad C)\,\frac{1}{6} \quad D)\,-\frac{1}{6} \quad E)\,1$

Yechish: Viyet teoremasidan $x_1 + x_2 = -5$ va $x_1 \cdot x_2 = -6$ bo'lib, $x_1 = -6$, $x_2 = 1$ larni topamiz. Ko'rinib turibdiki, $-6 < 1$. Bundan kichik ildizining katta ildiziga nisbati $-6 : 1 = -6$.

Javob: $B)\,-6$.

7-misol. Agar $x^2 - 3x - 6 = 0$ tenglamaning ildizlari x_1 va x_2 bo'lsa, $\frac{1}{x_1^3} + \frac{1}{x_2^3}$ ni hisoblang.

$A)\ \frac{1}{3}$ $B)\ 0{,}5$ $C)\ -0{,}5$ $D)\ 0{,}375$ $E)\ -0{,}375$

Yechish: Viyet teoremasiga asosan $x_1 + x_2 = 3$ va $x_1 \cdot x_2 = -6$. U holda

$$\frac{1}{x_1^3} + \frac{1}{x_2^3} = \frac{x_1^3 + x_2^3}{(x_1x_2)^3} = \frac{(x_1 + x_2)^3 - 3x_1x_2(x_1 + x_2)}{(x_1x_2)^3} =$$

$$= \frac{3^3 - 3\cdot(-6)\cdot 3}{(-6)^3} = -\frac{81}{216} = -\frac{3}{8} = -0{,}375.$$

Javob: $E)-0{,}375$.

8-misol. Agar x_1 va x_2 lar $x^2 + x - 5 = 0$ tenglamaning ildizlari bo'lsa, $x_1^2x_2^4 + x_2^2x_1^4$ ning qiymatini toping.

$A)\ 225$ $B)\ 145$ $C)\ 125$ $D)\ 175$ $E)\ 275$

Yechish: Viyet teoremasiga asosan $x_1 + x_2 = -1$ va $x_1 \cdot x_2 = -5$. Bundan

$$x_1^2x_2^4 + x_2^2x_1^4 = (x_1x_2)^2(x_1^2 + x_2^2) = (x_1x_2)^2[(x_1 + x_2)^2 - 2x_1x_2] =$$

$$= (-5)^2\left[(-1)^2 - 2\cdot(-5)\right] = 25\cdot 11 = 275.$$

Javob: $E)\ 275$.

9-misol. Agar $x^2 - x + q = 0$ tenglamaning ildizlari a va b bo'lsa, $a^3 + b^3 + 3(a^3b + ab^3) + 6(a^3b^2 + a^2b^3)$ ifodaning qiymatini toping.

$A)\ 6q$ $B)\ 3q$ $C)\ 1 + 6q$ $D)\ 1$

Yechish: $x^2 - x + q = 0$ tenglamaning ildizlari $x_1 = a$ va $x_2 = b$ bo'lsa, Viyet teoremasiga asosan $a + b = 1$ va $ab = q$ o'rinli. U holda

$$a^3 + b^3 + 3(a^3b + ab^3) + 6(a^3b^2 + a^2b^3) = (a + b)(a^2 - ab + b^2) +$$

$$+3ab(a^2 + b^2) + 6a^2b^2(a + b) = (a + b)\left[(a + b)^2 - 3ab\right] +$$

$$+3ab\left[(a+b)^2-2ab\right]+6(ab)^2(a+b)=1\cdot(1-3q)+3q(1-2q)+$$
$$+6q^2=1-3q+3q-6q^2+6q^2=1.$$

Javob: $D)\,1$.

10-misol. $x^2-(14+m)x+m^2=0 (m>0)$ tenglamaning ildizlari orasida $x_1=9x_2$ munosabat o'rinli. Berilgan tenglamaning kichik ildizini toping.

$A)\,2$ $B)\,24$ $C)\,9$ $D)\,18$

Yechish: Viyet teoremasidan

$$\begin{cases} x_1+x_2=14+m \\ x_1x_2=m^2 \\ x_1=9x_2 \end{cases} \Rightarrow \begin{cases} 9x_2+x_2=14+m \\ x_1x_2=m^2 \\ x_1=9x_2 \end{cases} \Rightarrow \begin{cases} x_2=\dfrac{14+m}{10} \\ x_1x_2=m^2 \\ x_1=\dfrac{9}{10}(14+m) \end{cases}$$

x_1 va x_2 ning topilgan qiymatlarini sistemaning ikkinchi tenglamasiga qo'yamiz.

$$\frac{9}{10}(14+m)\cdot\frac{14+m}{10}=m^2 \Rightarrow \frac{9}{100}(14+m)^2=$$
$$=m^2 \Rightarrow \left(\frac{m}{14+m}\right)^2=\frac{9}{100} \Rightarrow \frac{m}{14+m}=\pm\frac{3}{10}.$$

$m>0$ bo'lganligi uchun

$$\frac{m}{14+m}=\frac{3}{10} \Rightarrow 10m=42+3m \Rightarrow 7m=42 \Rightarrow m=6.$$
$$x_1=\frac{9}{10}\cdot(14+6)=18;\ x_2=\frac{14+6}{10}=2.$$

Javob: $A)\,2$.

11. p ning qanday qiymatida $x^2-px+4=0$ tenglamaning ildizlaridan biri boshqasidan 3 ga katta?

$A)\pm5$ $B)\,4$ $C)\pm4$ $D)\pm6$

Yechish: x_1 va x_2 sonlari $x^2-px+4=0$ tenglamaning ildizlari bo'lsin. Masala shartiga asosan $x_1=x_2+3$ o'rinli. Viyet teoremasidan

$$\begin{cases} x_1 + x_2 = p \\ x_1 x_2 = 4 \\ x_1 - x_2 = 3 \end{cases} \Rightarrow \begin{cases} 2x_1 = p + 3 \\ x_1 x_2 = 4 \\ x_1 - x_2 = 3 \end{cases} \Rightarrow \begin{cases} x_1 = \dfrac{p+3}{2} \\ x_1 x_2 = 4 \\ x_2 = x_1 - 3 = \dfrac{p+3}{2} - 3 = \dfrac{p-3}{2} \end{cases}$$

Topilganlarni sistemaning ikkinchi tenglamasiga qo'yamiz:

$$\frac{p+3}{2} \cdot \frac{p-3}{2} = 4 \Rightarrow \frac{p^2 - 9}{4} = 4 \Rightarrow p^2 = 25 \Rightarrow p = \pm 5.$$

Javob: *A*) ± 5.

12. $x^2 + px + 4 = 0$ tenglama ildizlari ayirmasining kvadrati 48 ga teng bo'lsa, ildizlarining yig'indisini toping.

A) $\pm\sqrt{40}$ *B*) $\pm\sqrt{2}$ *C*) ± 6 *D*) -8 va 8

Yechish: x_1 va x_2 sonlari $x^2 + px + 4 = 0$ tenglama ildizlari va $(x_1 - x_2)^2 = 48$ bo'lsin. Viyet teoremasidan $\begin{cases} x_1 + x_2 = -p \\ x_1 x_2 = 4 \end{cases}$ o'rinli. Bundan

$$(x_1 - x_2)^2 = 48 \Rightarrow x_1^2 - 2x_1x_2 + x_2^2 = 48 \Rightarrow (x_1 + x_2)^2 - 4x_1x_2 = 48 \Rightarrow$$
$$\Rightarrow p^2 - 16 = 48 \Rightarrow p^2 = 64 \Rightarrow p = \pm 8.$$

Javob: *D*) -8 va 8.

Mustaqil yechish uchun misollar

6.1-misol. Quyidagi keltirilgan kvadrat tenglamalarning ildizlarini tanlash usuli bilan toping.

1. $x^2 + 4x - 5 = 0$ **2.** $x^2 - 4x - 5 = 0$

3. $x^2 - 6x + 5 = 0$ **4.** $x^2 - 7x + 12 = 0$

5. $x^2 - 8x + 15 = 0$ **6.** $x^2 - x - 12 = 0$

7. $x^2 + x - 12 = 0$ **8.** $x^2 - 8x + 16 = 0$

9. $x^2 + 11x + 30 = 0$ **10.** $x^2 + 2x - 15 = 0$

6.2-misol. Test topshiriqlari.

1. Kvadrat tenglama kichik ildizining katta ildiziga nisbatini toping:

$$x^2+5x+6=0.$$

$A)\ \frac{2}{3}$ $B)\ -\frac{1}{3}$ $C)\ \frac{3}{2}$ $D)\ -\frac{1}{2}$ $E)\ -3$

2. $x^2-18x+45=0$ tenglamaning katta ildizini toping.

$A)\ -3$ $B)\ 3$ $C)\ -15$ $D)\ 15$ $E)\ 5$

3. Agar a va b sonlari $x^2-8x+7=0$ kvadrat tenglamaning ildizlari bo'lsa, $\frac{1}{a^2}+\frac{1}{b^2}$ ni hisoblang.

$A)\ 1\frac{1}{49}$ $B)\ 1\frac{1}{50}$ $C)\ 2\frac{1}{15}$ $D)\ 1\frac{1}{10}$ $E)\ 2\frac{1}{49}$

4. Agar a va b sonlari $x^2-10x+9=0$ tenglamaning ildizlari bo'lsa, $\frac{1}{a^2}+\frac{1}{b^2}$ ni hisoblang.

$A)\ 1\frac{1}{49}$ $B)\ 1\frac{1}{100}$ $C)\ 1\frac{1}{81}$ $D)\ 1\frac{7}{81}$

5. Agar $x^2+x-1=0$ tenglamaning ildizlari x_1 va x_2 bo'lsa, $x_1^3+x_2^3$ ni hisoblang.

$A)\ 1$ $B)\ 3$ $C)\ 2$ $D)\ -2$ $E)\ -4$

6. Ushbu $x^2+4x-5=0$ tenglamaning ildizlari x_1 va x_2 bo'lsa, $x_1^3\cdot x_2^3$ ni hisoblang.

$A)\ 124$ $B)\ -125$ $C)\ 130$ $D)\ 5$ $E)\ -124$

7. Agar x_1 va x_2 lar $x^2+x-3=0$ tenglamaning ildizlari bo'lsa, $\frac{1}{x_1^2x_2^4}+\frac{1}{x_1^4x_2^2}$ ning qiymatini toping.

$A)\ \frac{5}{81}$ $B)\ \frac{7}{81}$ $C)\ \frac{11}{81}$ $D)\ \frac{4}{27}$ $E)\ \frac{3}{16}$

8. Agar x_1 va x_2 lar $x^2+3x-3=0$ tenglamaning ildizlari bo'lsa, $x_1^4+x_2^4$ ning qiymatini toping.

$A)\ 207$ $B)\ 192$ $C)\ 243$ $D)\ 168$ $E)\ 252$

9. Agar x_1 va x_2 lar $x^2+x-7=0$ tenglamaning ildizlari bo'lsa, $x_1^2x_2^4+x_2^2x_1^4$ ning qiymatini toping.

$A)\ 625 \quad B)\ 345 \quad C)\ 935 \quad D)\ 735$

10. $x^2-\frac{\sqrt{85}}{4}x+1\frac{5}{16}=0$ tenglamaning katta va kichik ildizlari kublarining ayirmasini toping.

$A)\ -2 \quad B)\ -1 \quad C)\ 2 \quad D)\ 1 \quad E)\ \frac{1}{2}(\sqrt{85}-6)$

11. x_1 va x_2 sonlari $x^2-ax+20=0$ tenglamaning ildizlari bo'lib, $\frac{1}{x_1}+\frac{1}{x_2}=\frac{9}{20}$ tenglikni qanoatlantirsa, a ning qiymatini toping.

$A)\ -1 \quad B)\ 9 \quad C)\ -3 \quad D)\ 3$

12. Agar $x^2+mx+m^2+c=0$ tenglamaning ildizlari a va b bo'lsa, a^2+ab+b^2+c ifodaning qiymatini toping.

$A)\ m+c \quad B)\ -m^2-c \quad C)\ 0 \quad D)\ mc$

13. Agar m va n sonlari $x^2+2mx-3n=0(m\cdot n\neq 0)$ tenglamaning ildizlari bo'lsa, $n-m$ ning qiymati nechaga teng bo'ladi?

$A)\ 6 \quad B)\ 24 \quad C)\ 18 \quad D)\ 12$

14. q ning qanday qiymatida $x^2-x-q=0$ tenglama ildizlari kublarining yig'indisi 16 ga teng bo'ladi?

$A)\ 6 \quad B)\ 5 \quad C)\ 7 \quad D)\ 4$

15. x_1 va x_2 lar $x^2+3x+m=0$ tenglamaning ildizlari. m ning qanday qiymatida tenglamaning ildizlari ayirmasi 6 ga teng bo'ladi?

$A)\ 6{,}75 \quad B)\ -6{,}75 \quad C)\ 6{,}5 \quad D)\ -4{,}75$

16. Agar x_1 va x_2 sonlari $x^2+3x+1=0$ tenglamaning ildizlari bo'lsa, $\left(\frac{x_1}{x_2+1}\right)^2+\left(\frac{x_2}{x_1+1}\right)^2$ ni toping.

$A)\ 18 \quad B)\ 9 \quad C)\ 27 \quad D)\ 36$

7. Ildizlari x_1 va x_2 bo'lgan kvadrat tenglama tuzish

Endi ildizlari x_1 va x_2 bo'lgan kvadrat tenglama tuzishni ko'ramiz. Bu tenglama yuqoridagi Viyet teoremasi yordamida tuziladi.

Teorema. Ildizlari x_1 va x_2 bo'lgan kvadrat tenglama quyidagi ko'rinishda bo'ladi:

$$x^2 - (x_1 + x_2)x + x_1x_2 = 0.$$

Isbot. x_1 va x_2 sonlari izlanayotgan $x^2 + px + q = 0$ kvadrat tenglamaning ildizlari bo'lsa, u holda Viyet teoremasiga asosan $x_1 + x_2 = -p$ va $x_1 \cdot x_2 = q$.. Agar p va q ning qiymatlarini $x^2 + px + q = 0$ tenglamaga qo'ysak, yuqoridagi $x^2 - (x_1 + x_2)x + x_1x_2 = 0$ tenglama hosil bo'ladi.

1-misol. Ildizlari $x_1 = -1$ va $x_2 = 3$ bo'lgan kvadrat tenglama tuzing.

Yechish: $x^2 - (-1+3) \cdot x + (-1) \cdot 3 = 0 \Rightarrow x^2 - 2x - 3 = 0$.

2-misol. Ildizlari $x_1 = -4$ va $x_2 = -5$ bo'lgan kvadrat tenglama tuzing.

Yechish: $x^2 - (-4-5)x + (-4) \cdot (-5) = 0 \Rightarrow x^2 + 9x + 20 = 0$.

Yuqoridagi formulani bilmagan holda ham kvadrat tenglama tuzish mumkin.

3-misol. Ildizlari $x_1 = -8$ va $x_2 = -11$ bo'lgan kvadrat tenglama tuzing.

Yechish: Izlanayotgan kvadrat tenglama $x^2 + px + q = 0$ ko'rinishida bo'lsin. Viyet teoremasidan p va q ning qiymatlarini topamiz:

$x_1 + x_2 = -p \Rightarrow p = -(x_1 + x_2) = -(-8-11) = 19$ va $q = x_1 \cdot x_2 = -8 \cdot (-11) = 88$ o'rniga qo'yamiz.

Izlanayotgan tenglama $x^2+19x+88=0$ ko'rinishida bo'ladi.

4-misol. Ildizlari $x^2-14x+24=0$ tenglamaning ildizlariga teskari bo'lgan kvadrat tenglama tuzing.

$A)\ 24x^2+14x+1=0$ $B)\ 24x^2-14x+1=0$

$C)\ 24x^2-14x-1=0$ $D)\ 24x^2+14x-1=0$

Yechish: $x^2-14x+24=0$ tenglama uchun Viyet teoremasiga asosan $x_1+x_2=14$ va $x_1\cdot x_2=24$ o'rinli. Izlangan kvadrat tenglamaning ildizlari $\frac{1}{x_1}$ va $\frac{1}{x_2}$ bo'ladi. Demak, quyidagi tenglamani hosil qilamiz:

$$x^2-\left(\frac{1}{x_1}+\frac{1}{x_2}\right)x+\frac{1}{x_1x_2}=0\Rightarrow x^2-\frac{x_1+x_2}{x_1x_2}x+\frac{1}{x_1x_2}=0\Rightarrow x^2-\frac{14}{24}x+\frac{1}{24}=$$

$$=0\Rightarrow 24x^2-14x+1=0$$

Javob: $B)\ 24x^2-14x+1=0$.

5-misol. Ildizlari $x^2-6x+1=0$ kvadrat tenglamaning ildizlaridan 2 marta katta bo'lgan kvadrat tenglama $x^2-bx+c=0$ ko'rinishida ifodalangan. bc ko'paytmani toping.

$A)\ 80$ $B)\ 48$ $C)\ 36$ $D)\ 24$

Yechish: $x^2-6x+1=0$ kvadrat tenglamaning ildizlari x_1 va x_2 bo'lsin.

U holda Viyet teoremasiga asosan $\begin{cases}x_1+x_2=6\\ x_1x_2=1\end{cases}$.

y_1 va y_2 lar $x^2-bx+c=0$ kvadrat tenglamaning ildizlari bo'lsin. Masala shartiga asosan $y_1=2x_1$; $y_2=2x_2$. U holda Viyet teoremasiga asosan,

$$\begin{cases}y_1+y_2=b\\ y_1y_2=c\end{cases}\Rightarrow\begin{cases}2x_1+2x_2=b\\ 2x_1\cdot 2x_2=c\end{cases}\Rightarrow\begin{cases}2(x_1+x_2)=b\\ 4x_1x_2=c\end{cases}\Rightarrow\begin{cases}b=2\cdot 6=12\\ c=4\cdot 1=4\end{cases}$$

Natijada $bc=12\cdot 4=48$.

Javob: $B)\ 48$.

6-misol. x_1 va x_2 sonlari $x^2-10x+21=0$ tenglamanıng ildizlari bo‘lsa, ildizlari $x_1^2+x_2^2$ va x_1x_2 bo‘lgan kvadrat tenglama tuzing.

$A)\ x^2-79x+1218=0$ $\quad B)\ x^2+79x+1218=0$

$C)\ x^2-58x+1218=0$ $\quad D)\ x^2-79x-1218=0$

Yechish: x_1 va x_2 sonlari $x^2-10x+21=0$ tenglamaning ildizlari bo‘lsa, u holda Viyet teoremasiga asosan $\begin{cases} x_1+x_2=10 \\ x_1x_2=21 \end{cases}$.

Ildizlari $x_1^2+x_2^2$ va x_1x_2 bo‘lgan kvadrat tenglama

$$x^2-(x_1^2+x_2^2+x_1x_2)x+x_1x_2(x_1^2+x_2^2)=0$$

ko‘rinishida bo‘ladi.

$$x^2-((x_1+x_2)^2-x_1x_2)x+x_1x_2((x_1+x_2)^2-2x_1x_2)=0$$

$$x^2-(10^2-21)x+21\cdot(10^2-2\cdot21)=0$$

$$x^2-79x+1218=0$$

Javob: $A)\ x^2-79x+1218=0$.

7-misol. Ildizlaridan biri $\dfrac{\sqrt{3}-\sqrt{5}}{\sqrt{3}+\sqrt{5}}$ ga teng bo‘lgan ratsional koeffitsiyentli kvadrat tenglama tuzing.

Yechish: Izlanayotgan kvadrat tenglama $x^2+px+q=0$ (p va q – ratsional sonlar) ko‘rinishida bo‘lsin.

$\dfrac{\sqrt{3}-\sqrt{5}}{\sqrt{3}+\sqrt{5}}=\dfrac{(\sqrt{3}-\sqrt{5})^2}{(\sqrt{3}+\sqrt{5})(\sqrt{3}-\sqrt{5})}=\dfrac{8-2\sqrt{15}}{3-5}=-4+\sqrt{15}$ soni tenglamaning ildizi bo‘lganligi uchun uni qanoatlantiradi:

$$(-4+\sqrt{15})^2+p(-4+\sqrt{15})+q=0.$$

Bundan

$$(31-4p+q)+(p-8)\sqrt{15}=0$$

p va q lar ratsional son bo‘lganligi uchun yuqoridagi tenglik bir vaqtda $31-4p+q=0$ va $p-8=0$ bo‘lganda bajariladi. p va q ning qiymatlarini tenglamalar sistemasini yechib topamiz:

$$\begin{cases} 31-4p+q=0 \\ p-8=0 \end{cases} \Rightarrow p=8;\ q=1.$$

Topilganlarni $x^2+px+q=0$ ga qo'yib, izlanayotgan tenglama $x^2+8x+1=0$ ekanligini topamiz.

Javob: $x^2+8x+1=0$.

8-misol. Ratsional koeffitsiyentli $x^2+px+q=0$ tenglama $x_1=1-\sqrt{3}$ ildizga ega. Bu tenglamaning ikkinchi ildizini toping.

$A)\ -1+\sqrt{3}$ $B)\ 2-\sqrt{3}$ $C)\ 1-\sqrt{3}$ $D)\ -1-\sqrt{3}$

Yechish: $x_1=1-\sqrt{3}$ berilgan tenglamaning ildizi bo'lganligi uchun uni qanoatlantirishi kerak, ya'ni

$$(1+\sqrt{3})^2+p(1+\sqrt{3})+q=0$$

$$4+2\sqrt{3}+p+p\sqrt{3}+q=0$$

$$(p+2)\sqrt{3}+(p+q+4)=0$$

$\sqrt{3}$ irratsional son, p va q lar ratsional sonlar bo'lganligi uchun oxirgi tenglikdagi qo'shiluvchilardan har biri bir vaqtda 0 ga teng bo'lganda bajariladi, ya'ni

$$\begin{cases} p+2=0 \\ p+q+4=0 \end{cases} \Rightarrow \begin{cases} p=-2 \\ q=-2 \end{cases}$$

Demak, kvadrat tenglama $x^2-2x-2=0$ ko'rinishga ega. Viyet teoremasidan

$$x_1+x_2=2 \Rightarrow 1+\sqrt{3}+x_2=2 \Rightarrow x_2=1-\sqrt{3}.$$

Xulosa. Agar $x^2+px+q=0$ tenglamaning

1) $x_1=a+b\sqrt{c}$ ildizi bo'lsa, u holda uning qo'shmasi $x_2=a-b\sqrt{c}$ ham ildizi bo'ladi.

2) $x_1=a-b\sqrt{c}$ ildizi bo'lsa, u holda uning qo'shmasi $x_2=a+b\sqrt{c}$ ham ildizi bo'ladi.

Javob: $C)\ 1-\sqrt{3}$.

Mustaqil yechish uchun misollar

7.1-misol. Ildizlari x_1 va x_2 bo‘lgan kvadrat tenglama tuzing.

1. $x_1=-3;\ x_2=4$ **2.** $x_1=5;\ x_2=6$ **3.** $x_1=-7;\ x_2=-8$

4. $x_1=-3;\ x_2=6$ **5.** $x_1=2;\ x_2=3$ **6.** $x_1=-8;\ x_2=10$

7.2-misol. Test topshiriqlari.

1. 3 va -2 sonlari qaysi tenglamaning ildizlari ekanligini ko‘rsating.

$A)\ x^2-x=6$ $B)\ x^2+x=6$

$C)\ x^2+6=x$ $D)\ x^2+6=-x$ $E)\ x^2+1=6x$

2. Ildizlari $3+\sqrt{2}$ va $3-\sqrt{2}$ bo‘lgan kvadrat tenglama tuzing.

$A)\ x^2-6x-7=0$ $B)\ x^2+6x-7=0$

$C)\ x^2-6x+7=0$ $D)\ x^2+6x+7=0$

3. $x_1=\frac{a}{a-b},\ x_2=\frac{b}{b-a}$ ildizlar bo‘yicha kvadrat tenglama tuzing.

$A)\ x^2-2x-\frac{ab}{(a-b)^2}=0$ $B)\ x^2+x-\frac{ab}{(a-b)^2}=0$

$C)\ x^2+x+\frac{ab}{(a-b)^2}=0$ $D)\ x^2-x-\frac{ab}{(a-b)^2}=0$

4. Ildizlari $6x^2+x-7=0$ tenglamaning ildizlariga qarama-qarshi sonlar bo‘lgan kvadrat tenglama tuzing.

$A)\ 6x^2-7x+1=0$ $B)\ 6x^2+x-6=0$

$C)\ 6x^2-x+7=0$ $D)\ 6x^2-x-7=0$

5. $ax^2+bx+c=0$ kvadrat tenglamaning ildizlari x_1 va x_2 bo‘lsa, ildizlari $\frac{1}{x_1}$ va $\frac{1}{x_2}$ bo‘lgan kvadrat tenglama tuzing.

$A)\ cx^2-bx+a=0$ $B)\ cx^2+bx+a=0$

$C)\ cx^2-bx-a=0$ $D)\ -cx^2+bx-a=0$

6. Ildizlaridan biri $3+\frac{\sqrt{2}}{2}$ ga teng bo'lgan ratsional koeffitsiyentli kvadrat tenglama tuzing.

$A)\ x^2-3x+9=0$ $B)\ x^2-6x+17=0$ $C)\ x^2-12x+9=0$

$D)\ 2x^2+12x-17=0$ $E)\ 2x^2-12x+17=0$

7. Ildizlaridan biri $\frac{1}{6+\sqrt{2}}$ ga teng bo'lgan ratsional koeffitsiyentli kvadrat tenglama tuzing.

$A)\ 34x^2-12x+1=0$ $B)\ x^2-12x+1=0$ $C)\ 34x^2-12x-1=0$

$D)\ x^2-12x+34=0$ $E)\ 34x^2+12x-1=0$

8. x_1 va x_2 lar $3x^2-5x+2=0$ kvadrat tenglamaning ildizlari. Ildizlari $\frac{x_1}{3x_2-x_1}$ va $\frac{x_2}{3x_1-x_2}$ ga teng bo'lgan kvadrat tenglama tuzing.

$A)\ 3x^2-7x+4=0$ $B)\ 7x^2+9x-2=0$ $C)\ 7x^2+9x+2=0$

$D)\ 7x^2-9x+2=0$ $E)\ 3x^2+7x-4=0$

8. Kvadrat uchhadni chiziqli ko'paytuvchilarga ajratish

Ta'rif. ax^2+bx+c *ko'phad kvadrat uchhad deyiladi.*

Kvadrat uchhadni nolga aylantiradigan x ning qiymatlari uning *ildizlari* deyiladi. Demak, kvadrat uchhadning nollarini topish uchun $ax^2+bx+c=0$ kvadrat tenglamani yechish yetarli.

Teorema. Agar x_1 va x_2 lar $ax^2+bx+c=0$ kvadrat tenglamaning ildizlari bo'lsa, u holda barcha x lar uchun quyidagi tenglik o'rinli bo'ladi:

$$ax^2+bx+c=a(x-x_1)(x-x_2).$$

Bu tenglik kvadrat uchhadni chiziqli ko'paytuvchilarga ajratish formulasi deyiladi.

Isbot. Agar x_1 va x_2 lar $ax^2+bx+c=0$ tenglamaning ildizlari bo'lsa, u holda Viyet teoremasiga asosan $x_1+x_2=-\frac{b}{a}$ va $x_1\cdot x_2=\frac{c}{a}$ tengliklar o'rinli bo'ladi. Bundan $b=-a(x_1+x_2)$ va $c=ax_1x_2$ ni hosil qilamiz.

$$ax^2+bx+c=ax^2-a(x_1+x_2)x+ax_1x_2=ax^2-ax_1x-ax_2x+$$
$$+ax_1x_2=ax(x-x_1)-ax_2(x-x_1)=a(x-x_1)(x-x_2).$$

Isbot tugadi.

Kvadrat uchhadni chiziqli ko'paytuvchilarga ajratish mos kvadrat tenglamaning diskriminantining ishorasiga bog'-liq.

1-hol. *Agar $D>0$ bo'lsa, kvadrat uchhad ikkita x_1 va x_2 ildizlarga ega bo'lib, kvadrat uchhad chiziqli ko'paytuv-chilarga ajraladi:*

$$ax^2+bx+c=a(x-x_1)(x-x_2).$$

1-misol. $2x^2+5x-3$ kvadrat uchhadni chiziqli ko'pay-tuvchilarga ajrating.

Yechish: $2x^2+5x-3=0$ kvadrat tenglama ildizlarini to-pamiz.

$$D=b^2-4ac=5^2-4\cdot2\cdot(-3)=49>0\Rightarrow x_{1,2}=$$
$$=\frac{-b\pm\sqrt{D}}{2a}=\frac{-5\pm7}{4}\Rightarrow x_1=-3;\ x_2=\frac{1}{2}$$

$$2x^2+5x-3=2(x-(-3))(x-\frac{1}{2})=(x+3)(2x-1).$$

2-hol. *Agar $D=0$ bo'lsa, u holda kvadrat uchhad yago-na $x_1=x_2$ ildizlarga ega bo'ladi. Bunda*

$$ax^2+bx+c=a(x-x_1)^2.$$

2-misol. $36x^2+12x+1$ ni chiziqli ko'paytuvchilarga ajrating.

Yechish: $D=12^2-4\cdot36\cdot1=0$ mos kvadrat tenglama ya-gona yechimga ega.

$$x_1 = x_2 = -\frac{b}{2a} = -\frac{12}{72} = -\frac{1}{6} \Rightarrow 36x^2 + 12x + 1 = 36 \cdot \left(x + \frac{1}{6}\right)^2 = (6x+1)^2$$

3-hol. *Agar* $D < 0$ *bo'lsa, kvadrat uchhad chiziqli ko'paytuvchilarga ajralmaydi.*

3-misol. $4x^2 - 4x + 15$ ni chiziqli ko'paytuvchilarga ajrating.

Yechish: $D = (-4)^2 - 4 \cdot 4 \cdot 15 = -224 < 0$, mos kvadrat tenglama yechimga ega emas. Demak, bu kvadrat uchhad chiziqli ko'paytuvchilarga ajralmaydi.

Yuqoridagi formulani keltirilgan kvadrat tenglama uchun ham yozamiz. Bu holda $a = 1$ deb qaraladi.

$x^2 + px + q -$ kvadrat uchhadning ildizlari x_1 va x_2 bo'lsa, u holda

$$x^2 + px + q = (x - x_1)(x - x_2).$$

Bu formula $x^2 + px + q$ kvadrat uchhadni chiziqli ko'paytuvchilarga ajratish formulasidir.

4-misol. $x^2 + 5x + 6$ kvadrat uchhadni chiziqli ko'paytuvchilarga ajrating.

Yechish: Tanlash usuli bilan $x^2 + 5x + 6 = 0$ kvadrat tenglama ildizlarini topamiz.

$$\begin{cases} x_1 + x_2 = -5 \\ x_1 \cdot x_2 = 6 \end{cases} \Rightarrow x_1 = -2 \text{ ; } x_2 = -3 \Rightarrow x^2 + 5x + 6 = (x+2)(x+3).$$

5-misol. $(a^2 - b^2)x^2 - 4abx - (a^2 - b^2)$ ni ko'paytuvchilarga ajrating.

$A)$ $\big((a-b)x + (a+b)\big)\big((a+b)x - (a-b)\big)$

$B)$ $\big((a+b)x + (a-b)\big)\big((a-b)x - (a+b)\big)$

$C)$ $\big((a+b)x - (a-b)\big)\big((a-b)x + (a+b)\big)$

$D)$ $\big((a+b)x + (a-b)\big)\big((a-b)x + (a+b)\big)$

Yechish: $D = (-4ab)^2 + 4(a^2 - b^2)^2 = 4(4a^2b^2 + a^4 -$

$-2a^2b^2 + b^4) = 4(a^4 + 2a^2b^2 + b^4) = 4(a^2 + b^2)^2;$

$$x_{1,2}=\frac{4ab\pm2(a^2+b^2)}{2(a^2-b^2)}=\frac{2ab\pm(a^2+b^2)}{a^2-b^2};$$

$$x_1=\frac{2ab-(a^2+b^2)}{a^2-b^2}=-\frac{a^2-2ab+b^2}{a^2-b^2}=-\frac{(a-b)^2}{(a-b)(a+b)}=-\frac{a-b}{a+b};$$

$$x_2=\frac{2ab+(a^2+b^2)}{a^2-b^2}=\frac{a^2+2ab+b^2}{a^2-b^2}=\frac{(a+b)^2}{(a-b)(a+b)}=\frac{a+b}{a-b};$$

$$(a^2-b^2)x^2-4abx-(a^2-b^2)=(a-b)(a+b)\left(x+\frac{a-b}{a+b}\right)\cdot$$

$$\cdot\left(x-\frac{a+b}{a-b}\right)=(a+b)\left(x+\frac{a-b}{a+b}\right)(a-b)\left(x-\frac{a+b}{a-b}\right)=$$

$$=\big((a+b)x+(a-b)\big)\big((a-b)x-(a+b)\big).$$

Javob: $B)\ ((a+b)x+(a-b))((a-b)x-(a+b))$.

6-misol. $(x^2+x)^2-14(x^2+x)+24$ ko'phadni ratsional koeffitsiyentli ko'paytuvchilarga ajrating.

$A)\ (x^2+3)(x^2+8)$ $\qquad B)\ (x^2+x+12)(x^2+x-2)$

$C)\ (x^2+x-2)(x^2+x-12)$ $\qquad D)\ (x^2+x+2)(x^2+x+12)$

Yechish: $x^2+x=t$ belgilash kiritamiz va $t^2-14t+24$ kvadrat uchhadni hosil qilamiz. Uning ildizlarini Viyet teoremasi yordamida topamiz:

$$\begin{cases}t_1+t_2=14\\ t_1t_2=24\end{cases}\Rightarrow t_1=2;\ t_2=12$$

$$t^2-14t+24=(t-2)(t-12)=(x^2+x-2)(x^2+x-12).$$

Javob: $C)\ (x^2+x-2)(x^2+x-12)$.

7-misol. $\dfrac{x^2+6x-7}{x^2-7x+6}$ kasrni qisqartiring.

Yechish: Kasrning surat va mahrajini ko'paytuvchilarga ajratamiz. $x^2+6x-7=0$ tenglama ildizlarini topamiz.

$$\begin{cases}x_1+x_2=-6\\ x_1\cdot x_2=-7\end{cases}\Rightarrow x_1=1;\ x_2=-7\Rightarrow x^2+6x-7=(x-1)(x+7).$$

$x^2-7x+6=0$ tenglama ildizlarini topamiz.

$$\begin{cases} x_1 + x_2 = 7 \\ x_1 \cdot x_2 = 6 \end{cases} \Rightarrow x_1 = 1;\ x_2 = 6 \Rightarrow x^2 - 7x + 6 = (x-1)(x-6).$$

Endi topilganlarni o'rniga qo'yamiz:

$$\frac{x^2+6x-7}{x^2-7x+6} = \frac{(x-1)(x+7)}{(x-1)(x-6)} = \frac{x+7}{x-6}.$$

Javob: $\dfrac{x+7}{x-6}$.

8-misol. $\dfrac{2a^2+6ab+4b^2}{a^2+5ab+6b^2}$ ni soddalashtiring.

A) $\dfrac{a+2b}{2(a-b)}$ *B*) $\dfrac{2(a-b)}{a+2b}$ *C*) $\dfrac{a+2b}{a+3b}$ *D*) $\dfrac{2a+2b}{a+3b}$

Yechish: 1) $2a^2+6ab+4b^2 = 2(a^2+3ab+2b^2) = 2b^2\left(\left(\dfrac{a}{b}\right)^2 + 3\cdot\dfrac{a}{b} + 2\right)$;

$\left(\dfrac{a}{b}\right)^2 + 3\cdot\dfrac{a}{b} + 2$ ifodada $\dfrac{a}{b} = y$ belgilash kiritamiz va y^2+3y+2 kvadrat uchhadni hosil qilib, uni ko'paytuvchilarga ajratamiz.

$$\begin{cases} y_1 + y_2 = -3 \\ y_1 y_2 = -2 \end{cases} \Rightarrow y_1 = -1;\ y_2 = -2 \Rightarrow y^2 + 3y + 2 = (y+1)(y+2).$$

$$2b^2\left(\left(\frac{a}{b}\right)^2 + 3\cdot\frac{a}{b} + 2\right) = 2b^2\left(\frac{a}{b}+1\right)\left(\frac{a}{b}+2\right) = 2b^2\cdot$$

$$\cdot\frac{(a+b)(a+2b)}{b^2} = 2(a+b)(a+2b).$$

2) $a^2+5ab+6b^2 = b^2\left(\left(\dfrac{a}{b}\right)^2 + 5\cdot\dfrac{a}{b} + 6\right)$;

$\left(\dfrac{a}{b}\right)^2 + 5\cdot\dfrac{a}{b} + 6$ ifodada $\dfrac{a}{b} = y$ belgilash kiritamiz va y^2+5y+6 kvadrat uchhadni hosil qilib, uni ko'paytuvchilarga ajratamiz.

$$\begin{cases} y_1 + y_2 = -5 \\ y_1 \cdot y_2 = 6 \end{cases} \Rightarrow y_1 = -2;\ y_2 = -3 \Rightarrow y^2 + 5y + 6 = (y+2)(y+3).$$

$$b^2\left(\left(\frac{a}{b}\right)^2+5\cdot\frac{a}{b}+6\right)=b^2\left(\frac{a}{b}+2\right)\left(\frac{a}{b}+3\right)=b^2\cdot$$

$$\cdot\frac{(a+2b)(a+3b)}{b^2}=(a+2b)(a+3b).$$

3) $\frac{2(a+b)(a+2b)}{(a+2b)(a+3b)}=\frac{2a+2b}{a+3b}.$

2-usul. Kasrning surat va maxrajini a ga nisbatan kvadrat uchhad sifatida qarab; ko'paytuvchilarga ajratamiz:

1) $2a^2+6ab+4b^2=2(a^2+3b\cdot a+2b^2)$,Viyet teoremasidan

$$\begin{cases} a_1+a_2=-3b=-b+(-2b) \\ a_1\cdot a_2=2b^2=-b\cdot(-2b) \end{cases} \Rightarrow a_1=-b,\ a_2=-2b;$$

$$2(a^2+3b\cdot a+2b^2)=2(a+b)(a+2b).$$

2) $a^2+5ab+6b^2=a^2+5b\cdot a+6b^2$ Viyet teoremasidan

$$\begin{cases} a_1+a_2=-5b=-2b+(-3b) \\ a_1\cdot a_2=6b^2=-2b\cdot(-3b) \end{cases} \Rightarrow a_1=-2b,\ a_2=-3b;$$

$$a^2+5b\cdot a+6b^2=(a+2b)(a+3b).$$

3) $\frac{2a^2+6ab+4b^2}{a^2+5ab+6b^2}=\frac{2(a+b)(a+2b)}{(a+2b)(a+3b)}=\frac{2a+2b}{a+3b}.$

Javob: $D)\ \frac{2a+2b}{a+3b}.$

9-misol. $\frac{x^3-x^2-4x+4}{x^2+mx+6}$ kasr qisqarishi mumkin bo'lgan m ning eng katta va eng kichik qiymatlari farqini toping.

A) 18 *B*) 17 *C*) 12 *D*) 15

Yechish:

$$x^3-x^2-4x+4=x^2(x-1)-4(x-1)=(x-1)(x^2-4)=$$
$$=(x-1)(x-2)(x+2).$$

Berilgan kasr qisqarishi uchun x^2+mx+6 kvadrat uchhadning ildizlaridan biri 1; 2; -2 ga teng bo‘lishi lozim.

1) x^2+mx+6 kvadrat uchhadning ildizlaridan biri $x_1=1$ bo‘lsin. U holda

$$\begin{cases} x_1+x_2=-m \\ x_1x_2=6 \end{cases} \Rightarrow x_1=1 \Rightarrow x_2=6 \Rightarrow m=-(x_1+x_2)=-7.$$

2) x^2+mx+6 kvadrat uchhadning ildizlaridan biri $x_1=2$ bo‘lsin. U holda

$$\begin{cases} x_1+x_2=-m \\ x_1x_2=6 \end{cases} \Rightarrow x_1=2 \Rightarrow x_2=3 \Rightarrow m=-(x_1+x_2)=-5.$$

3) x^2+mx+6 kvadrat uchhadning ildizlaridan biri $x_1=-2$ bo‘lsin. U holda

$$\begin{cases} x_1+x_2=-m \\ x_1x_2=6 \end{cases} \Rightarrow x_1=-2 \Rightarrow x_2=-3 \Rightarrow m=-(x_1+x_2)=5.$$

Demak, m parametr $-7; -5; 5$ qiymatlarni qabul qilishi, eng katta va eng kichik qiymatlari farqi $5-(-7)=12$ ga teng bo‘lishini topamiz.

Javob: $C)\,12$.

Mustaqil yechish uchun misollar

8.1-misol. Kvadrat uchhadni chiziqli ko‘paytuvchilarga ajrating.

1. $2x^2+7x-4$	**2.** $6x^2+7x+1$	**3.** $2x^2-3x+5$
4. $16x^2+8x+1$	**5.** x^2+4x-5	**6.** $x^2+5x-24$
7. x^2+x-42	**8.** x^2+4x+4	**9.** x^2+x+1
10. $x^2+11x+28$	**11.** x^2+x-56	**12.** x^2-x-56

8.2-misol. Kasrni qisqartiring.

1. $\frac{x-8}{x^2-x-56}$ 2. $\frac{2x^2-3x-2}{4x^2-1}$ 3. $\frac{x^2-8x-9}{x^2+9x+8}$

4. $\frac{36+5x-x^2}{x^2-x-20}$ 5. $\frac{2x^2-x-1}{3x^2-5x+2}$ 6. $\frac{x^3+4x^2-21x}{x^4-9x^2}$

8.3-misol. Test topshiriqlari.

1. Kvadrat uchhadni chiziqli ko'paytuvchilarga ajrating: x^2-3x+2.

$A)\ (x-1)(x+2)$ $B)(x-2)(x+1)$ $C)\ (x-1)(x-2)$
$D)\ (x+1)(x+2)$ $E)\ (1-x)(x+2)$

2. Kvadrat uchhadni chiziqli ko'paytuvchilarga ajrating: x^2+x-2.

$A)\ (x-1)(x-2)$ $B)(x-1)(x+2)$ $C)\ (1-x)(x+2)$
$D)\ (x+1)(x-2)$ $E)\ (x+1)(x+2)$

3. Ushbu x^2-x-2 kvadrat uchhadni chiziqli ko'paytuvchilarga ajrating.

$A)\ (x-1)(x+2)$ $B)(x-1)(x-2)$ $C)\ (x+1)(x+2)$
$D)\ (x+1)(x-2)$ $E)\ (1-x)(x+2)$

4. x^2-x-6 kvadrat uchhadni chiziqli ko'paytuvchilarga ajrating.

$A)\ (x+3)(x-2)$ $B)\ (x-3)(x+2)$
$C)(x+3)(2-x)$ $D)\ (x+2)(3-x)$

5. x^2+x-12 kvadrat uchhadni chiziqli ko'paytuvchilarga ajrating.

$A)\ (x-3)(x+4)$ $B)\ (x+3)(x-4)$
$C)(x-3)(4-x)$ $D)\ (x+3)(4-x)$

6. Kasrni qisqartiring: $\frac{x^2-3x+2}{x^2-1}$.

$A)\ \frac{x+2}{x-1}$ $B)\ \frac{x+2}{x+1}$ $C)\ \frac{x-2}{x-1}$ $D)\ \frac{x-2}{x+1}$ $E)\ \frac{x+3}{x-1}$

7. Kasrni qisqartiring: $\frac{x^2-16}{x^2-5x+4}$.

$A)\ \frac{4+x}{1-x}$ $B)\ \frac{4-x}{x+1}$ $C)\ \frac{x+4}{x+1}$ $D)\ \frac{x-4}{x+1}$ $E)\ \frac{x+4}{x-1}$

8. Kasrni qisqartiring: $\frac{y^2-3y-4}{y^2-1}$.

$A)\ \frac{y+4}{y+1}$ $B)\ \frac{4-y}{y-1}$ $C)\ \frac{y+4}{y-1}$ $D)\ \frac{y-4}{y+1}$ $E)\ \frac{y-4}{y-1}$

9. Kasrni qisqartiring: $\frac{n^2-7n+6}{n^2-1}$.

$A)\ \frac{n+6}{n-1}$ $B)\ \frac{n-6}{n+1}$ $C)\ \frac{n+6}{n+1}$ $D)\ \frac{n-6}{n-1}$ $E)\ \frac{n-3}{n+1}$

10. $\frac{x^2-8x+15}{x^2+7x-30}$ kasrni qisqartiring.

$A)\ \frac{x-5}{x+10}$ $B)\ \frac{x+5}{x-10}$ $C)\ \frac{x+5}{x+10}$ $D)\ \frac{x-5}{x-10}$

11. $\frac{12x^2-x-1}{21x^2-19x+4}$ kasrni qisqartiring.

$A)\ \frac{4x+1}{7x-4}$ $B)\ \frac{4x+1}{7x+4}$ $C)\ \frac{4x-1}{7x+4}$ $D)\ \frac{4x+1}{4-7x}$

12. $\frac{a^2-5ab+6b^2}{a^2-2ab-8b^2}:\frac{a^2-2ab-3b^2}{a^2-3ab-4b^2}$ ni soddalashtiring.

$A)\ 1$ $B)\ \frac{a-2b}{a+3b}$ $C)\ \frac{a-3b}{a+2b}$ $D)\ \frac{a-2b}{a+2b}$

13. m ning qanday butun qiymatida quyidagi ifodani qisqartirish mumkin?

$$\frac{36+mx+x^2}{7+8x+x^2}$$

$A)\ 35$ $B)\ -37$ $C)\ -35$ $D)\ 37$

14. $\frac{(2p-q)^2+2q^2-3pq}{2p^{-1}+q^2}:\frac{4p^2-3pq}{2+pq^2}$ ifodani soddalashtiring va uning son qiymatini toping. $p=0,78$, $q=7/25$.

$A)\ 1$ $B)\ 0,25$ $C)\ 0,5$ $D)\ -1$

3-§. KVADRAT TENGLAMAGA KELTIRILADIGAN TENGLAMALAR

1. Bikvadrat tenglamalar

Ta'rif. $ax^4+bx^2+c=0$ *ko'rinishidagi tenglamalar bikvadrat tenglamalar deyiladi.* Bu yerda a, b, c – biror haqiqiy sonlar va $a \neq 0$.

Bikvadrat tenglamani yechish uchun $x^2=y \geq 0$ va $x^4=(x^2)^2=y^2$ almashtirishni bajarib, $ay^2+by+c=0$ kvadrat tenglamani yechishga, keyin $x^2=y_1$ va $x^2=y_2$ tenglamalarni yechishga keltiramiz.

y_1 va y_2 ildizlarning ishoralariga qarab tenglama ildizlari soni yoki ildizga ega emasligi aniqlanadi. Bunda 4 ta hol o'rinli.

1-hol. Agar $y_1 \geq 0$ va $y_2 \geq 0$ bo'lsa, u holda bikvadrat tenglama to'rtta haqiqiy ildizga ega bo'ladi:

a) $x^2=y_1 \Rightarrow x_{1,2}=\pm\sqrt{y_1}$ b) $x^2=y_2 \Rightarrow x_{3,4}=\pm\sqrt{y_2}$

2-hol. Agar $y_1<0$ va $y_2<0$ bo'lsa, u holda bikvadrat tenglama haqiqiy ildizlarga ega emas.

a) $x^2=y_1<0 \Rightarrow x\in\varnothing$ b) $x^2=y_2<0 \Rightarrow x\in\varnothing$.

3-hol. Agar $y_1 \geq 0$ va $y_2<0$ bo'lsa, u holda bikvadrat tenglama ikkita haqiqiy ildizga ega bo'ladi:

a) $x^2=y_1 \Rightarrow x_{1,2}=\pm\sqrt{y_1}$ b) $x^2=y_2<0 \Rightarrow x\in\varnothing$.

4-hol. Agar $y_1<0$ va $y_2 \geq 0$ bo'lsa, u holda bikvadrat tenglama ikkita haqiqiy ildizga ega bo'ladi:

a) $x^2=y_1<0 \Rightarrow x\in\varnothing$ b) $x^2=y_2 \Rightarrow x_{1,2}=\pm\sqrt{y_2}$.

1-misol. $5x^4-16x^2+3=0$ tenglamani yeching.

Yechish: $x^2 = y \geq 0$ almashtirishni bajargandan keyin $5y^2 - 16y + 3 = 0$ kvadrat tenglamani hosil qilamiz. $D = 256 - 60 = 196$ ni topib, ildizlarini aniqlaymiz:

$$y_{1,2} = \frac{16 \pm 14}{10} \Rightarrow y_1 = \frac{1}{5}; \quad y_2 = 3;$$

$$1)\ x^2 = \frac{1}{5} \Rightarrow x_{1,2} = \pm\frac{\sqrt{5}}{5} \quad 2)\ x^2 = 3 \Rightarrow x_{3,4} = \pm\sqrt{3}.$$

Javob: $\left\{\pm\frac{\sqrt{5}}{5}; \pm\sqrt{3}\right\}$.

2-misol. $x^4 + 3x^2 + 2 = 0$ tenglama ildizlarini toping.

Yechish: $x^2 = y \geq 0$ deb belgilaymiz va $y^2 + 3y + 2 = 0$ kvadrat tenglamani Viyet toeremasi yordamida yechamiz:

$$\begin{cases} y_1 + y_2 = -3 \\ y_1 \cdot y_2 = 2 \end{cases} \Rightarrow y_1 = -1; \ y_2 = -2.$$

Ko'rinib turibdiki, bikvadrat tenglama haqiqiy ildizga ega emas.

$$1)\ x^2 = -1 < 0 \Rightarrow x \in \varnothing \quad 2)\ x^2 = -2 < 0 \Rightarrow x \in \varnothing.$$

Javob: $x \in \varnothing$.

3-misol. $x^4 - 13x^2 + 36 = 0$ tenglama ildizlari yig'indisini toping.

A) 13 *B*) 5 *C*) 0 *D*) 36 *E*) 1

Yechish: $x^2 = y \geq 0$ almashtirishni bajargandan keyin $y^2 - 13y + 36 = 0$ kvadrat tenglamani hosil qilamiz. Viet teoremasidan

$$\begin{cases} y_1 + y_2 = 13 \\ y_1 \cdot y_2 = 36 \end{cases} \Rightarrow y_1 = 4; \ y_2 = 9$$

Bikvadrat tenglama to'rtta haqiqiy ildizga ega:

$$1)\ x^2 = 4 \Rightarrow x_{1,2} = \pm\sqrt{4} = \pm 2 \quad 2)\ x^2 = 9 \Rightarrow x_{3,4} = \pm\sqrt{9} = \pm 3.$$

$$x_1 + x_2 + x_3 + x_4 = -2 + 2 + (-3) + 3 = 0.$$

Bundan xulosa qilish mumkinki, agar bikvadrat tenglama ildizga ega bo'lsa, uning ildizlari yig'indisi 0 ga teng bo'lar ekan.

Javob: $C)\ 0$.

4-misol. $13x^4 - 5x^2 - 17 = 0$ tenglamaning barcha ildizlari yig'indisining barcha ildizlari ko'paytmasiga nisbatini toping.

$A)\ 1$ $\quad B)\ 0$ $\quad C)\ \frac{3}{2}$ $\quad D)\ \frac{2}{3}$ $\quad E)$ aniqlab bo'lmaydi

Yechish: $x^2 = y \geq 0$ belgilash kiritamiz va $13y^2 - 5y - 17 = 0$ kvadrat tenglamani yechamiz. $D = 25 + 884 = 909$ ni topib, ildizlarini aniqlaymiz:

$$y_{1,2} = \frac{5 \pm \sqrt{909}}{26} = \frac{5 \pm 3\sqrt{101}}{26} \Rightarrow y_1 = \frac{5 - 3\sqrt{101}}{26};\ y_2 = \frac{5 + 3\sqrt{101}}{26}.$$

1) $x^2 = \frac{5 - 3\sqrt{101}}{26} < 0$ bo'lib, haqiqiy ildizlarga ega emas.

2) $x^2 = \frac{5 + 3\sqrt{101}}{26} > 0 \Rightarrow x_{1,2} = \pm\sqrt{\frac{5 + 3\sqrt{101}}{26}}$.

Tenglamaning ildizlari qarama-qarshi ishorali, bundan $x_1 + x_2 = 0$ bo'ladi. Demak, $\frac{x_1 + x_2}{x_1 \cdot x_2} = 0$ ekanligi kelib chiqadi.

Javob: $B)\ 0$.

5-misol. Tenglamaning haqiqiy ildizlari ko'paytmasini toping:

$$y^4 - 2y^2 - 8 = 0.$$

$A)\ 4$ $\quad B)\ -16$ $\quad C)\ 16$ $\quad D)\ -4$ $\quad E)\ 64$

Yechish: $y^2 = t$ deb belgilaymiz va $t^2 - 2t - 8 = 0$ kvadrat tenglamani hosil qilib, uning ildizlarini Viyet teoremasi yordamida topamiz:

$$\begin{cases} t_1 + t_2 = 2 \\ t_1 \cdot t_2 = -8 \end{cases} \Rightarrow t_1 = -2;\ t_2 = 4.$$

1) $y^2 = -2 < 0$ bo'lib, haqiqiy ildizlarga ega emas.

2) $y^2=4 \Rightarrow y_{1,2}=\pm 2$. Demak, $y_1 \cdot y_2=-2\cdot 2=-4$.

Javob: $D)\ -4$.

Mustaqil yechish uchun misollar

1.1-misol. Bikvadrat tenglamalarning haqiqiy ildizlarini toping.

1. $2x^4-5x^2+2=0$

2. $2x^4-19x^2+9=0$

3. $x^4-50x^2+49=0$

4. $x^4-25x^2+144=0$

5. $x^4-29x^2+100=0$

6. $x^4-5x^2+4=0$

7. $x^4-11x^2+18=0$

8. $x^4-3x^2-4=0$

9. $x^4+x^2-20=0$

10. $x^4-5x^2+7=0$.

1.2-misol. Test topshiriqlari.

1. Tenglamaning ildizlari yig'indisini toping: $x^4-17x^2+16=0$.

A) 17 *B*) 0 *C*) -16 *D*) -17 *E*) 4

2. Tenglamaning eng katta va kichik ildizlari ayirmasini toping:

$$x^4-10x^2+9=0.$$

A) 1 *B*) 8 *C*) 2 *D*) 4 *E*) 6

3. Tenglamaning ildizlari yig'indisini toping:

$2x^4-7x^2+2=0$.

A) 7 *B*) 3,5 *C*) 0 *D*) 2 *E*) aniqlab bo'lmaydi

4. Tenglamaning barcha ildizlari yig'indisini toping:

$$5x^4-8x^2+1=0.$$

A) 1,6 *B*) 0 *C*) 8 *D*) $\frac{1}{5}$ *E*) aniqlab bo'lmaydi

5. $3x^4-5x^2+2=0$ tenglamaning eng kichik va eng katta ildizlari ayirmasini toping.

A) 2 *B*) $\frac{2\sqrt{6}}{3}$ *C*) $-\frac{2\sqrt{6}}{3}$ *D*) -2 *E*) $\frac{5}{3}$

6. $x^4-4x^2+3=0$ tenglamani yeching.

$A)\pm 1$ $B)\pm 1;\pm\sqrt{3}$ $C)\pm\sqrt{2}$ $D)\pm 2$

7. $a^4-3a^2-4=0$ tenglamaning haqiqiy ildizlari ko'paytmasini toping.

$A)\,4$ $B)\,3$ $C)-3$ $D)-4$

8. $x^4-(\sqrt{5}+\sqrt{3})x+\sqrt{15}=0$ tenglamaning ildizlari sonini toping.

$A)\,2$ $B)\,4$ $C)\,1$ $D)\,0$ $E)\,3$

2. Tenglamalarni ko'paytuvchilarga ajratib yechish

Bu turdagi tenglamalarni yechish uchun uning chap qismini ko'paytuvchilarga ajratamiz va har bir ko'paytuvchini nolga tenglab tenglamani yechamiz.

1-misol. $x^3-3x^2-x+3=0$ tenglamani yeching.

Yechish: Tenglamaning chap qismini ko'paytuvchilarga ajratamiz:

$$x^3-3x^2-x+3=x^2(x-3)-(x-3)=(x-3)(x^2-1)=$$
$$=(x-3)(x-1)(x+1).$$

Bundan

$$(x-3)(x-1)(x+1)=0.$$

1) $x-3=0\Rightarrow x_1=3$ 2) $x-1=0\Rightarrow x_2=1$

3) $x+1=0\Rightarrow x_3=-1$

Javob: $\{-1;1;3\}$.

2-misol. $(x+1)(x^2+2)+(x+2)(x^2+1)=2$ tenglamaning haqiqiy ildizlarini toping.

Yechish: Tenglamaning chap qismidagi qavslarni ochib, ko'paytuvchilarga ajratamiz:

$$(x+1)(x^2+2)+(x+2)(x^2+1)=2$$
$$x^3+2x+x^2+2+x^3+x+2x^2+2=2$$
$$2x^3+3x^2+3x+2=0$$
$$2x^3+2+3x^2+3x=0$$
$$2(x+1)(x^2-x+1)+3x(x+1)=0$$
$$(x+1)(2x^2+x+2)=0$$

1) $x+1=0 \Rightarrow x_1=-1$

2) $2x^2+x+2=0$. $D=1-16=-15<0$, haqiqiy ildizlarga ega emas.

Javob: $\{-1\}$.

3-misol. $9x^3-13x-6=0$ tenglamani yeching.

Yechish: Tenglamaning ikkala tomonini 3 ga ko'paytiramiz.

$$27x^3-39x-18=0$$
$$(3x)^3-13\cdot(3x)-18=0$$

$3x=t$ deb belgilash kiritib, $t^3-13t-18=0$ tenglamani hosil qilamiz.

Tenglamaning chap tomonini ko'paytuvchilarga ajratamiz:

$$t^3-13t-18=t^3+8-13t-26=(t+2)(t^2-2t+4)-13(t+2)=$$
$$=(t+2)(t^2-2t-9)$$
$$(t+2)(t^2-2t-9)=0.$$

1) $t+2=0 \Rightarrow t_1=-2 \Rightarrow 3x=-2 \Rightarrow x_1=-\frac{2}{3}$.

2) $t^2-2t-9=0$, $D=4+36=40$.

$$t_{1,2}=\frac{2\pm 2\sqrt{10}}{2}=1\pm\sqrt{10} \Rightarrow 3x=1\pm\sqrt{10} \Rightarrow x_{2,3}=\frac{1\pm\sqrt{10}}{3}.$$

Javob: $\left\{-\frac{2}{3}; \frac{1\pm\sqrt{10}}{3}\right\}$.

4-misol. $x^4+5x^3+15x+75=0$ tenglamani yeching.

Yechish: Tenglamaning chap qismini ko‘paytuvchilarga ajratamiz:

$$x^4+5x^3+15x+75=x^3(x+5)+15(x+5)=(x+5)(x^3+15);$$

$$(x+5)(x+\sqrt[3]{15})(x^2-\sqrt[3]{15}x+\sqrt[3]{225})=0.$$

Har bir qavs ichidagi ifodani 0 ga tenglaymiz:

1) $x+5=0 \Rightarrow x_1=-5$ 2) $x+\sqrt[3]{15}=0 \Rightarrow x_2=-\sqrt[3]{15}$

3) $x^2-\sqrt[3]{15}x+\sqrt[3]{225}=0$. $D=\sqrt[3]{225}-4\sqrt[3]{225}=-3\sqrt[3]{225}<0$, tenglama haqiqiy ildizlarga ega emas.

Javob: $\{-5;-\sqrt[3]{15}\}$.

5-misol. $x^5+x^4-2x^3-2x^2+x+1=0$ tenglamani yeching.

Yechish: Ko‘paytuvchilarga ajratamiz:

$$x^5+x^4-2x^3-2x^2+x+1=x^4(x+1)-2x^2(x+1)+(x+1)=$$
$$=(x+1)(x^4-2x^2+1)=(x+1)(x^2-1)^2=(x+1)[(x-1)(x+1)]^2=$$
$$=(x+1)^3(x-1)^2;$$

Demak, $(x+1)^3(x-1)^2=0$.

1) $(x+1)^3=0 \Rightarrow x+1=0 \Rightarrow x_1=-1$;

2) $(x-1)^2=0 \Rightarrow x-1=0 \Rightarrow x_2=1$.

Javob: $\{-1;\ 1\}$.

6-misol. Agar $x^4+6x^3+13x^2+12x+\frac{15}{4}=0$ bo‘lsa, $2x+3$ ifodaning qiymatini toping.

Yechish: Tenglamadagi qo‘shiluvchilarni guruhlaymiz:

$$(x^4+6x^3+9x^2)+(4x^2+12x)+\frac{15}{4}=0 \Leftrightarrow (x^2+3x)^2+4(x^2+3x)+\frac{15}{4}=0.$$

Oxirgi tenglamadan $x^2+3x=y$ belgilash kiritib, $y^2+4y+\frac{15}{4}=0$ kvadrat tenglamani hosil qilamiz va $y_1=-2{,}5;\ y_2=-1{,}5$ ni topamiz. Bundan

$$\left[\begin{array}{l} x^2+3x=-2{,}5 \\ x^2+3x=-1{,}5 \end{array}\right. \Rightarrow \left[\begin{array}{l} x^2+3x+2{,}5=0 \\ x^2+3x+1{,}5=0 \end{array}\right. \Rightarrow \left[\begin{array}{l} x\in\varnothing \\ x_{1,2}=\dfrac{-3\pm\sqrt{3}}{2}. \end{array}\right.$$

Qidirilayotgan qiymat $2x+3=2\cdot\frac{-3\pm\sqrt{3}}{2}+3=-3\pm\sqrt{3}+3=\pm\sqrt{3}$ bo'ladi.

Javob: $\pm\sqrt{3}$.

7-misol. $x^2(x-1)^2+(x-2)^3=76$ tenglamani yeching.

Yechish: Berilgan tenglamani quyidagicha almashtiramiz:

$$x^2(x-1)^2-7^2+(x-2)^3-3^3=0.$$

$a^2-b^2=(a+b)(a-b)$ va $a^3-b^3=(a-b)(a^2+ab+b^2)$ formulalarni qo'llaymiz. Natijada

$$x^2(x-1)^2-7^2=[x(x-1)]^2-7^2=(x^2-x-7)(x^2-x+7);$$

$$(x-2)^3-3^3=(x-5)[((x-2)^2+3(x-2)+9)]=(x-5)(x^2-x+7)$$

larni hosil qilamiz. Topilganlarni tenglamaga qo'yamiz va

$$(x^2-x-7)(x^2-x+7)+(x-5)(x^2-x+7)=0$$

$$(x^2-x+7)(x^2-12)=0.$$

1) $x^2-x+7>0$, $\forall x\in R$ da, chunki $D<0$. Shuning uchun $x^2-x+7=0$ tenglama haqiqiy ildizlarga ega emas.

2) $x^2-12=0\Rightarrow x_{1,2}=\pm\sqrt{12}=\pm2\sqrt{3}$.

Javob: $\{-2\sqrt{3};\ 2\sqrt{3}\}$.

8-misol. $(x^2+2x-1)^2+2x^2+3x=3$ tenglamani yeching.

Yechish: Tenglamani quyidagi teng kuchli tenglamalarga almashtiramiz:

$$(x^2+2x-1)^2+2x^2+3x=3$$

$$(x^2+2x-1)^2+2(x^2+2x-1)+1=x+2$$

$$[(x^2+2x-1)+1]^2=x+2$$

$$[x(x+2)]^2-(x+2)=0$$

$$(x+2)[x^2(x+2)-1]=0$$

$$(x+2)(x^3+2x^2-1)=0$$

$$(x+2)(x+1)(x^2+x-1)=0$$

1) $x+2=0\Rightarrow x_1=-2$ 2) $x+1=0\Rightarrow x_2=-1$

3) $x^2+x-1=0 \Rightarrow x_{3,4}=\frac{-1\pm\sqrt{5}}{2}$.

Javob: $\left\{-2;\,-1;\,\frac{-1-\sqrt{5}}{2};\frac{-1+\sqrt{5}}{2}\right\}$.

9-misol. $5(5+x)(5+2x)(5+3x)=3(3+x)(3+2x)(3+3x)$ tenglamani yeching.

Yechish: Tenglamaning chap va o'ng qismlarini shakl almashtiramiz:

$$5(5+3x)(5+x)(5+2x)=(15x+25)(2x^2+15x+25)=$$
$$=(15x+25)^2+2x^2(15x+25),$$
$$3(3+3x)(3+x)(3+2x)=(9x+9)(2x^2+9x+9)=$$
$$=(9x+9)^2+2x^2(9x+9).$$

Topilganlarni tenglamaga qo'yamiz:

$$(15x+25)^2+2x^2(15x+25)=(9x+9)^2+2x^2(9x+9)$$
$$(15x+25)^2-(9x+9)^2+2x^2[(15x+25)-(9x+9)]=0$$
$$(6x+16)(24x+34)+2x^2(6x+16)=0$$
$$(6x+16)(2x^2+24x+34)=0$$
$$(3x+8)(x^2+12x+17)=0$$

Oxirgi tenglamadan

$$\left[\begin{array}{l}3x+8=0\\ x^2+12x+17=0\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x_1=-\frac{8}{3}\\ x_{2,3}=-6\pm\sqrt{19}\end{array}\right.$$

ni topamiz.

Javob: $\left\{-\frac{8}{3};\,-6\pm\sqrt{19}\right\}$.

Mustaqil yechish uchun misollar

2.1-misol. Tenglamalarni ko'paytuvchilarga ajratish usuli bilan yeching.

1. $x^3+x^2-x-1=0$ **2.** $x^3+4x^2+4x+1=0$

3. $x^3-4x^2-x+4=0$ **4.** $x^3-6x^2+11x-6=0$

5. $x^3 - 7x^2 + 16x - 12 = 0$

6. $x^3 + 3x^2 - 4 = 0$

7. $x^3 - 3x + 2 = 0$

8. $x^3 + x^2 - 2 = 0$

9. $25x^3 - 34x - 15 = 0$

10. $27x^3 + 9x^2 - 48x + 20 = 0$

11. $x^3 + x + 3\sqrt{2} = 0$

12. $(x-1)^3 + (2x+3)^3 = 27x^3 + 8$

13. $(1+x)(1+2x)(1+3x) = 4(4+x)(4+2x)(4+3x)$

14. $(3x+5)^2 + (x+6)^3 = 4x^2 + 1$

15. $(x-5)^2 + (x-4)^3 + (x-3)^4 = 2$

2.2-misol. Tenglamaning haqiqiy ildizlarini toping.

1. $x^4 + x^3 + x + 1 = 0$

2. $x^4 + x^3 - x - 1 = 0$

3. $x^4 - 6x^3 + 7x^2 + 6x - 8 = 0$

4. $4x^4 + 4x^3 + 3x^2 - x - 1 = 0$

5. $6x^4 - 9x^3 - 8x^2 + 3x + 2 = 0$

6. $(x^2 + x + 1)^2 = 3(x^4 + x^2 + 1)$

7. $x^5 - 2x^3 - 3x^2 + 6 = 0$

8. $x^5 - 2x^4 - 6x^3 + 12x^2 + x - 2 = 0$

9. $4x^4 + 12x^3 + 5x^2 - 6x - 15 = 0$

3. Kubik tenglama uchun Viyet teoremasi

Ta'rif. $ax^3 + bx^2 + cx + d = 0$ *ko'rinishdagi tenglamalar kubik tenglamalar deyiladi. Bu yerda* a, b, c, d *lar – haqiqiy sonlar.*

Teorema (Viyet teoremasi). x_1, x_2 va x_3 sonlari $ax^3 + bx^2 + cx + d = 0$ kubik tenglamaning ildizlari bo'lsa, u holda

$$\begin{cases} x_1 + x_2 + x_3 = -\dfrac{b}{a} \\ x_1 \cdot x_2 + x_2 \cdot x_3 + x_1 \cdot x_3 = \dfrac{c}{a} \\ x_1 + x_2 + x_3 = -\dfrac{d}{a} \end{cases} \quad (1)$$

tenglik o'rinli.

Isbot. x_1, x_2 va x_3 sonlari $ax^3 + bx^2 + cx + d = 0$ kubik tenglamaning ildizlari bo'lsa, $ax^3 + bx^2 + cx + d == a(x - x_1)(x - x_2)(x - x_3)$ tenglik o'rinli. Qavslarni ochamiz:

$$ax^3 + bx^2 + cx + d = a[x^2 - (x_1 + x_2)x + x_1x_2](x - x_3)$$

$$ax^3 + bx^2 + cx + d = a(x^3 - (x_1 + x_2)x^2 + x_1x_2x - x_3x^2 -$$

$$-(x_1 + x_2)x_3x - x_1x_2x_3)ax^3 + bx^2 + cx + d = ax^3 - a(x_1 + x_2 + x_3)x^2 +$$

$$+a(x_1x_2 + x_2x_3 + x_1x_3)x - ax_1x_2x_3$$

$$\begin{cases} -a(x_1 + x_2 + x_3) = b \\ a(x_1x_2 + x_2x_3 + x_1x_3) = c \\ -ax_1x_2x_3 = d \end{cases} \Rightarrow \begin{cases} x_1 + x_2 + x_3 = -\dfrac{b}{a} \\ x_1x_2 + x_2x_3 + x_1x_3 = \dfrac{c}{a} \\ x_1x_2x_3 = -\dfrac{d}{a} \end{cases}$$

Isbot tugadi.

Endi keltirilgan kubik tenglama uchun Viyet teoremasini ko'rib chiqamiz.

Ta'rif. $x^3 + px^2 + qx + r = 0$ *ko'rinishdagi tenglamalar keltirilgan kubik tenglamalar deyiladi.*

Teorema (Viyet teoremasi). x_1, x_2 va x_3 sonlari $x^3 + px^2 + qx + r = 0$ keltirilgan kubik tenglamaning ildizlari bo'lsa, u holda

$$\begin{cases} x_1 + x_2 + x_3 = -p \\ x_1 \cdot x_2 + x_2 \cdot x_3 + x_1 \cdot x_3 = q \\ x_1 + x_2 + x_3 = -r \end{cases} \quad (2)$$

tenglik o'rinli.

Haqiqiy sonlar to'plamida yuqoridagi kubik tenglamaga Viyet teoremasi o'rinli bo'lishi uchun tenglama uchta haqiqiy ildizga ega bo'lishi lozim. Aks holda formula o'rinli bo'lmaydi.

1-misol. $7x^3 - 3x^2 - 3x - 1 = 0$ tenglamaning barcha haqiqiy ildizlari yig'indisini toping.

$A)\ 7 \qquad B)\ -\dfrac{3}{7} \qquad C)\ \dfrac{3}{7} \qquad D)\ 1$

Yechish: Yuqoridagi (1) formulaga asosan tenglama ildizlari yig'indisi $x_1 + x_2 + x_3 = \frac{3}{7}$ va ildizlari ko'paytmasi $x_1 \cdot x_2 \cdot x_3 = \frac{1}{7}$ ga teng bo'lishi kerak.

Lekin kubik tenglama nechta haqiqiy ildizga ega ekanligini bilmaymiz. Shuning uchun tenglamani ko'paytuvchilarga ajratish usuli bilan yechib ko'raylik.

$$7x^3 - 3x^2 - 3x - 1 = 7x^3 - 7x^2 + 4x^2 - 4x + x - 1 = 7x^2(x-1) +$$
$$+4x(x-1) + (x-1) = (x-1)(7x^2 + 4x + 1)$$

Natijada $(x-1)(7x^2 + 4x + 1) = 0$ tenglamaning ildizlarini topamiz.

1) $x - 1 = 0 \Rightarrow x_1 = 1$

2) $7x^2 + 4x + 1 = 0$ bo'lib, bu kvadrat tenglamaning diskriminanti $D = -24 < 0$ bo'lib, haqiqiy ildizlarga ega emasligini bildiradi.

Demak, berilgan kubik tenglama yagona $x = 1$ haqiqiy ildizga ega va uning ildizlari yig'indisi ham 1 ga teng bo'ladi.

Javob: $D)\,1$.

2-misol. $x^3 + 5x^2 - 4x - 20 = 0$ tenglama ildizlari ko'paytmasini toping.

$A)\ -10$ $B)\ 20$ $C)\ -4$ $D)\ -20$ $E)\ 16$

Yechish: Tenglamaning chap tomonini ko'paytuvchilarga ajratamiz:

$$x^3 + 5x^2 - 4x - 20 = x^2(x+5) - 4(x+5) =$$
$$= (x+5)(x^2 - 4) = (x+5)(x-2)(x+2)$$
$$(x+5)(x-2)(x+2) = 0.$$

Har bir ko'paytuvchini 0 ga tenglaymiz.

1) $x + 5 = 0 \Rightarrow x_1 = -5$ 2) $x - 2 = 0 \Rightarrow x_2 = 2$

3) $x + 2 = 0 \Rightarrow x_3 = -2$

$$x_1 \cdot x_2 \cdot x_3 = -5 \cdot 2 \cdot (-2) = 20.$$

Javob: $B)20$.

3-misol. $x^3-13x+12=0$ tenglama haqiqiy ildizlarining o'rta arifmetigini toping.

$A)\ 2\frac{2}{3}$ $B)\ 1\frac{1}{3}$ $C)\ 0$ $D)\ -\frac{1}{2}$ $E)\ -1\frac{1}{3}$

Yechish: $x^3-13x+12=x^3-x-12x+12=x(x-1)(x+1)-$ $-12(x-1)=(x-1)(x^2+x-12)=(x-1)(x-3)(x+4)$.

Demak, $(x-1)(x-3)(x+4)=0$.

Bundan $x_1=1$, $x_2=3$, $x_3=-4$ ekanligini topamiz. Demak, tenglama ildizlarining o'rta arifmetigi $\frac{x_1+x_2+x_3}{3}=$ $=\frac{1+3+(-4)}{3}=0$ ga teng bo'ladi.

Javob: $C)0$.

4-misol. $10x^3-3x^2-2x+1=0$ tenglamaning barcha haqiqiy ildizlari yig'indisini toping.

$A)\ 0{,}3$ $B)\ -2$ $C)\ 2$ $D)\ -0{,}5$

Yechish: Biz berilgan tenglamaning nechta haqiqiy ildizga ega ekanligini bilmaymiz. Shuning uchun bu tenglamaning ildizlarini aniqlaymiz. Tenglamaning ikkala tomonini 100 ga ko'paytiramiz. Natijada

$$10x^3-3x^2-2x+1=0/\times 100$$

$$1000x^3-300x^2-200x+100=0$$

$$(10x)^3-3\cdot(10x)^2-2\cdot 10x+100=0$$

$10x=t$ deb belgilash kiritamiz. Natijada

$$t^3-3t^2-20t+100=0$$

$$t^3+5t^2-8t^2-40t+20t+100=0$$

$$t^2(t+5)-8t(t+5)+20(t+5)=0$$

$$(t+5)(t^2-8t+20)=0$$

1) $t+5=0 \Rightarrow t_1=-5 \Rightarrow 10x=-5 \Rightarrow x_1=-0{,}5$.

2) $t^2-8t+20=0$ kvadrat tenglama haqiqiy ildizga ega emas, chunki uning diskriminanti $D=-16<0$.

Demak, kubik tenglamaning barcha haqiqiy ildizlari yig'indisi $-0{,}5$ ga teng.

Javob: $D)\ -0{,}5$.

Mustaqil yechish uchun misollar

3.1-misol. Test topshiriqlari.

1. Tenglamaning ildizlari yig'indisini toping:
$x^3+2x^2-9x-18=0$.

$A)\ 9 \quad B)\ -2 \quad C)\ 6 \quad D)\ -18 \quad E)\ 2$

2. Tenglamaning ildizlari ko'paytmasini toping:
$x^3-3x^2-4x+12=0$.

$A)\ 6 \quad B)\ -4 \quad C)\ 12 \quad D)\ -12 \quad E)\ 24$

3. Tenglama ildizlarining yig'indisini toping:
$x^3+3x^2-4x-12=0$.

$A)\ -3 \quad B)\ -7 \quad C)\ 4 \quad D)\ 12 \quad E)\ 0$

4. $x^3-3x^2-2x+6=0$ tenglamaning ildizlari ko'paytmasini toping.

$A)\ 3 \quad B)\ -6 \quad C)\ 6 \quad D)\ -3 \quad E)\ 1$

5. $x^3-7x-6=0$ tenglamaning barcha haqiqiy ildizlari o'rta geometrigini toping.

$A)\ \sqrt{6} \quad B)\ \sqrt[3]{6} \quad C)\ -\sqrt[3]{6} \quad D)\ 2\sqrt{2} \quad E)\ -2$

6. $x^3-5x^2-2x+10=0$ tenglama ildizlarining ko'paytmasini toping.

$A)\ 10 \quad B)\ -10 \quad C)\ 20 \quad D)\ 5 \quad E)\ -5$

7. $x^3+3x^2-4=2x+2$ tenglamaning ildizlari ko'paytmasini toping.

$A)\ 6 \quad B)\ -4 \quad C)\ 12 \quad D)\ -12 \quad E)\ 24$

8. $x^3-6x^2+12=3x^2+2x-6$ tenglamaning ildizlari yig'indisini toping.

$A)\ 9 \quad B)\ -2 \quad C)\ 6 \quad D)\ -18 \quad E)\ 2$

9. $x^3+2x^2+7=8x+23$ tenglamaning ildizlari ko'paytmasini toping.

$A)\ -10 \quad B)\ 20 \quad C)\ -4 \quad D)\ -20 \quad E)\ 16$

10. $6x^3-11x^2-3x+8=0$ tenglamaning barcha haqiqiy ildizlari yig'indisini toping.

$A)\ \frac{5}{4} \quad B)\ \frac{4}{5} \quad C)\ \frac{11}{6} \quad D)\ \frac{3}{4}$

11. $2x^3-x^2+x+1=0$ tenglamaning barcha haqiqiy ildizlari yig'indisini toping.

$A)\ \frac{1}{2} \quad B)\ -\frac{1}{2} \quad C)\ -1 \quad D)\ 1$

12. $x^3+9x^2+23x+13=-2$ tenglamaning barcha haqiqiy ildizlari yig'indisini toping.

$A)\ 7 \quad B)\ -3 \quad C)\ -5 \quad D)\ -9$

4. Uch hadli tenglamalar

Ta'rif. $ax^{2n}+bx^n+c=0(a\neq 0, n\geq 2, n\in N)$ (1) *ko'rinishidagi tenglamalar uch hadli tenglamalar deyiladi.*

$n=2$ da uch hadli tenglama *bikvadrat tenglama* deb ataladi.

Uch hadli tenglamani yechish uchun $x^n=y$ deb belgilash kiritiladi va

$$ay^2+by+c=0 \quad (2)$$

kvadrat tenglama hosil qilinadi. (2) kvadrat tenglamaning ildizlariga qarab (1) tenglamaning ildizlari topiladi.

1-hol. Agar (2) tenglama ikkita turli y_1 va y_2 ildizlarga ega bo'lsa, (1) tenglamaning ildizlari quyidagi tenglamalar juftligidan topiladi:

$$\left[\begin{array}{l} x^n=y_1 \\ x^n=y_2 \end{array}\right.$$

2-hol. Agar (2) tenglama yagona $y_1 = y_2$ ildizlarga ega bo'lsa, (1) tenglamaning ildizlari $x^n = y_1$ tenglikdan topiladi.

3-hol. Agar $ay^2 + by + c = 0$ tenglama yechimga ega bo'lmasa, uch hadli tenglama ham yechimga ega bo'lmaydi.

1-misol. $x^8 - 17x^4 + 16 = 0$ tenglamani yeching.

Yechish: $x^4 = y \geq 0$ belgilash kiritamiz va $y^2 - 17y + 16 = 0$ kvadrat tenglamani hosil qilamiz. Viyet teoremasidan

$$\begin{cases} y_1 + y_2 = 17 \\ y_1 \cdot y_2 = 16 \end{cases} \Rightarrow y_1 = 1;\ y_2 = 16.$$

Natijada quyidagi yechimlarni hosil qilamiz:

$$\left[\begin{array}{l} x^4 = 1 \\ x^4 = 16 \end{array}\right. \Rightarrow \left[\begin{array}{l} x = \pm\sqrt[4]{1} \\ x = \pm\sqrt[4]{16} \end{array}\right. \Rightarrow \left[\begin{array}{l} x_{1,2} = \pm 1 \\ x_{3,4} = \pm 2 \end{array}\right.$$

Javob: $\{\pm 1; \pm 2\}$.

Umumiy holda uch hadli tenglamalar

$$a \cdot [f(x)]^2 + b \cdot [f(x)] + c = 0,\ a \neq 0 \quad (3)$$

ko'rinishida beriladi. Bu yerda $f(x)$ – berilgan funksiya bo'lib, tenglama $f(x) = y$ belgilash kiritish yordamida yechiladi.

2-misol. $(x^2 + x - 2)(x^2 + x - 3) = 12$ tenglamani yeching.

Yechish: $x^2 + x = y$ belgilash kiritib, $(y-2)(y-3) = 12$ ni hosil qilamiz. Hosil bo'lgan $y^2 - 5y - 6 = 0$ tenglamaning ildizlarini Viyet teoremasidan topamiz:

$$\begin{cases} y_1 + y_2 = 5 \\ y_1 \cdot y_2 = -6 \end{cases} \Rightarrow y_1 = -1;\ y_2 = 6.$$

1) $x^2 + x = -1 \Rightarrow x^2 + x + 1 = 0$, $D = 1 - 4 = -3 < 0$. Haqiqiy ildizlarga ega emas.

2) $x^2 + x = 6 \Rightarrow x^2 + x - 6 = 0$. Bundan $x_1 = -3;\ x_2 = 2$ ekanligini topamiz.

Javob: $\{-3; 2\}$.

3-misol. $(x^2 + 2x)^2 - 4(x+1)^2 + 7 = 0$ tenglamani yeching.

Yechish: $(x^2+2x)^2-4(x^2+2x+1)+7=0 \Rightarrow (x^2+2x)^2-4(x^2+2x)+3=0$.

$x^2+2x=y$ belgilash kiritib, $y^2-4y+3=0$ kvadrat tenglamani hosil qilamiz. Viyet teoremasidan

$$\begin{cases} y_1+y_2=4 \\ y_1\cdot y_2=3 \end{cases} \Rightarrow y_1=1;\ y_2=3$$

ni topamiz. Berilgan tenglama quyidagi tenglamalarga teng kuchli bo'ladi:

$$\left[\begin{array}{l} x^2+2x=1 \\ x^2+2x=3 \end{array}\right.$$

1) $x^2+2x=3 \Rightarrow x^2+2x-3=0$. Viyet teoremasidan

$$\begin{cases} x_1+x_2=-2 \\ x_1\cdot x_2=-3 \end{cases} \Rightarrow x_1=-3;\ x_2=1$$

2) $x^2+2x=1 \Rightarrow x^2+2x-1=0$, $D=4+4=8>0$,

$x_{3,4}=\dfrac{-2\pm2\sqrt{2}}{2}=-1\pm\sqrt{2}$.

Javob: $\{-1\pm\sqrt{2};-3;1\}$.

4-misol. $(6x+5)^2(3x+2)(x+1)=35$ tenglamani yeching.

Yechish: Tenglamaning ikkala tomonini 12 ga ko'paytiramiz:

$$(6x+5)^2\cdot2\cdot(3x+2)\cdot6\cdot(x+1)=35\cdot12$$
$$(6x+5)^2(6x+4)(6x+6)=420$$

$6x+5=y$ deb belgilash kiritib, quyidagi bikvadrat tenglamani hosil qilamiz:

$$y^2(y-1)(y+1)=420$$
$$y^4-y^2-420=0$$

$y^2=t\geq0$ deb belgilash kiritib, hosil bo'lgan $t^2-t-420=0$ kvadrat tenglamaning ildizlarini Viyet teoremasidan topamiz:

$$\begin{cases} t_1 + t_2 = 1 \\ t_1 \cdot t_2 = -420 \end{cases} \Rightarrow t_1 = -20;\ t_2 = 21.$$

Bundan

$$\left[\begin{array}{l} y^2 = -20 < 0 \\ y^2 = 21 \end{array}\right. \Rightarrow \left[\begin{array}{l} y \in \varnothing \\ y_{1,2} = \pm\sqrt{21} \end{array}\right. \Rightarrow 6x + 5 = \pm\sqrt{21} \Rightarrow x_{1,2} = \frac{-5 \pm \sqrt{21}}{6}.$$

Javob: $\left\{\frac{-5 \pm \sqrt{21}}{6}\right\}$.

5-misol. $x = 1 - 5(1 - 5x^2)^2$ tenglamani yeching.

Yechish: $1 - 5x^2 = y$ deb belgilash kiritib, quyidagi tenglamalar sistemasini hosil qilamiz:

$$\begin{cases} x = 1 - 5y^2 \\ y = 1 - 5x^2 \end{cases}$$

Sistemaning 1-tenglamasidan 2-tenglamasini hadma-had ayiramiz:

$$x - y = 5(x^2 - y^2) \Leftrightarrow (x - y) - 5(x - y)(x + y) =$$
$$= 0 \Leftrightarrow (x - y)(1 - 5x - 5y) = 0$$

Demak, $x - y = 0$ yoki $1 - 5x - 5y = 0$ bo'ladi.

1) $x - y = 0 \Rightarrow x - (1 - 5x^2) = 0 \Rightarrow 5x^2 + x - 1 = 0 \Rightarrow x_{1,2} = \frac{-1 \pm \sqrt{21}}{10}$.

2) $1 - 5x - 5y = 0 \Rightarrow 1 - 5x - 5(1 - 5x^2) = 0 \Rightarrow 25x^2 - 5x - 4 = 0$, $D = 25 + 400 = 425$,

$$x_{3,4} = \frac{5 \pm \sqrt{425}}{50} = \frac{5 \pm 5\sqrt{17}}{50} = \frac{1 \pm \sqrt{17}}{10}.$$

Javob: $\left\{\frac{-1 \pm \sqrt{21}}{10}; \frac{1 \pm \sqrt{17}}{10}\right\}$.

6-misol. $(x^2 + 3x - 4)^3 + (2x^2 - 5x + 3)^3 = (3x^2 - 2x - 1)^3$ tenglamani yeching.

Yechish: $x^2 + 3x - 4 = a$ va $2x^2 - 5x + 3 = b$ belgilashlar kiritamiz. Bundan $a + b = (x^2 + 3x - 4) + (2x^2 - 5x + 1) = 3x^2 - 2x - 1$ ni hosil qilamiz. Berilgan tenglama $a^3+b^3=(a+b)^3$ ko'rinishiga keladi va $3ab(a+b)=0$ o'rinli.

Demak, berilgan tenglama $(x^2+3x-4)(2x^2-5x+3)\cdot$ $\cdot(3x^2-2x-1)=0$ tenglamaga teng kuchli.

1) $x^2+3x-4=0$ tenglamaning ildizlarini topamiz:

$$\begin{cases} x_1+x_2=-3 \\ x_1\cdot x_2=-4 \end{cases} \Rightarrow x_1=-4;\ x_2=1.$$

2) $2x^2-5x+3=0$ tenglamaning ildizlari:

$$x_{3,4}=\frac{5\pm1}{4}\Rightarrow x_3=1;\ x_4=\frac{3}{2}.$$

3) $3x^2-2x-1=0$ tenglamaning ildizlari:

$$x_{5,6}=\frac{2\pm4}{6}\Rightarrow x_5=-\frac{1}{3};\ x_6=1.$$

Javob: $\left\{-4;-\frac{1}{3};1;\frac{3}{2}\right\}$.

7-misol. $x^3-3x=27+\frac{1}{27}$ tenglamani yeching.

Yechish: $a^3+b^3=(a+b)^3-3ab(a+b)$ formuladan foydalanamiz:

$$27+\frac{1}{27}=3^3+\frac{1}{3^3}=\left(3+\frac{1}{3}\right)^3-3\cdot3\cdot\frac{1}{3}\left(3+\frac{1}{3}\right)=\left(3+\frac{1}{3}\right)^3-3\cdot\left(3+\frac{1}{3}\right).$$

Bundan tenglama quyidagi ko'rinishga keladi va yechimni osonlikcha topamiz.

$$x^3-3x=\left(3+\frac{1}{3}\right)^3-3\cdot\left(3+\frac{1}{3}\right) \text{ va } x=3+\frac{1}{3}=\frac{10}{3}.$$

Tenglamaning boshqa ildizlari yo'qligini tekshirib ko'rish mumkin.

Javob: $\left\{\frac{10}{3}\right\}$.

8-misol. $(1+x^2)^2=4x(1-x^2)$ tenglamani yeching.

Yechish: **1-usul.** Qavslarni ochamiz va

$$(1+x^2)^2=4x(1-x^2)$$

$$1+2x^2+x^4=4x-4x^3$$
$$(x^4+4x^3+4x^2)-2x^2-4x+1=0$$
$$(x^2+2x)^2-2(x^2+2x)+1=0$$
$$(x^2+2x-1)^2=0;x^2+2x-1=0$$
$$D=8;\ x_{1,2}=\frac{-2\pm2\sqrt{2}}{2}=-1\pm\sqrt{2}$$

2-usul. $x^4+4x^3+2x^2-4x+1=0$. $x=0$ tenglamaning ildizi emas.

Tenglamaning ikkala tomonini x^2 ga bo‘lamiz. Natijada
$$x^2+4x+2-\frac{4}{x}+\frac{1}{x^2}=0$$
$$\left(x^2+\frac{1}{x^2}\right)+4\left(x-\frac{1}{x}\right)+2=0$$

$x-\frac{1}{x}=t$ belgilash kiritamiz. $\left(x-\frac{1}{x}\right)^2=t^2\Rightarrow x^2+\frac{1}{x^2}=t^2+2$.

$t^2+2+4t+2=0\Rightarrow t^2+4t+4=0\Rightarrow(t+2)^2=0\Rightarrow t=-2$.

Belgilashga qaytamiz.
$$x-\frac{1}{x}=2\Rightarrow x^2-2x-1=0\Rightarrow x_{1,2}=-1\pm\sqrt{2}.$$

3-usul. Tenglamaning ikkala tomonidan $4x^2$ ni ayiramiz.
$$(1+x^2)^2=4x(1-x^2)$$
$$(1+x^2)^2-4x^2=4x(1-x^2)-4x^2$$
$$(1-x^2)^2=4x(1-x^2)-4x^2$$

Oxirgi tenglikning ikkala tomonini $x(1-x^2)\neq0$ ga bo‘lamiz. Natijada $\frac{1-x^2}{x}=4-4\cdot\frac{x}{1-x^2}$ ni hosil qilamiz. $\frac{1-x^2}{x}=t$ belgilash kiritamiz va

$$t=4-\frac{4}{t}\ \text{yoki}\ t^2-4t+4=0\Rightarrow(t-2)^2=0\Rightarrow t=2.$$
$$\frac{1-x^2}{x}=2\Rightarrow x^2+2x-1=0\Rightarrow x_{1,2}=-1\pm\sqrt{2}.$$

Javob: $x_{1,2}=-1\pm\sqrt{2}$.

Mustaqil yechish uchun misollar

4.1-misol. Uch hadli tenglamalarni yeching.

1. $x^6+7x^3-8=0$ **2.** $x^6-3x^3+2=0$

3. $2x^8+5x^4-7=0$ **4.** $4x^8-5x^4+1=0$

4.2-misol. Quyidagi tenglamalarni yeching.

1. $(x^2+2x)^2-(x+1)^2=55$

2. $(x^2+x+1)^2-3x^2-3x-1=0$

3. $(x^2-5x)^2-30(x^2-5x)-216=0$

4. $(x^2-5x)^2+10(x^2-5x)+24=0$

5. $(x^2+5x)^2-2(x^2+5x)-24=0$

6. $(x^2+5x+8)^2-6(x^2+5x+8)+8=0$

7. $(x^2-6x)^2-2(x-3)^2=81$

8. $(x+3)^6-9(x+3)^3+8=0$

9. $3(2x+7)^4-7(2x+7)^2+2=0$

4.3-misol. Tenglamalarni belgilash kiritib yeching.

1. $(8x+7)^2(4x+3)(x+1)=\frac{9}{2}$ **2.** $2(6x+5)^2(3x+2)(x+1)=1$

3. $16(2x+1)^2x(x+1)=3$ **4.** $(6x+7)^2(3x+4)(x+1)=1$

4.4-misol. Tenglamalarni yeching.

1. $(x^2-x-1)^3+(x^2-3x+2)^3=(2x^2-4x+1)^3$

2. $(x-a)^3+(x-b)^3=(2x-a-b)^3$

3. $(3-x)^4+(2-x)^4=(5-2x)^4$

4. $x^3-3x=8\frac{1}{8}$

5. $\frac{x}{5}+\frac{5}{x}=3\frac{1}{3}$

6. $x=2+3(2+3x^3)^3$

7. $(x^2-x-1)^3+(2x^2-x-7)^3=(3x^2-2x-8)^3$.

4.5-misol. Test topshiriqlari.

1. Tenglamaning haqiqiy ildizlari yig'indisini toping: $x^6 - 65x^3 = -64$.

$A)\ 5 \quad B)\ 65 \quad C)\ 64 \quad D)\ 16 \quad E)\ 1$

2. $x^6 - 9x^3 + 8 = 0$ tenglamaning haqiqiy ildizlari yig'indisini toping.

$A)\ 3 \quad B)\ 9 \quad C)\ -9 \quad D)\ 8 \quad E)\ 4$

3. $x^8 = \frac{5x^4 + 1}{3}$ tenglamaning barcha haqiqiy ildizlari yig'indisini toping.

$A)\ 0 \quad B)\ 1 \quad C)\ 2 \quad D)\ 2{,}5 \quad E)$ aniqlab bo'lmaydi

4. $(x^2 + x + 1)(x^2 + x + 2) = 12$ tenglamaning haqiqiy ildizlari ko'paytmasini toping.

$A)\ -12 \quad B)\ 6 \quad C)\ -2 \quad D)\ 8 \quad E)\ 2$

5. $(x^2 + x - 4)(x^2 + x + 4) = 9$ tenglama ildizlarining ko'paytmasini toping.

$A)\ 16 \quad B)\ 4 \quad C)\ -4 \quad D)\ 5 \quad E)\ -5$

6. Ushbu $(x^2 + 1)^4 - 3(x^2 + 1)^2 - 4 = 0$ tenglamaning nechta ildizi bor?

$A)\ 6 \quad B)\ 4 \quad C)\ 3 \quad D)\ 2 \quad E)\ 5$

7. $(x^2 + 7x + 5)(x^2 + 7x + 9) = 21$ tenglamaning haqiqiy ildizlari yig'indisini toping.

$A)\ -7 \quad B)\ 14 \quad C)\ 21 \quad D)\ -14$

8. $(x - 1)^2(x^2 - 2x) = 12$ tenglamaning haqiqiy ildizlari yig'indisini toping.

$A)\ 0 \quad B)\ 2 \quad C)\ 3 \quad D)\ 4$

5. $(x-a)(x-b)(x-c)(x-d)=A$ ko'rinishidagi tenglamalar

$(x-a)(x-b)(x-c)(x-d)=A$, bu yerda $a<b<c<d$ va $b-a=d-c$ shartlar bajarilsa, bu ko'rinishidagi tenglamalar yangi o'zgaruvchi

$$y=\frac{1}{4}[(x-a)+(x-b)+(x-c)+(x-d)]=x-\frac{a+b+c+d}{4}$$

kiritish orqali yechiladi.

1-misol. $(x+2)(x+4)(x-6)(x-8)=2925$ tenglamani yeching.

Yechish: Tenglamaning shartlari $-4<-2<6<8$ va $-2-(-4)=8-6$ bajariladi. Yangi $y=\frac{1}{4}[(x+2)+(x+4)+$ $+(x-6)+(x-8)]=x-2$ o'zgaruvchi kiritamiz va bundan $x=y+2$ ni hosil qilamiz. Bu ifodani tenglamaning dastlabki ko'rinishiga qo'yamiz. Natijada

$$(y+4)(y+6)(y-4)(y-6)=2925$$

$$(y^2-16)(y^2-36)=2925$$

$$y^4-52y^2+576=2925$$

$$(y^2-26)^2-100=2925$$

$$(y^2-26)^2=3025$$

$$y^2-26=\pm 55$$

1) $y^2-26=-55 \Rightarrow y^2=-29<0$ yechimga ega emas.

2) $y^2-26=55 \Rightarrow y^2=81 \Rightarrow y_{1,2}=\pm 9$,

$$y_1=-9 \Rightarrow x_1=-9+2=-7$$

$$y_2=9 \Rightarrow x_2=9+2=11$$

Bu tenglamani quyidagi usulda ham yechish mumkin: Ko'paytuvchilarni guruhlaymiz.

$$(x+2)(x-6)(x+4)(x-8)=2925$$

$$(x^2-4x-12)(x^2-4x-32)=2925$$

Yangi $x^2-4x-12=y$ o'zgaruvchi kiritamiz, u holda tenglama quyidagi ko'rinishga ega bo'ladi:

$$y(y-20)=2925$$

$$y^2-20y-2925=0$$

Viyet teoremasidan

$$\begin{cases} y_1+y_2=20 \\ y_1\cdot y_2=-2925 \end{cases} \Rightarrow y_1=-45;\ y_2=65.$$

Topilganlarni $x^2-4x-12=y$ ga qo'yib, berilgan tenglamaning ildizlarini topamiz.

1) $x^2-4x-12=-45 \Rightarrow x^2-4x+33=0,\ D=16-132=-116<0.$

Demak, tenglama haqiqiy ildizlarga ega emas.

2) $x^2-4x-12=65 \Rightarrow x^2-4x-77=0$. Viyet teoremasidan

$$\begin{cases} x_1+x_2=4 \\ x_1\cdot x_2=-77 \end{cases} \Rightarrow x_1=-7;\ x_2=11.$$

Javob: $\{-7;\ 11\}$.

2-misol. $(12x-1)(6x-1)(4x-1)(3x-1)=5$ tenglamani yeching.

Yechish: Tenglamani quyidagi ko'rinishda yozib olamiz:

$$\left(x-\frac{1}{12}\right)\left(x-\frac{1}{6}\right)\left(x-\frac{1}{4}\right)\left(x-\frac{1}{3}\right)=\frac{5}{3\cdot4\cdot6\cdot12}.$$

$\frac{1}{12}<\frac{1}{6}<\frac{1}{4}<\frac{1}{3}$ va $\frac{1}{6}-\frac{1}{12}=\frac{1}{3}-\frac{1}{4}$ shartlar bajarilganligi uchun, yangi belgilash $y=\frac{1}{4}\left[\left(x-\frac{1}{12}\right)+\left(x-\frac{1}{6}\right)+\left(x-\frac{1}{4}\right)+\left(x-\frac{1}{3}\right)\right]=$ $=x-\frac{5}{24}$ ni kiritamiz.

$x=y+\frac{5}{24}$ ni dastlabki tenglamaga qo'yamiz.

$$\left(y+\frac{3}{24}\right)\left(y+\frac{1}{24}\right)\left(y-\frac{1}{24}\right)\left(y-\frac{3}{24}\right)=\frac{5}{3\cdot4\cdot6\cdot12}$$

$$\left[y^2-\left(\frac{1}{24}\right)^2\right]\cdot\left[y^2-\left(\frac{3}{24}\right)^2\right]=\frac{5}{3\cdot4\cdot6\cdot12}$$

Bundan $y^2=\frac{49}{24^2}$, ya'ni $y_1=-\frac{7}{24}$ va $y_2=\frac{7}{24}$ larni topamiz. Berilgan tenglamaning ildizlari

$$y_1=-\frac{7}{24}\Rightarrow x_1=-\frac{7}{24}+\frac{5}{24}=-\frac{1}{12};\ y_2=\frac{7}{24}\Rightarrow x_2=\frac{7}{24}+\frac{5}{24}=\frac{1}{2}$$

ga teng bo'ladi.

Javob: $\left\{-\frac{1}{12};\frac{1}{2}\right\}$.

3-misol. $(x+1)(x+2)(x+4)(x+5)=40,\ x\in R$ tenglamaning ildizlari yig'indisini toping.

$A)\ -6 \qquad B)\ 0 \qquad C)\ -5 \qquad D)\ 6 \qquad E)\ 7$

Yechish: Tenglamadagi ko'paytuvchilarni guruhlaymiz:

$$(x+1)(x+5)(x+2)(x+4)=40$$

$$(x^2+6x+5)(x^2+6x+8)=40$$

Yangi $x^2+6x=y$ o'zgaruvchi kiritamiz, u holda tenglama quyidagi ko'rinishga ega bo'ladi:

$$(y+5)(y+8)=40\Rightarrow y^2+13y=0\Rightarrow y(y+13)=0\Rightarrow y_1=0;$$

$$y_2=-13.$$

Topilganlarni $x^2+6x=y$ ga qo'yib, berilgan tenglamaning ildizlarini topamiz.

1) $x^2+6x=0\Rightarrow x(x+6)=0\Rightarrow x_1=0;\ x_2=-6$.

2) $x^2+6x=-13\Rightarrow x^2+6x+13=0,\ D=36-52=-16<0$.

Tenglama haqiqiy ildizlarga ega emas.

Berilgan tenglama ikkita haqiqiy ildizga ega bo'lib, $x_1+x_2=-6$.

Javob: $A)-6$.

Mustaqil yechish uchun misollar

5.1-misol. Quyidagi tenglamalarning haqiqiy ildizlarini toping.

1. $x(x+2)(x+3)(x+5)=72$

2. $x(x+1)(x+2)(x+3)=24$

3. $x(x+1)(x-1)(x+2)=24$

4. $(x+1)(x+3)(x+5)(x+7)+15=0$

5. $(x+1)(x+2)(x+3)(x+4)=1$

6. $(x-1)x(x+2)(x+1)=3$

7. $(2x-1)(2x+3)(3x-2)(3x-8)+25=0$

8. $(2x-1)(2x+3)(3x-2)(3x+4)=35$

9. $(x-1)(x-3)(x-5)(x-7)(x-9)(x-11)+225=0$

5.2-misol. Test topshiriqlari.

1. $(x-1)(x+2)(x+4)(x+7)=-56(x\in R)$ tenglamaning ildizlari yig'indisini toping.

$A)\ -6$ $B)\ 0$ $C)\ -5$ $D)\ -12$

2. $(x-4)(x-5)(x-6)(x-7)=1680$ tenglamani yeching.

$A)\ x_1=5;\ x_2=6$ $B)\ x_1=x_2=1$

$C)\ x_1=-1;\ x_2=12$ $D)\ x_1=4;\ x_2=7$

6. $(ax^2+b_1x+c)(ax^2+b_2x+c)=Ax^2$ ko'rinishidagi tenglamalar

Bu ko'rinishdagi tenglamalarda $c\neq 0$ va $A\neq 0$ bo'lib, $x=0$ tenglamaning ildizi emas. Tenglamaning ikkala tomonini x^2 ga bo'lib, berilgan tenglamaga teng kuchli bo'lgan quyidagi tenglamani hosil qilamiz:

$$\left(ax+\frac{c}{x}+b_1\right)\left(ax+\frac{c}{x}+b_2\right)=A.$$

$y = ax + \frac{c}{x}$ belgilash kiritilib, kvadrat tenglamaga keltiriladi va tenglama yechiladi.

1-misol. $(2x^2 - 3x + 1)(2x^2 + 5x + 1) = 9x^2$ tenglamani yeching.

Yechish: $x = 0$ berilgan tenglamaning ildizi emasligi ravshan. Shuning uchun tenglamaning ikkala tomonini x^2 ga bo'lamiz va

$$\left(2x - 3 + \frac{1}{x}\right)\left(2x + 5 + \frac{1}{x}\right) = 9$$

tenglamani hosil qilamiz. $2x + \frac{1}{x} = y$ deb belgilab, $(y-3)(y+5) = 9$ yoki $y^2 + 2y - 24 = 0$ tenglamaning $y_1 = -6$ va $y_2 = 4$ ildizlarini topamiz. Shunday qilib, berilgan tenglama quyidagi tenglamalarga teng kuchli:

$$2x + \frac{1}{x} = -6 \text{ yoki } 2x + \frac{1}{x} = 4.$$

1) $2x + \frac{1}{x} = -6 \Rightarrow 2x^2 + 6x + 1 = 0 \Rightarrow x_{1,2} = \frac{-6 \pm 2\sqrt{7}}{4} = \frac{-3 \pm \sqrt{7}}{2}$;

2) $2x + \frac{1}{x} = 4 \Rightarrow 2x^2 - 4x + 1 = 0 \Rightarrow x_{1,2} = \frac{4 \pm 2\sqrt{2}}{4} = \frac{2 \pm \sqrt{2}}{2}$.

Javob: $\left\{\frac{-3 \pm \sqrt{7}}{2}; \frac{2 \pm \sqrt{2}}{2}\right\}$.

2-misol. $(x^2 + 37x + 37)(x^2 + x + 37) = 37x^2$ tenglamaning haqiqiy ildizlari yig'indisini toping.

$A)\ -39 \qquad B)\ -38 \qquad C)\ -37 \qquad D)\ -36$

Yechish: $x = 0$ berilgan tenglamaning ildizi emas. Tenglamaning ikkala qismini x^2 ga bo'lamiz va

$$\left(x + 37 + \frac{37}{x}\right)\left(x + 1 + \frac{37}{x}\right) = 37$$

tenglamani hosil qilamiz. $x+\frac{37}{x}=y$ deb belgilash kiritamiz. Natijada $(y+37)(y+1)=37 \Rightarrow y^2+38y=0 \Rightarrow y(y+38)=0 \Rightarrow y_1=0$; $y_2=-38$ ni topamiz. Belgilashga qaytamiz:

1) $x+\frac{37}{x}=0 \Rightarrow x^2+37=0 \Rightarrow x^2=-37<0 \Rightarrow x \in \varnothing$.

2) $x+\frac{37}{x}=-38 \Rightarrow x^2+38x+37=0$. Hosil bo'lgan kvadrat tenglamaning diskriminanti musbat bo'lib, tenglama ikkita haqiqiy ildizga ega. Viyet teoremasiga asosan $x_1+x_2=-38$ ga teng.

Javob: $B)-38$.

Mustaqil yechish uchun misollar

6.1-misol. Quyidagi tenglamalarni yeching.

1. $(x^2+x+2)(x^2+2x+2)=2x^2$

2. $(x^2+2x-1)(x^2-3x-1)=-4x^2$

6.2-misol. Test topshiriqlari.

1. $(x^2+7x+7)(x^2+x+7)=7x^2$ tenglamaning haqiqiy ildizlari yig'indisini toping.

$A)-8$ $B)-9$ $C)\ -10$ $D)-7$

2. $(x^2+12x+12)(x^2+x+12)=12x^2$ tenglamaning haqiqiy ildizlari yig'indisini toping.

$A)-14$ $B)-15$ $C)\ 12$ $D)-13$

7. $(x-a)(x-b)(x-c)(x-d)=Ax^2$ ko'rinishidagi tenglamalar

$(x-a)(x-b)(x-c)(x-d)=Ax^2$ tenglamada $ab=cd$ shart bajarilsa, bunday tenglamalar yangi o'zgaruvchi

$y = x + \dfrac{ab}{x}$ kiritish yordamida ikkita kvadrat tenglamani yechishga keltiriladi.

Misol. $(x+2)(x+3)(x+8)(x+12)=4x^2$ tenglamani yeching.

Yechish: Tekshirib ko'rganimizda $(-2)\cdot(-12)=(-3)\cdot(-8)$ shart bajariladi. Qavslarni guruhlaymiz:

$$(x+2)(x+12)(x+3)(x+8)=4x^2$$

$$(x^2+14x+24)(x^2+11x+24)=4x^2$$

$x=0$ tenglamaning ildizi emas, shuning uchun tenglamaning ikkala tomonini x^2 ga bo'lamiz va berilgan tenglamaga teng kuchli tenglamani hosil qilamiz:

$$\left(x+14+\frac{24}{x}\right)\left(x+11+\frac{24}{x}\right)=4.$$

$x+\dfrac{24}{x}=y$ belgilash kiritamiz va $(y+14)(y+11)=4$ tenglamani, undan $y^2+25y+150=0$ kvadrat tenglamani hosil qilamiz. Viyet teoremasidan

$$\begin{cases} y_1+y_2=-25 \\ y_1\cdot y_2=150 \end{cases} \Rightarrow y_1=-15;\ y_2=-10.$$

Dastlabki tenglama quyidagi tenglamalarga teng kuchli:

$$\left[\begin{array}{l} x+\dfrac{24}{x}=-15 \\ x+\dfrac{24}{x}=-10 \end{array}\right. \Rightarrow \left[\begin{array}{l} x^2+15x+24=0 \\ x^2+10x+24=0. \end{array}\right.$$

1) $x^2+10x+24=0$ ning ildizlari:

$$\begin{cases} x_1+x_2=-10 \\ x_1\cdot x_2=24 \end{cases} \Rightarrow x_1=-6;\ x_2=-4.$$

2) $x^2+15x+24=0$ ning ildizlari: $x_{3,4}=\dfrac{-15\pm\sqrt{129}}{2}$

Javob: $\left\{\dfrac{-15\pm\sqrt{129}}{2};-6;-4\right\}.$

Mustaqil yechish uchun misollar

7.1-misol. Quyidagi tenglamalarni yeching:

1. $4(x+5)(x+6)(x+10)(x+12)-3x^2=0$
2. $(x-1)(x-2)(x-4)(x-8)=4x^2$
3. $(x-2)(x-4)(x+5)(x+10)=18x^2$

8. $(x-a)^4+(x-b)^4=A$ ko'rinishidagi tenglamalar

Bu ko'rinishdagi tenglamalar yangi o'zgaruvchi $y=\frac{1}{2}[(x-a)+(x-b)]=x-\frac{a+b}{2}$ kiritib yechiladi.

Bu turdagi tenglamalarni yechishda qisqa ko'paytirish formulalaridan foydalaniladi:

$$(a+b)^4=a^4+4a^3b+6a^2b^2+4ab^3+b^4$$

$$(a-b)^4=a^4-4a^3b+6a^2b^2-4ab^3+b^4$$

$$(a+b)^4+(a-b)^4=2a^4+12a^2b^2+2b^4$$

1-misol. $(6-x)^4+(8-x)^4=16$ tenglamani yeching.

Yechish: $y=\frac{1}{2}[(6-x)+(8-x)]=7-x$ yangi o'zgaruvchi, bundan $x=7-y$ ni topamiz. Demak, berilgan tenglama quyidagi ko'rinishga keladi:

$2y^4+12y^2-14=0$ yoki $y^4+6y^2-7=0$ bikvadrat tenglamani hosil qilamiz. $y^2=t\geq 0$ belgilash kiritib, $t^2+6t-7=0$ kvadrat tenglamani hosil qilamiz va uni Viyet teoremasi yordamida yechamiz.

$$\begin{cases} t_1+t_2=-6 \\ t_1\cdot t_2=7 \end{cases} \Rightarrow t_1=-7;\, t_2=1.$$

1) $y^2=-7<0$, tenglama haqiqiy ildizlarga ega emas.

2) $y^2=1 \Rightarrow y_{1,2}=\pm 1$

a) $y_1=-1 \Rightarrow x_1=7-(-1)=8$ b) $y_2=1 \Rightarrow x_2=7-1=6$.

Javob: $\{6;8\}$.

Endi huddi shu usulda yechiladigan yuqori darajali tenglamalarni ko'rib o'tamiz. Bu tenglamalarni yechishda quyidagi qisqa ko'paytirish formulalaridan foydalanamiz:

$$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

$$(a-b)^5 = a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5$$

$$(a+b)^5 + (a-b)^5 = 2a^5 + 20a^3b^2 + 10ab^4$$

2-misol. $(x-1)^5 + (x+3)^5 = 242(x+1)$ tenglamani yeching.

Yechish: $y = \frac{1}{2}[(x-1)+(x+3)] = x+1$ belgilash kiritib, undan $x = y-1$ topiladi. Topilganlarni tenglamaning dastlabki ko'rinishiga qo'yamiz:

$$(y-2)^5 + (y+2)^5 = 242y.$$

Yuqoridagi qisqa ko'paytirish formulasidan

$$2y^5 + 80y^3 + 160y = 242y$$

$$y^5 + 40y^3 - 41y = 0$$

$$y(y^4 + 40y^2 - 41) = 0$$

$$y(y^2 - 1)(y^2 + 41) = 0$$

Oxirgi tenglamadan

1) $y_1 = 0 \Rightarrow x_1 = -1$;

2) $y^2 - 1 = 0 \Rightarrow y_{2,3} = \pm 1 \Rightarrow a) y_2 = -1 \Rightarrow x_2 = -2;\ b) y_3 = 1 \Rightarrow x_3 = 0$;

3) $y^2 + 41 = 0 \Rightarrow y^2 = -41 < 0 \Rightarrow y \in \varnothing$.

Javob: $\{-2; -1; 0\}$.

Mustaqil yechish uchun misollar

8.1-misol. Quyidagi tenglamalarning haqiqiy ildizlarini toping.

1. $(x-2)^4 + (x-3)^4 = 1$ **2.** $(x+3)^4 + (x+5)^4 = 16$

3. $x^4 + (x-4)^4 = 82$ **4.** $(x+1)^4 + (x+3)^4 = 20$

5. $(x-a)^4 + (x-b)^4 = (2x-a-b)^4$

8.2-misol. Ushbu yuqori darajali tenglamalarni yeching.

1. $(x+1)^5+(x-1)^5=32x$ **2.** $x^5+(6-x)^5=1056$

3. $(x-2)^6+(x-4)^6=64$ **4.** $(x+1)^6+(x+2)^6+(x+3)^6=2$

9. Tenglamalarni turli xil usullarda yechish

Quyidagi turdagi tenglamalarni ko'rib o'taylik:

1. $F^{2n}=G^{2n} \Leftrightarrow \left[\begin{array}{l} F=G \\ F=-G. \end{array}\right.$

2. $F^{2n-1}=G^{2n-1} \Leftrightarrow F=G$.

1-misol. $x^4+4x-1=0$ tenglamani yeching.

Yechish: Tenglamada quyidagicha shakl almashtiramiz:

$$x^4+2x^2+1-2x^2+4x-2=0 \Leftrightarrow (x^2-1)^2-2(x^2-2x+1)=0 \Leftrightarrow$$
$$\Leftrightarrow (x^2-1)^2=2(x-1)^2 \Leftrightarrow x^2-1=\pm\sqrt{2}(x-1).$$

Demak, berilgan tenglama $x^2-1=-\sqrt{2}(x-1)$ va $x^2-1=\sqrt{2}(x-1)$ tenglamalarni yechishga keladi.

1) $x^2-1=-\sqrt{2}(x-1) \Leftrightarrow x^2+\sqrt{2}x+1-\sqrt{2}=0$, $D=2-4(1-\sqrt{2})=$ $=4\sqrt{2}-2>0$.

$$x_{1,2}=\frac{-\sqrt{2}\pm\sqrt{4\sqrt{2}-2}}{2}.$$

2) $x^2-1=\sqrt{2}(x-1) \Leftrightarrow x^2-\sqrt{2}x+1+\sqrt{2}=0$, $D=2-4(1+\sqrt{2})=$ $=-2-4\sqrt{2}<0$.

Tenglama haqiqiy ildizlarga ega emas.

Bu tenglamani ko'paytuvchilarga ajratish orqali ham yechsa bo'ladi.

$$(x^2-1)^2=2(x-1)^2 \Leftrightarrow (x^2-1-\sqrt{2}(x-1))(x^2-1+\sqrt{2}(x-1))=$$
$$0 \Leftrightarrow (x^2-\sqrt{2}x+1+\sqrt{2})(x^2+\sqrt{2}x+1-\sqrt{2})=0.$$

Oxirgi tenglikdagi har bir qavsni 0 ga tenglasak, yuqoridagi ildizlarni hosil qilamiz.

Javob: $\left\{\frac{-\sqrt{2}\pm\sqrt{4\sqrt{2}-2}}{2}\right\}$.

2-misol. $x^4-2x^2-400x=9999$ tenglamani yeching.

Yechish: Tenglamaning ikkala tomoniga $4x^2+400x+1$ ifodani qo'shamiz va to'liq kvadratga keltiramiz.

$$x^4-2x^2-400x+4x^2+400x+1=9999+4x^2+400x+1$$

$$x^4+2x^2+1=4(x^2+100x+2500)$$

$$(x^2+1)^2=[2(x+50)]^2$$

$$x^2+1=\pm 2(x+50).$$

1) $x^2+1=-2(x+50)\Rightarrow x^2+2x+101=0$. $D<0$ bo'lib, tenglama haqiqiy ildizlarga ega emas.

2) $x^2+1=2(x+50)\Rightarrow x^2-2x-99=0$. Viyet teoremasidan

$$\begin{cases} x_1+x_2=2 \\ x_1\cdot x_2=-99 \end{cases}\Rightarrow x_1=-9;\ x_2=11.$$

Javob: $\{-9;11\}$.

3-misol. $x^4-2\sqrt{3}x^2-x+3-\sqrt{3}=0$ tenglamani yeching.

Yechish: Berilgan tenglamani $\sqrt{3}$ ga nisbatan kvadrat tenglamaga keltirib yechamiz: $(\sqrt{3})^2-(2x^2+1)\sqrt{3}+x^4-x=0$.

$$D=(2x^2+1)^2-4(x^4-x)=4x^4+4x^2+1-4x^4+4x=$$

$$=4x^2+4x+1=(2x+1)^2.$$

$$\left[\begin{array}{l}\sqrt{3}=\dfrac{(2x^2+1)+(2x+1)}{2} \\ \sqrt{3}=\dfrac{(2x^2+1)-(2x+1)}{2}\end{array}\right.\Leftrightarrow\left[\begin{array}{l}x^2+x+1-\sqrt{3}=0 \\ x^2-x-\sqrt{3}=0\end{array}\right.\Leftrightarrow\left[\begin{array}{l}x_{1,2}=\dfrac{-1\pm\sqrt{4\sqrt{3}-3}}{2} \\ x_{3,4}=\dfrac{1\pm\sqrt{1+4\sqrt{3}}}{2}.\end{array}\right.$$

Javob: $\left\{\frac{-1\pm\sqrt{4\sqrt{3}-3}}{2};\frac{1\pm\sqrt{1+4\sqrt{3}}}{2}\right\}$.

4-misol. $x^4-x^2+2x-1=0$ tenglamani yeching.

Yechish: $x^4-1=t$ parametrni kiritamiz. U holda berilgan tenglama $x^2-2x-t=0$ ko'rinishga keladi. Bu tenglamaning ildizlari x_1 va x_2 ga teng bo‘lsa, tenglama $(x-x_1)(x-x_2)=0$ ko‘rinishiga keladi.

$D=(-2)^2-4\cdot(-t)=4+4t=4(1+t)$ va $x_{1,2}=\dfrac{2\pm2\sqrt{1+t}}{2}=1\pm\sqrt{1+t}$.

$x^4-1=t$ dan $1+t=x^4$ ni topamiz va $x_1=1-\sqrt{1+t}=1-x^2$, $x_2=1+\sqrt{1+t}=1+x^2$ larga ega bo‘lamiz. Topilganlarni $(x-x_1)(x-x_2)=0$ ga qo‘yamiz va berilgan tenglama

$$[x-(1-x^2)][x-(1+x^2)]=0$$

$$(x^2+x-1)(x^2-x+1)=0$$

ko‘rinishga keladi. Bundan

$$(x^2+x-1)(x^2-x+1)=0\Leftrightarrow\left[\begin{array}{l}x^2+x-1=0\\x^2-x+1=0\end{array}\right.\Leftrightarrow\left[\begin{array}{l}x_{1,2}=\dfrac{-1\pm\sqrt{5}}{2}\\x\in\varnothing.\end{array}\right.$$

Javob: $\left\{\dfrac{-1\pm\sqrt{5}}{2}\right\}$.

5-misol. $x^3+x^2+x=-\frac{1}{3}$ tenglamani yeching.

Yechish: Tenglamaning ikkala tomonini 3 ga ko‘paytiramiz va $3x^3+3x^2+3x+1=0$ tenglamani hosil qilamiz. Bundan

$$3x^3+3x^2+3x+1=0\Rightarrow x^3+3x^2+3x+1=-2x^3\Rightarrow(x+1)^3=$$
$$=(-\sqrt[3]{2}\,x)^3\Rightarrow x+1=-\sqrt[3]{2}\,x\Rightarrow(\sqrt[3]{2}+1)x=-1\Rightarrow x=-\frac{1}{\sqrt[3]{2}+1}.$$

Javob: $\left\{-\dfrac{1}{\sqrt[3]{2}+1}\right\}$.

6-misol. $(2x^2+3)^2=x(4x^2+35x+6)$ tenglamani yeching.

Yechish: $x=0$ tenglamaning ildizi emas, tenglamaning ikkala tomonini x^2 ga bo‘lamiz. Natijada

$$\left(\frac{2x^2+3}{x}\right)^2=\frac{4x^2+35x+6}{x}\Rightarrow\left(\frac{2x^2+3}{x}\right)^2=2\cdot\frac{2x^2+3}{x}+35.$$

$\frac{2x^2+3}{x}=y$ belgilash kiritamiz va $y^2-2y-35=0$ kvadrat tenglamani hosil qilib, $y_1=-5$; $y_2=7$ ildizlarini topamiz.

Demak, berilgan tenglama quyidagi tenglamalarni yechishga keltiriladi:

$$\left[\begin{array}{l}\frac{2x^2+3}{x}=-5 \\ \frac{2x^2+3}{x}=7\end{array}\right. \Rightarrow \left[\begin{array}{l}2x^2+5x+3=0 \\ 2x^2-7x+3=0\end{array}\right. \Rightarrow \left[\begin{array}{l}x_1=-\frac{3}{2};\ x_2=-1 \\ x_3=\frac{1}{2};\ x_4=3.\end{array}\right.$$

Javob: $\left\{-\frac{3}{2};-1;\frac{1}{2};3\right\}$.

7-misol. $(2x+2)(5-2x)(4x^2+8x+11)=10(2x+3)^2$ tenglamani yeching.

Yechish: Berilgan tenglamada quyidagicha shakl almashtiramiz:

$$(-4x^2+6x+11)(4x^2+8x+11)=10(2x+3)^2.$$

$x=-1,5$ tenglamaning ildizi emas, shuning uchun tenglamaning ikkala tomonini $(2x+3)^2$ ga bo‘lamiz va teng kuchli tenglama hosil qilamiz.

$$\frac{-4x^2+6x+11}{2x+3}\cdot\frac{4x^2+8x+11}{2x+3}=10.$$

$\frac{-4x^2+6x+11}{2x+3}=y$ va $\frac{4x^2+8x+11}{2x+3}=z$ bo‘lsin, u holda

$y+z=\frac{14x+21}{2x+3}=\frac{7(2x+3)}{2x+3}=7$ o‘rinli. Bundan quyidagi sistemani hosil qilamiz:

$$\left\{\begin{array}{l}yz=10 \\ y+z=7\end{array}\right. \Leftrightarrow \left[\begin{array}{l}\left\{\begin{array}{l}y=2 \\ z=5\end{array}\right. \\ \left\{\begin{array}{l}y=5 \\ z=2.\end{array}\right.\end{array}\right.$$

Hosil bo‘lgan sistemalarning har biridan bittadan tenglamani ko‘rish yetarli, ya’ni

1) $\frac{4x^2+8x+11}{2x+3}=5 \Rightarrow 2x^2-x-2=0 \Rightarrow x_{1,2}=\frac{1\pm\sqrt{17}}{4}$;

2) $\frac{4x^2+8x+11}{2x+3}=2 \Rightarrow 4x^2+4x+5=0 \Rightarrow x\in\varnothing$.

Javob: $\frac{1\pm\sqrt{17}}{4}$.

8-misol. Tenglamani yeching:

$$(1+x+\ldots+x^7)(1+x+\ldots+x^5)=(1+x+\ldots+x^6)^2.$$

Yechish: $x=1$ tenglamaning ildizi emas. Haqiqatdan ham $x=1$ da tenglamaning o'ng tomoni 48 ga, chap tomoni 49 ga teng.

Tenglamaning ikkala tomonini $(x-1)^2$ ga ko'paytiramiz va $x^n-1=(x-1)(x^{n-1}+x^{n-2}+\ldots+1)$, $n\in N$ formulani qo'llaymiz. Ko'rinib turibdiki, $(1+x+\ldots+x^5)(x-1)=x^6-1$; $(1+x+\ldots+x^6)(x-1)=x^7-1$ va $(1+x+\ldots+x^7)(x-1)=x^8-1$ o'rinli. Bundan berilgan tenglama $(x^8-1)(x^6-1)=(x^7-1)^2$ ko'rinishga keladi. Qavslarni ochamiz:

$$x^{14}-x^8-x^6+1=x^{14}-2x^7+1$$

$$x^8+x^6=2x^7$$

$$x^6(x-1)^2=0$$

$$x_1=0;\ x_2=1.$$

Demak, $x=0$ yagona ildiz.

Tenglamani geometrik progressiyaning dastlabki n ta hadi yig'indisi formulasidan foydalanib yechsa ham bo'ladi.

Javob: $x=0$.

9-misol. $x^4-2x^3+3x^2-4x+5=0$ tenglama ildizga ega emasligini isbotlang.

Yechish: Tenglamaning chap tomonida quyidagicha shakl almashtiramiz:

$$x^4-2x^3+3x^2-4x+5=(x^4-2x^3+x^2)+(2x^2-4x+2)+3=$$
$$=(x^2-x)^2+2(x-1)^2+3\geq 3.$$

Demak, tenglamaning chap tomoni musbat bo‘lib, tenglama ildizga ega emasligini bildiradi.

Mustaqil yechish uchun misollar

9.1-misol. Tenglamalarni yeching.

1. $x^4 + 8x - 7 = 0$

2. $x^4 - 2x^2 - 12x - 8 = 0$

3. $x^4 - 3x^2 + 4x - 3 = 0$

4. $x^3 - (\sqrt{2} + 1)x^2 + 2 = 0$

5. $x^3 - 3x^2 - 3x - 1 = 0$

6. $2(3x^2 + 1)^2 = x(15x^2 + 52x + 5)$

7. $(x^2 + 4x + 8)^2 + 3x^3 + 14x^2 + 24x = 0$

8. $8x^3 + 36x^2 + 54x + 33 = 0$

9. $(6x - 15)^7 = (x - 1)^{14}$

10. $(1 + x + x^2)(1 + x + x^2 + \ldots + x^{10}) = (1 + x + \ldots + x^6)^2$

11. $(2x^3 + x - 3)^3 = 3 - x^3$.

9.2-misol. Progressiyaga doir tenglamalarni yeching.

1. $1 + 4 + 7 + \cdots + x = 117$

2. $1 + 7 + 13 + \cdots + x = 280$

3. $(x + 1) + (x + 4) + (x + 7) + \cdots + (x + 28) = 155$

4. $(2x + 1) + (2x + 4) + (2x + 7) + \ldots + (2x + 46) = 472$.

5. $\frac{x-1}{x} + \frac{x-2}{x} + \ldots + \frac{1}{x} = 3$.

4-§. QAYTMA TENGLAMALAR

1. Simmetrik tenglamalar

Qaytma yoki simmetrik tenglama deb istalgan darajadagi shunday algebraik tenglamaga aytiladiki, ularning boshidan va oxiridan teng uzoqlikda bo'lgan hadlarining koeffitsiyentlari teng bo'ladi.

Ta'rif. *Birinchi darajali simmetrik tenglama deb* $ax+a=0$ *tenglamaga aytiladi.*

Uning yechimi $a\neq 0$ bo'lganligi sababli $x+1=0$ bo'ladi va yagona $x=-1$ ildizga ega.

1-misol. $4x+4=0$ dan $x=-1$. Demak, tenglama bitta $x=-1$ ildizga ega.

Ta'rif. *Ikkinchi darajali simmetrik tenglama deb* $ax^2+bx+a=0$ *tenglamaga aytiladi.*

Uning yechimlari $x_1=x_0,\, x_2=\dfrac{1}{x_0}$ ko'rinishida bo'ladi. Bu tenglama $D\geq 0$ bo'lganda har doim yechimga ega.

2-misol. $3x^2-10x+3=0$ tenglamaning ildizlarini toping.

Yechish: $D=(-10)^2-4\cdot 3\cdot 3=100-36=64\geq 0$ bo'ladi.

$$x_{1,2}=\frac{10\pm\sqrt{64}}{6}=\frac{10\pm 8}{6}\Rightarrow x_1=\frac{1}{3};\ x_2=3$$

Javob: $x_1=\dfrac{1}{3};\ x_2=3$.

Yuqori darajali simmetrik va qaytma tenglamalarni yechishda quyidagi qisqa ko'paytirish formulalaridan foydalanamiz:

$$a^2-b^2=(a-b)(a+b)\,;\quad a^3+b^3=(a+b)(a^2-ab+b^2)$$

$$a^3-b^3=(a-b)(a^2+ab+b^2)\,;\ a^4-b^4=(a-b)(a+b)(a^2+b^2)$$

$$a^5+b^5=(a+b)(a^4-a^3b+a^2b^2-ab^3+b^4)$$

$$a^5-b^5=(a-b)(a^4+a^3b+a^2b^2+ab^3+b^4)$$

$$a^6-b^6=(a-b)(a^5+a^4b+a^3b^2+a^2b^3+ab^4+b^5)$$

Ta'rif. *Uchinchi darajali simmetrik tenglama deb* $ax^3+bx^2+bx+a=0$ *ko'rinishidagi tenglamalarga aytiladi.*

Bu tenglama chap tomonini ko'paytuvchilarga ajratish yo'li bilan yechiladi.

Yechish: Tenglamaning chap tomonini guruhlaymiz: $a(x^3+1)+bx(x+1)=0$. Bundan $a(x+1)(x^2-x+1)+bx(x+1)=0$ yoki $(x+1)(ax^2-ax+ba+a)=0$ ni hosil qilamiz. Demak, $x+1=0$ va $ax^2-(a+b)x+a=0$ (1) tenglamalarni yechishga kelamiz. Birinchi tenglamaning ildizi $x_1=-1$.

Ikkinchi tenglama uchun $D=a^2-2ab+b^2-4a^2=b^2-$ $-2ab-3a^2$ bo'ladi. Agar $D\geq 0$ bo'lsa, $a\neq 0$ bo'lganligi uchun

$$x_{3,4}=\frac{(a-b)\pm\sqrt{b^2-2ab-3a^2}}{2a}$$

haqiqiy ildizlari hosil bo'ladi. Agar $D<0$ bo'lsa, (1) tenglamaning haqiqiy ildizlari mavjud emas, kompleks ildizlarga ega bo'ladi.

3-misol. $2x^3+7x^2+7x+2=0$ tenglamaning ildizlarini toping.

Yechish: Tenglamaning o'ng tomonini ko'paytuvchilarga ajratamiz:

$$2x^3+7x^2+7x+2=2(x^3+1)+7x(x+1)=2(x+1)(x^2-x+1)+$$
$$+7x(x+1)=(x+1)(2x^2-2x+2+7x)=(x+1)(2x^2+5x+2)$$

$(x+1)(2x^2+5x+2)=0$ tenglamani yechamiz.

1) $x+1=0$ dan $x_1=-1$ ni topamiz.

2) $2x^2+5x+2=0$ dan $D=5^2-4\cdot 2\cdot 2=9\geq 0$ bo'ladi. Bundan

$$x_{2,3}=\frac{-5\pm\sqrt{9}}{4}=\frac{-5\pm 3}{4}\Rightarrow x_2=-\frac{1}{2},$$ $x_3=-2$ ni hosil qilamiz.

Javob: $\left\{-2\,;-1\,;-\frac{1}{2}\right\}$.

Ta'rif. *To'rtinchi darajali birinchi jins simmetrik tenglamalar deb* $ax^4 + bx^3 + cx^2 + bx + a = 0$ *ko'rinishidagi tenglamalarga aytiladi.*

$x = 0$ tenglamaning ildizi bo'lmaganligi uchun uning ikkala tomonini x^2 ga bo'lib,

$$ax^2 + bx + c + \frac{b}{x} + \frac{a}{x^2} = 0$$

tenglamani hosil qilamiz. Koeffitsiyentlari bir xil bo'lgan hadlarini guruhlaymiz:

$$a\left(x^2 + \frac{1}{x^2}\right) + b\left(1 + \frac{1}{x}\right) + c = 0 \quad (2)$$

$x + \frac{1}{x} = y$ deb belgilash kiritib, uning ikkala tomonini kvadratga oshiramiz:

$$x^2 + 2x \cdot \frac{1}{x} + \frac{1}{x^2} = y^2 \text{ yoki } x^2 + \frac{1}{x^2} = y^2 - 2.$$

Bularni (2) ga qo'ysak, simmetrik tenglama y ga nisbatan kvadrat tenglamaga keladi:

$$a(y^2 - 2) + by + c = 0 \text{ yoki } ay^2 + by + (c - 2a) = 0.$$

4-misol. $8x^4 - 54x^3 + 101x^2 - 54x + 8 = 0$ tenglamani yeching.

Yechish: $8x^4 - 54x^3 + 101x^2 - 54x + 8 = 0 \quad / : x^2$

$$8x^2 - 54x + 101 - \frac{54}{x} + \frac{8}{x^2} = 0$$

$$8\left(x^2 + \frac{1}{x^2}\right) - 54\left(x + \frac{1}{x}\right) + 101 = 0$$

$x + \frac{1}{x} = y$ deb belgilash kiritib, $x^2 + \frac{1}{x^2} = y^2 - 2$ ni oxirgi tenglamaga qo'yamiz:

$8(y^2 - 2) - 54y + 101 = 0 \Rightarrow 8y^2 - 54y + 85 = 0$ kvadrat tenglamani yechamiz.

$$D = (-54)^2 - 4 \cdot 8 \cdot 85 = 2916 - 2720 = 196 \geq 0$$

$$y_{1,2}=\frac{54\pm\sqrt{196}}{16}=\frac{54\pm14}{16}\Rightarrow y_1=\frac{5}{2};y_2=\frac{17}{4}$$

y_1 va y_2 ning qiymatlarini yuqoridagi belgilashga qo'yamiz:

1) $x+\frac{1}{x}=\frac{5}{2}$ yoki $2x^2-5x+2=0$. Bundan $x_1=\frac{1}{2}$ va $x_2=2$ bo'ladi.

2) $x+\frac{1}{x}=\frac{17}{4}$ yoki $4x^2-17x+4=0$. Bundan $x_3=\frac{1}{4}$ va $x_4=4$ bo'ladi.

Javob: $\left\{\frac{1}{4};\frac{1}{2};2;4\right\}$.

Eslatma: *Belgilashlarga ko'ra,* $x+\frac{1}{x}=y$ *bo'lganligi uchun to'rtinchi darajali birinchi jins simmetrik tenglama ildizlarining har bir jufti qiymat jihatidan bir-biriga teskari bo'ladi.*

Ta'rif. *To'rtinchi darajali ikkinchi jins simmetrik tenglamalar deb* $ax^4+bx^3+cx^2-bx+a=0$ *ko'rinishidagi tenglamalarga aytiladi.*

Bu tenglamani yechish uchun ham ikkala tomonini x^2 ga bo'lib,

$$ax^2+bx+c-\frac{b}{x}+\frac{a}{x^2}=0$$

tenglamani hosil qilamiz. Koeffitsiyentlari bir xil bo'lgan hadlarini guruhlaymiz:

$$a\left(x^2+\frac{1}{x^2}\right)+b\left(1-\frac{1}{x}\right)+c=0 \quad (3)$$

$x-\frac{1}{x}=y$ deb belgilash kiritib, uning ikkala tomonini kvadratga oshiramiz:

$$x^2-2x\cdot\frac{1}{x}+\frac{1}{x^2}=y^2 \text{ yoki } x^2+\frac{1}{x^2}=y^2+2.$$

Bularni (3) ga qo‘ysak, simmetrik tenglama y ga nisbatan kvadrat tenglamaga keladi:

$$a(y^2+2)+by+c=0 \text{ yoki } ay^2+by+(c+2a)=0.$$

5-misol. $6x^4+7x^3-36x^2-7x+6=0$ tenglamani yeching.

Yechish: $6x^4+7x^3-36x^2-7x+6=0$ ikkala tomonini x^2 ga bo‘lamiz va $6x^2+7x-36-\frac{7}{x}+\frac{6}{x^2}=0$ yoki $6\left(x^2+\frac{1}{x^2}\right)+$ $+7\left(x-\frac{1}{x}\right)-36=0$ ni hosil qilamiz. Bunda $x-\frac{1}{x}=y$ deb belgilaymiz va $x^2+\frac{1}{x^2}=y^2+2$ munosabatni aniqlaymiz. Demak, $6(y^2+2)+7y-36=0$ yoki $6y^2+7y-24=0$. y ga nisbatan kvadrat tenglamani yechib, $y_1=-\frac{8}{3}$ va $y_2=\frac{3}{2}$ larni topamiz. Bu qiymatlarni belgilangan ifodaga qo‘yib, mos tenglamalarni yechamiz:

1) $x-\frac{1}{x}=-\frac{8}{3}$ yoki $3x^2+8x-3=0$. Bundan $x_1=-3$ va $x_2=\frac{1}{3}$ bo‘ladi.

2) $x-\frac{1}{x}=\frac{3}{2}$ yoki $2x^2-3x-2=0$. Bundan $x_3=-\frac{1}{2}$ va $x_4=2$ bo‘ladi.

Javob: $\left\{-3;-\frac{1}{2};\frac{1}{3};2\right\}$.

Eslatma: $x-\frac{1}{x}=y$ *bo‘lganligi uchun ikkinchi jins to‘rtinchi darajali simmetrik tenglama ildizlarining har bir jufti qiymat jihatidan bir-biriga teskari, ishora jihatidan qarama-qarshi bo‘ladi.*

Ta’rif. *Beshinchi darajali simmetrik tenglama deb* $ax^5+bx^4+cx^3+cx^2+bx+a=0$ *ko‘rinishidagi tenglamalarga aytiladi.*

Bu tenglamaning chap tomoni ko'paytuvchilarga ajratilib, ikkita tenglama yechishga keltiriladi. Bu tenglamalarning biri birinchi darajali, ikkinchisi to'rtinchi darajali simmetrik tenglama bo'ladi.

6-misol. $2x^5+5x^4-13x^3-13x^2+5x+2=0$ tenglamani yeching.

Yechish: Tenglamaning chap tomonini ko'paytuvchilarga ajratamiz:

$$2x^5+5x^4-13x^3-13x^2+5x+2=(2x^5+2)+(5x^4+5x)-$$
$$-(13x^3+13x^2)=2(x^5+1)+5x(x^3+1)-13x^2(x+1)=$$
$$=2(x+1)(x^4-x^3+x^2-x+1)+5x(x+1)(x^2-x+1)-$$
$$-13x^2(x+1)=(x+1)[2x^4-2x^3+2x^2-2x+2+5x^3-$$
$$-5x^2+5x-13x^2]=(x+1)(2x^4+3x^3-16x^2+3x+2)$$

Demak, $(x+1)(2x^4+3x^3-16x^2+3x+2)=0$ tenglamani yechamiz.

1) $x+1=0$ dan $x_1=-1$ ni topamiz.

2) $2x^4+3x^3-16x^2+3x+2=0$ to'rtinchi darajali simmetrik tenglamani yechamiz:

$$2x^2+3x-16+\frac{3}{x}+\frac{2}{x^2}=0 \text{ yoki } 2\left(x^2+\frac{1}{x^2}\right)+3\left(x+\frac{1}{x}\right)-16=0.$$

$x+\frac{1}{x}=y$ deb belgilash kiritib, $x^2+\frac{1}{x^2}=y^2-2$ ni oxirgi tenglamaga qo'yamiz:

$$2(y^2-2)+3y-16=0 \text{ yoki } 2y^2+3y-20=0.$$

Kvadrat tenglamani y ga nisbatan yechamiz: $D=3^2-4\cdot2\cdot(-20)=169$.

$$y_{1,2}=\frac{-3\pm\sqrt{169}}{4}=\frac{-3\pm13}{4}\Rightarrow y_1=-4;\ y_2=\frac{5}{2}.$$

a) $x+\frac{1}{x}=-4\Rightarrow x^2+4x+1=0\Rightarrow x_{2,3}=-2\pm\sqrt{3}$;

b) $x+\frac{1}{x}=\frac{5}{2}\Rightarrow 2x^2-5x+2=0\Rightarrow x_4=\frac{1}{2};\ x_5=2$.

Javob: $\left\{-1; -2\pm\sqrt{3};\ \frac{1}{2};2\right\}$.

Ta'rif. $ax^6+bx^5+cx^4+dx^3+cx^2+bx+a=0$ *ko'rinishidagi tenglamalar oltinchi darajali simmetrik tenglama deyiladi.*

Bu tenglamani yechish uchun tenglamaning ikkala tomonini x^3 ga bo'lamiz va $ax^3+bx^2+cx+d+\frac{c}{x}+\frac{b}{x^2}+\frac{a}{x^3}=0$ ni hosil qilamiz. Tenglamadagi hadlarni guruhlasak,

$$a\left(x^3+\frac{1}{x^3}\right)+b\left(x^2+\frac{1}{x^2}\right)+c\left(x+\frac{1}{x}\right)+d=0 \quad (4)$$

Bu yerda $x+\frac{1}{x}=y$ deb belgilab, quyidagilarni aniqlaymiz:

$$\left(x+\frac{1}{x}\right)^2=y^2 \Leftrightarrow x^2+2x\cdot\frac{1}{x}+\frac{1}{x^2}=y^2 \Leftrightarrow x^2+\frac{1}{x^2}=y^2-2$$

$$\left(x+\frac{1}{x}\right)^3=y^3 \Leftrightarrow x^3+\frac{1}{x^3}+3x\cdot\frac{1}{x}\cdot\left(x+\frac{1}{x}\right)=y^3 \Leftrightarrow x^3+\frac{1}{x^3}=y^3-3y$$

Topilganlarni (4) ga qo'ysak, $a(y^3-3y)+b(y^2-2)+cy+d=0$ hosil bo'ladi. Qavslarni ochib soddalashtiramiz:

$$ay^3+by^2-(3a-c)y-(2b-d)=0$$

Bu yangi tenglama simmetrik tenglama bo'lmaydi. Agar bu kubik tenglama yechilsa (ildizlari topilsa), u holda berilgan oltinchi darajali simmetrik tenglama ham yechiladi.

7-misol. $x^6-6x^5+14x^4-18x^3+14x^2-6x+1=0$ tenglamani yeching.

Yechish: Tenglamaning ikkala tomonini x^3 ga bo'lamiz:

$$x^3-6x^2+14x-18+\frac{14}{x}-\frac{6}{x^2}+\frac{1}{x^3}=0$$

$$\left(x^3+\frac{1}{x^3}\right)-6\left(x^2+\frac{1}{x^2}\right)+14\left(x+\frac{1}{x}\right)-18=0$$

$$x+\frac{1}{x}=y;\quad x^2+\frac{1}{x^2}=y^2-2\ ;\quad x^3+\frac{1}{x^3}=y^3-3y$$

$$y^3-3y-6(y^2-2)+14y-18=0$$

$$y^3-3y-6y^2+12+14y-18=0$$
$$y^3-6y^2+11y-6=0$$

Kubik tenglamaning ayrim hadlarini qo‘shiluvchilarga ajratamiz:

$$y^3-3y^2-3y^2+9y+2y-6=0$$

Guruhlaymiz: $y^2(y-3)-3y(y-3)+2(y-3)=0$.

Ko‘paytuvchilarga ajratamiz: $(y-3)(y^2-3y+2)=0$.

Bundan, $y_1=3;\ y_2=1;\ y_3=2$ ni topamiz. Topilgan qiymatlarni y ning o‘rniga qo‘yamiz.

1) $x+\frac{1}{x}=3$ tenglama ildizlari $x_1=\frac{3-\sqrt{5}}{2}$ va $x_2=\frac{3+\sqrt{5}}{2}$;

2) $x+\frac{1}{x}=2$ tenglama ildizlari $x_{3,4}=1$;

3) $x+\frac{1}{x}=1$ tenglama yechimga ega emas (haqiqiy ildizga ega emas).

Javob: $\left\{\frac{3\pm\sqrt{5}}{2};1\right\}$.

8-misol. $x^8-7x^7+4x^6-21x^5+6x^4-21x^3+4x^2-7x+1=0$ tenglamani yeching.

Yechish: Berilgan tenglamaning sakkizinchi darajali simmetrik tenglama ekanligi ko‘rinib turibdi. $x=0$ tenglamaning ildizi bo‘lmaganligi uchun uning ikkala tomonini x^4 ga bo‘lamiz va

$$x^4-7x^3+4x^2-21x+6-\frac{21}{x}+\frac{4}{x^2}-\frac{7}{x^3}+\frac{1}{x^4}=0$$

$$\left(x^4+\frac{1}{x^4}\right)-7\left(x^3+\frac{1}{x^3}\right)+4\left(x^2+\frac{1}{x^2}\right)-21\left(x+\frac{1}{x}\right)+6=0$$

tenglamani hosil qilamiz. $x+\frac{1}{x}=y$ deb belgilash kiritamiz va

$$\left(x+\frac{1}{x}\right)^2=y^2\Rightarrow x^2+\frac{1}{x^2}=y^2-2;$$

$$\left(x+\frac{1}{x}\right)^3=y^3\Rightarrow x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)=y^3\Rightarrow x^3+\frac{1}{x^3}=y^3-3y;$$

$$(a+b)^4=a^4+4a^3b+6a^2b^2+4ab^3+b^4=$$
$$=a^4+b^4+4ab(a^2+b^2)+6(ab)^2$$

dan

$$\left(x+\frac{1}{x}\right)^4=y^4\Rightarrow x^4+\frac{1}{x^4}+4\left(x^2+\frac{1}{x^2}\right)+6=y^4\Rightarrow$$
$$\Rightarrow x^4+\frac{1}{x^4}=y^4-4(y^2-2)-6.$$

$x^4+\frac{1}{x^4}=y^4-4y^2+2$ ni topamiz. Topilganlarni tenglamaning dastlabki ko'rinishiga qo'yamiz. Natijada

$$(y^4-4y^2+2)-7(y^3-3y)+4(y^2-2)-21y+6=0$$
$$y^4-7y^3=0$$
$$y^3(y-7)=0$$
$$y_1=0;\ y_2=7.$$

Belgilashga qaytamiz:

$$\left[\begin{array}{l} x+\frac{1}{x}=0 \\ x+\frac{1}{x}=7 \end{array}\right.\Rightarrow\left[\begin{array}{l} x^2=-1<0 \\ x^2-7x+1=0 \end{array}\right.\Rightarrow\left[\begin{array}{l} x\in\varnothing \\ x_{1,2}=\frac{7\pm3\sqrt{5}}{2} \end{array}\right.$$

Javob: $\left\{\frac{7\pm3\sqrt{5}}{2}\right\}$.

Mustaqil yechish uchun misollar

1.1-misol. Simmetrik tenglamalarni yeching:

1. $3x^3-7x^2-7x+3=0$
2. $x^3-5x^2-5x+1=0$
3. $13x^3-9x^2-9x+13=0$
4. $x^3-x^2-x+1=0$
5. $-2x^3+3x^2+3x-2=0$
6. $4x^3+7x^2+7x+4=0$

1.2-misol. To'rtinchi darajali simmetrik tenglamalarni yeching:

1. $6x^4+5x^3-38x^2+5x+6=0$

2. $2x^4+x^3-11x^2+x+2=0$

3. $x^4-2x^3-x^2-2x+1=0$

4. $6x^4+7x^3-36x^2-7x+6=0$

5. $2x^4-3x^3+2x^2-3x+2=0$

6. $3x^4+5x^3-16x^2+5x+3=0$

7. $(x+1)^4=2(1+x^4)$

8. $x^2+\frac{x}{2}-\frac{1}{2x}+\frac{1}{x^2}=5$

1.3-misol. Simmetrik tenglamalarni yeching:

1. $2x^5+3x^4-5x^3-5x^2+3x+2=0$
2. $3x^5-7x^4+10x^3+10x^2-7x+3=0$
3. $x^5+2x^4-3x^3-3x^2+2x+1=0$
4. $12x^5+18x^4-45x^3-45x^2+18x+12=0$
5. $x^5+\frac{1}{x^5}=\frac{205}{16}(x+\frac{1}{x})$.

1.4-misol. Simmetrik tenglamalarni yeching:

1. $x^6-10x^5+27x^4-20x^3+27x^2-10x+1=0$
2. $x^6-3x^5+5x^4-6x^3+5x^2-3x+1=0$
3. $x^6-7x^5+10x^4-8x^3+10x^2-7x+1=0$
4. $2x^6-3x^5+x^4-6x^3+x^2-3x+2=0$

2. Qaytma tenglamalar

Ta'rif. $ax^4+bx^3+cx^2+\lambda bx+\lambda^2a=0$, $(a\neq 0\,,\lambda\neq 0)$ *ko'rinishidagi tenglamalar to'rtinchi darajali qaytma tenglamalar deyiladi.*

Simmetrik tenglamalar qaytma tenglamaning xususiy holi hisoblanadi. Masalan, $\lambda=1$ da to'rtinchi darajali qaytma tenglama to'rtinchi darajali simmetrik tenglamaga aylanadi.

To'rtinchi darajali qaytma tenglamani yechish uchun tenglamaning ikkala tomonini x^2 ga bo'lamiz:

$$ax^2+bx+c+\frac{\lambda b}{x}+\frac{\lambda^2a}{x^2}=0.$$

Bir xil koeffitsiyentli hadlarni guruhlaymiz:

$$a\left(x^2+\frac{\lambda^2}{x^2}\right)+b\left(x+\frac{\lambda}{x}\right)+c=0 \quad (1)$$

$x+\frac{\lambda}{x}=y$ deb belgilash kiritamiz va

$$\left(x+\frac{\lambda}{x}\right)^2=y^2 \Rightarrow x^2+2x\cdot\frac{\lambda}{x}+\frac{\lambda^2}{x^2}=y^2 \Rightarrow x^2+\frac{\lambda^2}{x^2}=y^2-2\lambda$$

ni hosil qilamiz. Bu qiymatlarni (1) ga qo'ysak, $a(y^2-2\lambda)+by+c=0$ yoki $ay^2+by+(c-2a\lambda)=0$. Bu kvadrat tenglamani y ga nisbatan yechamiz.

Agar $D\geq 0$ bo'lsa, y_1 va y_2 ildizlari topiladi. Bu qiymatlarni yuqoridagi belgilashga qo'yamiz:

1) $x+\frac{\lambda}{x}=y_1$ 2) $x+\frac{\lambda}{x}=y_2$.

Bu ikkita tenglamani yechib, to'rtinchi darajali qaytma tenglamaning ildizlarini topamiz.

1-misol. $3x^4-4x^3-3x^2-8x+12=0$ tenglamani yeching.

Yechish: $x=0$ berilgan tenglamaning ildizi emas. Tenglamaning ikkala tomonini x^2 ga bo'lamiz:

$$3x^2-4x-3-\frac{8}{x}+\frac{12}{x^2}=0 \Rightarrow 3\left(x^2+\frac{4}{x^2}\right)-4\left(x+\frac{2}{x}\right)-3=0.$$

$x+\frac{2}{x}=y$ belgilash kiritamiz. Bundan $x^2+\frac{4}{x^2}=y^2-4$ ekanligini topamiz va oxirgi tenglamaga qo'yamiz:

$$3(y^2-4)-4y-3=0 \Rightarrow 3y^2-4y-15=0.$$

Kvadrat tenglamani yechib, ildizlari $y_1=-\frac{5}{3}$ va $y_2=3$ ekanligini aniqlaymiz. Keyin belgilashga qaytamiz:

$$\left[\begin{array}{l} x+\frac{2}{x}=-\frac{5}{3} \\ x+\frac{2}{x}=3 \end{array}\right. \Rightarrow \left[\begin{array}{l} 3x^2+5x+6=0 \\ x^2-3x+2=0 \end{array}\right. \Rightarrow \left[\begin{array}{l} x\in\varnothing \\ x_1=1;\ x_2=2 \end{array}\right. \Rightarrow x_1=1;\ x_2=2.$$

Javob: $\{1; 2\}$.

$\frac{20}{x}$ $\frac{1}{x}$ ($\frac{16}{x^2}$) ($\frac{4}{x}$)

2

Kvadrat tenglamaning ildizlarini topamiz: $D=25-4=21>0$.

$$y_{1,2}=\frac{-5\pm\sqrt{21}}{2}\Rightarrow y_1=\frac{-5-\sqrt{21}}{2};\ y_2=\frac{-5+\sqrt{21}}{2}.$$

Demak, tenglama quyidagi tenglamalar juftligiga teng kuchli:

$$\left[\begin{array}{l} x+\dfrac{4}{x}=\dfrac{-5-\sqrt{21}}{2} \\ x+\dfrac{4}{x}=\dfrac{-5+\sqrt{21}}{2} \end{array}\right. \Rightarrow \left[\begin{array}{l} 2x^2+(5+\sqrt{21})x+8=0 \\ 2x^2+(5-\sqrt{21})x+8=0. \end{array}\right.$$

Ikkinchi tenglama haqiqiy ildizlarga ega emas. Birinchisining ildizlari

$$x_1=\frac{-5-\sqrt{21}-\sqrt{10\sqrt{21}-18}}{4};\ x_2=\frac{-5-\sqrt{21}+\sqrt{10\sqrt{21}-18}}{4}.$$

Javob: $\left\{2;\ \dfrac{-5-\sqrt{21}\pm\sqrt{10\sqrt{21}-18}}{4}\right\}$.

Mustaqil yechish uchun misollar

2.1-misol. Quyidagi qaytma tenglamalarni yeching:

1. $9x^4-6x^3-18x^2-2x+1=0$
2. $4x^4-20x^3-15x^2+60x+36=0$
3. $2x^4+3x^3-11x^2-9x+18=0$
4. $16x^4+32x^3-369x^2-96x+144=0$
5. $18x^4-3x^3-25x^2+2x+8=0$
6. $x^5-3x^4-2x^3-4x^2-24x+32=0$
7. $4x^6+5x^5-3x^4+50x^3-9x^2+45x+108=0$

2.2-misol. Test topshiriqlari.

1. $x^4-2x^3-18x^2-6x+9=0$ tenglamaning barcha haqiqiy ildizlari yig‘indisini toping.

$A)\ -4$ $B)\ 2$ $C)\ 6$ $D)\ 10$

2. $x^4-3x^3-8x^2+12x+16=0$ tenglamaning barcha haqiqiy ildizlari ko'paytmasini toping.

$A)\ -8 \qquad B)\ -16 \qquad C)\ 3 \qquad D)\ 16$

3. $12x^2+\frac{1}{3x^2}+10\left(2x+\frac{1}{3x}\right)+11=0$ tenglama nechta haqiqiy ildizga ega?

$A)\ 2 \qquad B)\ 1 \qquad C)\ 4 \qquad D)\ \varnothing$

5-§. KASR-RATSIONAL TENGLAMALAR

1. Sodda ratsional tenglamalar

Ta'rif. $\frac{P(x)}{Q(x)}=0$ ko'rinishidagi tenglamalarga ratsional tenglamalar deyiladi. Bu yerda $P(x), Q(x)$ ko'phadlar va $Q(x)\neq 0$.

Bu tenglamani yechish quyidagi $\begin{cases} P(x)=0 \\ Q(x)\neq 0 \end{cases}$ sistemani yechishga keltiriladi.

$\frac{P(x)}{Q(x)}=\frac{R(x)}{S(x)}$ ko'rinishidagi kasr-ratsional tenglamalar proporsiyaning asosiy xossasidan foydalanib yechiladi:

$$\begin{cases} P(x)\cdot S(x)=R(x)\cdot Q(x) \\ Q(x)\neq 0 \\ S(x)\neq 0 \end{cases}$$

Bu yerda $P(x), Q(x), R(x), S(x)$ – lar ko'phadlar.

1-misol. $\frac{x}{x^2-1}=\frac{4}{x+4}$ tenglamani yeching.

Yechish: Proporsiyaning asosiy xossasidan foydalanamiz va tenglamaning aniqlanish sohasini hisobga olgan holda tenglamani yechamiz:

$$\begin{cases} x(x+4)=4(x^2-1) \\ x^2-1\neq 0 \\ x+4\neq 0 \end{cases} \Rightarrow \begin{cases} 3x^2-4x-4=0 \\ x\neq \pm 1 \\ x\neq -4 \end{cases} \Rightarrow \begin{cases} x_1=-\frac{2}{3};\ x_2=2 \\ x\neq \pm 1 \\ x\neq -4 \end{cases} \Rightarrow x_1=-\frac{2}{3};\ x_2=2.$$

Javob: $\left\{-\frac{2}{3};\ 2\right\}$.

2-misol. $3\left(x+\frac{1}{x^2}\right)-7\left(1+\frac{1}{x}\right)=0$ tenglamani yeching.

Yechish: $3\cdot\frac{x^3+1}{x^2}-7\cdot\frac{x+1}{x}=0 \Rightarrow 3\cdot\frac{(x+1)(x^2-x+1)}{x^2}-7\cdot\frac{x+1}{x}=0 \Rightarrow$

$$\Rightarrow \frac{x+1}{x^2}\cdot\left[3(x^2-x+1)-7x\right]=0 \Rightarrow \frac{x+1}{x^2}\cdot(3x^2-10x+3)=0.$$

1) $x+1=0 \Rightarrow x_1=-1$.

2) $3x^2-10x+3=0,\ D=100-36=64>0 \Rightarrow x_{2,3}=\frac{10\pm8}{6} \Rightarrow x_2=\frac{1}{3};\ x_3=3.$

Javob: $\left\{-1;\ \frac{1}{3};\ 3\right\}$.

3-misol. $\frac{2}{x^2-4}-\frac{1}{x(x+2)}+\frac{x-4}{x(x+2)}=0$ tenglamani yeching.

Yechish: Tenglamani quyidagi ko'rinishga keltiramiz:

$$\frac{2}{(x-2)(x+2)}-\frac{1}{x(x+2)}+\frac{x-4}{x(x+2)}=0.$$

Aniqlanish sohasi $x\neq0$ va $x\neq\pm2$ dan iborat. Ko'rinib turibdiki, tenglamada qatnashgan kasrlarning umumiy maxraji $x(x-2)(x+2)$ ga teng va oxirgi tenglamaning ikkala tomonini shu umumiy maxrajga ko'paytiramiz. Natijada,

$$2x-(x+2)+(x-4)(x-2)=0$$

$$x^2-5x+6=0$$

$$x_1=2;\ x_2=3$$

ni topamiz. $x_1=2$ chet ildiz, aniqlanish sohasiga kirmaydi (maxraj 0 ga teng bo'lib qoladi). Demak, yechim $x=3$.

Javob: $\{3\}$.

4-misol. $\frac{(x-1)(x-2)(x-3)(x-4)}{(x+1)(x+2)(x+3)(x+4)}=1$ tenglamani yeching.

Yechish: Kasrning surat va maxrajidagi qavslarni soddalashtiramiz.

$$(x-1)(x-2)(x-3)(x-4)=(x^2-3x+2)(x^2-7x+12)=$$
$$=x^4-10x^3+35x^2-50x+24;$$

$$(x+1)(x+2)(x+3)(x+4)=(x^2+3x+2)(x^2+7x+12)=$$
$$=x^4+10x^3+35x^2+50x+24.$$

Topilganlarni tenglamaning dastlabki ko‘rinishiga qo‘yamiz:

$$x^4-10x^3+35x^2-50x+24=x^4+10x^3+35x^2+50x+24 \Leftrightarrow$$

$$\Leftrightarrow x^3+5x=0 \Leftrightarrow x(x^2+5)=0.$$

1) $x_1=0$ 2) $x^2+5=0 \Rightarrow x^2=-5<0 \Rightarrow x\in\varnothing$.

Javob: $\{0\}$.

5-misol. Tenglamani yeching:

$$\frac{1}{x(x+1)}+\frac{1}{(x+1)(x+2)}+\frac{1}{(x+2)(x+3)}+\frac{1}{(x+3)(x+4)}=2.$$

Yechish: Aniqlanish sohasi $x\neq 0;-1;-2;-3;-4$ dan iborat. Berilgan tenglama quyidagi tenglamaga teng kuchli:

$$\left(\frac{1}{x}-\frac{1}{x+1}\right)+\left(\frac{1}{x+1}-\frac{1}{x+2}\right)+\left(\frac{1}{x+2}-\frac{1}{x+3}\right)+\left(\frac{1}{x+3}-\frac{1}{x+4}\right)=2.$$

Qavslarni ochib, soddalashtirgandan keyin $\frac{1}{x}-\frac{1}{x+4}=2$ tenglama hosil bo‘ladi. Maxrajdan qutulib, $x^2+4x-2=0$ kvadrat tenglamani hosil qilamiz va $x_{1,2}=-2\pm\sqrt{6}$ ildizlarini topamiz.

Javob: $\{-2\pm\sqrt{6}\}$.

Mustaqil yechish uchun mashqlar

1.1-misol. Quyidagi ratsional tenglamalarni yeching:

1. $\frac{40}{x-20}-\frac{40}{x}=1$

2. $\frac{x+5}{x+2}+\frac{1}{(x+1)(x+2)}=\frac{1}{x+1}$

3. $\frac{5}{x-1}+\frac{3x+3}{2x+2}=\frac{2x^2+8}{x^2-1}$

4. $\frac{2}{x^2-x+1}=\frac{1}{x+1}+\frac{2x-1}{x^3+1}$

5. $\frac{x}{x+1}+\frac{x+1}{x+2}+\frac{x+2}{x}=3$

6. $\frac{11}{5x-5}+\frac{x+3}{x^2-2x+1}=\frac{7x+6}{5x^2-10x+5}-\frac{5}{2-2x}$

7. $\frac{6-x}{1-x^2}-\frac{x+3}{x-x^2}=\frac{x+5}{x+x^2}$

8. $\frac{x^3-8}{2x-4}=12x-18$

9. $\frac{x^2+1}{x+1}+\frac{x^2+2}{x-2}=-2$ 10. $\frac{x^2+1}{3x^2+2}=\frac{4x^2-5}{x^2+6}$

11. $\frac{u^2}{2-u^2}+\frac{u}{2-u}=2$ 12. $\frac{x+2}{x-2}-\frac{x(x-4)}{x^2-4}=\frac{x-2}{x+2}-\frac{4(x+3)}{4-x^2}$

13. $\frac{3}{x^2-9}-\frac{1}{9-6x+x^2}=\frac{3}{2x^2+6x}$ 14. $\frac{x}{3(x^2-1)}+\frac{2x}{3(1-x^4)}=\frac{1}{x(1+x^2)}$.

1.2-misol. Tenglamalarni yeching.

1. $\frac{3}{2x+1}-\frac{x+1}{x+1/2}-1=\frac{x}{2}\left(\frac{3}{2(x+1/2)}-\frac{2x+2}{2x+1}-1\right)$

2. $\frac{7(x-2)(x-3)(x-4)}{(2x-7)(x+2)(x-6)}=-2$

3. $\frac{x^2+x+2}{3x^2+5x-14}=\frac{x^2+x+6}{3x^2+5x-10}$

4. $\left(\frac{x^2+15}{x^2-9}\right)^2=\left(\frac{8x}{9-x^2}\right)^2$

1.3-misol. Tenglamalarning ildizlarini toping.

1. $\frac{1}{6}+\frac{1}{10}+\frac{1}{15}+\frac{1}{x}=\frac{14}{33}$ 2. $\frac{10}{3+\frac{x+2}{2+\frac{x-11}{2}}}=5$ 3. $\frac{6}{1+\frac{x-2}{1+\frac{3-x}{2}}}=3$

4. $\frac{1}{x}+\frac{2}{x^2-1}+\frac{4}{x^2-4}+\frac{6}{x^2-9}+\ldots+\frac{18}{x^2-81}=$
$=10\left(\frac{1}{(x-1)(x+9)}+\frac{1}{(x-2)(x+8)}+\ldots+\frac{1}{(x-9)(x+1)}+1\right)$

5. Agar $x=1+\frac{1}{2+\frac{1}{3+\frac{1}{x}}}$ bo‘lsa, $(7x-4)^2$ ning qiymatini toping.

2. Belgilash kiritib yechiladigan ratsional tenglamalar

Bu turdagi ratsional tenglamalarni yangi o'zgaruvchi kiritib yechamiz.

1-misol. $\frac{x^2+x-5}{x}+\frac{3x}{x^2+x-5}=4$ tenglamani yeching.

Yechish: $\frac{x^2+x-5}{x}=y$ belgilash kiritib $y+\frac{3}{y}=4$ tenglamani hosil qilamiz.

Almashtirishlardan keyin:

$$y+\frac{3}{y}-4=0 \Rightarrow \frac{y^2-4y+3}{y}=0 \Rightarrow \begin{cases} y^2-4y+3=0 \\ y\neq 0 \end{cases}$$

$y^2-4y+3=0$ kvadrat tenglama ildizlarini Viyet teoremasi yordamida topamiz:

$$\begin{cases} y_1+y_2=4 \\ y_1\cdot y_2=3 \end{cases} \Rightarrow y_1=1;\ y_2=3.$$

Shunday qilib, berilgan tenglama $\frac{x^2+x-5}{x}=1$ yoki $\frac{x^2+x-5}{x}=3$ tenglamalar juftligiga teng kuchli.

1) $\frac{x^2+x-5}{x}-1=0 \Rightarrow \frac{x^2-5}{x}=0 \Rightarrow \begin{cases} x^2-5=0 \\ x\neq 0 \end{cases} \Rightarrow x_{1,2}=\pm\sqrt{5}.$

2) $\frac{x^2+x-5}{x}-3=0 \Rightarrow \frac{x^2-2x-5}{x}=0 \Rightarrow \begin{cases} x^2-2x-5=0 \\ x\neq 0 \end{cases} \Rightarrow x_{3,4}=1\pm\sqrt{6}.$

Javob: $\{1-\sqrt{6};\ 1+\sqrt{6};\ -\sqrt{5};\ \sqrt{5}\}$.

2-misol. $(x+2)^2+\frac{24}{x^2+4x}=18$ tenglamani ildizlarini toping.

Yechish: Tenglamaning aniqlanish sohasi:

$$x^2+4x\neq 0 \Rightarrow x(x+4)\neq 0 \Rightarrow x\neq 0,\ x\neq -4.$$

Qavslarni ochib, tenglamani quyidagicha yozib olamiz:

$$x^2+4x+4+\frac{24}{x^2+4x}=18.$$

$x^2+4x=y$ belgilash kiritib,

$$y+4+\frac{24}{y}=18\Leftrightarrow y+\frac{24}{y}-14=0\Leftrightarrow\frac{y^2-14y+24}{y}=0$$

ni hosil qilamiz. Bundan

$$\begin{cases}y^2-14y+24=0\\ y\neq 0\end{cases}\Rightarrow y_1=2;\ y_2=12.$$

1) $x^2+4x=2\Rightarrow x^2+4x-2=0,\ D=16+8=24,\ x_{1,2}=\frac{-4\pm2\sqrt{6}}{2}=-2\pm\sqrt{6}$..

2) $x^2+4x=12\Rightarrow x^2+4x-12=0\Rightarrow x_3=-6;\ x_4=2$.

Javob: $\{-2-\sqrt{6};\ -2+\sqrt{6};\ -6;\ 2\}$.

3-misol. $\frac{x-6}{x-12}-\frac{x-12}{x-6}=\frac{5}{6}$ tenglamani yeching.

Yechish: Tenglamaning aniqlanish sohasi:

$$\begin{cases}x-6\neq 0\\ x-12\neq 0\end{cases}\Rightarrow\begin{cases}x\neq 6\\ x\neq 12\end{cases}$$

$\frac{x-6}{x-12}=t$ belgilash kiritamiz va $t-\frac{1}{t}=\frac{5}{6}\Rightarrow\frac{6t^2-5t-6}{t}=0\Rightarrow$

$\Rightarrow\begin{cases}6t^2-5t-6=0\\ t\neq 0\end{cases}$ ni hosil qilamiz. Sistemaning birinchi tenglamasini yechamiz:

$$t_{1,2}=\frac{5\pm13}{12}\Rightarrow t_1=-\frac{2}{3};\ t_2=\frac{3}{2}.$$

Topilganlarni belgilashga qo'yamiz va berilgan tenglama

$$\frac{x-6}{x-12}=-\frac{2}{3}$$

yoki $\frac{x-6}{x-12}=\frac{3}{2}$ tenglamalarga ekvivalent bo'ladi.

1) $\frac{x-6}{x-12}=-\frac{2}{3}\Rightarrow\begin{cases}3(x-6)=-2(x-12)\\ x-12\neq 0\end{cases}\Rightarrow\begin{cases}5x=42\\ x\neq 12\end{cases}\Rightarrow\begin{cases}x=8,4\\ x\neq 12\end{cases}\Rightarrow x=8,4;$

2) $\frac{x-6}{x-12}=\frac{3}{2}\Rightarrow\begin{cases}2(x-6)=3(x-12)\\x-12\neq 0\end{cases}\Rightarrow\begin{cases}x=24\\x\neq 12\end{cases}\Rightarrow x=24.$

Hosil qilingan ildizlar aniqlanish sohasiga tegishli.

Javob: $\{8{,}4\,;24\}$.

4-misol. $\frac{(x-1)^2x}{(x^2-x+1)^2}=\frac{2}{9}$ tenglamani yeching.

Yechish: Berilgan tenglamani maxrajidan qutulib, soddalashtirilsa, 4-darajali qaytma tenglama hosil bo'ladi. Buni boshqacha usulda amalga oshirsa ham bo'ladi. Tenglamani surat va maxrajini x^2 ga bo'lamiz:

$$\frac{\frac{(x^2-2x+1)x}{x^2}}{\frac{(x^2-x+1)^2}{x^2}}=\frac{2}{9}\Rightarrow\frac{x+\frac{1}{x}-2}{\left(x+\frac{1}{x}-1\right)^2}=\frac{2}{9}.$$

$x+\frac{1}{x}=y$ belgilash kiritamiz va $\frac{y-2}{(y-1)^2}=\frac{2}{9}$ ni hosil qilamiz.

Bundan $9(y-2)=2(y^2-2y+1)\Rightarrow 2y^2-13y+20=0$

$$y_{1,2}=\frac{13\pm 3}{4}\Rightarrow y_1=\frac{5}{2}\,;\,y_2=4$$

ni topamiz. Topilganlarni belgilashga qo'yamiz:

1) $x+\frac{1}{x}=\frac{5}{2}\Rightarrow 2x^2-5x+2=0\Rightarrow x_{1,2}=\frac{5\pm 3}{4}\Rightarrow x_1=2\,;\,x_2=\frac{1}{2}$;

2) $x+\frac{1}{x}=4\Rightarrow x^2-4x+1=0\Rightarrow x_{3,4}=\frac{4\pm 2\sqrt{3}}{2}=2\pm\sqrt{3}\Rightarrow x_3=$
$=2-\sqrt{3}\,;\,x_4=2+\sqrt{3}$.

Javob: $\left\{2\,;\frac{1}{2}\,;2\pm\sqrt{3}\right\}$.

5-misol. $x^2+\frac{36}{x^2}=\frac{112}{5}\left(\frac{x}{2}-\frac{3}{x}\right)$ tenglamani yeching.

Yechish: Aniqlanish sohasi: $x \neq 0$. $\frac{x}{2}-\frac{3}{x}=t$ deb belgilash kiritamiz. Bundan $\left(\frac{x}{2}-\frac{3}{x}\right)^2=t^2 \Rightarrow \frac{x^2}{4}-2\cdot\frac{x}{2}\cdot\frac{3}{x}+\frac{9}{x^2}=t^2 \Rightarrow$

$\Rightarrow \frac{x^2}{4}+\frac{9}{x^2}=t^2+3$ ni topamiz.

Oxirgi tenglikning ikkala tomonini 4 ga ko'paytiramiz va $x^2+\frac{36}{x^2}=4t^2+12$ ni topib, tenglamani dastlabki ko'rinishiga qo'yamiz. Natijada $4t^2+12=\frac{112}{5}t$ yoki $5t^2-28t+15=0$ tenglamani hosil qilamiz.

$$t_{1,2}=\frac{28\pm 22}{10} \Rightarrow t_1=\frac{3}{5};\ t_2=5.$$

Belgilashga qaytamiz:

1) $\frac{x}{2}-\frac{3}{x}=\frac{3}{5} \Rightarrow 5x^2-6x-30=0 \Rightarrow x_{1,2}=\frac{6\pm 2\sqrt{159}}{10}=\frac{3\pm\sqrt{159}}{5}$;

2) $\frac{x}{2}-\frac{3}{x}=5 \Rightarrow x^2-10x-6=0 \Rightarrow x_{1,2}=\frac{10\pm 2\sqrt{31}}{2}=5\pm\sqrt{31}$.

Javob: $\left\{\frac{3\pm\sqrt{159}}{5};\ 5\pm\sqrt{31}\right\}$.

6-misol. $x^2+3x+2=15\cdot\frac{x^2+5x+10}{x^2+7x+12}$ tenglamani yeching.

Yechish: Aniqlanish sohasi:

$$x^2+7x+12=(x+3)(x+4)\neq 0 \Rightarrow x\neq -3;\ x\neq -4.$$

Tenglamani quyidagicha shakl almashtiramiz:

$$(x^2+3x+2)(x^2+7x+12)=15(x^2+5x+10)$$

$$(x+1)(x+2)(x+3)(x+4)=15(x^2+5x+10)$$

$$[(x+1)(x+4)][(x+2)(x+3)]=15(x^2+5x+10)$$

$$(x^2+5x+4)(x^2+5x+6)=15(x^2+5x+10)$$

$x^2+5x=y$ belgilash kiritib, $(y+4)(y+6)=15(y+10)$ tenglamani hosil qilamiz. Bundan

$$y^2+10y+24=15y+150 \Rightarrow y^2-5y-126=0.$$

Viyet teoremasidan

$$\begin{cases} y_1+y_2=5 \\ y_1\cdot y_2=-126 \end{cases} \Rightarrow y_1=-9\,;\, y_2=14.$$

1) $x^2+5x=-9 \Leftrightarrow x^2+5x+9=0,\ D=25-36=-11<0 \Rightarrow x\in\varnothing$.

2) $x^2+5x=14 \Leftrightarrow x^2+5x-14=0$.

Viyet teoremasidan

$$\begin{cases} x_1+x_2=-5 \\ x_1\cdot x_2=-14 \end{cases} \Rightarrow x_1=-7\,;\, x_2=2.$$

Javob: $\{-7\,;\,2\}$.

Mustaqil yechish uchun mashqlar

2.1-misol. Quyidagi ratsional tenglamalarni yeching:

1. $\dfrac{x}{10-x}+\dfrac{10-x}{x}=25$ **2.** $\dfrac{1}{x^2+4}+\dfrac{1}{x^2+5}=\dfrac{11}{30}$

3. $\dfrac{1}{x^2+2x+4}-\dfrac{1}{x^2+2x+5}=\dfrac{1}{12}$ **4.** $\dfrac{x^2+1}{x}+\dfrac{x}{x^2+1}=-2{,}5$

5. $\dfrac{x^2-3x}{x-2}+\dfrac{x-2}{x^2-3x}=2{,}5$ **6.** $\dfrac{x-3}{x^2+4x+9}+\dfrac{x^2+4x+9}{x-3}=-2$

7. $\dfrac{x^2+x+2}{x^2+x+1}+\dfrac{x^2+x+6}{x^2+x+3}=4$ **8.** $\dfrac{2}{(x-2)^2}-(x-2)^2-2=0$

9. $\dfrac{3}{(x+1)(x+2)}+\dfrac{4}{(x-1)(x+4)}=\dfrac{1}{2}$ **10.** $\dfrac{5}{x(x+4)}+\dfrac{8}{(x+1)(x+3)}=2$

2.2-misol. Tenglamalarning haqiqiy ildizlarini toping.

1. $\dfrac{(x^2+1)x}{(x^2-x+1)^2}=\dfrac{10}{9}$ **2.** $\dfrac{(x^2+1)^2}{x(x+1)^2}=\dfrac{625}{112}$

3. $\dfrac{(x^2+x+1)^2}{(x+1)^2(x^2+1)}=\dfrac{49}{45}$ **4.** $\dfrac{1+x^4}{(1+x)^4}=\dfrac{17}{81}$

5. $\dfrac{x^2}{3}+\dfrac{48}{x^2}=10\left(\dfrac{x}{3}-\dfrac{4}{x}\right)$ **6.** $\dfrac{24}{x^2-2x}=\dfrac{12}{x^2-x}+x^2-x$

7. $\frac{1}{x^3}-\frac{1}{(x+1)^3}=\frac{7}{8}$

8. $\frac{1}{6x^2-7x+2}+\frac{1}{12x^2-17x+6}=4x^2-5x$

9. $\frac{1}{6x^2-5x+1}+\frac{1}{2x^2-3x+1}=3x^2-4x$

10. $\frac{x^6}{9}+\frac{1}{x^2}+\frac{2x^2}{3}=\frac{4x^4+12}{9x}$

11. $2\left(\frac{2}{x}-\frac{x}{3}\right)=\frac{2}{x^2}+\frac{x^2}{18}+\frac{4}{3}$.

3. Maxsus yo'llar bilan yechiladigan ratsional tenglamalar

3.1. $\frac{a_1}{x+b_1}+\frac{a_2}{x+b_2}+\dots+\frac{a_n}{x+b_n}=A$ ko'rinishidagi tenglamalar

Bu turdagi tenglamalarni yechish uchun tenglamanin[g] hadlari ikkitadan qilib guruhlanadi va ular ustida algebra[ik] almashtirishlar bajariladi.

1-misol. $\frac{1}{x-7}+\frac{1}{x-5}+\frac{1}{x-6}+\frac{1}{x-4}=0$ tenglamani yeching.

Yechish: Bu tenglamani maxrajidan qutqarib, kvadr[at] tenglamaga keltirib yechish mumkin. Bunda ishimiz juda mu[ra]-rakkablashib ketadi. Mos kasrlarni guruhlasak, tenglama so[d]-da holga keladi:

$$\left(\frac{1}{x-4}+\frac{1}{x-7}\right)+\left(\frac{1}{x-5}+\frac{1}{x-6}\right)=0\Rightarrow\frac{2x-11}{(x-4)(x-7)}+\frac{2x-11}{(x-5)(x-6)}=0.$$

Ko'pchilik oxirgi tenglamani ikkala tomonini $2x-11$ g[a] bo'lib yuboradi. Bu qo'pol xato hisoblanib, $2x-11=0\Rightarrow x=5$[,5] tenglamaning ildizi hisoblanadi.

$$(2x-11)\left(\frac{1}{(x-4)(x-7)}+\frac{1}{(x-5)(x-6)}\right)=0\Rightarrow\frac{(2x-11)(x^2-11x+29)}{(x-4)(x-5)(x-6)(x-7)}=0$$

1) $2x-11=0\Rightarrow x_1=5{,}5$.

2) $x^2-11x+29=0, D=121-116=5>0 \Rightarrow x_{2,3}=\frac{11\pm\sqrt{5}}{2}$.

Javob: $\left\{5{,}5;\frac{11-\sqrt{5}}{2};\frac{11+\sqrt{5}}{2}\right\}$.

3.2. $\frac{a_1x+b_1}{x+c_1}+\frac{a_2x+b_2}{x+c_2}+\dots+\frac{a_nx+b_n}{x+c_n}=B$ ko'rinishidagi tenglamalar

Bu korinishdagi tenglamalarni yechishda tenglamada qatnashgan har bir kasrni butun va kasr qismlarini ajratib olamiz va 3.1 ko'rinishidagi tenglama kabi yechamiz:

$$\alpha_1+\frac{\beta_1}{x+c_1}+\alpha_2+\frac{\beta_2}{x+c_2}+\dots+\alpha_n+\frac{\beta_n}{x+c_n}=B.$$

2-misol. $31\left(\frac{24-5x}{x+1}+\frac{5-6x}{x+4}\right)+370=29\left(\frac{17-7x}{x+2}+\frac{8x+55}{x+3}\right)$

tenglamani yeching.

Yechish: Tenglamada qatnashgan har bir kasrni sodda kasrga keltiramiz:

$$\frac{24-5x}{x+1}=\frac{29-5(x+1)}{x+1}=\frac{29}{x+1}-5;\ \frac{5-6x}{x+4}=\frac{29-6(x+4)}{x+4}=\frac{29}{x+4}-6$$

$$\frac{17-7x}{x+2}=\frac{31-7(x+2)}{x+2}=\frac{31}{x+2}-7;\ \frac{8x+55}{x+3}=\frac{31+8(x+3)}{x+3}=\frac{31}{x+3}+8$$

Topilganlarni tenglamaning dastlabki ko'rinishiga qo'yamiz:

$$31\left(\frac{29}{x+1}-5+\frac{29}{x+4}-6\right)+370=29\left(\frac{31}{x+2}-7+\frac{31}{x+3}+8\right)$$

$$31\cdot29\cdot\left(\frac{1}{x+1}+\frac{1}{x+4}\right)-341+370=29+29\cdot31\cdot\left(\frac{1}{x+2}+\frac{1}{x+3}\right)$$

Soddalashtirishlardan so'ng tenglama yuqoridagi 1-misolni yechish usuliga keladi:

$$\frac{1}{x+1}+\frac{1}{x+4}=\frac{1}{x+2}+\frac{1}{x+3}$$

$$\frac{1}{x+1}-\frac{1}{x+2}=\frac{1}{x+3}-\frac{1}{x+4}$$

$$\frac{1}{(x+1)(x+2)}=\frac{1}{(x+3)(x+4)}$$

$$(x+1)(x+2)=(x+3)(x+4)$$

$$4x+10=0$$

$$x=-2,5$$

Javob: $\{-2,5\}$.

3.3. $\frac{a_1x+b_1}{p_1x^2+q_1x+r_1}+\frac{a_2x+b_2}{p_2x^2+q_2x+r_2}+\dots+\frac{a_nx+b_n}{p_nx^2+q_nx+r_n}=A$

ko'rinishidagi tenglamalar

Bu ko'rinishdagi tenglamalarni yechishda tenglamada qatnashgan har bir kasrni sodda kasrlar yig'indisi ko'rinishiga keltiramiz va 3.1 ko'rinishidagi tenglama kabi yechamiz:

$$\frac{a_ix+b_i}{p_ix^2+q_ix+r_i}=\frac{A_i}{x+\alpha_i}+\frac{B_i}{x+\beta_i},\ i=\overline{1,n}.$$

3-misol. $\frac{x+1}{x^2+2x}+\frac{x+6}{x^2+12x+35}=\frac{x+2}{x^2+4x+3}+\frac{x+5}{x^2+10x+24}$

tenglamani yeching.

Yechish: Tenglamada qatnashgan har bir kasrning maxrajlarini ko'paytuvchilarga ajratamiz:

$$x^2+2x=x(x+2)\ ;\ x^2+12x+35=(x+5)(x+7);$$

$$x^2+4x+3=(x+1)(x+3);\ x^2+10x+24=(x+4)(x+6).$$

Har bir kasrning suratini 2 ga ko'paytirib, quyidagilarni hosil qilamiz:

$$2(x+1)=x+x+2;\ 2(x+6)=x+5+x+7;$$

$$2(x+2)=x+1+x+3;\ 2(x+5)=x+4+x+6.$$

Topilganlarni tenglamaga qo'yamiz:

$$\frac{x+x+2}{x(x+2)}+\frac{x+5+x+7}{(x+5)(x+7)}=\frac{x+1+x+3}{(x+1)(x+3)}+\frac{x+4+x+6}{(x+4)(x+6)}$$

$$\frac{1}{x}+\frac{1}{x+2}+\frac{1}{x+5}+\frac{1}{x+7}=\frac{1}{x+1}+\frac{1}{x+3}+\frac{1}{x+4}+\frac{1}{x+6}$$

Tenglamaning mos hadlarini guruhlaymiz va quyidagi tenglamani hosil qilamiz:

$$\left(\frac{1}{x}+\frac{1}{x+7}\right)+\left(\frac{1}{x+2}+\frac{1}{x+5}\right)=\left(\frac{1}{x+1}+\frac{1}{x+6}\right)+\left(\frac{1}{x+3}+\frac{1}{x+4}\right)$$

$$\frac{2x+7}{x^2+7x}+\frac{2x+7}{x^2+7x+10}=\frac{2x+7}{x^2+7x+6}+\frac{2x+7}{x^2+7x+12}$$

Demak, berilgan tenglama quyidagi ikkita tenglamaga teng kuchli bo'ladi.

1) $2x+7=0 \Rightarrow x_1=-3,5$.

2) $\frac{1}{x^2+7x}+\frac{1}{x^2+7x+10}=\frac{1}{x^2+7x+6}+\frac{1}{x^2+7x+12}$.

Ikkinchi tenglamani yechish uchun $x^2+7x+6=u$ deb belgilash kiritamiz:

$$\frac{1}{u-6}+\frac{1}{u+4}=\frac{1}{u}+\frac{1}{u+6}\Rightarrow\frac{1}{u-6}-\frac{1}{u+6}=\frac{1}{u}-\frac{1}{u+4}\Rightarrow$$

$$\Rightarrow\frac{3}{u^2-36}=\frac{1}{u^2+4u}\Rightarrow\frac{u^2+6u+18}{u(u+4)(u-6)(u+6)}=0.$$

$u^2+6u+18=0$ tenglama haqiqiy ildizga ega emas, chunki $D=36-72=-36<0$. Bundan ikkinchi tenglamaning yechimga ega emasligi kelib chiqadi.

Javob: $\{-3,5\}$.

3.4. $\frac{a_1x^2+b_1x+c_1}{\alpha_1x+\beta_1}+\frac{a_2x^2+b_2x+c_2}{\alpha_2x+\beta_2}+\ldots\frac{a_nx^2+b_nx+c_n}{\alpha_nx+\beta_n}=A$

ko'rinishidagi tenglamalar

Bu turdagi tenglamalarni yechish uchun tenglamaning o'ng tomonidagi har bir hadi quyidagicha yozib olinadi:

$$\frac{a_ix^2+b_ix+c_i}{\alpha_ix+\beta_i}=\gamma_ix+\delta_i+\frac{B_i}{\alpha_ix+\beta_i}.$$

4-misol. $\frac{x^2+2x+2}{x+1}+\frac{x^2+8x+20}{x+4}=\frac{x^2+4x+6}{x+2}+\frac{x^2+6x+12}{x+3}$

tenglamani yeching.

Yechish: Tenglamada qatnashgan har bir kasrni sodda kasrga keltiramiz:

$$\frac{x^2+2x+2}{x+1}=\frac{(x+1)^2+1}{x+1}=x+1+\frac{1}{x+1};$$

$$\frac{x^2+8x+20}{x+4}=\frac{(x+4)^2+4}{x+4}=x+4+\frac{4}{x+4};$$

$$\frac{x^2+4x+6}{x+2}=\frac{(x+2)^2+2}{x+2}=x+2+\frac{2}{x+2};$$

$$\frac{x^2+6x+12}{x+3}=\frac{(x+3)^2+3}{x+3}=x+3+\frac{3}{x+3}$$

Natijani tenglamaning dastlabki ko'rinishiga qo'yamiz:

$$x+1+\frac{1}{x+1}+x+4+\frac{4}{x+4}=x+2+\frac{2}{x+2}+x+3+\frac{3}{x+3}$$

$$\frac{1}{x+1}+\frac{4}{x+4}=\frac{2}{x+2}+\frac{3}{x+3}$$

$$\frac{4}{x+4}-\frac{3}{x+3}=\frac{2}{x+2}-\frac{1}{x+1}$$

$$\frac{x}{(x+3)(x+4)}=\frac{x}{(x+1)(x+2)}$$

1) $x=0\Rightarrow x_1=0$ 2) $(x+1)(x+2)=(x+3)(x+4)\Rightarrow 4x=-10\Rightarrow x_2=-2,5$

Javob: $\{-2,5\,;0\}$.

3.5. $\frac{Ax}{ax^2+b_1x+c}+\frac{Bx}{ax^2+b_2x+c}=D$ ko'rinishidagi tenglamalar

Bu yerda $ABD\neq 0$ va $ac\neq 0$. Bu tenglama $y=ax+\frac{c}{x}$ belgilash kiritish yordamida $\frac{A}{y+b_1}+\frac{B}{y+b_2}=D$ ko'rinishga keltirib yechiladi.

5-misol. $\frac{4x}{4x^2-8x+7}+\frac{3x}{4x^2-10x+7}=1$ tenglamani yeching.

Yechish: $x=0$ berilgan tenglamaning ildizi bo'lmaganligi sababli, har bir kasrning surat va maxrajini x ga bo'lamiz va berilgan tenglamaga teng kuchli tenglamani hosil qilamiz:

$$\frac{4}{4x+\frac{7}{x}-8}+\frac{3}{4x+\frac{7}{x}-10}=1.$$

$4x+\frac{7}{x}=y$ belgilash kiritib, $\frac{4}{y-8}+\frac{3}{y-10}=1$ tenglamani hosil qilamiz. Bundan

$$\frac{4}{y-8}+\frac{3}{y-10}-1=0 \Rightarrow \frac{y^2-25y+144}{(y-8)(y-10)}=0 \Rightarrow$$

$$\Rightarrow \begin{cases} y^2-25y+144=0 \\ (y-8)(y-10)\neq 0 \end{cases} \Rightarrow y_1=9\ ;\ y_2=16.$$

1) $4x+\frac{7}{x}=9 \Leftrightarrow 4x^2-9x+7=0,\ D=81-112=-31<0 \Rightarrow x\in\varnothing;$

2) $4x+\frac{7}{x}=16 \Leftrightarrow 4x^2-16x+7=0,\ D=256-112=144>0.$

$$x_{1,2}=\frac{16\pm 12}{8} \Rightarrow x_1=\frac{1}{2};\ x_2=\frac{7}{2}.$$

Javob: $\left\{\frac{1}{2};\frac{7}{2}\right\}$.

3.6. $\frac{ax^2+b_1x+c}{ax^2+b_2x+c}\pm\frac{ax^2+b_3x+c}{ax^2+b_4x+c}=A$ **va** $\frac{ax^2+b_1x+c}{ax^2+b_2x+c}=\frac{Ax}{ax^2+b_3x+c}$, $(A\neq 0,\ ac\neq 0)$ **ko'rinishidagi tenglamalar**

Bu turdagi tenglamalar ham yuqoridagi tenglamalar kabi $y=ax+\frac{c}{x}$ belgilash kiritish yordamida yechiladi.

6-misol. $\frac{x^2+5x+4}{x^2-7x+4}-\frac{x^2-x+4}{x^2+x+4}+\frac{13}{3}=0$ tenglamani yeching.

Yechish: Tenglamada qatnashgan kasrlarning surat va maxrajini $x\neq 0$ ga bo'lamiz. Natijada

$$\frac{x+\frac{4}{x}+5}{x+\frac{4}{x}-7}+\frac{x+\frac{4}{x}-1}{x+\frac{4}{x}+1}+\frac{13}{3}=0$$

ni hosil qilamiz. $x+\frac{4}{x}=t$ belgilash kiritib $\frac{t+5}{t-7}+\frac{t-1}{t+1}+\frac{13}{3}=0$ yoki

$$\frac{3(t+5)(t+1)+3(t-1)(t-7)+13(t+1)(t-7)}{(t+1)(t-7)}=0\Rightarrow\frac{19t^2-84t-55}{(t+1)(t-7)}=0\Rightarrow$$

$$\Rightarrow\begin{cases}19t^2-84t-55=0\\ t\neq-1,\, t\neq7\end{cases}\Rightarrow t_{1,2}=\frac{84\pm106}{38}\Rightarrow t_1=-\frac{11}{19};\, t_2=5.$$

Belgilashga qaytamiz:

1) $x+\frac{4}{x}=-\frac{11}{19}\Rightarrow 19x^2+11x+76=0;\ D=11^2-76^2<0\Rightarrow x\in\varnothing;$

2) $x+\frac{4}{x}=5\Rightarrow x^2-5x+4=0\Rightarrow x_1=1;\ x_2=4.$

Javob: $\{1;4\}$.

Mustaqil yechish uchun mashqlar

3.1-misol. Tenglamalarning haqiqiy ildizlarini toping.

1. $\frac{1}{x-1}+\frac{2}{x-2}+\frac{3}{x-3}=\frac{6}{x+6}$

2. $\frac{1}{x}+\frac{1}{x+1}+\frac{1}{x+2}+\frac{1}{x+3}+\frac{1}{x+4}=0$

3. $\frac{1}{x-8}+\frac{1}{x-6}+\frac{1}{x+6}+\frac{1}{x+8}=0$

4. $\frac{2}{x+8}+\frac{5}{x+9}=\frac{3}{x+15}+\frac{4}{x+6}$

5. $\frac{5}{x-1}+\frac{4}{x+2}+\frac{21}{x-3}=\frac{5}{x+1}+\frac{4}{x-2}+\frac{21}{x+3}$

3.2-misol. Tenglamalarni yeching.

1. $\frac{2x-1}{x+1}+\frac{3x-1}{x+2}=\frac{x-7}{x-1}+4$

2. $\frac{x+1}{x-1}+\frac{x+5}{x-5}=\frac{x+3}{x-3}+\frac{x+4}{x-4}$

3. $\frac{x+1}{x-1}+\frac{x-2}{x+2}+\frac{x-3}{x+3}+\frac{x+4}{x-4}=4$

4. $112+19\left(\frac{8-3x}{x+3}+\frac{3-2x}{x+7}\right)=17\left(\frac{15-x}{x+4}+\frac{31+2x}{x+6}\right)$

5. $\frac{x+4}{x-1}+\frac{x-4}{x+1}=\frac{x+8}{x-2}+\frac{x-8}{x+2}-\frac{8}{3}$

6. $\frac{x+2}{x+1}+\frac{x+6}{x+3}+\frac{x+10}{x+5}=6$

3.3-misol. Tenglamalarni yeching.

1. $\frac{x^2+4x+4}{x+4}-\frac{2x+6}{x+2}=\frac{x^2+x+1}{x+1}-\frac{2x+9}{x+3}$

2. $\frac{x^2+x+1}{x+1}+\frac{x^2+2x+2}{x+2}-\frac{x^2+3x+3}{x+3}-\frac{x^2+4x+4}{x+4}=0$

3.4-misol. Ratsional tenglamalarni yeching.

1. $\frac{4x}{x^2+x+3}+\frac{5x}{x^2-5x+3}=-\frac{3}{2}$

2. $\frac{2x}{3x^2-x+2}-\frac{7x}{3x^2+5x+2}=1$

3. $\frac{3x}{2x^2-4x+1}+\frac{2x}{2x^2-6x+1}=3$

4. $\frac{2x}{x^2-2x+7}+\frac{3x}{x^2+2x+7}=\frac{7}{8}$

3.5-misol. Quyidagi tenglamalarni yeching.

1. $\frac{x^2-13x+15}{x^2-14x+15}-\frac{x^2-15x+15}{x^2-16x+15}=-\frac{1}{12}$

2. $\frac{x^2-10x+15}{x^2-6x+15}=\frac{3x}{x^2-8x+15}$

4. To‘la kvadratga keltiriladigan ratsional tenglamalar

1-misol. $x^2+\frac{9x^2}{(x-3)^2}=7$ tenglamani yeching.

Yechish: Tenglamaning chap tomonida yig‘indining to‘la kvadratini hosil qilamiz:

$$x^2+2x\cdot\frac{3x}{x-3}+\left(\frac{3x}{x-3}\right)^2-2x\cdot\frac{3x}{x-3}=7$$

$$\left(x+\frac{3x}{x-3}\right)^2-\frac{6x^2}{x-3}-7=0,\ \left(\frac{x^2}{x-3}\right)^2-6\cdot\frac{x^2}{x-3}-7=0$$

$\frac{x^2}{x-3}=y$ belgilash kiritamiz va $y^2-6y-7=0$ kvadrat tenglamani hosil qilamiz. Viyet teoremasidan bu kvadrat tenglamaning ildizlarini topamiz:

$$\begin{cases} y_1+y_2=6 \\ y_1\cdot y_2=-7 \end{cases} \Rightarrow y_1=-1;\ y_2=7.$$

Berilgan tenglama quyidagi tenglamalar juftligiga ekvivalent bo'ladi:

$$\left[\begin{array}{l} \frac{x^2}{x-3}=-1 \\ \frac{x^2}{x-3}=7 \end{array}\right. \Rightarrow \begin{cases} \left[\begin{array}{l} x^2+x-3=0 \\ x^2-7x+21=0 \end{array}\right. \\ x\neq 3 \end{cases}$$

1) $x^2+x-3=0$, $D=1+12=13>0\Rightarrow x_{1,2}=\frac{-1\pm\sqrt{13}}{2}$;

2) $x^2-7x+21=0$, $D=49-84=-35<0\Rightarrow x\in\varnothing$.

Javob: $\left\{\frac{-1-\sqrt{13}}{2};\frac{-1+\sqrt{13}}{2}\right\}$.

2-misol. $\left(\frac{x}{x+1}\right)^2+\left(\frac{x}{x-1}\right)^2=a(a-1)$ tenglamani yeching.

Yechish: Tenglamaning chap tomonida to'la kvadrat hosil qilamiz:

$$\left(\frac{x}{x+1}\right)^2+2\cdot\frac{x}{x+1}\cdot\frac{x}{x-1}+\left(\frac{x}{x-1}\right)^2-2\cdot\frac{x}{x+1}\cdot\frac{x}{x-1}=a(a-1)$$

$$\left(\frac{x}{x+1}+\frac{x}{x-1}\right)^2-\frac{2x^2}{x^2-1}=a(a-1),$$

$$\left(\frac{2x^2}{x^2-1}\right)^2-\frac{2x^2}{x^2-1}-a(a-1)=0$$

$\frac{2x^2}{x^2-1}=y$ yangi o'zgaruvchi kiritamiz va $y^2-y+a(1-a)=0$ kvadrat tenglamani hosil qilamiz. Viyet teoremasi yordamida bu tenglamaning ildizlarini topamiz:

$$\begin{cases} y_1+y_2=1 \\ y_1\cdot y_2=a(1-a) \end{cases} \Rightarrow y_1=a\,;\, y_2=1-a\,.$$

Bundan berilgan tenglama $\frac{2x^2}{x^2-1}=a$ yoki $\frac{2x^2}{x^2-1}=1-a$ tenglamalarga teng kuchli bo'ladi.

1) $\frac{2x^2}{x^2-1}=a \Rightarrow (a-2)x^2=1$, bundan $a\neq 2$ da $x_{1,2}=\pm\sqrt{\frac{a}{a-2}}$ bo'ladi.

$a=2$ da tenglama $y^2-y-2=0$ ko'rinishga ega bo'lib, $y_1=-1;\, y_2=2$ ni hosil qilamiz.

a) $\frac{2x^2}{x^2-1}=-1 \Rightarrow 3x^2=1 \Rightarrow x=\pm\sqrt{\frac{1}{3}}=\pm\frac{\sqrt{3}}{3}$.

b) $\frac{2x^2}{x^2-1}=2$ tenglama yechimga ega bo'lmaydi.

2) $\frac{2x^2}{x^2-1}=1-a \Rightarrow (a+1)x^2=a-1$, bundan $a\neq -1$ da $x_{3,4}=\pm\sqrt{\frac{a-1}{a+1}}$ bo'ladi.

$a=-1$ da ham tenglama $y^2-y-2=0$ ko'rinishga ega bo'lib, $y_1=-1;\, y_2=2$ ni hosil qilamiz. Bunda yuqoridagi 1-hol takrorlanadi.

a) $\frac{2x^2}{x^2-1}=-1 \Rightarrow 3x^2=1 \Rightarrow x\pm\sqrt{\frac{1}{3}}=\pm\frac{\sqrt{3}}{3}$;

b) $\frac{2x^2}{x^2-1}=2$ tenglama yechimga ega bo'lmaydi.

Javob: $a=2$ va $a=-1$ da tenglama $x=\pm\frac{\sqrt{3}}{3}$ ildizlarga ega. $a\neq 2$ va $a\neq -1$ larda tenglama $x_{1,2}=\pm\sqrt{\frac{a}{a-2}}$, $x_{3,4}=\pm\sqrt{\frac{a-1}{a+1}}$ ildizlarga ega bo'ladi.

Mustaqil yechish uchun mashqlar

4.1-misol. Quyidagi tenglamalarni yeching:

1. $x^2+\frac{25x^2}{(x+5)^2}=11$ **2.** $x^2+\frac{81x^2}{(x+9)^2}=40$

3. $x^2+\frac{4x^2}{(x+2)^2}=5$ **4.** $x^2+\frac{25x^2}{(5+2x)^2}=\frac{74}{49}$

5. $x^2+\frac{x^2}{(x+1)^2}=\frac{40}{9}$ **6.** $x^2+\left(\frac{x}{x-1}\right)^2=8$

7. $\frac{1}{x^2}-\frac{1}{(x+1)^2}=1$

4.2-misol. Tenglamalarni yeching.

1. $\left(\frac{x}{x+1}\right)^2+\left(\frac{x}{x-1}\right)^2=90$ **2.** $\left(\frac{x}{x-1}\right)^2+\left(\frac{x}{x+1}\right)^2=\frac{45}{16}$

5. Bir jinsli tenglamalar

Ta'rif. $a_0p^n(x)+a_1p^{n-1}(x)q(x)+\cdots+a_{n-1}p(x)q^{n-1}(x)+$ $+a_nq^n(x)=0$ (1) *ko'rinishidagi tenglamalarga bir jinsli tenglamalar deyiladi.*

Bu yerda $a_0\neq 0$ va $a_1, a_2, \ldots, a_n$ koeffitsiyentlardan kamida bittasi noldan farqli. $p(x)$ va $q(x)$ lar biror funksiyalar.

Bu tenglama quyidagi usulda yechiladi:

1) $\begin{cases} p(x)=0 \\ q(x)=0 \end{cases}$ sistema yechimga ega bo'lsa, bu yechimlar berilgan (1) tenglamaning yechimi bo'ladi.

2) Tenglamaning ikkala tomonini $q^n(x)$ ga bo'lamiz va ushbu

$$a_0\left(\frac{p(x)}{q(x)}\right)^n + a_1\left(\frac{p(x)}{q(x)}\right)^{n-1} + \cdots + a_{n-1}\left(\frac{p(x)}{q(x)}\right) + a_n = 0 \quad (2)$$

tenglamani hosil qilamiz.

$\frac{p(x)}{q(x)} = y$ belgilash kiritgandan keyin (2) tenglama quyidagi ko'rinishga keladi:

$$a_0y^n + a_1y^{n-1} + \cdots + a_{n-1}y + a_n = 0 \quad (3)$$

(3) tenglama ko'pi bilan n ta ildizga ega bo'ladi, ya'ni $y_1, y_2, \ldots, y_m$ ildizlar bo'lib, bu yerda $m \le n$. Endi berilgan (1) tenglamaning ildizlari quyidagi tenglamalarning yechimlaridan iborat bo'ladi:

$$\frac{p(x)}{q(x)} = y_1, \ \frac{p(x)}{q(x)} = y_2, \ \ldots, \ \frac{p(x)}{q(x)} = y_m \quad (4)$$

1-misol. $(x^2 - x + 1)^4 - 6x^2(x^2 - x + 1)^2 + 5x^4 = 0$ tenglamani yeching.

Yechish: Berilgan tenglama $p(x) = x^2 - x + 1$ va $q(x) = x$ ko'phadlarga nisbatan bir jinsli tenglama hisoblanadi.

1. $\begin{cases} x^2 - x + 1 = 0 \\ x = 0 \end{cases}$ sistema yechimga ega emas.

2. $x=0$ berilgan tenglamaning ildizi bo'lmaganligi uchun uning ikkala tomonini x^4 ga bo'lamiz va natijada ushbu

$$\left(\frac{x^2 - x + 1}{x}\right)^4 - 6\cdot\left(\frac{x^2 - x + 1}{x}\right)^2 + 5 = 0$$

tenglamani hosil qilamiz.

$\left(\frac{x^2-x+1}{x}\right)^2=y$ belgilash kiritib, $y^2-6y+5=0$ tenglamaning ildizlarini Viet teoremasi yordamida topamiz:

$$\begin{cases} y_1+y_2=6 \\ y_1\cdot y_2=5 \end{cases} \Rightarrow y_1=1;\ y_2=5.$$

1) $\left(\frac{x^2-x+1}{x}\right)^2=1>0 \Rightarrow \frac{x^2-x+1}{x}=\pm 1.$

a) $\frac{x^2-x+1}{x}=1 \Rightarrow x^2-x+1=x \Rightarrow x^2-2x+1=$
$=0 \Rightarrow (x-1)^2=0 \Rightarrow x-1=0 \Rightarrow x_{1,2}=1$

b) $\frac{x^2-x+1}{x}=-1 \Rightarrow x^2-x+1=-x \Rightarrow x^2=-1<0 \Rightarrow x\in\varnothing.$

2) $\left(\frac{x^2-x+1}{x}\right)^2=5>0 \Rightarrow \frac{x^2-x+1}{x}=\pm 5.$

a) $x^2-x+1=\sqrt{5}\,x \Rightarrow x^2-(\sqrt{5}+1)x+1=0,\ D=(\sqrt{5}+1)^2-4=$
$=2+2\sqrt{5}>0$

$$x_{3,4}=\frac{\sqrt{5}+1\pm\sqrt{2+2\sqrt{5}}}{2}.$$

b) $x^2-x+1=-\sqrt{5}\,x \Rightarrow x^2+(\sqrt{5}-1)x+1=0,$

$$D=(\sqrt{5}+1)^2-4=2-2\sqrt{5}<0 \Rightarrow x\in\varnothing.$$

Javob: $\left\{1;\ \frac{\sqrt{5}+1\pm\sqrt{2+2\sqrt{5}}}{2}\right\}.$

2-misol. $x^2(x-1)^2-8(x-1)^2+x^2=0$ tenglamani yeching.

Yechish: Tenglamani bir jinsli tenglama ko'rinishiga keltirib olamiz.

$$x^2(x-1)^2-8(x-1)^2+x^2=0$$
$$x^2(x^2-2x+1)+x^2-8(x-1)^2=0$$
$$x^4+x^2(-2x+1)+x^2-8(x-1)^2=0$$
$$x^4-2x^2(x-1)-8(x-1)^2=0$$

Oxirgi tenglama $p(x)=x^2$ va $q(x)=x-1$ ko'phadlarga nisbatan bir jinsli tenglama hisoblanadi. Ko'rinib turibdiki, $\begin{cases} x^2=0 \\ x-1=0 \end{cases}$ sistema yechimga ega emas. $x=1$ berilgan tenglamaning ildizi bo'lmaganligi uchun uning ikkala tomonini $(x-1)^2$ ga bo'lamiz va natijada ushbu

$$\left(\frac{x^2}{x-1}\right)^2-2\cdot\frac{x^2}{x-1}-8=0$$

tenglamani hosil qilamiz. $\frac{x^2}{x-1}=t$ belgilash kiritib, $t^2-2t-8=0$ tenglamaning ildizlarini Viet teoremasi yordamida topamiz:

$$\begin{cases} t_1+t_2=2 \\ t_1\cdot t_2=-8 \end{cases} \Rightarrow t_1=-2\,;\,t_2=4.$$

1) $\frac{x^2}{x-1}=-2 \Rightarrow x^2=-2x+2 \Rightarrow x^2+2x-2=0,$

$$x_{1,2}=\frac{-2\pm 2\sqrt{3}}{2}=-1\pm\sqrt{3} \Rightarrow x_1=-1-\sqrt{3}\,;\,x_2=-1+\sqrt{3}.$$

2) $\frac{x^2}{x-1}=4 \Rightarrow x^2=4x-4 \Rightarrow x^2-4x+4=0 \Rightarrow (x-2)^2=$
$=0 \Rightarrow x-2=0 \Rightarrow x_{3,4}=2.$

Javob: $\{2\,;-1\pm\sqrt{3}\}$.

3-misol. $(x^2+x-4)^2+3x(x^2+x-4)+2x^2=0$ tenglamani yeching.

Yechish: Berilgan tenglama bir jinsli tenglama bo'lib, uni ko'paytuvchilarga ajratish yo'li bilan yechsa ham bo'ladi.

$x^2+x-4=y$ belgilash kiritamiz, natijada $y^2+3xy+2x^2=0$ tenglamani hosil qilamiz. Bundan

$$(y+x)(y+2x)=0 \Leftrightarrow \left[\begin{array}{l} y+x=0 \\ y+2x=0 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x^2+2x-4=0 \\ x^2+3x-4=0 \end{array}\right. \Rightarrow \left[\begin{array}{l} x_{1,2}=-1\pm\sqrt{5} \\ x_3=-4;\,x_4=1. \end{array}\right.$$

Javob: $\{-4;1;-1\pm\sqrt{5}\}$.

4-misol. $5\left(\frac{x-2}{x+1}\right)^2-44\left(\frac{x+2}{x-1}\right)^2+12\cdot\frac{x^2-4}{x^2-1}=0$ tenglamani yeching.

Yechish: Tenglamaning aniqlanish sohasi: $x^2-1\neq 0\Rightarrow$ $x\neq\pm1$.

$\frac{x^2-4}{x^2-1}=\frac{x-2}{x+1}\cdot\frac{x+2}{x-1}$ tenglik o'rinli. Demak, tenglama $\frac{x-2}{x+1}=u$ va $\frac{x+2}{x-1}=v$ funksiyalarga nisbatan bir jinsli ekan. Bundan

$$1)\ \begin{cases}\frac{x-2}{x+1}=0\\ \frac{x+2}{x-1}=0\end{cases}\Rightarrow\begin{cases}x=2\\ x=-2\Rightarrow x\in\varnothing.\\ x\neq\pm1\end{cases}$$

2) Berilgan tenglama $5u^2+12uv-44v^2=0$ ko'rinishga kelib, uning ikkala tomonini v^2 ga bo'lamiz va $5\left(\frac{u}{v}\right)^2+12\left(\frac{u}{v}\right)-44=0$ tenglamani hosil qilamiz.

$\frac{u}{v}=t$ belgilash kiritib, $5t^2+12t-44=0$ kvadrat tenglamani yechamiz:

$$D=144+4\cdot5\cdot44=1024=32^2,\ t_{1,2}=\frac{-12\pm32}{10}\Rightarrow t_1=-\frac{22}{5};\ t_2=2.$$

a) $\frac{u}{v}=-\frac{22}{5}\Rightarrow\frac{(x-1)(x-2)}{(x+1)(x+2)}=-\frac{22}{5}\Rightarrow5(x^2-3x+2)+22(x^2+3x+2)=0\Rightarrow$

$\Rightarrow 27x^2+51x+54=0\Rightarrow9x^2+17x+18=0,$

$D=289-4\cdot9\cdot18=-359<0\Rightarrow x\in\varnothing$

b) $\frac{u}{v}=2\Rightarrow\frac{(x-1)(x-2)}{(x+1)(x+2)}=2\Rightarrow x^2-3x+2=2(x^2+3x+2)\Rightarrow x^2+9x+2=0,$

$$D=81-8=73>0,\ x_{1,2}=\frac{-9\pm\sqrt{73}}{2}.$$

Javob: $\left\{\frac{-9\pm\sqrt{73}}{2}\right\}$.

3. $x(7-x)(7+x^2)=12(x+1)^2$ tenglamani yeching.

Yechish: $x=-1$ tenglamaning ildizi emas. Tenglamaning ikkala tomonini $(x+1)^2$ ga bo'lamiz va

$$\frac{x(7-x)(7+x^2)}{(x+1)^2}=12 \quad \text{yoki} \quad x\left(\frac{7-x}{x+1}\right)\cdot\left(x+\frac{7-x}{x+1}\right)=12$$

ni hosil qilamiz.

$x\left(\frac{7-x}{x+1}\right)=u$ va $x+\frac{7-x}{x+1}=v$ deb belgilash kiritamiz. Bundan

$$uv=12 \quad \text{va} \quad u+v=x\left(\frac{7-x}{x+1}\right)+x+\frac{7-x}{x+1}=\frac{7-x}{x+1}(x+1)+x=7-x+x=7$$

o'rinli.

Quyidagi $\begin{cases} u+v=7 \\ uv=12 \end{cases}$ sistemani hosil qilamiz. Viyet teoremasidan $\begin{cases} u_1=3 \\ v_1=4 \end{cases}$ va $\begin{cases} u_2=4 \\ v_2=3 \end{cases}$ larni topamiz. Belgilashga qaytamiz.

1) $x\left(\frac{7-x}{x+1}\right)=3 \Rightarrow x^2-4x+3=0 \Rightarrow x_1=1;\ x_2=3;$

2) $x\left(\frac{7-x}{x+1}\right)=4 \Rightarrow x^2-3x+4=0 \Rightarrow D=(-3)^2-4\cdot4\cdot1=-7<0 \Rightarrow x\in\varnothing.$

Javob: $\{1;3\}$.

Mustaqil yechish uchun mashqlar

5.1-misol. Tenglamalarning ildizlarini toping.

1. $(x-2)^2(x+1)^2-(x-2)(x^2-1)-2(x-1)^2=0$
2. $(x^2+1)^2+5(x^4-1)-6(x^2-1)^2=0$
3. $(x^2-x+3)^2-3(x^2-x+3)(2x^2-x+2)+2(2x^2-x+2)^2=0$
4. $2(x^2+6x+1)^2+5(x^2+6x+1)(x^2+1)+2(x^2+1)^2=0$
5. $(x^2-16)(x-3)^2+9x^2=0$
6. $(x^2+4x+8)^2+3x^3+14x^2+24x=0$

7. $(x^2+x+1)^2 = x^2(3x^2+x+1)$

8. $x^4+5x^2(x+1)=6(x+1)^2$

9. $x^4-(x-1)(5x^2-4x+4)=0$.

5.2-misol. Tenglamani yeching.

1. $20\left(\frac{x-2}{x+1}\right)^2-5\left(\frac{x+2}{x-1}\right)^2+48\cdot\frac{x^2-4}{x^2-1}=0$

2. $\left(\frac{x+1}{x-2}\right)^2+\frac{x+1}{x-4}=12\left(\frac{x-2}{x-4}\right)^2$

6. Turli usullarda yechiladigan ratsional tenglamalar

Endi turli usullarda yechiladigan ratsional tenglamalar ko'rib o'tamiz.

1-misol. $\frac{x^2+x+2}{3x^2+5x-14}=\frac{x^2+x+6}{3x^2+5x-10}$ tenglamani yeching.

Yechish: $x^2+x+2=a$ va $3x^2+5x-14=b$ belgilashlar kiritamiz. Natijada $\frac{a}{b}=\frac{a+4}{b+4}\Leftrightarrow ab+4a=ab+4b\Leftrightarrow a=b$ o'rinli.

Demak, berilgan tenglama $x^2+x+2=3x^2+5x-14$ tenglamaga teng kuchli. Bundan $x^2+2x-8=0$ kvadrat tenglama hosil qilib,

$$\begin{cases} x_1+x_2=-2 \\ x_1\cdot x_2=-8 \end{cases}\Rightarrow x_1=-4;\ x_2=2$$

ildizlarini topamiz.

Javob: $\{-4;2\}$.

2-misol. $\left(\frac{x^3+x}{3}\right)^3+\frac{x^3+x}{3}=3x$ tenglamani yeching.

Yechish: $y=\frac{x^3+x}{3}$ belgilash kiritamiz, natijada $\begin{cases} y^3+y=3x \\ x^3+x=3y \end{cases}$ sistemani hosil qilamiz. Sistemaning birinchi tenglamasidan ikkinchisini ayiramiz:

$$(y-x)(y^2+xy+x^2)+(y-x)=3(x-y) \Leftrightarrow (y-x)(y^2+xy+x^2)+$$
$$+3(y-x)=0 \Leftrightarrow (y-x)(y^2+xy+x^2+3)=0 \Leftrightarrow \left[\begin{array}{l} y=x \\ y^2+xy+x^2=-3. \end{array}\right.$$

$y=x$ dan $x^3+x=3x \Rightarrow x(x^2-2)=0$ tenglamani hosil qilib, $x_1=0$ va $x_{2,3}=\pm\sqrt{2}$ ildizlarini topamiz.

$y^2+xy+x^2=-3$ tenglama yechimga ega emas, chunki ikkita sonning chala kvadrati har doim nomanfiy qiymatlar qabul qiladi.

Javob: $\{0; \pm\sqrt{2}\}$.

3-misol. $\frac{x-49}{50}+\frac{x-50}{49}=\frac{49}{x-50}+\frac{50}{x-49}$ tenglamani yeching.

Yechish: Aniqlanish sohasi: $x \neq 49;\ x \neq 50$ dan iborat.

$$\frac{x-49}{50}-\frac{50}{x-49}=\frac{49}{x-50}-\frac{x-50}{49} \Rightarrow \frac{(x-49)^2-50^2}{50(x-49)}=\frac{49^2-(x-50)^2}{49(x-50)} \Rightarrow$$
$$\Rightarrow \frac{(x-99)(x+1)}{50(x-49)}=\frac{(99-x)(x-1)}{49(x-10)} \Rightarrow \frac{(x-99)(x+1)}{50(x-49)}+\frac{(x-99)(x-1)}{49(x-50)}=0 \Rightarrow$$
$$\Rightarrow (x-99)\left[\frac{x+1}{50(x-49)}+\frac{x-1}{49(x-50)}\right]=0.$$

1) $x-99=0 \Rightarrow x_1=99$.

2) $\frac{x+1}{50(x-49)}+\frac{x-1}{49(x-50)}=0 \Rightarrow \frac{49(x+1)(x-50)+50(x-1)(x-49)}{2450(x-49)(x-50)}=$
$$=0 \Rightarrow 49(x^2-49x-50)+50(x^2-50x+49)=0 \Rightarrow$$
$$\Rightarrow 99x^2-4901x=0 \Leftrightarrow x(99x-4901)=0 \Rightarrow$$
$$\Rightarrow x_2=0;\ x_3=\frac{4901}{99}=49\frac{50}{99}.$$

Javob: $\left\{0; 99; 49\frac{50}{99}\right\}$.

4-misol. $4x^2+\frac{10}{3x}=\frac{61}{9}$ tenglamani yeching.

Yechish: Berilgan tenglamada quyidagicha shakl al mashtiramiz:

$$4x^2+\frac{10}{3x}=\frac{61}{9}\Rightarrow 4x^2+\frac{10}{3x}=4+\frac{25}{9}\Rightarrow 4x^2-\frac{25}{9}=4-\frac{10}{3x}\Rightarrow$$

$$\Rightarrow\left(2x-\frac{5}{3}\right)\left(2x+\frac{5}{3}\right)=\frac{2}{x}\left(2x-\frac{5}{3}\right)\Rightarrow\left(2x-\frac{5}{3}\right)\left(2x+\frac{5}{3}-\frac{2}{x}\right)=0;$$

Oxirgi tenglikning har bir qavslari ichidagi ifodani 0 g tenglaymiz:

1) $2x-\frac{5}{3}=0\Rightarrow 2x=\frac{5}{3}\Rightarrow x_1=\frac{5}{6}$;

2) $2x-\frac{5}{3}-\frac{2}{x}=0\Rightarrow 6x^2-5x-6=0;\ D=25+144=169=13^2$;

$$x_{1,2}=\frac{5\pm13}{12}\Rightarrow x_1=-\frac{2}{3};\ x_2=\frac{3}{2}$$

Javob: $\left\{-\frac{2}{3};\frac{3}{2};\frac{5}{6}\right\}$.

Mustaqil yechish uchun mashqlar

6.1-misol. Tenglamalarni yeching.

1. $(x-2)^2=6-\frac{8}{x^2}-2x$ **2.** $\frac{x}{x^2+7x+a}=\frac{x^2+8x+a}{x^2+6x+a},\ a\in R$

3. $\frac{1}{x^2}+\frac{1}{(x+2)^2}=\frac{10}{9}$ **4.** $\frac{x-9}{10}+\frac{x-10}{9}=\frac{10}{x-9}+\frac{9}{x-10}$

5. $x\left(\frac{5-x}{x+1}\right)\left(x+\frac{5-x}{x+1}\right)=6$ **6.** $\frac{x^{21}-1}{1-x^{19}}=\frac{1-x^{19}}{x^{17}-1}$

7. $\frac{x^4+324}{x^2+6x+18}=43-6x$.

7. Ratsional tenglamalar mavzusiga doir test topshiriqlari

1-misol. $\frac{3x^2+8x-3}{x+3}=x^2-x+2$ tenglamaning ildizlari yig'indisini toping.

$A)\ -8 \quad B)\ -6 \quad C)\ -4 \quad D)\ 4 \quad E)\ 6$

Yechish: Aniqlanish sohasi: $x+3\neq 0 \Rightarrow x\neq -3$. Tenglamaning chap tomonidagi kasrning suratini ko'paytuvchilarga ajratib, tenglamani yechamiz:

$$\frac{(3x-1)(x+3)}{x+3}=x^2-x+2 \Rightarrow 3x-1=x^2-x+2 \Rightarrow x^2-4x+3=0.$$

Viyet teoremasidan

$$\begin{cases} x_1+x_2=4 \\ x_1\cdot x_2=3 \end{cases} \Rightarrow x_1=1;\ x_2=3.$$

Demak, ikkala ildiz ham aniqlanish sohasiga tegishli. Bundan $x_1+x_2=4$ ekanligi kelib chiqadi.

Javob: $D)4$.

2-misol. $x^2+x-2=\frac{x^2+x-2}{x^2-1}$ tenglama ildizlari ko'paytmasini toping.

$A)\ 4 \quad B)\ -2 \quad C)\ 6 \quad D)\ -4$

Yechish: Aniqlanish sohasi $x^2-1\neq 0 \Rightarrow x\neq \pm 1$.

Berilgan tenglamani

$$(x-1)(x+2)=\frac{(x-1)(x+2)}{(x-1)(x+1)}$$

ko'rinishida yozib olamiz. $x\neq \pm 1$ ni hisobga olsak,

$$(x-1)(x+2)=\frac{x+2}{x+1} \Rightarrow (x+2)\left(x-1-\frac{1}{x+1}\right)=$$

$$0 \Rightarrow \frac{(x+2)(x^2-2)}{x+1}=0 \Rightarrow (x+2)(x^2-2)=0.$$

1) $x+2=0 \Rightarrow x_1=-2$;

2) $x^2-2=0$ dan $x_{2,3}=\pm\sqrt{2}$ bo'lib, $x_1\cdot x_2\cdot x_3=-2\cdot(-2)=4$.

Javob: $A)\ 4$.

3-misol. $x^2-\frac{27}{x^2}+x-\frac{27}{x}=0$ tenglama ildizlari ko'paytmasini toping.

$A)\ 1$ $B)\ 0$ $C)\ -2$ $D)\ -3$

Yechish: $x(x+1)-\frac{27}{x}\left(\frac{1}{x}+1\right)=0\Rightarrow x(x+1)-\frac{27}{x}\cdot\frac{x+1}{x}=$

$=0\Rightarrow(x+1)\left[x-\frac{27}{x^2}\right]=0$

$1)\ x+1=0\Rightarrow x_1=-1;$

$2)\ x-\frac{27}{x^2}=0\Rightarrow\frac{x^3-27}{x^2}=0\Rightarrow x^3-27=0\Rightarrow x_2=3;\ x_1\cdot x_2=-1\cdot3=-3$

Javob: $D)-3$.

4-misol. $\dfrac{1+\dfrac{1+\dfrac{\cdots}{5}}{5}}{5}+1=x$ tenglamadan x ni toping.

$A)\ \frac{11}{6}$ $B)\ \frac{7}{5}$ $C)\ \frac{4}{5}$ $D)\ \frac{5}{4}$

Yechish: $1+\dfrac{1+\dfrac{1+\dfrac{\cdots}{5}}{5}}{5}=x\Rightarrow1+\frac{x}{5}=x\Rightarrow5+x=5x\Rightarrow4x=5\Rightarrow x=\frac{5}{4}$

Javob: $D)\ \frac{5}{4}$.

5-misol. n ning qanday natural qiymatida quyidagi tenglik o'rinli?

$$\left(1-\frac{1}{n}\right)\left(1-\frac{1}{n-1}\right)\left(1-\frac{1}{n-2}\right)\left(1-\frac{1}{n-3}\right)\ldots\left(1-\frac{1}{3}\right)\left(1-\frac{1}{2}\right)=\frac{1}{100}.$$

$A)\ 50$ $B)\ 1000$ $C)\ 100$ $D)\ 200$

Yechish: 1) $\left(1-\frac{1}{3}\right)\left(1-\frac{1}{2}\right)=\frac{2}{3}\cdot\frac{1}{2}=\frac{1}{3};$

2) $\left(1-\frac{1}{4}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{2}\right)=\frac{3}{4}\cdot\frac{2}{3}\cdot\frac{1}{2}=\frac{1}{4};$

3) $\left(1-\frac{1}{5}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{2}\right)=\frac{4}{5}\cdot\frac{3}{4}\cdot\frac{2}{3}\cdot\frac{1}{2}=\frac{1}{5}$; ......

4) $\left(1-\frac{1}{n}\right)\left(1-\frac{1}{n-1}\right)\left(1-\frac{1}{n-2}\right)\left(1-\frac{1}{n-3}\right)\ldots\left(1-\frac{1}{3}\right)\left(1-\frac{1}{2}\right)=\frac{1}{n}$.

Demak, $\frac{1}{n}=\frac{1}{100}\Rightarrow n=100$.

Javob: $C)\,100$.

6-misol. $x^2+3x+\frac{6}{2-3x-x^2}=1$ tenglama butun ildizlari yig'indisini toping.

$A)\,-3$ $B)\,1$ $C)\,-5$ $D)\,3$ $E)\,4$

Yechish: Aniqlanish sohasi:

$$2-3x-x^2\neq 0\Rightarrow x^2+3x-2\neq 0\Rightarrow x\neq\frac{-3\pm\sqrt{17}}{2}.$$

Tenglamani yangi $x^2+3x=y$ o'zgaruvchi kiritib yechamiz:

$$x^2+3x+\frac{6}{2-3x-x^2}=1\Rightarrow x^2+3x+\frac{6}{2-(x^2+3x)}=1\Rightarrow$$

$$\Rightarrow y-\frac{6}{y-2}-1=0\Rightarrow\frac{y^2-3y-4}{y-2}=0.$$

$y^2-3y-4=0\Rightarrow y_1=-1$; $y_2=4$. Topilganlarni belgilashga qo'yamiz.

1) $x^2+3x=-1\Rightarrow x^2+3x+1=0\Rightarrow x_{1,2}=\frac{-3\pm\sqrt{5}}{2}$, ildizlari irratsional.

2) $x^2+3x=4\Rightarrow x^2+3x-4=0\Rightarrow x_3=1$; $x_4=-4$, ildizlari butun.

Tenglamaning barcha ildizlari aniqlanish sohasiga tegishli va butun ildizlari yig'indisi $x_3+x_4=-3$ ga teng.

Javob: $A)\,-3$.

7-misol. $\dfrac{\frac{7}{x+25}+\frac{5}{x-50}}{\frac{2}{x-50}-\frac{3}{x+25}}=17$ tenglamani yeching.

$A)\,125$ $B)\,25$ $C)\,100$ $D)\,50$

Yechish: $\dfrac{1}{x+25}=u$ va $\dfrac{1}{x-50}=v$ deb belgilash kiritamiz. Natijada

$$\frac{7u+5v}{2v-3u}=17\Rightarrow 17-\frac{7u+5v}{2v-3u}=0\Rightarrow\frac{17(2v-3u)-7u-5v}{2v-3u}=0\Rightarrow$$

$$\Rightarrow\frac{29v-58u}{2v-3u}=0\Rightarrow 29v-58u=0\Rightarrow v=2u.$$

Demak, $\dfrac{1}{x-50}=\dfrac{2}{x+25}\Rightarrow\dfrac{2}{x+25}-\dfrac{1}{x-50}=0\Rightarrow\dfrac{2x-100-x-25}{(x+25)(x-50)}=$

$$=0\Rightarrow\frac{x-125}{(x+25)(x-50)}=0\Rightarrow x-125=0\Rightarrow x=125.$$

Javob: $A)\,125$.

8-misol. $\left(\dfrac{x}{x-1}\right)^4-3\left(\dfrac{x}{x-1}\right)^2+2=0$ tenglama ildizlari ko'paytmasini toping.

$A)\,2$ $\quad B)\,1$ $\quad C)\,4\frac{1}{2}$ $\quad D)\,3\frac{1}{6}$

Yechish: $\left(\dfrac{x}{x-1}\right)^2=t\geq 0$ deb belgilash kiritamiz. Natijada $t^2-3t+2=0$ kvadrat tenglamani hosil qilamiz va $t_1=1;\ t_2=2$ ildizlarini topamiz.

Belgilashga qaytamiz:

1) $\left(\dfrac{x}{x-1}\right)^2=1\Rightarrow\dfrac{x}{x-1}=\pm 1$ bo'lib,

$a)\,\dfrac{x}{x-1}=-1\Rightarrow x=-x+1\Rightarrow x=\dfrac{1}{2};$ $\quad b)\,\dfrac{x}{x-1}=1\Rightarrow x=x-1\Rightarrow x\in\varnothing.$

2) $\left(\dfrac{x}{x-1}\right)^2=2\Rightarrow\dfrac{x}{x-1}=\pm\sqrt{2}$ ni topamiz va

$$a)\quad\frac{x}{x-1}=-\sqrt{2}\Rightarrow x=-\sqrt{2}x+\sqrt{2}\Rightarrow(\sqrt{2}+1)x=\sqrt{2}\Rightarrow$$

$$\Rightarrow x=\frac{\sqrt{2}}{\sqrt{2}+1}=\frac{\sqrt{2}(\sqrt{2}-1)}{2-1}=2-\sqrt{2};$$

$b)\ \frac{x}{x-1}=\sqrt{2}\Rightarrow x=\sqrt{2}x-\sqrt{2}\Rightarrow(1-\sqrt{2})x=-\sqrt{2}\Rightarrow$

$$\Rightarrow x=\frac{\sqrt{2}}{\sqrt{2}-1}=\frac{\sqrt{2}(\sqrt{2}+1)}{2-1}=2+\sqrt{2}.$$

Topilgan uchchala $x_1=\frac{1}{2};\ x_2=2+\sqrt{2};\ x_3=2-\sqrt{2}$ ildizlar tenglamani qanoatlantiradi va $x_1\cdot x_2\cdot x_3=1$.

Javob: $B)\,1$.

9-misol. $\frac{3ab+1}{a}x=\frac{3ab}{a+1}+\frac{2a+1}{a(a+1)^2}x+\frac{a^2}{(a+1)^3}$ tenglamani yeching.

$A)\ \frac{a}{a-1}$ $\quad B)\ 1$ $\quad C)\ \frac{a}{a+1}$ $\quad D)\ \frac{a}{a^2+1}$

Yechish: $\left[\frac{3ab+1}{a}-\frac{2a+1}{a(a+1)^2}\right]x=\frac{3ab}{a+1}+\frac{a^2}{(a+1)^3}$ o'rinli.

$$1)\ \frac{3ab+1}{a}-\frac{2a+1}{a(a+1)^2}=\frac{(3ab+1)(a+1)^2-(2a+1)}{a(a+1)^2}=$$

$$=\frac{3ab(a+1)^2+(a+1)^2-(2a+1)}{a(a+1)^2}=$$

$$=\frac{3ab(a+1)^2+a^2}{a(a+1)^2}=\frac{a\left[3b(a+1)^2+a\right]}{a(a+1)^2}=\frac{3b(a+1)^2+a}{(a+1)^2};$$

$$2)\ \frac{3ab}{a+1}+\frac{a^2}{(a+1)^3}=\frac{3ab(a+1)^2+a^2}{(a+1)^3}=\frac{a\left[3b(a+1)^2+a\right]}{(a+1)^3};$$

$$3)\ \frac{3b(a+1)^2+a}{(a+1)^2}x=\frac{a\left[3b(a+1)^2+a\right]}{(a+1)^3}\Rightarrow x=\frac{a}{a+1}$$

Javob: $C)\ \frac{a}{a+1}$.

10-misol. $\frac{a+x}{a^2+ax+x^2}-\frac{a-x}{ax-x^2-a^2}=\frac{3a}{x(a^4+a^2x^2+x^4)}$ tenglamani yeching.

$A)\ \frac{5}{2a^2}$ $\quad B)\ \frac{a^2}{3}$ $\quad C)\ \frac{3}{2a^2}$ $\quad D)\ 1$

Yechish: Berilgan tenglamani quyidagi ko‘rinishda yozib olamiz:

$$\frac{a+x}{a^2+ax+x^2}+\frac{a-x}{a^2+ax+x^2}=\frac{3a}{x(a^4+a^2x^2+x^4)}$$

Bu yerda $(a^2+ax+x^2)(a^2-ax+x^2)=(a^2+x^2)^2-(ax)^2=$
$=a^4+a^2x^2+x^4$.

Bundan $\dfrac{a+x}{a^2+ax+x^2}+\dfrac{a-x}{a^2-ax+x^2}=\dfrac{a^3+x^3+a^3-x^3}{a^4+a^2x^2+x^4}=\dfrac{2a^3}{a^4+a^2x^2+x^4}$.

Natijada

$$\frac{2a^3}{a^4+a^2x^2+x^4}-\frac{3a}{x(a^4+a^2x^2+x^4)}=0\Rightarrow\frac{2a^3x-3a}{x(a^4+a^2x^2+x^4)}=0$$

$$\Rightarrow 2a^3x-3a=0\Rightarrow x=\frac{3a}{2a^3}=\frac{3}{2a^2}.$$

Javob: $C)\ \dfrac{3}{2a^2}$.

Mustaqil yechish uchun mashqlar

7.1-misol. Test topshiriqlari.

1. Tenglamani yeching: $\dfrac{2x^2-5x+3}{(10x-5)(x-1)}=0$.

$A)\ 1\quad B)\ 1.;\ \frac{3}{2}\quad C)\ \frac{3}{2}\quad D)\ 5\quad E)\ \frac{1}{2}$

2. Tenglamaning ildizlari nechta? $\dfrac{x^2-x-2}{x^2+x}=0$.

$A)\ 2\quad B)\ 4\quad C)\ 1\quad D)\ 3\quad E)\ \varnothing$

3. Tenglama ildizlarining yig‘indisini toping:

$$\frac{2}{3-x}+\frac{1}{2}=\frac{6}{x(3-x)}.$$

$A)\ 4\quad B)\ 7\quad C)\ 3\quad D)\ 10\quad E)\ 0$

4. Tenglamani yeching: $\dfrac{2}{x-3}=\dfrac{x+5}{x^2-9}$.

$A)\ -2\quad B)\ 2\quad C)\ 1\quad D)\ -1\quad E)\ 1{,}5$

5. $\frac{x+8}{3}=x-\frac{x-3}{x}$ tenglama ildizlari ayirmasining modulini toping.

$A)\ 5{,}5$ $B)\ 5$ $C)\ 3{,}5$ $D)\ 4$ $E)\ 2{,}5$

6. $\frac{x^3-8}{x-2}=6x+1$ tenglamaning ildizlari yig'indisini toping.

$A)\ 6$ $B)\ 4$ $C)\ -1$ $D)\ 3$ $E)\ -2$

7. $\frac{3x^2+4x-4}{x+2}=x^2-4x+4$ tenglama ildizlarining yig'indisini toping.

$A)\ 10$ $B)\ -5$ $C)\ -4$ $D)\ 8$ $E)\ 7$

8. $\frac{1}{x^2-3x-3}+\frac{5}{x^2-3x+1}=2$ tenglamaning ildizlari yig'indisini toping.

$A)\ 6$ $B)\ 5$ $C)\ 4$ $D)\ 3$ $E)\ 2$

9. $\frac{26}{5(x+x^{-1})}=1$ tenglama ildizlari ko'paytmasini toping.

$A)\ 1$ $B)\ 5$ $C)\ 2$ $D)\ 2{,}4$ $E)\ 4{,}8$

10. Tenglamaning yechimlari ko'paytmasini toping:

$$\left(x^2+\frac{1}{x^2}\right)-4\left(x+\frac{1}{x}\right)+5=0.$$

$A)\ 3$ $B)\ 2\sqrt{3}$ $C)\ 6$ $D)\ -2\sqrt{3}$ $E)\ 1$

11. $$\frac{1}{\dfrac{1}{\dfrac{1}{\frac{1}{x}+\frac{1}{2}}+\dfrac{1}{\frac{1}{x}+\frac{1}{2}}}+\dfrac{1}{\dfrac{1}{\frac{1}{x}+\frac{1}{2}}+\dfrac{1}{\frac{1}{x}+\frac{1}{2}}}}=\frac{x}{36}$$ tenglamani yeching.

$A)\ 60$ $B)\ 70$ $C)\ 36$ $D)\ 1$

12. $1+\cfrac{x}{1+\cfrac{x}{1+\cfrac{x}{\dots}}}=4$ tenglamadan x ni toping.

$A)\ 12$ $B)\ 14$ $C)\ 6$ $D)\ 16$

13. $\left(1-\frac{1}{a}\right)\left(1-\frac{1}{a-1}\right)\left(1-\frac{1}{a-2}\right)\cdots\left(1-\frac{1}{3}\right)\left(1-\frac{1}{2}\right)=\frac{1}{30};\ a=?$

$A)\ 30 \qquad B)\ 10 \qquad C)\ 6 \qquad D)\ 15$

14. $\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\frac{1}{4\cdot 5}+\ldots+\frac{1}{(n+1)(n+2)}=\frac{80}{64}$ bo‘lsa, $n=?$

$A)\ 160 \qquad B)\ 20 \qquad C)\ 40 \qquad D)\ 80$

15. $\frac{4x^2-7x-2}{x^2-5x+6}=0$ tenglamani yeching.

$A)\ -\frac{1}{4};\ 2 \qquad B)\ -\frac{1}{4} \qquad C)\ 2 \qquad D)\ 3$

16. $\frac{x^2+x-2}{x^2-1}=x^2+5x+6$ tenglama ildizlari ko‘paytmasini toping.

$A)\ -6 \qquad B)\ -12 \qquad C)\ -8 \qquad D)\ -4$

17. $\frac{x-4}{x-5}+\frac{6x-30}{x-4}=5$ tenglamani yeching.

$A)\ 6{,}5;\ 6 \qquad B)\ 5{,}5;\ 6 \qquad C)\ -5{,}5;\ -6 \qquad D)\ -5{,}5;\ 6$

18. $x^2+x+1=\frac{15}{x^2+x+3}$ tenglama haqiqiy ildizlari ko‘paytmasini toping.

$A)\ -2 \qquad B)\ 1 \qquad C)\ 2 \qquad D)\ -1$

19. $\frac{3x^2}{x+2}+\frac{x+2}{x^2}=4$ tenglamaning ildizlari yig‘indisini toping.

$A)\ \frac{1}{3} \qquad B)\ 2 \qquad C)\ 1 \qquad D)\ \frac{4}{3}$

20. $\frac{x^2-x}{x^2-x-1}-1=\frac{x^2-x+2}{x^2-x-2}$ tenglama ildizlari o‘rta arifmetigini toping.

$A)\ 2 \qquad B)\ 1 \qquad C)\ 0{,}5 \qquad D)\ 1{,}5$

21. $\frac{x-1}{n-1}+\frac{2n^2(1-x)}{(n^2-1)(n^2+1)}=\frac{2x-1}{1-n^4}-\frac{1-x}{1+n}$ tenglamani yeching.

$A)\ \frac{6}{8}$ $\quad B)\ \frac{1}{2}$ $\quad C)\ \frac{1}{4}$ $\quad D)\ \frac{5}{8}$

22. $\frac{a^2+x}{b^2-x}-\frac{a^2-x}{b^2+x}=\frac{4abx+2a^2-2b^2}{b^4-x^2}$ tenglamani yeching.

$A)\ a+b$ $\quad B)\ \frac{a+b}{a-b}$ $\quad C)\ \frac{a-b}{a+b}$ $\quad D)\ 1$

6-§. KO'PHADLAR

1. Ko'phadlar ustida amallar

Ta'rif. Quyidagi $P(x)=a_0x^n+a_1x^{n-1}+a_2x^{n-2}+\ldots+a_{n-1}x+a_n$ ifodaga x o'zgaruvchiga nisbatan n – darajali ko'phad deyiladi.

Bu yerda $a_0, a_1, a_2, \ldots, a_{n-1}, a_n \in R$ va $a_0 \neq 0$ bo'lib, *ko'phadning koeffitsiyentlari* deb ataladi va $n \in N$. a_0 – *bosh koeffitsiyent*, a_n esa *ozod had* deb nomlanadi.

Ko'phadning darajasi bir bo'lsa, birinchi darajali ko'phad yoki chiziqli ko'phad deyiladi. Ikkinchi darajali ko'phad – kvadrat ko'phad, uchinchi darajali ko'phad – kub ko'phad deb ataladi.

Har bir qo'shiluvchilar *ko'phadning hadlari* deb ataladi.

1-misol. 1) $P(x)=3x+2$ – chiziqli ko'phad bo'lib, bu yerda $a_0=3, a_1=2$, $n=1$. Bunda birinchi darajali ko'phad $P(x)=a_0x+a_1$ ko'rinishga ega.

2) $P(x)=-2x^2+5x-7$ – kvadrat uchhad, $a_0=-2$, $a_1=5$, $a_2=-7, n=2$.

Chunki kvadrat uchhad $P(x)=a_0x^2+a_1x+a_2$ ko'rinishga ega.

3) $P(x)=-x^5+6x^4-2x^2-8x+9$ – beshinchi darajali ko'phad bo'lib, bunda $a_0=-1, a_1=6, a_2=0, a_3=-2$, $a_4=-8$, $a_5=9$, $n=5$. Chunki beshinchi darajali ko'phad $P(x)=a_0x^5+a_1x^4+a_2x^3+a_3x^2+a_4x+a_5$ ko'rinishga ega.

Ta'rif. $P(x)=a_0x^n+a_1x^{n-1}+a_2x^{n-2}+\ldots+a_{n-1}x+a_n$ va

$$Q(x)=b_0x^n+b_1x^{n-1}+b_2x^{n-2}+\ldots+b_{n-1}x+b_n$$

ko'phadlar teng deyiladi, agar bir xil darajalar oldidagi koeffitsiyentlar teng bo'lsa, ya'ni

$$a_0=b_0,\ a_1=b_1,\ a_2=b_2,\ \ldots,\ a_{n-1}=b_{n-1},\ a_n=b_n.$$

Ta'rif. $P(x)=a_0x^n+a_1x^{n-1}+a_2x^{n-2}+\ldots+a_{n-1}x+a_n$ va

$$Q(x)=b_0x^n+b_1x^{n-1}+b_2x^{n-2}+\ldots+b_{n-1}x+b_n$$

ko'phadlarning yig'indisi deb, x o'zgaruvchining mos darajalari oldidagi koeffitsiyentlari yig'indisidan iborat $S(x)$ ko'phadga aytiladi, ya'ni

$$S(x)=P(x)+Q(x)=(a_0+b_0)x^n+(a_1+b_1)x^{n-1}+\ldots$$
$$+(a_{n-1}+b_{n-1})x+(a_n+b_n).$$

2-misol. $P(x)=x^4-7x^3+2x^2+x+3$ va $Q(x)=2x^3-5x^2-$ $-x+2$ ko'phadlar yig'indisini toping.

Yechish: $S(x)=P(x)+Q(x)=(x^4-7x^3+2x^2+x+3)+$ $+(2x^3-5x^2-x+2)=x^4+(-7+2)x^3+(2-5)x^2+(1-1)x+(3+2)=$ $=x^4+(-7+2)x^3+(2-5)x^2+(1-1)x+(3+2)=x^4-5x^3-3x^2+5$

Javob: $x^4-5x^3-3x^2+5$.

Ta'rif. $P(x)=a_0x^n+a_1x^{n-1}+a_2x^{n-2}+\ldots+a_{n-1}x+a_n$ ko'phadga qarama-qarshi ko'phad deb, $-P(x)=-a_0x^n-a_1x^{n-1}-$ $-a_2x^{n-2}-\ldots-a_{n-1}x-a_n$ ko'phadga aytiladi va ular yi'gindisi $P(x)+(-P(x))=0$.

Ta'rif. $P(x)$ va $Q(x)$ ko'phadlarning ayirmasi deb, $P(x)$ bilan $Q(x)$ ga qarama – qarshi ko'phadning yig'indisiga aytiladi, ya'ni

$$L(x)=P(x)+(-Q(x))=P(x)-Q(x).$$

Ta'rif. n – darajali $P(x)=a_0x^n+a_1x^{n-1}+a_2x^{n-2}+\ldots+a_{n-1}x+a_n$ ko'phad va m – darajali $Q(x)=b_0x^m+b_1x^{m-1}+b_2x^{m-2}+\ldots+b_{m-1}\cdot$ $x+b_m$ ko'phadlarning ko'paytmasi deb, $n+m$ darajali $M(x)$ ko'phadga aytiladi.

Hosil bo'lgan $M(x)$ ko'phadning $c_0, c_1, c_2, \ldots, c_{n+m-1}$, c_{n+m} koeffitsiyentlari quyidagi formulalar yordamida hisoblanadi:

$$c_0 = a_0 \cdot b_0\,,\; c_1 = a_1 \cdot b_1\,,\; c_2 = a_2 \cdot b_2\,,\ldots,$$

$$c_k = a_k \cdot b_0 + a_{k-1} \cdot b_1 + \cdots + a_1 \cdot b_{k-1} + a_0 \cdot b_0\,,\;\ldots\,,\; c_{n+m} = a_n \cdot b_m$$

Demak, $M(x) = P(x) \cdot Q(x)$ bo'lar ekan.

3-misol. $P(x) = x^3 + 2x^2 - 5x + 6$ va $Q(x) = x^4 + 2x + 3$ ko'phadlar ko'paytmasini toping.

Yechish: $M(x) = P(x) \cdot Q(x) = (x^3 + 2x^2 - 5x + 6) \cdot (x^4 + 2x + 3) =$

$$= x^7 + 2x^4 + 3x^3 + 2x^6 + 4x^3 + 6x^2 - 5x^5 - 10x^2 - 15x + 6x^4 +$$

$$+ 12x + 18 = x^7 + 2x^6 - 5x^5 + 8x^4 + 7x^3 - 4x^2 - 3x + 18$$

Javob: $x^7 + 2x^6 - 5x^5 + 8x^4 + 7x^3 - 4x^2 - 3x + 18$.

4-misol. Ushbu $(x-1)(2-x) + (2x-3)^2$ ifodani ko'phadning standart shakliga keltiring.

$A)\ 5x^2 + 9x - 7$ $B)\ 3x^2 - 8$ $C)\ 3x^2 - 9x + 7$

$D)\ 12x + 4 - x^2$ $E)\ 5x^2 - 10x + 1$

Yechish: Qavslarni ochib, o'xshash hadlarni ixchamlaymiz:

$$(x-1)(2-x) + (2x-3)^2 = 2x - x^2 - 2 + x + 4x^2 - 12x + 9 = 3x^2 - 9x + 7.$$

Javob: $C)\ 3x^2 - 9x + 7$.

5-misol. Ushbu $(y^4 - y^2 + 1)(y^2 + 1) + (y-1)(y+1)$ ifodani soddalashtirgandan keyin uning nechta hadi bo'ladi?

$A)\ 3$ $B)\ 4$ $C)\ 2$ $D)\ 5$ $E)\ 6$

Yechish: Ifodadagi birinchi qo'shiluvchiga kublar yig'indisi $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ ni va ikkinchi qo'shiluvchiga kvadratlar ayirmasi $a^2 - b^2 = (a-b)(a+b)$ formulalarini qo'llaymiz. Natijada

$$(y^4 - y^2 + 1)(y^2 + 1) + (y-1)(y+1) = (y^2)^3 + 1^3 + y^2 - 1^2 = y^6 + y^2$$

ni hosil qilamiz. Demak, ko'phad ikkita haddan iborat ekan.

Javob: $C)\ 2$.

Yuqoridagilardan ixtiyoriy ikkita ko'phadning yig'indisi, ayirmasi va ko'paytmasi yana ko'phad bo'lishi kelib chiqadi.

Endi ko'phadlar tengligiga doir misollar ko'rib o'taylik.

6-misol. $x^2-4x+2=a(x-1)^2+b(x-1)+c$ tenglik o'rinli bo'lsa, a,b,c larni toping.

Yechish: Tenglikni o'ng tomonidagi qavslarni ochib, standart shaklga keltiramiz.

$$x^2-4x+2=a(x-1)^2+b(x-1)+c=ax^2-2ax+a+bx-b+c$$

$$x^2-4x+2=ax^2+(b-2a)x+(a-b+c)$$

Ikkita ko'phad teng bo'ladi, agar bir xil darajalar oldidagi koeffitsiyentlar teng bo'lsa, ya'ni

$$\begin{cases} a=1 \\ b-2a=-4 \\ a-b+c=2 \end{cases} \Leftrightarrow \begin{cases} a=1 \\ b-2=-4 \\ 1-b+c=2 \end{cases} \Leftrightarrow \begin{cases} a=1 \\ b=-2 \\ c=-1. \end{cases}$$

Javob: $a=1;\ b=-2;\ c=-1.$

7-misol. Agar $(ax^2-bx)+(bx^2+ax)=-12x$ ayniyat bo'lsa, a va b ning qiymatlarini toping.

$A)\ a=-6; b=-6$ $\quad B)\ a=8; b=-8$ $\quad C)\ a=-6; b=6$

$D)\ a=6; b=-6$ $\quad E)\ a=6; b=6$

Yechish:

$$(ax^2-bx)+(bx^2+ax)=-12x \Leftrightarrow (a+b)x^2+(a-b)x=0\cdot x^2-12x;$$

$$\begin{cases} a+b=0 \\ a-b=-12 \end{cases} \Leftrightarrow \begin{cases} a=-b \\ -2b=-12 \end{cases} \Leftrightarrow \begin{cases} a=-6 \\ b=6 \end{cases}$$

Javob: $C)\ a=-6; b=6.$

8-misol. Ushbu $(-3x+\alpha y)(\beta x-2y)=\gamma x^2+7xy+2y^2$ ayniyatdagi noma'lum koeffitsiyentlardan biri β ni toping.

$A)\ 1$ $\quad B)\ -1$ $\quad C)\ 2$ $\quad D)\ -2$ $\quad E)\ 3$

Yechish: Tenglikning chap tomonidagi qavslarni ochamiz:

$$(-3x+\alpha y)(\beta x-2y)=\gamma x^2+7xy+2y^2$$

$$-3\beta x^2+6xy+\alpha\beta xy-2\alpha y^2=\gamma x^2+7xy+2y^2$$

$$-3\beta x^2+(6+\alpha\beta)xy-2\alpha y^2=\gamma x^2+7xy+2y^2.$$

Oxirgi tenglikdan mos koeffitsiyentlarni tenglaymiz. Natijada

$$\begin{cases} -3\beta=\gamma \\ 6+\alpha\beta=7 \\ -2\alpha=2 \end{cases} \Leftrightarrow \begin{cases} -3\beta=\gamma \\ 6-\beta=7 \\ \alpha=-1 \end{cases} \Leftrightarrow \begin{cases} \gamma=3 \\ \beta=-1 \\ \alpha=-1. \end{cases}$$

Javob: $B)\ -1$.

Mustaqil yechish uchun mashqlar

1.1-misol. Quyidagi ko‘phadlar yig‘indisi, ayirmasi va ko‘paytmasini toping: $P(x)\pm Q(x)=?$ $P(x)\cdot Q(x)=?$

1. $P(x)=2x^3+5x^2+x-2$, $Q(x)=x^2+3x$;

2. $P(x)=2x^4-5x^3-x^2+3x+1$, $Q(x)=x^3-5x^2+3x+1$;

3. $P(x)=x^4-2x^3+4x^2-6x+7$, $Q(x)=2x^2-9x-12$;

4. $P(x)=x^2-1$, $Q(x)=x^5-1$;

5. $P(x)=x^5-3x^4+4x^3+x^2+x+4$, $Q(x)=x^7-x^6+6x^5+x^3+$ $+2x+7$.

1.2-misol. Test topshiriqlari.

1. Ushbu $(4x-3)^2-x(4x+1)$ ifodani ko‘phadning stand shakliga keltiring.

$A)\ 2x^2+x-9$ $B)\ 12x^2-25x+9$ $C)\ 4x^2-13x$

$D)\ 8x^2-x+7$ $E)\ 12x^2-23x+9$

2. $(x^2+1)(x^4-x^2+1)+(x^3-1)^2$ ni soddalashtirgandan keyin ho sil bo‘lgan ko‘phadning nechta hadi bo‘ladi?

$A)\ 5$ $B)\ 4$ $C)\ 3$ $D)\ 6$ $E)\ 2$

3. Soddalashtiring: $12^2-(x+7)^2-(5-x)(19+x)$.

$A)\ 0$ $B)\ 50$ $C)\ 140$ $D)\ 90$ $E)\ 85$

4. Agar $ax^2+kx+kx^2-ax=x^2-17x$ ayniyat bo‘lsa, k ni qiymati qanchaga teng bo‘ladi?

$A)\ -6$ $B)\ -8$ $C)\ -7$ $D)\ -9$ $E)\ 8$

5. Agar $a(x-1)^2+b(x-1)+c=2x^2-3x+5$ ayniyat bo‘lsa, $a+b+c$ yig‘indi nimaga teng?

$A)\ 7 \quad B)\ 8 \quad C)\ 6 \quad D)\ 4 \quad E)\ 5$

6. Ushbu $(\alpha x+2y)(3x+\beta y)=\gamma x^2+7xy+y^2$ ayniyatdagi noma’lum koeffitsiyentlardan biri α ni toping.

$A)\ 3 \quad B)\ 2 \quad C)\ 4 \quad D)\ \frac{3}{2} \quad E)\ \frac{5}{2}$

2. Ko‘phadni ko‘phadga bo‘lish

Ta’rif. Agar berilgan $P(x)$ va $Q(x)$ ko‘phadlar uchun $P(x)=Q(x)\cdot G(x)$ bo‘ladigan $G(x)$ ko‘phad mavjud bo‘lsa, $P(x)$ ko‘phad $Q(x)$ ko‘phadga bo‘linadi deyiladi..

$Q(x)$ ko‘phad $P(x)$ ko‘phadning *bo‘luvchisi*, $G(x)$ ko‘phad $P(x)$ va $Q(x)$ ko‘phadlarning *bo‘linmasi* deyiladi.

$Q(x)$ ko‘phad $P(x)$ ko‘phadning bo‘luvchisi bo‘lsa, u holda $G(x)$ ko‘phad ham $P(x)$ ko‘phadning bo‘luvchisi bo‘ladi.

1-xossa. Agar $P(x)$ ko‘phad $K(x)$ ko‘phadga, $K(x)$ ko‘phad esa $Q(x)$ ko‘phadga bo‘linsa, u holda $P(x)$ ko‘phad ham $Q(x)$ ko‘phadga bo‘linadi.

2-xossa. Agar $P(x)$ va $K(x)$ ko‘phadlar $Q(x)$ ko‘phadga bo‘linsa, u holda ularning yig‘indisi va ayirmasi ham $Q(x)$ ko‘phadga bo‘linadi.

3-xossa. Agar $P(x)$ ko‘phad $Q(x)$ ko‘phadga bo‘linsa, u holda $P(x)$ ko‘phadning istalgan $K(x)$ ko‘phadga ko‘paytmasi ham $Q(x)$ ko‘phadga bo‘linadi.

4-xossa. Agar $P(x)$ va $Q(x)$ ko‘phadlar uchun $P(x)=c\cdot Q(x)$, $(c\neq 0,\ c=const)$ bo‘lganda va faqat shundagina bir-biriga bo‘linadi.

Agar $P(x)$ ko‘phad $Q(x)$ ko‘phadga bo‘linmasa, u hol-da qoldiqli bo‘lish amali kiritiladi.

Ta’rif. $P(x)$ ko‘phadni $Q(x)$ ko‘phadga qoldiqli bo‘-lish

$$P(x)=Q(x)\cdot G(x)+R(x)$$

bo‘ladigan $G(x)$ va $R(x)$ ko‘phadlarni topish demakdir.

Bu yerda $P(x)$ – *bo‘linuvchi*, $Q(x)$– *bo‘luvchi*, $G(x)$– *bo‘linma*, $R(x)$– *qoldiq* deb ataladi. Qoldiqli bo‘lishda $R(x)$ ko‘phadning darajasi $Q(x)$ ko‘phadning darajasidan kichik bo‘lishi lozim.

$P(x)$ ko‘phadni $Q(x)$ ko‘phadga qoldiqli bo‘lishdagi $P(x)=Q(x)\cdot G(x)+R(x)$ ifodani quyidagi ko‘rinishda ham yozish mumkin:

$$\frac{P(x)}{Q(x)}=G(x)+\frac{R(x)}{Q(x)}$$

Ko‘phadni ko‘phadga bo‘lishda, odatda, «burchak qilib bo‘lish» usulidan foydalaniladi.

1-misol. $P(x)=x^4+x^3+3x^2+3x+2$ ko‘phadni $Q(x)=x^2+x+1$ ko‘phadga bo‘ling.

Yechish: «Burchak» usulida bo‘lamiz.

$$\begin{array}{r|l}
-\,x^4+x^3+3x^2+3x+2 & x^2+x+1 \\ \cline{2-2}
\underline{x^4+x^3+x^2} & x^2+2 \\
-\,2x^2+3x+2 & \\
\underline{2x^2+2x+2} & \\
x &
\end{array}$$

Demak, $P(x)=x^4+x^3+3x^2+3x+2$ ko‘phadni $Q(x)=x^2+x+1$ ko‘phadga bo‘lganda, bo‘linma $G(x)=x^2+2$ ga, qoldiq $R(x)=x$ ga teng bo‘lar ekan. Bundan $x^4+x^3+3x^2+3x+2=$ $=(x^2+x+1)(x^2+2)+x$ ni hosil qilamiz.

Oxirgi tenglikni quyidagi ko‘rinishda ham yozish mumkin:

$$\frac{x^4+x^3+3x^2+3x+2}{x^2+x+1}=x^2+2+\frac{x}{x^2+x+1}$$

2-misol. $P(x)=3x^5+x^4-19x^2-13x-10$ ko‘phadni $Q(x)=x^3+x^2-1$ ko‘phadga bo‘ling.

Yechish:

$$\begin{array}{r|l}
-\;3x^5+x^4-19x^2-13x-10 & x^3+x^2-1 \\
\underline{3x^5+3x^4-3x^2} & 3x^2-2x+2 \\
-\;-2x^4-16x^2-13x-10 & \\
\underline{-2x^4-2x^3+2x} & \\
-\;2x^3-16x^2-15x-10 & \\
\underline{2x^3+2x^2-2} & \\
-18x^2-15x-8 &
\end{array}$$

Demak, $G(x)=3x^2-2x+2$, $R(x)=-18x^2-15x-8$ va $3x^5+x^4-19x^2-13x-10=(x^3+x^2-1)\cdot(3x^2-2x+2)+(-18x^2-15x-8)$ yoki

$$\frac{3x^5+x^4-19x^2-13x-10}{x^3+x^2-1}=3x^2-2x+2-\frac{18x^2+15x+8}{x^3+x^2-1}.$$

Mustaqil yechish uchun mashqlar

2.1-misol. $P(x)$ ko‘phadni $Q(x)$ ko‘phadga bo‘ling.

1. $P(x)=x^2-3x+2$, $Q(x)=x-1$;
2. $P(x)=x^3-3x^2+x+1$, $Q(x)=x-2$;
3. $P(x)=x^4-3x^3+2x^2+x-5$, $Q(x)=x^2+2x-5$;
4. $P(x)=x^4+3x^3+6x^2+6x+4$, $Q(x)=x^2+x+1$;
5. $P(x)=x^4-9x^3-10x^2+2x+5$, $Q(x)=x+2$;
6. $P(x)=x^5-2x^4-13x^3+2x^2-1$, $Q(x)=x^3-1$;
7. $P(x)=x^6+x^5+4x^4+x^3+2x^2+x+1$, $Q(x)=x^3+x^2+2x+1$.

3. Ratsional kasr ifodalarni sodda kasrlarga yoyish

Agar $n \geq m$ bo'lsa, n – darajali $P(x)$ ko'phadni m– darajali $Q(x)$ ko'phadga bo'lganda $G(x)$ bo'linma, $R(x)$ qoldiq hosil bo'lsa,

$$\frac{P(x)}{Q(x)} = G(x) + \frac{R(x)}{Q(x)} \quad (1)$$

tenglik o'rinli bo'ladi.

Bu yerda $\frac{R(x)}{Q(x)}$ to'g'ri kasr bo'ladi. $Q(x)$ ko'phad quyidagi ko'rinishda ko'paytuvchilarga ajratilgan bo'lsin:

$$Q(x) = (x-a)^k (x-b)^l \ldots (x^2+px+q)^r, \; k, l, \ldots r \in N.$$

U holda

$$\frac{R(x)}{Q(x)} = \frac{A_1}{x-a} + \frac{A_2}{(x-a)^2} + \ldots + \frac{A_k}{(x-a)^k} + \frac{B_1}{x-b} + \frac{B_2}{(x-b)^2} + \ldots$$
$$\frac{B_l}{(x-b)^l} + \ldots + \frac{C_1x+D_1}{x^2+px+q} + \frac{C_2x+D_2}{(x^2+px+q)^2} + \ldots + \frac{C_rx+D_r}{(x^2+px+q)^r} \quad (2)$$

ko'rinishidagi sodda kasrlar yig'indisiga ajraladi. Bu yerda x^2+px+q kvadrat uchhadda $D<0$ bo'lib, ko'paytuvchilarga ajralmaydi.

1-misol. $\frac{x}{(x+1)(x-1)(x+3)}$ kasrni sodda kasrlar yig'indisi ko'rinishida tasvirlang.

Yechish: Berilgan kasr to'g'ri kasr bo'lib, uni (2) ga asosan

$$\frac{x}{(x+1)(x-1)(x+3)} = \frac{a}{x+1} + \frac{b}{x-1} + \frac{c}{x+3}$$

ko'rinishida tasvirlaymiz. Tenglikning o'ng tomonini umumiy maxrajga keltiramiz:

$$\frac{x}{(x+1)(x-1)(x+3)} = \frac{a(x-1)(x+3)+b(x+1)(x+3)+c(x+1)(x-1)}{(x+1)(x-1)(x+3)}.$$

Ikkita kasrning maxrajlari teng, tenglik bajarilishi uchun ularning suratlari ham teng bo'lishi kerak. Ya'ni

$$x = a(x-1)(x+3)+b(x+1)(x+3)+c(x+1)(x-1)$$

$$x = a(x^2+2x-3)+b(x^2+4x+3)+c(x^2-1)$$

$$x = ax^2+2ax-3a+bx^2+4bx+3b+cx^2-c$$

$$0\cdot x^2+1\cdot x+0=(a+b+c)x^2+(2a+4b)x+(-3a+3b-c)$$

Ikkita ko'phad teng bo'lishi uchun bir xil darajalar oldidagi koeffitsiyentlari teng bo'lishi lozim. Bundan quyidagi sistemani hosil qilamiz:

$$\begin{cases} a+b+c=0 \\ 2a+4b=1 \\ -3a+3b-c=0 \end{cases} \Leftrightarrow \begin{cases} a+b+c=0 \\ 2a+4b=1 \\ -2a+4b=0 \end{cases} \Leftrightarrow \begin{cases} a+b+c=0 \\ 2a+4b=1 \\ 8b=1 \end{cases} \Leftrightarrow \begin{cases} a=\dfrac{1}{4} \\ b=\dfrac{1}{8} \\ c=-\dfrac{3}{8}. \end{cases}$$

Demak, $\dfrac{x}{(x+1)(x-1)(x+3)}=\dfrac{1}{4(x+1)}+\dfrac{1}{8(x-1)}-\dfrac{3}{8(x+3)}$ ko'rinishidagi sodda kasrlarga yoyilar ekan.

2-misol. $\dfrac{x^4+2}{(x-1)(x+3)(x-4)}$ ni sodda kasrlarga yoying.

Yechish: Berilgan kasr noto'g'ri kasr, uning suratini maxrajiga bo'lib, to'g'ri kasrga keltiramiz. $(x-1)(x+3)(x-4)=$ $=x^3-2x^2-11x+12$ bo'lib, burchak usulida bo'lishni bajaramiz:

$$\begin{array}{r|l} -\;x^4+2 & x^3-2x^2-11x+12 \\ \underline{x^4-2x^3-11x^2+12x} & x+2 \\ -\;2x^3+11x^2-12x+2 & \\ \underline{2x^3-4x^2-22x+24} & \\ 15x^2+10x-22 & \end{array}$$

Demak, $\dfrac{x^4+2}{(x-1)(x+3)(x-4)}=x+2+\dfrac{15x^2+10x-22}{x^3-2x^2-11x+12}$.

Hosil bo'lgan to'g'ri kasrni sodda kasrlarga yoyamiz:

$$\frac{15x^2+10x-22}{(x-1)(x+3)(x-4)}=\frac{a}{x-1}+\frac{b}{x+3}+\frac{c}{x-4}=$$

$$=\frac{a(x+3)(x-4)+b(x-1)(x-4)+c(x-1)(x+3)}{(x-1)(x+3)(x-4)}.$$

Kasrlarning maxrajlari teng, ularning suratlarini tenglaymiz:

$$15x^2+10x-22=a(x+3)(x-4)+b(x-1)(x-4)+c(x-1)(x+3).$$

Koeffitsiyentlarni hususiy qiymatlar berib topish qulayroq. Oxirgi tenglikdan,

$$x=1\Rightarrow 15+10-22=a(1+3)(1-4)\Rightarrow -12a=3\Rightarrow a=-\frac{1}{4};$$

$$x=-3\Rightarrow 15\cdot 9-30-22=b(-3-1)(-3-4)\Rightarrow 28b=83\Rightarrow b=\frac{83}{28};$$

$$x=4\Rightarrow 15\cdot 16+40-22=c(4-1)(4+3)\Rightarrow 21c=258\Rightarrow c=\frac{258}{21}=\frac{86}{7}.$$

Shunday qilib,

$$\frac{x^4+2}{(x-1)(x+3)(x-4)}=x+2-\frac{1}{4(x-1)}+\frac{83}{28(x+3)}+\frac{86}{7(x-4)}.$$

3-misol. $\frac{x^2-x+7}{(x-2)^2(x+5)}$ ni sodda kasrlarga yoying.

Yechish: (2) ga asosan $\frac{x^2-x+7}{(x-2)^2(x+5)}=\frac{a}{x-2}+\frac{b}{(x-2)^2}+\frac{c}{x+5}$.

Tenglikning o‘ng tomonini umumiy maxrajga keltiramiz:

$$\frac{x^2-x+7}{(x-2)^2(x+5)}=\frac{a(x-2)(x+5)+b(x+5)+c(x-2)^2}{(x-2)^2(x+5)}.$$

Suratlarini tenglaymiz va noma’lum koeffitsiyentlar usuli yordamida koeffitsiyentlarini aniqlaymiz.

$$x^2-x+7=a(x-2)(x+5)+b(x+5)+c(x-2)^2$$

$$x^2-x+7=ax^2+3ax-10a+bx+5b+cx^2-4cx+4c$$

$$x^2-x+7=(a+c)x^2+(3a+b-4c)x+(-10a+5b+4c).$$

Mos darajalar oldidagi koeffitsiyentlarni tenglaymiz.

$$\begin{cases} a+c=1 \\ 3a+b-4c=-1 \\ -10a+5b+4c=7 \end{cases} \Leftrightarrow \begin{cases} a=12/49 \\ b=9/7 \\ c=37/49 \end{cases}$$

Berilgan kasr quyidagicha yoyilmaga ega:

$$\frac{x^2-x+7}{(x-2)^2(x+5)}=\frac{12}{49(x-2)}+\frac{9}{7(x-2)^2}+\frac{37}{49(x+5)}.$$

4-misol. $\frac{2x^4-10x^3+7x^2+4x+3}{(x+2)(x-1)^2(x^2+1)}$ kasrni sodda kasrlar yig'indisi ko'rinishida tasvirlang.

Yechish: Berilgan kasr quyidagi yoyilmaga ega bo'lsin:

$$\frac{2x^4-10x^3+7x^2+4x+3}{(x+2)(x-1)^2(x^2+1)}=\frac{a}{x+2}+\frac{b}{x-1}+\frac{c}{(x-1)^2}+\frac{dx+l}{x^2+1}.$$

Tenglikning o'ng tomonini umumiy maxrajga keltirib, kasrlar suratlarini tenglashtiramiz.

$$2x^4-10x^3+7x^2+4x+3=a(x-1)^2(x^2+1)+b(x+2)(x-1)(x^2+1)+$$
$$+c(x+2)(x^2+1)+(dx+l)(x+2)(x-1)^2=a(x^4-2x^3+2x^2-2x+1)+$$
$$+b(x^4+x^3-x^2+x-2)+c(x^3+2x^2+x+2)+(dx+l)(x^3-3x+2)=$$
$$=(a+b+d)x^4+(-2a+b+c+l)x^3+(2a-b+2c-3d)x^2+$$
$$+(-2a+b+c+2d-3l)x+(a-2b+2c+2l).$$

Bir xil darajalar oldidagi koeffitsiyentlarni tenglashtiramiz.

$$\begin{cases} a+b+d=2 \\ -2a+b+c+l=-10 \\ 2a-b+2c-3d=7 \\ -2a+b+c+2d-3l=4 \\ a-2b+2c+2l=3 \end{cases} \Leftrightarrow \begin{cases} a=3 \\ b=-2 \\ c=1 \\ d=1 \\ l=-3. \end{cases}$$

Demak,

$$\frac{2x^4-10x^3+7x^2+4x+3}{(x+2)(x-1)^2(x^2+1)}=\frac{3}{x+2}-\frac{2}{x-1}+\frac{1}{(x-1)^2}+\frac{x-3}{x^2+1}.$$

5-misol. $\dfrac{3-x}{(x-1)(x^2+x+1)^2}$ kasrni sodda kasrlarga yoying.

Yechish: $$\frac{3-x}{(x-1)(x^2+x+1)^2}=\frac{A}{x-1}+\frac{B_1x+C_1}{x^2+x+1}+\frac{B_2x+C_2}{(x^2+x+1)^2}$$

ko'rinishida yoyamiz.

$$\frac{3-x}{(x-1)(x^2+x+1)^2}=\frac{A(x^2+x+1)^2+(B_1x+C_1)(x-1)(x^2+x+1)+(B_2x+C_2)(x-1)}{(x-1)(x^2+x+1)^2}$$

$$3-x=A(x^2+x+1)^2+(B_1x+C_1)(x-1)(x^2+x+1)+(B_2x+C_2)(x-1).$$

Agar $x=1$ bo'lsa, oxirgi tenglikdan $2=9A\Rightarrow A=\dfrac{2}{9}$ bo'ladi.

$$0\cdot x^4+0\cdot x^3+0\cdot x^2-x+3=A(x^4+2x^3+3x^2+2x+1)+(B_1x+C_1)$$
$$(x^3-1)+(B_2x+C_2)(x-1)=Ax^4+2Ax^3+3Ax^2+2Ax+A+B_1x^4+$$
$$+C_1x^3-B_1x-C_1+B_2x^2-B_2x+C_2x-C_2=(A+B_1)x^4+(2A+C_1)x^3+$$
$$+(3A+B_2)x^2+(2A-B_1-B_2+C_2)x+(A-C_1-C_2).$$

x ning bir xil darajalari oldidagi koeffitsiyentlarini tenglaymiz:

$$\begin{cases}A+B_1=0\\2A+C_1=0\\3A+B_2=0\\2A-B_1-B_2+C_2=-1\\A-C_1-C_2=3\end{cases}\Leftrightarrow\begin{cases}\frac{2}{9}+B_1=0\\2\cdot\frac{2}{9}+C_1=0\\3\cdot\frac{2}{9}+B_2=0\\2\cdot\frac{2}{9}-B_1-B_2+C_2=-1\\\frac{2}{9}-C_1-C_2=3\end{cases}\Leftrightarrow\begin{cases}A=\frac{2}{9}\\B_1=-\frac{2}{9}\\C_1=-\frac{4}{9}\\B_2=-\frac{2}{3}\\C_2=-\frac{7}{3}\end{cases}$$

$$\frac{3-x}{(x-1)(x^2+x+1)^2}=\frac{2}{9(x-1)}-\frac{2x+4}{9(x^2+x+1)}-\frac{2x+7}{3(x^2+x+1)^2}.$$

6-misol. a va b ning qanday qiymatida quyidagi tenglik ayniyat bo'ladi?

$$\frac{2}{x^2+x-6}=\frac{a}{x-2}+\frac{b}{x+3}.$$

$A)\ a=1;\ b=1$ $\quad B)\ a=\frac{2}{5};\ b=-\frac{2}{5}$ $\quad C)\ a=5;\ b=-5$

$D)\ a=-\frac{2}{5};\ b=\frac{2}{5}$ $\quad E)\ a=-\frac{1}{5};\ b=\frac{3}{5}$

Yechish: Ko'rinib turibdiki, $x^2+x-6=(x-2)(x+3)$ o'rinli. Ayniyatning o'ng tomonini umumiy maxrajga keltiramiz.

$$\frac{2}{x^2+x-6}=\frac{a(x+3)+b(x-2)}{(x-2)(x+3)}=\frac{(a+b)x+(3a-2b)}{x^2+x-6}.$$

$$2=(a+b)x+(3a-2b)\Leftrightarrow 0\cdot x+2=(a+b)x+(3a-2b).$$

$$\begin{cases}a+b=0\\3a-2b=2\end{cases}\Leftrightarrow\begin{cases}a=-b\\-3b-2b=2\end{cases}\Leftrightarrow\left|\begin{aligned}a&=\frac{2}{5}\\b&=-\frac{2}{5}.\end{aligned}\right.$$

Javob: $B)\ a=\frac{2}{5};\ b=-\frac{2}{5}$.

Mustaqil yechish uchun mashqlar

3.1-misol. Quyidagi kasrlarni sodda kasrlar yig'indisi ko'rinishida tasvirlang.

1. $\frac{2x}{(x+1)(x-2)}$ **2.** $\frac{x^2-4x-2}{(x-1)(x+2)(x+3)}$ **3.** $\frac{x^2-3}{(x-3)(x-5)(x-6)}$

3.2-misol. Quyidagi kasrlarni sodda kasrlar yig'indisi ko'rinishida tasvirlang.

1. $\frac{x^2-3}{(x-4)^2(x+3)}$ **2.** $\frac{x^3+x-2}{(x-4)^2(x^2-4)}$

3. $\frac{4-x}{x(3x^2-27)}$ **4.** $\frac{2x+3}{(x-3)^2(x+1)^2}$

5. $\frac{x^5+2x^3+4x+4}{x^4+2x^3+2x^2}$ **6.** $\frac{x-1}{(x+1)(x^2+x+1)}$

7. $\frac{x^2+9x+1}{x^4+6x^2+8}$ **8.** $\frac{x^2+2x+9}{(x+3)(x-3)^2(x^2+3)}$

9. $\dfrac{x^3}{x^3-8}$

10. $\dfrac{x^2-x+1}{(x-1)^2(x^2+x+5)^2}$

11. $\dfrac{1+x}{(x^4-16)(x^2+1)}$

12. $\dfrac{x^5-x+3}{(x^2+2x+7)(x+2)^2}$

3.3-misol. Soddalashtiring:

$$\frac{1}{x(x+4)}+\frac{1}{(x+4)(x+8)}+\frac{1}{(x+8)(x+12)}+\frac{1}{(x+12)(x+16)}+$$
$$+\frac{1}{(x+16)(x+20)}.$$

3.4-misol. Isbotlang:

$$\frac{1}{6\cdot 8}+\frac{1}{8\cdot 10}+\frac{1}{10\cdot 12}+\ldots+\frac{1}{(2n+4)(2n+6)}=\frac{n}{12\cdot(n+3)}.$$

3.5-misol. Test topshiriqlari.

1. a va b ning qanday qiymatida quyidagi tenglik ayniyat bo'ladi?

$$\frac{1}{x^2-5x-6}=\frac{a}{x-6}+\frac{b}{x+1}$$

$A)\ a=7;\ b=-1$ $\quad B)\ a=\frac{1}{7};\ b=-\frac{1}{7}$ $\quad C)\ a=1;\ b=1$

$D)\ a=-\frac{1}{7};\ b=\frac{1}{7}$ $\quad E)\ a=-1;\ b=7$

2. a va b ning qanday qiymatlarida $\dfrac{1}{4x^2-1}=\dfrac{a}{2x-1}-\dfrac{b}{2x+1}$ tenglik ayniyat bo'ladi?

$A)\ a=-\frac{1}{2};\ b=\frac{1}{2}$ $\quad B)\ a=1;\ b=-1$ $\quad C)\ a=-1;\ b=1$

$D)\ a=\frac{1}{2};\ b=-\frac{1}{2}$ $\quad E)\ a=\frac{1}{2};\ b=\frac{1}{2}$

3. $\dfrac{1}{(x+1)^2(x+2)}=\dfrac{a}{x+1}+\dfrac{b}{(x+1)^2}+\dfrac{c}{x+2}$ tenglikni qanoatlantiradigan a, b, c larni toping.

$A)\ a=1;\ b=1;\ c=1$ $\quad B)\ a=-1;\ b=-1;\ c=1$

$C)\ a=1;\ b=-1;\ c=1$ $\quad D)\ a=-1;\ b=1;\ c=1$

4. $\frac{1}{x^3+1}=\frac{a}{x+1}+\frac{bx+c}{x^2-x+1}$ tenglikni qanoatlantiradigan a, b, c larni toping.

$A)\ a=\frac{1}{3};\ b=\frac{2}{3};\ c=-\frac{1}{3}$ $\quad B)\ a=\frac{2}{3};\ b=-\frac{1}{3};\ c=\frac{1}{3}$

$C)\ a=b=c=\frac{1}{3}$ $\quad D)\ a=\frac{1}{3};\ b=-\frac{1}{3};\ c=\frac{2}{3}$

4. Ko'phad koeffitsiyentlarining yig'indisi

Ko'pincha ba'zi masalalarda yuqori darajali ko'phadlarning koeffitsiyentlari yig'indisini topish masalasi qo'yiladi.

n – darajali $P(x)=a_0x^n+a_1x^{n-1}+a_2x^{n-2}+\ldots+a_{n-1}x+a_n$ ko'phad berilgan bo'lsin. Bu ko'phadning barcha koeffitsiyentlari yig'indisini topish uchun, bu ko'phadning $x=1$ dagi qiymatini topish kifoya. Haqiqatdan ham,

$$P(1)=a_0+a_1+a_2+\ldots+a_{n-1}+a_n\,.$$

1-misol. $P(x)=(1+3x-5x^2)^{2013}(1-8x+6x^2)^{2012}$ ko'phadning barcha koeffitsiyentlari yig'indisini toping.

Yechish: Agar biz qavslarni ochib, ko'phadni standart shaklga keltirsak, natijaga erishishimiz ancha mushkul. Yuqoridagiga asosan koeffitsiyentlari yig'indisi

$$x=1\Rightarrow P(1)=(1+3\cdot 1-5\cdot 1^2)^{2013}(1-8\cdot 1+6\cdot 1^2)^{2012}=$$
$$=(-1)^{2013}\cdot(-1)^{2012}=-1\cdot 1=-1$$

ga teng bo'ladi.

Javob: -1.

$P(x)$ ko'phadning juft darajali hadlari oldidagi koeffitsiyentlari yig'indisi $\frac{1}{2}[P(1)+P(-1)]$ ga teng bo'ladi.

$P(x)$ ko'phadning toq darajali hadlari oldidagi koeffitsiyentlari yig'indisi $\frac{1}{2}[P(1)-P(-1)]$ ga teng bo'ladi.

$P(x)$ ko'phadning ozod hadini topish uchun berilgan ko'phadni $x=0$ dagi qiymatini, ya'ni $P(0)$ ni topish kifoya. Haqiqatan ham n – darajali $P(x)=a_0x^n+a_1x^{n-1}+a_2x^{n-2}+$ $+\ldots+a_{n-1}x+a_n$ ko'phadning ozod hadi

$$P(0)=a_0\cdot 0^n+a_1\cdot 0^{n-1}+a_2\cdot 0^{n-2}+\ldots+a_{n-1}\cdot 0+a_n=a_n$$

ga teng.

2-misol. $P(x)=(x^2+5x+1)^{100}+(x-1)^{50}+8x+12$ ko'phadning ozod hadini toping.

Yechish: Ko'rinib turibdiki, qavslar ochib chiqilsa, berilgan ko'phadning eng katta darajasi x^{200} bo'lgan, standart shaklga keladi. Agar shu tariqa yo'l tutsak, qiyinchiliklarga duch kelamiz. Berilgan ko'phadning ozod hadi $P(0)$ ga tengligini e'tiborga olsak,

$$P(0)=(0^2+5\cdot 0+1)^{100}+(0-1)^{50}+8\cdot 0+12=14$$

ga teng ekanligi kelib chiqadi.

Javob: 14.

3-misol. $P(x)=(x^4-4x+1)^3-5x^3$ ko'phadning juft darajali hadlari koeffitsiyentlari yig'indisini toping.

Yechish: Bu yig'indi $\frac{1}{2}[P(1)+P(-1)]$ ga teng bo'ladi.

$$P(1)=(1^4-4\cdot 1+1)^3-5\cdot 1^3=(-2)^3-5=-13;$$

$$P(-1)=((-1)^4-4\cdot(-1)+1)^3-5\cdot(-1)^3=6^3+5=221;$$

$$\frac{1}{2}[P(1)+P(-1)]=\frac{1}{2}(-13+221)=104.$$

Javob: 104.

4-misol. Ushbu $x^3-px^2-qx+4=0$ tenglamaning ildizlaridan biri 1 ga teng. Shu tenglama barcha koeffitsiyentlari yig'indisini toping.

$A)-1$ $B)\ 0$ $C)\ 1$ $D)\ 1{,}5$ $E)\ 2$

Yechish: $x=1$ soni berilgan tenglamaning ildizi bo'lsa, u holda $1^3-p\cdot1^2-q\cdot1+4=0$ o'rinli va $p+q=5$ ni hosil qilamiz. Tenglamaning barcha koeffitsiyentlari yig'indisi

$$P(1)=1-p-q+4=5-(p+q)=5-5=0.$$

Javob: $B)\,0$.

5-misol. $P(x)=(3x^2-4x+a)^5+x^3+3$ ko'phadning koeffitsiyentlari yig'indisi 36 ga teng bo'lsa, a ni toping.

Yechish: Ko'phadning koeffitsiyentlari yig'indisi $P(1)$ ga teng:

$$P(1)=(3\cdot1^2-4\cdot1+a)^5+1^3+3=36\Rightarrow(a-1)^5=32\Rightarrow a-1=2\Rightarrow a=3.$$

Javob: $a=3$.

6-misol. $P(x)=x^4-6x^3-x^2+mx+5$ ko'phadning koeffitsiyentlari yig'indisi 12 ga teng bo'lsa, m ni toping.

Yechish: Masala shartidan $P(1)=12$ ekanligi ravshan. Bundan

$$P(1)=1-6-1+m+5=12\Rightarrow m=13.$$

Javob: $m=13$.

Mustaqil yechish uchun mashqlar

4.1-misol. $P(x)=(x^3-ax+3)^3+26x+14$ ko'phadning koeffitsiyentlari yig'indisi 256 ga teng bo'lsa, a ni toping.

4.2-misol. $P(x)=ax^5+x^4-4x^3+x-4$ ko'phadning koeffitsiyentlari yig'indisi 18 ga teng bo'lsa, a ni toping.

4.3-misol. Test topshiriqlari.

1. Agar $(x-1)^2\cdot(x+1)^3+3x-1$ ifoda standart shakldagi ko'phad ko'rinishida yozilsa, koeffitsiyentlarining yig'indisi nechaga teng bo'ladi?

$A)\,10\quad B)\,4\quad C)\,2\quad D)\,3\quad E)\,1$

2. Agar $(x^3 - x + 1)^3 + x$ ifoda standart shakldagi ko'phad ko'rinishida yozilsa, x ning toq darajali hadlari oldidagi koeffitsiyentlarining yig'indisi nechaga teng bo'ladi?

$A)\ 1 \quad B)\ 7 \quad C)\ 4 \quad D)\ 5 \quad E)\ 3$

3. $f(x) = (x^3 + 2x^2 - 1)^2 - 3x^2$ ko'phadning juft darajali hadlari koeffitsiyentlarining yig'indisini toping.

$A)\ -6 \quad B)\ -2 \quad C)\ 3 \quad D)\ -3 \quad E)\ -1$

4. $P(x) = (x^2 - 7x + 3)(x^3 - 5) + x^2 + 4$ ko'phadning toq darajali hadlari koeffitsiyentlari yig'indisini toping.

$A)\ 39 \qquad B)\ -39 \qquad C)\ 22 \qquad D)\ -22$

5. Bezu teoremasi. Ko'phadning ildizlari

n – darajali $P(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n \ (a_0 \neq 0)$ ko'phad berilgan bo'lsin.

Teorema (Bezu). $P(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n$ ko'phadni $x - a$ ga bo'lgandagi chiqadigan R qoldiq, shu ko'phadni $x = a$ dagi qiymatiga teng, ya'ni $R = P(a)$.

Isbot. $P(x)$ ko'phadni $x - a$ ikkihadga bo'lsak, $Q(x)$ bo'linma va $R(x)$ qoldiq hosil bo'lsa; $P(x) = (x - a) \cdot Q(x) + R(x)$ tenglik o'rinli. $x = a$ da $P(a) = (a - a) \cdot Q(a) + R(a) = R(a) = R$ hosil bo'ladi. Isbot tugadi.

Ta'rif. Agar biror $x = c$ soni uchun $P(c) = 0$ tenglik bajarilsa, u holda c soni $P(x)$ ko'phadning *ildizi* deyiladi. Masalan, $P(x) = x^3 - 7x - 6$ ko'phadning ildizi $x = -1$ bo'ladi. Chunki, $P(-1) = (-1)^3 - 7 \cdot (-1) - 6 = 0$.

Demak, $P(x)$ ko'phadning ildizlarini topish uchun $P(x) = 0$ tenglamani yechish kerak.

1-teorema. Agar c soni $n-$ darajali $P(x)$ ko'phadning ildizi bo'lsa, u holda $P(x)$ ko'phad $x-c$ chiziqli ko'phadga qoldiqsiz bo'linadi.

Isbot. Bu holda $P(x)=(x-c)Q(x)+R(x)$ ko'rinishida bo'lib, $P(c)=R(c)=0$ va $R=0$ kelib chiqadi. Demak, $P(x)=(x-c)Q(x)$.

1-misol. $p(x)$ ko'phadni $x-1$ ga bo'lgandagi qoldiq 2 ga, $x-3$ ga bo'lgandagi qoldiq 1 ga teng. $p(x)$ ko'phadni x^2-4x+3 ga bo'lgandagi qoldiqni toping.

Yechish: $x^2-4x+3=(x-1)(x-3)$ o'rinli. $p(x)$ ko'phadni

$$p(x)=(x-1)(x-3)q(x)+ax+b$$

ko'rinishida tasvirlash mumkin. Bu yerda $ax+b$ izlanayotgan qoldiq.

Bezu teoremasiga asosan $p(1)=2$, $p(3)=1$. Oxirgi tenglikdan $x=1$ da $p(1)=a+b$ va $x=3$ da $p(3)=3a+b$. Quyidagi sistemani hosil qilamiz:

$$\begin{cases} a+b=2 \\ 3a+b=1 \end{cases} \Rightarrow \begin{cases} a+b=2 \\ -2a=1 \end{cases} \Rightarrow \begin{cases} a=-0,5 \\ b=2,5. \end{cases}$$

Demak, qoldiq $-0,5x+2,5$ ga teng ekan.

Javob: $-0,5x+2,5$.

2-misol. $p(x)=x^3+ax^2+bx+c$ ko'phad x^2-3x+2 ga qoldiqsiz bo'linadi, shu ko'phadni $x-4$ ga bo'lgandagi qoldiq 6 ga teng. a,b koeffitsiyentlarni toping.

Yechish: Bezu teoremasiga asosan $p(x)$ ko'phadni $x-4$ ga bo'lgandagi qoldiq $p(4)=6$ o'rinli. Bundan

$$p(4)=4^3+a\cdot4^2+4b+c=6 \Rightarrow 16a+4b+c=-58.$$

Ikkinchi tomondan $p(x)$ ko'phad $x^2-3x+2=(x-1)(x-2)$ ga bo'linadi. Bundan $x=1, x=2$ lar berilgan ko'phadning ildizi ekanligi kelib chiqadi.

Demak, $p(x)=x^3+ax^2+bx+c=(x-1)(x-2)q(x)$ ko‘rinishida bo‘lib, $p(1)=1+a+b+c=0$ va $p(2)=8+4a+2b+c=0$.

Bizda quyidagi uch noma’lumli uchta chiziqli tenglamalar sistemasi hosil bo‘ladi:

$$\begin{cases} a+b+c=-1 \\ 4a+2b+c=-8 \\ 16a+4b+c=-58 \end{cases} \Rightarrow \begin{cases} a+b+c=-1 \\ 3a+b=-7 \\ 12a+2b=-50 \end{cases} \Rightarrow \begin{cases} a+b+c=-1 \\ 3a+b=-7 \\ 6a+b=-25 \end{cases} \Rightarrow \begin{cases} a=-6 \\ b=11 \\ c=-6. \end{cases}$$

Javob: $a=-6, b=11, c=-6$.

3-misol. $p(x)=x^{243}+x^{81}+x^{27}+x^9+x^3+1$ ko‘phadni x^2-1 ikkihadga bo‘lgandagi qoldiqni toping.

Yechish: Bo‘lishni «Burchak» usulida bajarsak, ishimiz ancha qiyinlashadi. Berilgan ko‘phadni x^2-1 ga bo‘linadigan qo‘shiluvchilar ko‘rinishida tasvirlasak maqsadga oson erishamiz.

$$p(x)=x^{243}+x^{81}+x^{27}+x^9+x^3+1=(x^{243}-x)+(x^{81}-x)+$$
$$+(x^{27}-x)+(x^9-x)+(x^3-x)+(5x+1).$$

Hosil bo‘lgan birinchi beshta qo‘shiluvchilarning har biri Bezu teoremasiga asosan x^2-1 ga bo‘linadi. Natijada izlanayotgan qoldiq $5x+1$ ga teng bo‘lishi kelib chiqadi.

Javob: $5x+1$.

4-misol. x^3+2nx^2+mx+5 ko‘phad x^2-x-2 ga qoldiqsiz bo‘linadi. n ni toping.

$A)\ \frac{21}{12}$ $B)-\frac{21}{12}$ $C)\ \frac{12}{21}$ $D)-\frac{12}{21}$ $E)-2$

Yechish: $x^2-x-2=(x+1)(x-2)$ o‘rinli. Masala shartidan berilgan ko‘phadni

$$x^3+2nx^2+mx+5=(x+1)(x-2)q(x)$$

ko‘rinishida yozib olamiz. Bundan berilgan ko‘phadni $x=-1$ va $x=2$ ildizi ekanligi kelib chiqadi.

$x=-1$ da $(-1)^3+2n\cdot(-1)^2+m\cdot(-1)+5=0 \Rightarrow 2n-m=-4$,

$x=2$ da $2^3+2n\cdot 2^2+m\cdot 2+5=0 \Rightarrow 8n+2m=-13$.

Quyidagi sistemani hosil qilamiz:

$$\begin{cases}2n-m=-4\\ 8n+2m=-13\end{cases}\Rightarrow\begin{cases}2n-m=-4\\ 12n=-21\end{cases}\Rightarrow\begin{cases}n=-\dfrac{21}{12}=-\dfrac{7}{4}\\ m=\dfrac{1}{2}.\end{cases}$$

Javob: $B)-\frac{21}{12}$.

5-misol. $x^6+x^4-3x^2+5$ ko'phadni $x^2-\sqrt{3}$ ga bo'lgandagi qoldiqni toping.

$A)\,8\quad B)\,7\quad C)\,6\quad D)\,9\quad E)\,5$

Yechish: $x^2=y$ deb belgilash kiritamiz. Berilgan ko'phad y^3+y^2-3y+5 ko'rinishiga kelib, hosil bo'lgan ko'phadni $y-\sqrt{3}$ ga bo'lgandagi qoldiq Bezu teoremasiga asosan $y=\sqrt{3}$ dagi qiymatiga teng:

$$R=(\sqrt{3})^3+(\sqrt{3})^2-3\sqrt{3}+5=3\sqrt{3}+3-3\sqrt{3}+5=8.$$

Javob: $A)8$.

6-misol. $x^{99}+5x^3+2x+8$ ko'phadni x^2+1 bo'lgandagi qoldiqni toping.

$A)8x+8\qquad B)-2x+8\qquad C)-4x+8\qquad D)-3x+7$

Yechish: $P(x)=x^{99}+5x^3+2x+8$ bo'lsin. $P(x)$ ko'phadni

$$P(x)=(x^2)^{49}\cdot x+5x\cdot x^2+2x+8$$

ko'rinishida yozib olamiz. $x^2=t$ deb olaylik. U holda $P(t)=xt^{49}+5xt+2x+8$ bo'lib, x ni $P(t)$ ko'phadning koeffitsiyenti sifatida qaraymiz. $P(t)$ ko'phadni $t+1$ ga bo'lgandagi qoldiq Bezu teoremasiga asosan

$$r(x)=P(-1)=x\cdot(-1)^{49}+5x\cdot(-1)+2x+8=-4x+8$$

ga teng.

Javob: $C)-4x+8$.

Bu yerda quyidagi xulosalarga ega bo'lamiz:

1. $x^n - a^n$ ikkihad $x-a$ ga bo'linadi($n \in N$, $x \neq a$).

Haqiqatan, ham $R = P(a) = a^n - a^n = 0$.

2. $x^{2n} - a^{2n}$ ikkihad $x-a$ ga bo'linadi.

3. $x^{2n} - a^{2n}$ ikkihad $x+a$ ga bo'linadi.

4. $x^{2n+1} - a^{2n+1}$ ikkihad $x-a$ ga bo'linadi.

5. $x^{2n+1} + a^{2n+1}$ ikkihad $x+a$ ga bo'linadi.

Quyidagi ayniyatlar o'rinli:

1. $x^5 - a^5 = (x-a)(x^4 + ax^3 + a^2x^2 + a^3x + a^4)$;

2. $x^5 + a^5 = (x+a)(x^4 - ax^3 + a^2x^2 - a^3x + a^4)$;

3. $x^6 - a^6 = (x-a)(x^5 + ax^4 + a^2x^3 + a^3x^2 + a^4x + a^5)$;

4. $x^6 + a^6 = (x+a)(x^5 - ax^4 + a^2x^3 - a^3x^2 + a^4x - a^5)$;

5. $x^n - a^n = (x-a)(x^{n-1} + ax^{n-2} + a^2x^{n-3} + \ldots + xa^{n-2} + a^{n-1}), n \in N$.

6. $x^n + a^n = (x+a)(x^{n-1} - ax^{n-2} + \ldots + a^{n-1}), n \in N$, n – toq.

7. $x^n - a^n = (x-a)(x^{n-1} + ax^{n-2} + \ldots + a^{n-1}), n \in N$, n – toq.

Xulosa. Agar $P(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n$ ko'phadning barcha koeffitsiyentlar yig'ndisi 0 ga teng bo'lsa, u holda $x=1$ ko'phadning ildizi bo'ladi.

Haqiqatan ham $P(1) = a_0 + a_1 + a_2 + \ldots + a_{n-1} + a_n = 0$ o'rinli.

7-misol. $4x^3 - 3x^2 - 1 = 0$ tenglamani yeching.

Yechish: Agar ko'phadning barcha koeffitsiyentlari yig'indisi 0 ga teng bo'lsa, u holda $x = 1$ ko'phadning ildizi bo'ladi.

Berilgan tenglamaning $x = 1$ ildizi. Shu sababli $P(x) = 4x^3 - 3x^2 - 1$ ko'phad $x - 1$ ikkihadga qoldiqsiz bo'linadi. Bo'lishni bajaramiz:

$$\begin{array}{r|l} 4x^3-3x^2-1 & x-1 \\ \underline{4x^3-4x^2} & 4x^2+x+1 \\ x^2-1 & \\ \underline{x^2-x} & \\ x-1 & \\ \underline{x-1} & \\ 0 & \end{array} \Rightarrow 4x^3-3x^2-1=(x-1)(4x^2+x+1)=0$$

$$4x^2+x+1=0;\ D=-15<0 \Rightarrow x\in\varnothing.$$

Javob: $x=1$.

Ba'zi hollarda $P(x)$ ko'phad $x-c$ ko'phadgagina emas, balki uning biror yuqori darajasi $(x-c)^k$ ko'phadga ham bo'-linishi mumkin, bu yerda $k>1,\ k\in N$.

Ta'rif. Agar $P(x)$ ko'phad $(x-c)^k$ ko'phadga bo'linib, $(x-c)^{k+1}$ ko'phadga bo'linmasa, c soni $P(x)$ ko'phadning k *karrali ildizi* deyiladi.

Agar $P(x)$ ko'phad $x-c$ chiziqli ko'phadga bo'linib, lekin $(x-c)^2$ ga bo'linmasa, c soni $P(x)$ ko'phadning *oddiy ildizi* deyiladi.

8-misol. $P(x)=x^3-5x^2+8x-4$ ko'phadning ildizlarini toping.

Yechish: Ko'phadning ildizini ko'paytuvchilarga ajratish orqali topamiz:

$$P(x)=x^3-5x^2+8x-4=x^3-x^2-4x^2+8x-4=x^2(x-1)-4(x^2-2x+1)=$$
$$=x^2(x-1)-4(x-1)^2=(x-1)(x^2-4x+4)=(x-1)(x-2)^2.$$

Bundan $P(x)=(x-1)(x-2)^2=0 \Leftrightarrow x_1=1;\ x_2=x_3=2$. Demak, $x=2$ ikki karrali ildiz, $x=1$ oddiy ildiz ekan.

Javob: $\{1;2\}$.

9-misol. $x=-3$ soni $p(x)=x^3+x^2+ax+b$ ko'phadning ikki karrali ildizi bo'lsa, uchinchi ildizi va a,b koeffitsiyentlarni toping.

Yechish: $x=-3$ soni berilgan ko'phadning ikki karrali ildizi bo'lsa, $p(x)$ ko'phad $(x+3)^2=x^2+6x+9$ kvadrat uchhadga bo'linadi.

$$\begin{array}{r|l} x^3+x^2+ax+b & x^2+6x+9 \\ -\underline{x^3+6x^2+9x} & x-5 \\ -5x^2+(a-9)x+b & \\ -\underline{-5x^2-30x-45} & \\ (a+21)x+b+45 & \end{array}$$

Demak, $p(x)=x^3+x^2+ax+b=(x+3)^2(x-5)+(a+21)x+b+45$ bo'lib, qoldiq 0 ga teng bo'lishi kerak:

$$\begin{cases} a+21=0 \\ b+45=0 \end{cases} \Rightarrow \begin{cases} a=-21 \\ b=-45. \end{cases}$$

Bundan $p(x)=(x+3)^2(x-5)=0$ bo'lib, uchinchi ildiz $x_3=5$.

Bu misolni quyidagicha usulda yechish ham mumkin: $x=-3$ soni $p(x)=x^3+x^2+ax+b$ ko'phadning ikki karrali ildizi, uchinchi ildizi $x=c$ bo'lsin. U holda

$$x^3+x^2+ax+b=(x+3)^2(x-c)$$

$$x^3+x^2+ax+b=(x^2+6x+9)(x-c)$$

$$x^3+x^2+ax+b=x^3+(6-c)x^2+(9-6c)x-9c$$

Ko'phadlarning tengligidan

$$\begin{cases} 6-c=1 \\ 9-6c=a \\ -9c=b \end{cases} \Rightarrow \begin{cases} c=5 \\ a=-21 \\ b=-45. \end{cases}$$

Javob: $a=-21, b=-45, x_3=5$.

1-natija. n – darajali $P(x)$ ko'phadning c_1 soni k_1 karrali, c_2 soni k_2 karrali va hokazo c_m soni k_m karrali ildizi bo'lsa, u holda $P(x)$ ko'phadni

$$P(x)=(x-c_1)^{k_1}\cdot(x-c_2)^{k_2}\cdot\ldots\cdot(x-c_k)^{k_m},\ k_1+k_2+\ldots+k_m=n$$

ko'rinishida tasvirlash mumkin. Bu yerda $k_1, k_2, \ldots, k_m \in N$.

2-natija. Agar $P(x)$ ko'phad $x=a_1$ va $x=a_2$ ildizlarga ega bo'lsa, u holda $P(x)$ ko'phad $(x-a_1)(x-a_2)$ bo'linadi.

Isbot. Agar $a_1=a_2$ bo'lsa, karrali ildiz ta'rifiga asosan $P(x)$ ko'phad $(x-a_1)^2$ ga bo'linadi, chunki $x=a_1$ ikki karrali ildiz.

$a_1 \neq a_2$ bo'lsin. Shartga ko'ra $x=a_1$ soni $P(x)$ ko'phadning ildizi. Bundan $P(x)=(x-a_1)q(x)$ tenglik o'rinli, bu yerda $q(x)$ – ko'phad. $x=a_2$ ni $P(x)=(x-a_1)q(x)$ tenglikka qo'yamiz: $P(a_2)=(a_2-a_1)q(a_2)=0$.

Shartga asosan $a_2-a_1 \neq 0$ bo'lganligi uchun, yuqoridagi tenglikdan faqat $q(a_2)=0$ ekanligi kelib chiqadi. U holda $q(x)$ ko'phad $x-a_2$ ga bo'linadi, ya'ni $q(x)=(x-a_2)\varphi(x)$ tenglik o'rinli, bu yerda $\varphi(x)$ – ko'phad.

Demak, $P(x)=(x-a_1)q(x)=(x-a_1)(x-a_2)\varphi(x)$ ni hosil qilamiz. Bu $P(x)$ ko'phadni $(x-a_1)(x-a_2)$ ga bo'linishini bildiradi. Isbot tugadi.

Biz yuqorida ko'phadlarning ildizini «sun'iy» (ko'paytuvchilarga ajratish va hokazo) usullarda topdik. Quyida keltirilgan teorema ko'phadning ildizini qulay ravishda topishga imkon beradi.

2-teorema. $\frac{p}{q} (p \in Z, q \in N)$ qisqarmas kasr butun koeffitsiyentli $a_0x^n+a_1x^{n-1}+a_2x^{n-2}+\ldots+a_{n-1}x+a_n=0$, $a_0 \neq 0$ (1) tenglamaning ildizi bo'lishi uchun p soni a_n ozod hadning bo'luvchisi, q esa a_0 bosh koeffitsiyentning bo'luvchisi bo'lishi zarur.

Isbot. Haqiqatan ham $\frac{p}{q}$ soni (1) tenglamaning ildizi bo'lsin, u holda

$$a_0\left(\frac{p}{q}\right)^n + a_1\left(\frac{p}{q}\right)^{n-1} + a_2\left(\frac{p}{q}\right)^{n-2} + \ldots + a_{n-1}\cdot\frac{p}{q} + a_n = 0$$

o'rinli. Oxirgi tenglikning ikkala tomonini q^n ga ko'paytiramiz va

$$a_0p^n + a_1p^{n-1}q + a_2p^{n-2}q^2 + \ldots + a_{n-1}pq^{n-1} + a_nq^n = 0$$

ni hosil qilamiz. Bundan

$$a_nq^n = -a_0p^n - a_1p^{n-1}q - a_2p^{n-2}q^2 - \cdots - a_{n-1}pq^{n-1} = -p(a_0p^{n-1} -$$
$$-a_1p^{n-2}q - a_2p^{n-3}q^2 - \cdots - a_{n-1}q^{n-1}).$$

Tenglikning o'ng tomoni p ga bo'linadi. Demak, chap tomonidagi a_nq^n ham p ga bo'linishi kerak. Lekin, $\frac{p}{q}$ qisqarmas kasr, ya'ni p va q^n lar o'zaro tub. Demak, p soni a_n ning bo'luvchisi. Xuddi shunday q soni a_0 ning bo'luvchisi ekanligi isbot qilinadi.

3-teorema. Butun koeffitsiyentli

$$P(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n$$

ko'phadning noldan farqli $x = a$ butun ildizlari ozod had a_n ning bo'luvchilari bo'ladi.

Isbot. Haqiqatan, $x = a$ soni $P(x)$ ko'phadning ildizi bo'lsa, u holda

$$a_0a^n + a_1a^{n-1} + a_2a^{n-2} + \ldots + a_{n-1}a + a_n = 0$$

tenglik o'rinli. Bu tenglikdagi a_n dan tashqari barcha qo'shiluvchilar a ga bo'linadi, o'ng tomonidagi 0 ham a ga bo'linadi. Bundan ozod had a_n ham a ga bo'linishi kelib chiqadi.

Ta'rif. $P(x) = x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n$ ko'phad keltirilgan ko'phad deyiladi. Bu yerda $a_1, a_2, \ldots, a_n$ lar butun sonlar.

4-teorema. Agar $x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n = 0$ tenglama butun ildizga ega bo'lsa, u holda bu ildiz ozod had $a_n\,(a_n \neq 0)$ning bo'luvchisi bo'ladi.

Isbot. $x_1 \neq 0$ soni $x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n = 0$ tenglamaning butun ildizi bo'lsin. U holda $x_1^n + a_1x_1^{n-1} + a_2x_1^{n-2} + \ldots + a_{n-1}x_1 + a_n = 0$ o'rinli. Tenglikning ikkala tomonini x_1 ga bo'lamiz va

$$x_1^{n-1} + a_1x_1^{n-2} + a_2x_1^{n-3} + \ldots + a_{n-1} + \frac{a_n}{x_1} = 0$$

ni hosil qilamiz. Bundan

$$x_1^{n-1} + a_1x_1^{n-2} + a_2x_1^{n-3} + \ldots + a_{n-1} = -\frac{a_n}{x_1}.$$

Shartga ko'ra oxirgi tenglikning chap tomoni butun son, shuning uchun tenglikning o'ng tomoni $-\frac{a_n}{x_1}$ ham butun son bo'ladi. Demak, ozod had a_n soni x_1 ga qoldiqsiz bo'linadi.

10-misol. $x^5 + 4x^4 - 6x^3 - 24x^2 - 27x - 108 = 0$ tenglamani yeching.

Yechish: Tenglamaning chap tomoni beshinchi darajali ko'phaddan iborat va $P(x) = x^5 + 4x^4 - 6x^3 - 24x^2 - 27x - 108 = 0$ deb olaylik. Ozod had -108 ga teng. 4-teoremaga asosan tenglamaning butun ildizlarini ozod hadning bo'luvchilari $\pm 1; \pm 2; \pm 3; \pm 4; \pm 6; \pm 9; \pm 12;\ \pm 18; \pm 27; \pm 36; \pm 54; \pm 108$ lar ichidan qidiramiz. Bu ko'phadning nollari $x = 3; -3; -4$ bo'ladi:

$$P(3) = 3^5 + 4\cdot 3^4 - 6\cdot 3^3 - 24\cdot 3^2 - 27\cdot 3 - 108 = 0$$

$$P(-3) = (-3)^5 + 4\cdot(-3)^4 - 6\cdot(-3)^3 - 24\cdot(-3)^2 - 27\cdot(-3) - 108 = 0$$

$$P(4) = 4^5 + 4\cdot 4^4 - 6\cdot 4^3 - 24\cdot 4^2 - 27\cdot 4 - 108 = 0.$$

Demak, berilgan ko'phad

$$(x-3)(x+3)(x+4) = (x^2-9)(x+4) = x^3 + 4x^2 - 9x - 36$$

ko'phadga qoldiqsiz bo'linadi.

$$
\begin{array}{r|l}
x^5+4x^4-6x^3-24x^2-27x-108 & x^3+4x^2-9x-36 \\
\underline{x^5+4x^4-9x^3-36x^2} \qquad\qquad & x^2+3 \\
3x^3+12x^2-27x-108 & \\
\underline{3x^3+12x^2-27x-108} & \\
0 \qquad\qquad &
\end{array}
$$

Demak,

$$x^5+4x^4-6x^3-24x^2-27x-108=(x-3)(x+3)(x+4)(x^2+3)$$

bo'lib, $x^2+3=0$ haqiqiy ildizlarga ega emas.

Javob: $\{-4;\ -3;\ 3\}$.

11-misol. $4x^4+8x^3-x^2-8x-3=0$ tenglamaning barcha ratsional ildizlarini toping.

Yechish: Ozod had -3 ning bo'luvchilari $\pm1;\pm3$ va bosh koeffitsiyent 4 ning bo'luvchilari $\pm1;\pm2;\pm4$ dan iborat. 2-teoremaga asosan berilgan tenglamaning ratsional ildizlarini

$$\pm1;\pm3;\pm\frac{1}{2};\pm\frac{3}{2};\pm\frac{1}{4};\pm\frac{3}{4}$$

sonlari orasidan qidiramiz. Bevosita tekshirib ko'rish orqali $x=-1;\ x=1;\ \ x=-\frac{1}{2};\ x=-\frac{3}{2}$ sonlari tenglamaning ildizi ekanligiga ishonch hosil qilamiz.

Javob: $\left\{-\frac{3}{2};-\frac{1}{2};-1;1\right\}$.

Ba'zi tenglamalarni belgilash kiritish yordamida keltirilgan tenglamaga keltirib sodda usulda yechish mumkin.

12-misol. $4x^4-16x^3+11x^2+4x-3=0$ tenglamaning barcha ratsional ildizlarini toping.

Yechish: Berilgan tenglamaning ikkala tomonini 4 ga ko'paytiramiz:

$$16x^4-64x^3+44x^2+16x-12=0$$

$$(2x)^4-8\cdot(2x)^3+11\cdot(2x)^2+8\cdot(2x)-12=0$$

$2x=y$ deb belgilash kiritamiz va $y^4-8y^3+11y^2+8y-12=0$ keltirilgan tenglamani hosil qilamiz. 4-teoremaga asosan ildizlarini ozod had -12 ning bo‘luvchilari $\pm1; \pm2; \pm3; \pm4; \pm6; \pm12$ lar orasidan izlaymiz. Bevosita tekshirish natijasida $y_{1,2}=\pm1;\ y_3=2;\ y_4=6$ lar ildizi ekanligini topamiz. Bundan berilgan tenglamaning ildizlari $x_{1,2}=\pm\frac{1}{2};\ x_3=1;\ x_4=3$ bo‘ladi.

Javob: $\left\{\pm\frac{1}{2}; 1; 3\right\}$.

Mustaqil yechish uchun mashqlar

5.1-misol. Qoldiqlarni toping.

1. $p(x)$ ko‘phadni $x-2$ ga bo‘lgandagi qoldiq 5 ga, $x-3$ ga bo‘lgandagi qoldiq 7 ga teng. $p(x)$ ko‘phadni $(x-2)(x-3)$ ga bo‘lgandagi qoldiqni toping.

2. $p(x)=x^{100}-2x^{51}+1$ ko‘phadni x^2-1 ga bo‘lgandagi qoldiqni toping.

3. a va b ning qanday qiymatlarida $p(x)=ax^4+bx^3+1$ ko‘phad $(x-1)^2$ ga qoldiqsiz bo‘linadi.

4. p va q ning qanday qiymatlarida $6x^4-7x^3+px^2+3x+2$ ko‘phad x^2-x+q ga qoldiqsiz bo‘linadi.

5. $p(x)=x^{72}+1$ ko‘phadni $x^{16}-x^8+1$ ko‘phadga qoldiqsiz bo‘linishini isbotlang.

6. Biror ko‘phadni x^2+6x+8 kvadrat uchhadga bo‘lganda qoldiq $3x+2$ ga teng bo‘ladi. Shu ko‘phadni $x+2$ va $x+4$ ga bo‘lgandagi qoldiqni toping.

5.2-misol. Test topshiriqlari.

1. x^3+2nx^2+mx+5 ko‘phad x^2-1 ga qoldiqsiz bo‘linadi. $m+n$ ni toping.

$A)\ 5 \quad B)\ \frac{7}{2} \quad C)\ -\frac{7}{2} \quad D)\ -7 \quad E)\ -6$

2. mx^3+nx^2-x+2 ko'phad x^2-3x+2 ga qoldiqsiz bo'linadi. $m\cdot n$ ni toping.

$A)\ 2$ $B)-2$ $C)3$ $D)-1$

3. $P(x)=2x^3-5x^2+ax-b$ ko'phad $(x-1)^2$ ga qoldiqsiz bo'linsa, $a+b$ ni hisoblang.

$A)\ 5$ $B)\ 2{,}5$ $C)\ 7$ $D)\ 3$

4. $x^{2001}+3x^{2000}+3x+13$ ko'phadni $x+3$ ga bo'lgandagi qoldiqni toping.

$A)\ 4$ $B)\ 3$ $C)\ 5$ $D)\ 2$ $E)\ 1$

5. $P(x)=x^{2015}+4x^{2014}+5x+23$ ko'phadni $x+4$ bo'lgandagi qoldiqni toping.

$A)\ 4^{2014}+3$ $B)-2$ $C)\ 3$ $D)\ 43$

6. $P(x)=x^3+2x^2-x+2$ ko'phadni x^2+3 ga bo'lgandagi qoldiqni toping.

$A)-8$ $B)-8x-4$ $C)-4x-4$ $D)-2x+6$

7. $3x^5+6x^4+11x^3+4x^2+6x+4$ ni $3x^2+2$ ga bo'lgandagi qoldiqni toping.

$A)\ 6x+4$ $B)\ 4$ $C)\ 3x+4$ $D)\ 2$

8. $P(x)=2x^{13}+3x^8+3x^2-5x+3$ ko'phadni $x^3+\sqrt{2}$ ga bo'lgandagi qoldiqni toping.

$A)\ 9x^2+3x+3$ $B)\ 3x^2-13x+3$ $C)-5x+3$ $D)\ x^2-7x+4$

9. $P(x)=Q(x)\cdot(x^2-7x+6)+5x-4$ berilgan. $P(x)$ ko'phadni $x-6$ ga bo'lgandagi qoldiqni toping.

$A)$ Aniqlab bo'lmaydi $B)\ 34$ $C)\ 26$ $D)\ 30$

5.3-misol. Quyidagi tenglamalarning butun ildizlarini toping.

1. $x^3+2x^2-x-2=0$

2. $x^3-x^2-8x+12=0$

3. $x^3+6x^2-x-30=0$

4. $x^4+4x^3-7x^2-22x+24=0$

5. $x^5+3x^4-2x^3-9x^2-11x-6=0$

5.4-misol. Quyidagi tenglamalarning ratsional ildizlarini toping.

1. $3x^3 + x^2 + x - 2 = 0$
2. $2x^4 - x^3 + 2x^2 + 3x - 2 = 0$
3. $6x^4 + 7x^3 - 22x^2 - 28x - 8 = 0$
4. $2x^4 - 7x^3 - 3x^2 + 5x - 1 = 0$

6. Gorner sxemasi

$$p(x) = a_0x^n + a_1x^{n-1} + \cdots + a_{n-1}x + a_n, (a_0 \neq 0) \quad (1)$$

ko‘phad berilgan bo‘lsin. $p(x)$ ko‘phadni $(x-a)$ ikkihadga bo‘lishda Gorner sxemasi usulidan foydalaniladi. $p(x)$ ko‘phadni $(x-a)$ ikkihadga bo‘lganda $(n-1)$– darajali

$$q(x) = b_0x^{n-1} + b_1x^{n-2} + \ldots + b_{n-2}x + b_{n-1}, (b_0 \neq 0) \quad (2)$$

ko‘phad va r qoldiq hosil bo‘lsa,

$$p(x) = (x-a)q(x) + r \quad (3)$$

tenglikka ega bo‘lamiz. (1) va (2) ni (3) ga qo‘yamiz. Natijada

$$a_0x^n + a_1x^{n-1} + \cdots + a_{n-1}x + a_n = (x-a)(b_0x^{n-1} + b_1x^{n-2} + \ldots + b_{n-2}x + b_{n-1}) + r;$$

$$a_0x^n + a_1x^{n-1} + \cdots + a_{n-1}x + a_n = b_0x^n + b_1x^{n-1} + \ldots + b_{n-2}x^2 + b_{n-1}x - ab_0x^{n-1} -$$
$$-ab_1x^{n-2} - \ldots - ab_{n-2}x - ab_{n-1} + r$$

$$a_0x^n + a_1x^{n-1} + \cdots + a_{n-1}x + a_n = b_0x^n + (b_1 - ab_0)x^{n-1} + \ldots +$$
$$+(b_{n-1} - ab_{n-2})x + r - ab_{n-1}$$

Oxirgi tenglikning ikkala tomonidagi x ning bir xil darajalari oldidagi koeffitsiyentlarini tenglaymiz:

$a_0 = b_0$	$b_0 = a_0$
$a_1 = b_1 - ab_0$	$b_1 = ab_0 + a_1$
$a_2 = b_2 - ab_1$	$b_2 = ab_1 + a_2$
....	....
$a_{n-1} = b_{n-1} - ab_{n-2}$	$b_{n-1} = ab_{n-2} + a_{n-1}$
$a_n = r - ab_{n-1}$	$r = ab_{n-1} + a_n$

Mana shu usul yordamida bo'linma $q(x)$ ko'phadning $b_0, b_1, b_2,..., b_{n-1}$ koeffitsiyentlari va r qoldig'ini topishning Gorner sxemasi deb ataladi. Buni jadval ko'rinishida yozamiz:

	a_0	a_1	a_2	...	a_{n-1}	a_n
a	$b_0=a_0$	$b_1=ab_0+a_1$	$b_2=ab_1+a_2$	...	$b_{n-1}=ab_{n-2}+a_{n-1}$	$r=ab_{n-1}+a_n$

1-misol. $P(x)=4x^5-7x^4+5x^3-2x+1$ ko'phadni $x-3$ ga bo'ling.

Yechish: $P(x)=4x^5-7x^4+5x^3+0\cdot x^2-2x+1$ ko'rinishiga ega bo'lib, $a_0=4, a_1=-7, a_2=5, a_3=0, a_4=-2, a_5=1$ va $a=3$ ni aniqlaymiz.

$b_0=a_0=4, b_1=3\cdot4-7=5, b_2=3\cdot5+5=20, b_3=3\cdot20+0=60,$
$b_4=3\cdot60-2=178, r=3\cdot178+1=535$

Jadvalni to'ldiramiz:

	$a_0=4$	$a_1=-7$	$a_2=5$	$a_3=0$	$a_4=-2$	$a_5=1$
$a=3$	$b_0=4$	$b_1=5$	$b_2=20$	$b_3=60$	$b_4=178$	$r=535$

Bo'linma $q(x)=4x^4+5x^3+20x^2+60x+178$ va qoldiq $r=535$ ga teng.

$4x^5-7x^4+5x^3-2x+1=(x-3)(4x^4+5x^3+20x^2+60x+178)+535.$

2-misol. $p(x)=x^4-8x^3+15x^2+4x-20$ ko'phadni $x-2$ ikkihadga bo'lgandagi bo'linma va qoldiqni toping.

Yechish: $a_0=1, a_1=-8, a_2=15, a_3=4, a_4=-20$ va $a=2$.

$b_0=a_0=1, b_1=2\cdot1-8=-6, b_2=2\cdot(-6)+15=3,$
$b_3=2\cdot3+4=10, r=2\cdot10-20=0.$

Jadvalni to'ldiramiz:

	$a_0=1$	$a_1=-8$	$a_2=15$	$a_3=4$	$a_4=-20$
$a=2$	$b_0=1$	$b_1=-6$	$b_2=3$	$b_3=10$	$r=0$

Demak, qoldiq $r=0$ bo'lib, $p(x)$ ko'phad $x-2$ ikkihadga qoldiqsiz bo'linadi. Bunda bo'linma $q(x)=x^3-6x^2+3x+10$ ga teng.

$$P(x)=x^4-8x^3+15x^2+4x-20=(x-2)(x^3-6x^2+3x+10).$$

Bezu teoremasiga asosan $P(x)$ ko'phadni $x-a$ ikkihadga bo'lgandagi qoldiq $r=P(a)$ ga teng. Gorner sxemasi yordamida ko'phadning nuqtalardagi qiymatlarini ham topish mumkin.

3-misol. $P(x)=x^4+x^3-16x^2-4x+48$ ko'phadni $(x+4)\cdot(x-2)(x-3)$ ga qoldiqsiz bo'linishini isbotlang.

Yechish: Gorner sxemasi yordamida berilgan ko'phadni $x=-4$, $x=2$ va $x=3$ nuqtadagi qiymatlarini topamiz:

	1	1	−16	−4	48
−4	1	−3	−4	−12	0
2	1	3	−10	−24	0
3	1	4	−4	−16	0

Ko'rinib turibdiki, oxirgi ustundagi qoldiqlar 0 ga teng. Bundan berilgan ko'phad $(x+4)(x-2)(x-3)$ ga qoldiqsiz bo'linishi kelib chiqadi.

Mustaqil yechish uchun mashqlar

6.1-misol. Gorner sxemasidan foydalanib, $P(x)$ ko'phadni $a(x)$ ikkihadga bo'lgandagi to'liqsiz bo'linma va qoldiqni toping.

1. $P(x)=5x^3-26x^2+25x-4$; $a(x)=x-5$;
2. $P(x)=4x^3-x^2-27x-18$; $a(x)=x+2$;
3. $P(x)=x^4-3x^3+6x^2-10x+16$; $a(x)=x-4$;
4. $P(x)=x^5-2x^4+3x^3-7x^2+x-5$; $a(x)=x-2$;
5. $P(x)=x^6-4x^4+x^3-2x^2+5$; $a(x)=x+3$.

6.2-misol. Gorner sxemasidan foydalanib, $P(x)$ ko‘phadni $x=a$ nuqtadagi qiymatini toping.

1. $P(x)=x^3-6x^2+11x-6$, $a=2, a=3, a=-3$;

2. $P(x)=x^3-2x^2+7$, $a=4$;

3. $P(x)=x^4+2x^3-12x^2-38x-4$, $a=-4$;

4. $P(x)=2x^5+3x^4+6x^2-16$, $a=-2$;

5. $P(x)=x^5+x^4-6x^3-14x^2-11x-3$, $a=3$;

6. $P(x)=3x^6-5x^4-6x^3+x^2-1$, $a=2$;

7. $P(x)=x^7+2x^6+3x^5+3x^4+2x^3+x^2-x+3$, $a=4$;

7. Ko‘phadlarni ko‘paytuvchilarga ajratish

Ko‘phadlarni ko‘paytuvchilarga ajratishda bir necha usullardan foydalanish mumkin. Bular qisqa ko‘paytirish formulalari, ko‘phadni ildizini topish haqidagi teorema, ko‘phadni ko‘phadga bo‘lish, noma’lum koeffitsiyentlar va hokazo usullardan foydalaniladi.

1-misol. x^4+4 ni ko‘paytuvchilarga ajrating.

Yechish: $a^2-b^2=(a-b)(a+b)$ kvadratlar ayirmasi formulasini qo‘llaymiz:

$$x^4+4=(x^4+4x^2+4)-4x^2=(x^2+2)^2-(2x)^2=(x^2+2-2x)\cdot$$
$$\cdot(x^2+2+2x)=(x^2-2x+2)(x^2+2x+2).$$

Hosil bo‘lgan kvadrat uchhadlar chiziqli ko‘paytuvchilarga ajralmaydi, chunki ular haqiqiy ildizga ega emas.

Javob: $(x^2-2x+2)(x^2+2x+2)$.

2-misol. x^6+27 ni ko‘paytuvchilarga ajrating.

Yechish: $a^3+b^3=(a+b)(a^2-ab+b^2)$ kublar yig‘indisi formulasini qo‘llaymiz. Natijada

$$x^6+27=(x^2)^3+3^3=(x^2+3)(x^4-3x^2+9)=(x^2+3)\left[(x^4+6x^2+9)-9x^2\right]=$$
$$=(x^2+3)\left[(x^2+3)^2-(3x)^2\right]=(x^2+3)(x^2-3x+3)(x^2+3x+3).$$

Javob: $(x^2+3)(x^2-3x+3)(x^2+3x+3)$.

3-misol. $x^4+1982x^2+1981x+1982$ ni ko'paytuvchilarga ajrating.

Yechish: Quyidagicha shakl almashtirishlarni bajaramiz:

$$x^4+1982x^2+1981x+1982=x^4+x^3+x^2+1981(x^2+x+1)-(x^3-1)=$$
$$=x^2(x^2+x+1)+1981(x^2+x+1)-(x-1)(x^2+x+1)=$$
$$=(x^2+x+1)(x^2+1981-x+1)=(x^2+x+1)(x^2-x+1982).$$

Javob: $(x^2+x+1)(x^2-x+1982)$.

4-misol. x^5+x+1 ni ko'paytuvchilarga ajrating.

Yechish: $x^5+x+1=(x^5+x^4+x^3)-(x^4+x^3+x^2)+(x^2+x+1)=$
$=x^3(x^2+x+1)-x^2(x^2+x+1)+(x^2+x+1)=(x^2+x+1)(x^3-x^2+1)$.

Bu misolda berilgan ko'phaddan x^2 ni ayirib, qo'shsak ham shu natijaga kelamiz:

$$x^5+x+1=(x^5-x^2)+(x^2+x+1)=x^2(x^3-1)+(x^2+x+1)=$$
$$=x^2(x-1)(x^2+x+1)+(x^2+x+1)=(x^2+x+1)(x^2(x-1)+1)=$$
$$=(x^2+x+1)(x^3-x^2+1).$$

Javob: $(x^2+x+1)(x^3-x^2+1)$.

5-misol. x^8+x^7+1 ni ko'paytuvchilarga ajrating.

Yechish:

$$x^8+x^7+1=(x^8+x^7+x^6)-(x^6+x^5+x^4)+(x^5+x^4+x^3)-$$
$$-(x^3+x^2+x)+(x^2+x+1)=x^6(x^2+x+1)-x^4(x^2+x+1)+x^3\cdot$$
$$(x^2+x+1)-x(x^2+x+1)+(x^2+x+1)=(x^2+x+1)(x^6-x^4+x^3-x+1).$$

Javob: $(x^2+x+1)(x^6-x^4+x^3-x+1)$.

6-misol. $x^6+2x^5+9x^4+16x^3+24x^2+32x+16$ ko'phadni ko'paytuvchilarga ajrating.

Yechish: Quyidagicha shakl almashtiramiz:

$$x^6+2x^5+9x^4+16x^3+24x^2+32x+16=(x^6+2x^3\cdot x^2+x^4)+$$
$$(8x^4+16x^3+8x^2)+(16x^2+32x+16)=(x^3+x^2)^2+8x^2(x^2+$$
$$2x+1)+16(x^2+2x+1)=\left[x^2(x+1)\right]^2+8x^2(x+1)^2+16(x+1)^2=$$
$$(x+1)^2(x^4+8x^2+16)=(x+1)^2(x^2+4)^2=(x+1)^2(x^2-$$
$$-2x+2)^2(x^2+2x+2)^2.$$

Javob: $(x+1)^2(x^2-2x+2)^2(x^2+2x+2)^2$.

7-misol. $a^{10}+a^5+1$ ni ko'paytuvchilarga ajrating.

Yechish:

$$a^{10}+a^5+1=(a^{10}+a^9+a^8)-(a^9+a^8+a^7)+(a^7+a^6+a^5)-(a^6+a^5+a^4)+$$
$$+(a^5+a^4+a^3)-(a^3+a^2+a)+(a^2+a+1)=a^8(a^2+a+1)-a^7(a^2+a+1)+$$
$$+a^5(a^2+a+1)-a^4(a^2+a+1)+a^3(a^2+a+1)-a(a^2+a+1)+(a^2+a+1)=$$
$$=(a^2+a+1)(a^8-a^7+a^5-a^4+a^3-a+1).$$

Javob: $(a^2+a+1)(a^8-a^7+a^5-a^4+a^3-a+1)$.

8-misol. $x^3+9x^2+11x-21$ ni ko'paytuvchilarga ajrating.

Yechish: $x^3+9x^2+11x-21=x^3-x^2+10x^2-10x+21x-21=$

$$=x^2(x-1)+10x(x-1)+21(x-1)=(x-1)(x^2+10x+21)=$$
$$=(x-1)(x+3)(x+7).$$

Javob: $(x-1)(x+3)(x+7)$.

9-misol. $(x+1)(x+3)(x+5)(x+7)-1920$ ni ko'paytuvchilarga ajrating.

Yechish: Quyidagi shakl almashtirishni bajaramiz:

$$(x+1)(x+3)(x+5)(x+7)-1920=(x+1)(x+7)(x+3)(x+5)-1920=$$
$$=(x^2+8x+7)(x^2+8x+15)-1920.$$

$x^2+8x=y$ deb belgilash kiritamiz, natijada berilgan ko'phad kvadrat uchhad ko'rinishiga keladi:

$$(y+7)(y+15)-1920=y^2+22y+105-1920=y^2+22y-1815,$$

Viyet teoremasidan

$$\begin{cases} y_1+y_2=-22 \\ y_1\cdot y_2=-1815 \end{cases} \Rightarrow y_1=33;\ y_2=-55$$

ni topamiz.

Demak, $y^2+22y-1815=(y-33)(y+55)$ bo'lib, belgilashga qaytamiz.

Natijada, berilgan ko'phad

$$(x^2+8x-33)(x^2+8x+55)=(x-3)(x+11)(x^2+8x+55)$$

ko'rinishidagi ko'paytuvchilarga ajraladi.

Javob: $(x-3)(x+11)(x^2+8x+55)$.

10-misol. $(x+1)(x+2)(x+3)(x+4)+1$ to‘liq kvadrat ko‘rinishida tasvirlang.

Yechish: Berilgan ko‘phadni quyidagi ko‘rinishda yozib olamiz:

$$(x+1)(x+2)(x+3)(x+4)+1=(x+1)(x+4)(x+2)(x+3)+1=$$
$$=(x^2+5x+4)(x^2+5x+6)+1.$$

$x^2+5x=y$ belgilash kiritamiz va berilgan ko‘phad kvadrat uchhad ko‘rinishiga keladi va belgilashga qaytamiz:

$$(t+4)(t+6)+1=t^2+10t+25=(t+5)^2=(x^2+5x+5)^2.$$

Javob: $(x^2+5x+5)^2$.

11-misol. $P(x)=4(x+5)(x+6)(x+10)(x+12)-3x^2$ ni ko‘paytuvchilarga ajrating.

Yechish: $(x+5)(x+6)(x+10)(x+12)=(x+5)(x+12)(x+6)(x+10)=(x^2+17x+60)(x^2+16x+60)$. Bundan $x^2+16x+60=t$ deb belgilash kiritamiz. Natijada berilgan ko‘phad

$$P(x)=4(t+x)t-3x^2=4t^2+4tx+x^2-4x^2=$$
$$=(2t+x)^2-(2x)^2=(2t-x)(2t+3x)$$

ko‘rinishiga keladi. Belgilashga qaytamiz:

$$P(x)=(2(x^2+16x+60)-x)(2(x^2+16x+60)+3x) \text{ yoki}$$
$$P(x)=(2x^2+31x+60)(2x^2+35x+60).$$

Endi qavs ichidagi har bir kvadrat uchhadni ko‘paytuvchilarga ajratamiz:

1) $2x^2+31x+60$ da $D=961-960=1>0$ va $x_1=-8;\ x_2=-\frac{15}{2}$.

Bundan

$$2x^2+31x+60=2(x+8)\left(x+\frac{15}{2}\right).$$

2) $2x^2+35x+60$ da $D=1225-960=265>0$ va $x_{1,2}=\frac{-35\pm\sqrt{265}}{4}$.

Bundan

$$2x^2+31x+60=2\left(x+\frac{35+\sqrt{265}}{4}\right)\left(x+\frac{35-\sqrt{265}}{4}\right).$$

Demak,

$$P(x)=4(x+8)\left(x+\frac{15}{2}\right)\left(x+\frac{35-\sqrt{265}}{4}\right)\left(x+\frac{35+\sqrt{265}}{4}\right).$$

12-misol. $P(x)=x^4-6x^3+9x^2+4x-12$ ko'phadni ko'paytuvchilarga ajrating.

Yechish: Ozod had -12 ning butun bo'luvchilarini topamiz:

$$\pm 1;\ \pm 2;\ \pm 3;\ \pm 4;\ \pm 6;\ \pm 12.$$

Demak, $P(x)$ ko'phadning ildizlarini ozod hadning bo'luvchilari ichidan qidiramiz:

$$P(-1)=1+6+9-4-12=0$$

$$P(2)=16-6\cdot 8+9\cdot 4+4\cdot 2-12=0$$

$$P(3)=81-6\cdot 27+9\cdot 9+4\cdot 3-12=0$$

$x=-1;\ x=2;\ x=3$ sonlari ko'phadning ildizi ekan. Bundan berilgan ko'phad $(x+1)(x-2)(x-3)=(x+1)(x^2-5x+6)=$ $=x^3-4x^2+x+6$ ko'phadga qoldiqsiz bo'linishi kelib chiqadi.

$$\begin{array}{r|l} x^4-6x^3+9x^2+4x-12 & x^3-4x^2+x+6 \\ \cline{2-2} -\ \ x^4-4x^3+x^2+6x & \quad x-2 \\ \cline{1-1} -2x^3+8x^2-2x-12 & \\ -\ \ -2x^3+8x^2-2x-12 & \\ \cline{1-1} 0 & \end{array}$$

Demak, berilgan ko'phad $P(x)=(x+1)(x-2)(x-3)(x-2)=$ $=(x+1)(x-2)^2(x-3)$ ko'rinishida ko'paytuvchilarga ajralar ekan. Bu yerda $x=2$ ikki karrali ildiz ekanligi ma'lum bo'ldi.

Javob: $(x+1)(x-2)^2(x-3)$.

Mustaqil yechish uchun mashqlar

7.1-misol. Ko'paytuvchilarga ajrating.

1. x^3+3x^2-4 **2.** x^4+1

3. x^4+64 **4.** x^4-x^2+1

5. x^4-5x^2+4 **6.** x^4+5x^2+9

7. x^8+x^4+1 **8.** x^8+x+1

9. x^8+3x^4+1 **10.** $(x^2+x+1)(x^2+x+2)-12$

11. $(x+1)^4+2(x+1)^3+x(x+2)$ **12.** $(4x+1)(12x-1)(3x+2)(x+1)-4$

13. x^7+x^5+1 **14.** x^8-1

15. x^5+x-1.

8. Noma'lum koeffitsiyentlar usuli

Endi noma'lum koeffitsiyentlar usuli yordamida ko'phadlarni ko'paytuvchilarga ajratishga doir misollarni ko'rib o'tamiz.

1-misol. $P(x)=x^3+9x^2+23x+15$ ko'phadni ko'paytuvchilarga ajrating.

Yechish: $x=-1$ soni ko'phadning ildizi, chunki $P(-1)=0$. Bundan berilgan ko'phad $x+1$ ga qoldiqsiz bo'linadi. U holda bo'linma x^2+ax+b ko'rinishida bo'ladi, chunki x^3 oldidagi bosh koeffitsiyent 1 ga teng.

Demak,

$$P(x)=x^3+9x^2+23x+15=(x+1)(x^2+ax+b)=x^3+ax^2+$$
$$+bx+x^2+ax+b=x^3+(a+1)x^2+(a+b)x+b.$$
$$x^3+9x^2+23x+15=x^3+(a+1)x^2+(a+b)x+b.$$

Ko'phadlar teng bo'lishi uchun bir xil darajalar oldidagi koeffitsiyentlar teng bo'lishi lozim.

$$\begin{cases} a+1=9 \\ a+b=23 \\ b=15 \end{cases} \Rightarrow \begin{cases} a=8 \\ b=15 \end{cases}$$

Bundan $P(x)=(x+1)(x^2+8x+15)$ ko‘rinishga keladi. $x^2+8x+15$ kvadrat uchhadning ildizlari $x_1=-3; x_2=-5$ ni topib,

$$x^2+8x+15=(x+3)(x+5)$$

ni hosil qilamiz. Natijada $P(x)=(x+1)(x+3)(x+5)$ ko‘rinishidagi ko‘paytuvchilarga ajraladi.

Javob: $(x+1)(x+3)(x+5)$.

2-misol. $P(x)=3x^4+5x^3-x^2-5x-2$ ko‘phadni ko‘paytuvchilarga ajrating.

Yechish: $x=1$ soni ko‘phadning ildizi, chunki ko‘phadning koeffitsiyentlari yig‘indisi 0 ga teng. Bundan berilgan ko‘phad $x-1$ ga qoldiqsiz bo‘linadi. U holda bo‘linma $3x^3+ax^2+bx+c$ ko‘rinishida bo‘ladi, chunki x^4 oldidagi bosh koeffitsiyent 3 ga teng. Demak,

$$P(x)=3x^4+5x^3-x^2-5x-2=(x-1)(3x^3+ax^2+bx+c)=3x^4+ax^3+$$
$$+bx^2+cx-3x^3-ax^2-bx-c=3x^4+(a-3)x^3+(b-a)x^2+(c-b)x-c.$$
$$3x^4+5x^3-x^2-5x-2=3x^4+(a-3)x^3+(b-a)x^2+(c-b)x-c.$$

Bir xil darajalar oldidagi koeffitsiyentlarni tenglaymiz:

$$\begin{cases} a-3=5 \\ b-a=-1 \\ c-b=-5 \\ -c=-2 \end{cases} \Rightarrow \begin{cases} a=8 \\ b=7 \\ c=2 \end{cases}$$

Bundan $P(x)=(x-1)(3x^3+8x^2+7x+2)$ ko‘rinishiga keladi. $x=-1$ soni $3x^3+8x^2+7x+2$ ko‘phadning ildizi, bu ko‘phadga noma'lum koeffitsiyentlar usulini yana bir marta qo‘llaymiz:

$$3x^3+8x^2+7x+2=(x+1)(3x^2+mx+n)=3x^3+mx^2+$$
$$+nx+3x^2+mx+n=3x^3+(m+3)x^2+(m+n)x+n.$$

$$\begin{cases} m+3=8 \\ m+n=7 \\ n=2 \end{cases} \Rightarrow \begin{cases} m=5 \\ n=2. \end{cases}$$

Demak, $3x^3+8x^2+7x+2=(x+1)(3x^2+5x+2)$ o‘rinli.

$3x^2+5x+2$ kvadrat uchhadning ildizlari $x_1=-1; x_2=-\frac{2}{3}$ ni topib,

$$3x^2+5x+2=3(x+1)\left(x+\frac{2}{3}\right)=(x+1)(3x+2)$$

ni hosil qilamiz.

Natijada

$3x^3+8x^2+7x+2=(x+1)(x+1)(3x+2)=(x+1)^2(3x+2)$ va $P(x)$ ko‘phad $P(x)=(x-1)(3x+2)(x+1)^2$ ko‘rinishidagi ko‘paytuvchilarga ajraladi.

Javob: $(x-1)(3x+2)(x+1)^2$.

Ba’zi ko‘rinishdagi to‘rtinchi darajali ko‘phadlarni quyidagi ayniyat $(x^2+ax+1)(x^2-ax+1)=x^4+(2-a^2)x+1$ yordamida juda sodda usulda ko‘paytuvchilarga ajratish mumkin.

Masalan, $a=1$ da $x^4+x^2+1=(x^2+x+1)(x^2-x+1)$;

$a=\sqrt{3}$ da $x^4-x^2+1=(x^2+x\sqrt{3}+1)(x^2-x\sqrt{3}+1)$.

3-misol. a ning qanday butun qiymatlarida $P(x)=x^4-3x^3+ax^2-9x-2$ ko‘phadni ikkita butun koeffitsiyentli kvadrat uchhad ko‘paytmasi ko‘rinishida tasvirlash mumkin.

Yechish: Ikkita kvadrat uchhadning x^2 hadi oldidagi koeffitsiyentlari 1 ga teng, chunki berilgan ko‘phadning x^4 darajasi oldidagi bosh koeffitsiyenti ham 1 ga teng. Ozod hadlari ko‘paytmasi -2 ga teng bo‘lib, kvadrat uchhadlarning ozod hadlari 2 va -1 yoki 1 va -2 ga teng bo‘ladi.

Ikkita holni ko‘rib o‘tamiz:

1) Kvadrat uchhadlarning ozod hadlari 2 va -1 bo‘lsin. U holda

$$x^4-3x^3+ax^2-9x-2=(x^2+px+2)(x^2+qx-1)$$

o‘rinli va $p,q\in Z$. Tenglikning o‘ng tomonidagi qavslarni ochib chiqamiz:

$$x^4-3x^3+ax^2-9x-2=x^4+(p+q)x^3+(1+pq)x^2+(2q-p)x-2.$$

Mos darajalar oldidagi koeffitsiyentlarni tenglaymiz:

$$\begin{cases} p+q=-3 \\ 1+pq=a \\ 2q-p=-9 \end{cases} \Rightarrow \begin{cases} p+q=-3 \\ 1+pq=a \\ 3q=-12 \end{cases} \Rightarrow \begin{cases} p=1 \\ q=-4 \\ a=-3 \end{cases}$$

2) Kvadrat uchhadlarning ozod hadlari 1 va -2 bo'lsin. U holda

$$x^4-3x^3+ax^2-9x-2=(x^2+px+1)(x^2+qx-2)$$

o'rinli va $p,q\in Z$. Tenglikning o'ng tomonidagi qavslarni ochib chiqamiz:

$$x^4-3x^3+ax^2-9x-2=x^4+(p+q)x^3+(pq-1)x^2+(q-2p)x-2.$$

Mos darajalar oldidagi koeffitsiyentlarni tenglaymiz:

$$\begin{cases} p+q=-3 \\ pq-1=a \\ q-2p=-9 \end{cases} \Rightarrow \begin{cases} p+q=-3 \\ pq-1=a \\ 3p=6 \end{cases} \Rightarrow \begin{cases} p=2 \\ q=-5 \\ a=-11. \end{cases}$$

Javob: $a=-3$; $a=-11$.

4-misol. Qaysi ko'phadni kubga ko'targanda

$$x^6+9x^5+30x^4+45x^3+30x^2+9x+1$$

ko'phad hosil bo'ladi?

Yechish: Berilgan ko'phadning bosh koeffitsiyenti (x^6 oldidagi koeffitsiyent) 1 ga teng. Shuning uchun berilgan ko'phadni

$$x^6+9x^5+30x^4+45x^3+30x^2+9x+1=(x^2+px+q)^3$$

ko'rinishida tasvirlaymiz.

Tenglikni o'ng tomonini kubga oshirib, keyin mos darajalar oldidagi koeffitsiyentlarni tenglab, p va q larni topsak, ishimiz qiyinlashadi.

Tenglikdagi o'zgaruvchilarga hususiy qiymatlar berib, noma'lum koeffitsiyentlarni aniqlaymiz:

$x=0$ da

$$0^6+9\cdot 0^5+30\cdot 0^4+45\cdot 0^3+30\cdot 0^2+9\cdot 0+1=$$
$$=(0^2+p\cdot 0+q)^3 \Rightarrow q^3=1 \Rightarrow q=1,$$

$x=1$ da

$$1^6+9\cdot 1^5+30\cdot 1^4+45\cdot 1^3+30\cdot 1^2+9\cdot 1+1=(1^2+p\cdot 1+q)^3 \Rightarrow (1+p+q)^3=125$$

ni va $1+p+q=5 \Rightarrow p+q=4 \Rightarrow p+1=4 \Rightarrow p=3$ ni topamiz.

Demak, berilgan ko'phad,

$$x^6+9x^5+30x^4+45x^3+30x^2+9x+1=(x^2+3x+1)^3$$

ga teng.

Javob: $(x^2+3x+1)^3$.

5-misol. $x^4+x^3-5x^2+5x+12$ ko'phadlarni ikkita butun koeffitsiyentli kvadrat uchhad ko'paytmasi ko'rinishida tasvirlang.

Yechish: $x^4+x^3+5x^2+5x+12$ ko'phadni ozod hadi 12 ga teng. 12 sonini $12=3\cdot 4=-3\cdot(-4)=2\cdot 6=-2\cdot(-6)$ ko'rinishida tasvirlash mumkin. Bundan berilgan ko'phadni $12=3\cdot 4$ ko'paytma uchun ko'raylik:

$$x^4+x^3+5x^2+5x+12=(x^2+px+3)(x^2+qx+4)$$

$$x^4+x^3+5x^2+5x+12=(x^2+px+3)(x^2+qx+4)=$$

$$=x^4+qx^3+4x^2+px^3+pqx^2+4px+3x^2+3qx+12=$$

$$=x^4+(p+q)x^3+(pq+7)x^2+(4p+3q)x+12.$$

$$x^4+x^3+5x^2+5x+12=x^4+(p+q)x^3+$$

$$+(pq+7)x^2+(4p+3q)x+12.$$

$$\begin{cases} p+q=1 \\ pq+7=5 \\ 4p+3q=5 \end{cases} \Rightarrow \begin{cases} p+q=1 \\ pq+7=0 \\ q=-1 \end{cases} \Rightarrow \begin{cases} p=2 \\ q=-1 \end{cases}$$

$$x^4+x^3+5x^2+5x+12=(x^2+2x+3)(x^2-x+4).$$

Bevosita tekshirish orqali $12=-3\cdot(-4)=2\cdot 6=-2\cdot(-6)$ ko'paytma uchun tenglik o'rinli emas.

Javob: $(x^2+2x+3)(x^2-x+4)$.

6-misol. a va b ning qanday qiymatlarida x^4+ax^3+ $+bx^2-8x+1$ ko'phad biror kvadrat uchhadning kvadratidan iborat bo'ladi?

Yechish: Berilgan ko‘phadning bosh koeffitsiyenti 1 ga teng bo‘lib, hosil bo‘lgan kvadrat uchhadning x^2 hadi oldidagi koeffitsiyent ham 1 ga teng bo‘ladi. Berilgan ko‘phadning ozod hadi 1 ga teng. 1 sonini $1=1^2=(-1)^2$ ko‘rinishida tasvirlash mumkin. Bundan berilgan ko‘phadni ikkita holda kvadrat uchhadning kvadrati ko‘rinishida tasvirlash mumkin:

$$x^4+ax^3+bx^2-8x+1=(x^2+px+1)^2$$
$$x^4+ax^3+bx^2-8x+1=(x^2+px-1)^2$$

1-hol.

$$x^4+ax^3+bx^2-8x+1=(x^2+px+1)^2=(x^2+px)^2+2(x^2+px)+1=$$
$$=x^4+2px^3+p^2x^2+2x^2+2px+1=x^4+2px^3+(p^2+2)x^2+2px+1.$$
$$x^4+ax^3+bx^2-8x+1=x^4+2px^3+(p^2+2)x^2+2px+1$$

Bundan

$$\begin{cases}2p=a\\ p^2+2=b\\ 2p=-8\end{cases}\Rightarrow\begin{cases}p=-4\\ a=-8\\ b=18\end{cases}$$

2-hol.

$$x^4+ax^3+bx^2-8x+1=(x^2+px-1)^2=(x^2+px)^2-2(x^2+px)+1=$$
$$=x^4+2px^3+p^2x^2-2x^2-2px+1=x^4+2px^3+(p^2-2)x^2-2px+1.$$
$$x^4+ax^3+bx^2-8x+1=x^4+2px^3+(p^2-2)x^2-2px+1$$

Bundan

$$\begin{cases}2p=a\\ p^2-2=b\\ -2p=-8\end{cases}\Rightarrow\begin{cases}p=4\\ a=8\\ b=14\end{cases}$$

2-usul. Bu misolni umumiy holda yechamiz.

$x^4+ax^3+bx^2-8x+1=(x^2+cx+d)^2$ deb olamiz. Bundan

$$x^4+ax^3+bx^2-8x+1=(x^2+cx)^2+2d(x^2+cx)+d^2=x^4+2cx^3+$$
$$+c^2x^2+2dx^2+2cdx+d^2=x^4+2cx^3+(c^2+2d)x^2+2cdx+d^2$$

$$\begin{cases} 2c = a \\ c^2 + 2d = b \\ 2cd = -8 \\ d^2 = 1 \end{cases} \Rightarrow \begin{cases} d = \pm 1 \\ cd = -4 \\ a = 2c \\ b = c^2 + 2d \end{cases} \Rightarrow \begin{cases} d_1 = -1 \\ c_1 = 4 \\ a_1 = 8 \\ b_1 = 14 \end{cases} \text{yoki} \begin{cases} d_2 = 1 \\ c_2 = -4 \\ a_2 = -8 \\ b_2 = 18 \end{cases}$$

Javob: $a = 8; b = 14$ yoki $a = -8; b = 18$.

Mustaqil yechish uchun mashqlar

8.1-misol. Noma'lum koeffitsiyentlar usuli yordamida ko'phadlarni ko'paytuvchilarga ajrating.

1. $P(x) = 2x^3 - x^2 - 5x - 2$

2. $P(x) = x^4 - 9x^3 + 30x^2 - 44x + 24$

8.2-misol. $x^4 - 2x^3 - 2x + 15$ ko'phadni ikkita butun koeffitsiyentli kvadrat uchhad ko'paytmasi ko'rinishida tasvirlang.

8.3-misol. a va b ning qanday qiymatlarida ko'phad biror kvadrat uchhadning kvadratidan iborat bo'ladi?

1. $x^4 + 24x^3 + ax^2 + 1992x + b$ **2.** $x^4 + x^3 + 2x^2 + ax + b$

8.4-misol. a, b va c ning qanday qiymatlarida $x^3 + ax^2 + bx + a$ ko'phad $x + c$ ikkihadning kubiga teng bo'ladi?

9. Ko'phadning ildizlari

Ko'phadlarning ildizlarini topish bo'yicha quyidagi tasdiqlar o'rinli:

1-tasdiq. Agar ratsional koeffitsiyentli $P(x)$ ko'phad uchun $P(a + b\sqrt{c}) = p + q\sqrt{c}$ o'rinli bo'lsa, u holda $P(a - b\sqrt{c}) = p - q\sqrt{c}$ bo'lishini isbotlang.

Bu yerda $a, b, c, p, q \in Q$ va $c > 0$, $\sqrt{c}$ – irratsional son.

Isbot. Matematik induksiya usulida isbotlaymiz.

$P(x)=a_0x^n+a_1x^{n-1}+a_2x^{n-2}+\ldots+a_{n-1}x+a_n$ ko'phad berilgan bo'lsin. Biz bu xossani $x^n(n\in N)$ uchun isbotlasak yetarli. $n=1$ da yuqoridagi tasdiq bajarilishi ko'rinib turibdi.

Bu tasdiq $n=2$ da ham o'rinli:

$$(a+b\sqrt{c})^2=(a^2+b^2c)+2ab\sqrt{c}$$

$$(a-b\sqrt{c})^2=(a^2+b^2c)-2ab\sqrt{c}$$

Faraz qilaylik, bu tasdiq $n=k$ uchun o'rinli bo'lsin, ya'ni

$$(a+b\sqrt{c})^k=p+q\sqrt{c}\Rightarrow(a-b\sqrt{c})^k=p-q\sqrt{c}.$$

U holda $n=k+1$ uchun

$$(a+b\sqrt{c})^{k+1}=(a+b\sqrt{c})^k\cdot(a+b\sqrt{c})=(p+q\sqrt{c})(a+b\sqrt{c})=$$
$$=(ap+bcq)+(bp+aq)\sqrt{c},$$

$$(a-b\sqrt{c})^{k+1}=(a-b\sqrt{c})^k\cdot(a-b\sqrt{c})=(p-q\sqrt{c})(a-b\sqrt{c})=$$
$$=(ap+bcq)-(bp+aq)\sqrt{c}$$

o'rinli. Bundan yuqoridagi tasdiq $\forall n\in N$ uchun o'rinli ekanligi kelib chiqadi.

2-tasdiq. Agar $x=a+b\sqrt{c}$ soni ratsional koeffitsiyentli $P(x)$ ko'phadning ildizi bo'lsa, u holda $x=a-b\sqrt{c}$ soni ham $P(x)$ ko'phadning ildizi bo'ladi.

Bu yerda $a,b,c,p,q\in Q$ va $c>0$, $\sqrt{c}$ – irratsional son.

Isbot. 1-tasdiqqa asosan $P(a+b\sqrt{c})=p+q\sqrt{c}\Rightarrow P(a-b\sqrt{c})=$ $=p-q\sqrt{c}$.

Ikkinchi tomondan $P(a+b\sqrt{c})=p+q\sqrt{c}=0$ bo'lib, tenglik faqat $p=q=0$ bo'lganda o'rinli. U holda $P(a-b\sqrt{c})=$ $=p-q\sqrt{c}=0-0\cdot\sqrt{c}=0$ bo'ladi. Bundan $x=a-b\sqrt{c}$ ham $P(x)$ ko'phadning ildizi bo'lishi kelib chiqadi.

1-misol. $P(x)=x^3+8x^2+ax+b$ ko'phadning ildizlaridan biri $2-\sqrt{5}$ ga teng bo'lsa, tenglamaning qolgan ildizlari va a, b ni aniqlang.

Yechish: **1-usul.** $x=2-\sqrt{5}$ ko'phadning ildizi bo'lganligi uchun ko'phadning shu nuqtadagi qiymati 0 ga teng bo'ladi:

$$(2-\sqrt{5})^3+8\cdot(2-\sqrt{5})^2+a\cdot(2-\sqrt{5})+b=0$$

$$8-12\sqrt{5}+30-5\sqrt{5}+72-32\sqrt{5}+2a-a\sqrt{5}+b=0$$

$$(-a-49)\sqrt{5}+(2a+b+110)=0$$

Oxirgi tenglik bajarilishi uchun

$$\begin{cases} -a-49=0 \\ 2a+b+110=0 \end{cases} \Rightarrow \begin{cases} a=-49 \\ b=-12 \end{cases}$$

o'rinli bo'ladi. Bundan $P(x)=x^3+8x^2-49x-12$ ko'rinishga ega. $x_1=2-\sqrt{5}$ ko'phadning ildizi va yuqoridagi 2-tasdiqqa asosan $x_2=2+\sqrt{5}$ ham ko'phadning ildizi bo'ladi.

Demak, berilgan ko'phad

$$(x-x_1)(x-x_2)=(x-2+\sqrt{5})(x-2-\sqrt{5})=(x-2)^2-5=x^2-4x-1$$

kvadrat uchhadga qoldiqsiz bo'linadi.

$$\begin{array}{r|l} x^3+8x^2-49x-12 & x^2-4x-1 \\ \cline{2-2} -\ \underline{x^3-4x^2-x\quad\quad} & x+12 \\ 12x^2-48x-12 & \\ -\ \underline{12x^2-48x-12} & \\ 0 & \end{array}$$

Bundan $P(x)=(x+12)(x^2-4x-1)$ bo'lib, uchinchi ildizi $x_3=-12$ ga teng.

2-usul. $x_1=2-\sqrt{5}$ ko'phadning ildizi va yuqoridagi 2-tasdiqqa asosan $x_2=2+\sqrt{5}$ ham ko'phadning ildizi bo'ladi. U holda berilgan ko'phad

$$(x-x_1)(x-x_2)=(x-2+\sqrt{5})(x-2-\sqrt{5})=(x-2)^2-5=x^2-4x-1$$

kvadrat uchhadga qoldiqsiz bo'linadi.

$$\begin{array}{r|l} -\begin{array}{l} x^3+8x^2+ax+b \\ x^3-4x^2-x \end{array} & \begin{array}{l} x^2-4x-1 \\ \hline x+12 \end{array} \\ \hline -\begin{array}{l} 12x^2+(a+1)x+b \\ 12x^2-48x-12 \end{array} & \\ \hline (a+49)x+b+12 & \end{array}$$

$P(x)=(x+12)(x^2-4x-1)+(a+49)x+b+12$ bo'lib, qoldiq 0 ga teng bo'lishi lozim:

$$\begin{cases} a+49=0 \\ b+12=0 \end{cases} \Rightarrow \begin{cases} a=-49 \\ b=-12 \end{cases}$$

Demak, $P(x)=(x+12)(x^2-4x-1)$ bo'lib, $x_3=-12$ ga teng.

Javob: $x_{1,2}=2\pm\sqrt{5}$, $x_3=-12$; $a=-49, b=-12$.

2-misol. Koeffitsiyentlari butun sonlardan iborat bo'lgan

$$P(x)=x^4+ax^3+bx^2+6x+2$$

ko'phadning ildizlaridan biri $1+\sqrt{3}$ ga teng bo'lsa, tenglamaning qolgan ildizlari va a, b ni aniqlang.

Yechish: 2-tasdiqqa asosan berilgan tenglamaning $x_1=1+\sqrt{3}$ ildizi bo'lsa, $x_2=1-\sqrt{3}$ ham ildizi bo'ladi. U holda berilgan ko'phad

$$(x-x_1)(x-x_2)=(x-1-\sqrt{3})(x-1+\sqrt{3})=(x-1)^2-3=x^2-2x-2$$

ga qoldiqsiz bo'linadi.

$P(x)$ ko'phadni x^2-2x-2 ko'phadga bo'lamiz:

$$\begin{array}{l|l} -\begin{array}{l} x^4+ax^3+bx^2+6x+2 \\ x^4-2x^3-2x^2 \end{array} & \begin{array}{l} x^2-2x-2 \\ \hline x^2+(a+2)x+2a+b+6 \end{array} \\ \hline -\begin{array}{l} (a+2)x^3+(b+2)x^2+6x+2 \\ (a+2)x^3-2(a+2)x^2-2(a+2)x \end{array} & \\ \hline -\begin{array}{l} (2a+b+6)x^2+(2a+10)x+2 \\ (2a+b+6)x^2-2(2a+b+6)x-2(2a+b+6) \end{array} & \\ \hline (6a+2b+22)x+2(2a+b+7) & \end{array}$$

$$P(x)=x^4+ax^3+bx^2+6x+2=(x^2-2x-2)(x^2+(a+2)x+$$
$$+2a+b+6)+(6a+2b+22)x+2(2a+b+7)$$

Demak, qoldiq $(6a+2b+22)x+2(2a+b+7)=0$ bo‘lishi kerak. Bu faqat

$$\begin{cases}6a+2b+22=0\\2a+b+7=0\end{cases}\Rightarrow\begin{cases}b=1\\a=-4\end{cases}$$

bo‘lsa o‘rinli. Bundan a va b ni qiymatlarini qo‘ysak

$$P(x)=(x^2-2x-2)(x^2-2x-1)$$

ga teng. x^2-2x-1 kvadrat uchhadning ildizlari $x=1\pm\sqrt{2}$ bo‘ladi.

Bu misolni noma’lum koeffitsiyentlar usulida ham yechsa bo‘ladi.

Yuqorida $P(x)=x^4+ax^3+bx^2+6x+2$ ko‘phadni x^2-2x-2 kvadrat uchhadga bo‘linishini ko‘rdik. U holda

$$x^4+ax^3+bx^2+6x+2=(x^2-2x-2)(x^2+cx-1)$$
$$x^4+ax^3+bx^2+6x+2=x^4+(c-2)x^3-(2c+3)x^2+2(1-c)x+2$$

$$\begin{cases}c-2=a\\-(2c+3)=b\\2(1-c)=6\end{cases}\Rightarrow\begin{cases}a=-4\\b=1\\c=-2.\end{cases}$$

Natijada $P(x)=x^4-4x^3+x^2+6x+2=(x^2-2x-2)(x^2-2x-1)$ bo‘ladi.

Javob: $x_{1,2}=1\pm\sqrt{3}$; $x_{3,4}=1\pm\sqrt{2}$; $a=-4, b=1$.

3-misol. Ildizlaridan biri 1) $2-\sqrt{6}$ 2) $\sqrt{5}+\sqrt{6}$ 3) $\sqrt[3]{6}$ 4) $1-\sqrt[3]{4}$ 5) $\sqrt[4]{3}-2$

bo‘lgan eng kichik darajali butun koeffitsiyentli ko‘phadni toping.

Yechish: 1) Izlalanayotgan ko‘phadning ildizi $x=2-\sqrt{6}$ bo‘lsin. U holda

$x-2=-\sqrt{6}\Rightarrow(x-2)^2=(-\sqrt{6})^2\Rightarrow x^2-4x+4=6\Rightarrow x^2-4x-2=0.$

2) $x=\sqrt{5}+\sqrt{6}\Rightarrow x^2=(\sqrt{5}+\sqrt{6})^2\Rightarrow x^2=11+2\sqrt{30}\Rightarrow(x^2-11)^2=$

$$=(2\sqrt{30})^2\Rightarrow x^4-22x^2+121=120\Rightarrow x^4-22x^2+1=0.$$

3) $x=\sqrt[3]{6} \Rightarrow x^3=6 \Rightarrow x^3-6=0$.

4) $x=1-\sqrt[3]{4} \Rightarrow (x-1)^3=(-\sqrt[3]{4})^3 \Rightarrow x^3-3x^2+3x+3=0$.

5) $x=\sqrt[4]{3}-2 \Rightarrow (x+2)^4=(\sqrt[4]{3})^4 \Rightarrow (x+2)^4-3=0$.

3-tasdiq. Butun koeffitsiyentli $P(x)=a_0x^n+a_1x^{n-1}+a_2x^{n-2}+$ $+\ldots+a_{n-1}x+a_n$ ko'phad berilgan bo'lsin. U holda $\forall c,d\in Z$ sonlari uchun $P(c)-P(d)$ butun son $c-d$ soniga bo'linadi.

Isbot: Bu tasdiqni to'g'riligini isbotlaymiz:

$$P(c)-P(d)=(a_0c^n+a_1c^{n-1}+a_2c^{n-2}+\ldots+a_{n-1}c+a_n)-(a_0d^n+a_1d^{n-1}+$$

$$+a_2d^{n-2}+\ldots+a_{n-1}d+a_n)=a_0(c^n-d^n)+a_1(c^{n-1}-d^{n-1})+\cdots+a_{n-1}(c-d).$$

Ixtiyoriy $k\in N$ uchun quyidagi formula o'rinli:

$$c^k-d^k=(c-d)(c^{k-1}+c^{k-2}d+\cdots+cd^{k-2}+d^{k-1}).$$

Shuning uchun $P(c)-P(d)$ ifodaning o'ng tomonidagi har bir qo'shiluvchi $c-d$ ga bo'linadi va bundan yig'indi ham $c-d$ ga bo'linishi kelib chiqadi.

4-misol. $P(7)=11$, $P(11)=13$ shartni qanoatlantiruvchi butun koeffitsiyentli $P(x)$ ko'phad mavjudmi?

Yechish: 3 – tasdiqqa asosan $P(11)-P(7)=13-11=2$ soni 11–7=4 soniga bo'linishi kerak. 2 soni 4 ga bo'linmaydi va bunday ko'phad mavjud emas.

Javob: mavjud emas.

5-misol. $P(0)=19$, $P(1)=85$, $P(2)=1985$ shartni qanoatlantiruvchi butun koeffitsiyentli $P(x)$ ko'phad mavjudmi?

Yechish: $P(x)$ ko'phadni $P(x)=ax(x-1)+bx+c$ ko'rinishida izlasak qulay bo'ladi. Bunda $x=0, x=1$ va $x=2$ qiymatlarni qo'yamiz va

$$\begin{cases} P(0)=19 \\ P(1)=85 \\ P(2)=1985 \end{cases} \Rightarrow \begin{cases} c=19 \\ b+c=85 \\ 2a+2b+c=1985 \end{cases} \Rightarrow \begin{cases} c=19 \\ b=66 \\ a=917 \end{cases}$$

ni topamiz. Bundan

$$P(x)=917x(x-1)+66x+19=917x^2-851x+19.$$

Javob: $P(x)=917x^2-851x+19$.

Umumiy holda $n+1$ ta $x=c_1,c_2,\dots,c_{n+1}$ nuqtalarda ma'lum qiymatlarni qabul qiluvchi darajasi n dan oshmaydigan ko'phadni

$$P(x)=b_0+b_1(x-c_1)+b_2(x-c_1)(x-c_2)+$$
$$+\dots+b_n(x-c_1)(x-c_2)\dots(x-c_{n+1})$$

ko'rinishida qidirish maqsadga muvofiq bo'ladi. Bunda $x=c_1,c_2,\dots,c_{n+1}$ ni ko'phadga qo'ysak, $n+1$ ta $b_0,b_1,\dots,b_n$ noma'lumli tenglamalar sistemasi hosil bo'ladi va $P(x)$ ko'phadning noma'lum koeffitsiyentlari aniqlanadi.

Mustaqil yechish uchun mashqlar

9.1-misol. Ildizlaridan biri 1) $3-\sqrt{5}$ 2) $\sqrt{2}+\sqrt{3}$ 3) $\sqrt[3]{5}$ 4) $2-\sqrt[3]{3}$ 5) $\sqrt[4]{2}-1$ bo'lgan eng kichik darajali butun koeffitsiyentli ko'phadni toping.

9.2-misol. $3-\sqrt{2}$ soni $x^4-7x^3+ax^2+bx-14$ ko'phadning ildizi bo'lsa, qolgan ildizlarini toping.

9.3-misol. $1+\sqrt{2}$ soni $x^5+ax^3+bx^2+5x+2$ ko'phadning ildizi bo'lsa, qolgan ildizlarini toping.

9.4-misol. a va b sonlari $P(x)=x^3+px+q$ ko'phadning ildizlari bo'lsin. Agar $a+b+ab=0$ bo'lsa, ko'phadning uchinchi ildizini toping.

9.5-misol. Quyidagi shartlarni qanoatlantiruvchi butun koeffitsiyentli ko'phad mavjudmi?

1) $P(1)=11,\ P(19)=85$ 2) $P(4)=18,\ P(12)=66$

9.6-misol. Quyidagi shartlarni qanoatlantiruvchi butun koeffitsiyentli ko'phadni toping.

1. $P(0)=1,\ P(1)=2,\ P(2)=6$
2. $P(0)=0,\ P(1)=3,\ P(3)=33,\ P(4)=72$

10. $f(f(x))=x$ ko‘rinishidagi tenglamalar

Ko‘pincha $f(f(x))=x$ ko‘rinishiga keladigan murakkab tenglamalarni yechishga to‘g‘ri keladi. Bunda $f(f(x))=x$ tenglamani yechish, soddaroq $f(x)=x$ tenglamani yechishga keladi.

1-tasdiq. $f(x)=x$ tenglamaning ixtiyoriy ildizi $f(f(x))=x$ tenglamaning ildizi bo‘ladi.

Isbot. x_0 soni $f(x)=x$ tenglamaning ildizi bo‘lsa, u holda $f(x_0)=x_0$ tenglik o‘rinli bo‘ladi. Bundan $f(f(x_0))=$ $=f(x_0)=x_0$, ya’ni $f(f(x_0))=x_0$. Bu x_0 soni $f(f(x))=x$ tenglamaning ildizi ekanligini bildiradi. Bu turdagi tenglamalarni yechishda quyidagi teorema o‘rinli.

2-tasdiq. Agar $y=f(x)$ qat‘iy o‘suvchi funksiya bo‘lsa, u holda

$$f(x)=x \quad (1) \text{ va } f(f(x))=x \qquad (2)$$

tenglamalar ekvivalent bo‘ladi.

Isbot. (2) tenglama (1) tenglamaning natijasi ekanligi ravshan. Chunki, agar (1) tenglamaning x_0 ixtiyoriy ildizi bo‘lsa, $f(x_0)=x_0$ bo‘ladi, u holda $f(f(x_0))=f(x_0)=x_0$ o‘rinli. Endi (2) tenglamaning ixtiyoriy ildizi (1) tenglamaning ham ildizi ekanligini isbotlaymiz. x_0 ni shunday olaylikki, $f(f(x_0))=x_0$ bo‘lsin. Faraz qilaylik, $f(f(x_0))\neq x_0$, aniqlik uchun $f(x_0)>x_0$ bo‘lsin. U holda $f(f(x_0))>f(x_0)>x_0$ bo‘ladi, bu $f(f(x_0))=x_0$ shartga zid. Isbot tugadi.

Izoh. Agar $y=f(x)$ qat‘iy o‘suvchi funksiya bo‘lsa, u holda $\forall k\in N$ uchun $\underbrace{f(f(\ldots f(x)\ldots)}_{k}$ va $f(x)=x$ tenglamalar ekvivalent bo‘ladi.

1-misol. $\left(\dfrac{x^3+6}{7}\right)^3=7x-6$ tenglamani yeching.

Yechish: Berilgan tenglamani $\dfrac{\left(\dfrac{x^3+6}{7}\right)^3+6}{7}=x$ (3) ko'rinishida yozib olamiz. $f(x)=\dfrac{x^3+6}{7}$ funksiya $\forall x\in R$ da qat'iy o'suvchi. Bundan (3) tenglama $f(f(x))=x$ ko'rinishiga ega bo'ladi. Yuqoridagi 2-tasdiqqa asosan $f(f(x))=x \Leftrightarrow f(x)=x$ ni hosil qilamiz. Natijada

$$\frac{x^3+6}{7}=x \Rightarrow x^3-7x+6=0 \Rightarrow x_1=1;\ x_2=2;\ x_3=-3.$$

Javob: $\{1;\ 2;-3\}$.

Endi $f(f(x))=x$ tenglama berilgan va $f(x)$ funksiya qat'iy o'suvchi bo'lmagan holda yechish usulini ko'rib o'tamiz.

2-misol. $(x^2+3x-2)^2+3\cdot(x^2+3x-2)-2=x$ tenglamani yeching.

Yechish: Agar $f(x)=x^2+3x-2$ deb olsak, berilgan tenglamani $f(f(x))=x$ ko'rinishida yozish mumkin. $f(x)$ funksiya R da aniqlangan, lekin qat'iy o'suvchi emas. Bundan yuqoridagi 1-tasdiqqa asosan $f(x)=x$ tenglamani yechamiz. Natijada

$$x^2+3x-2=x \Rightarrow x^2+2x-2=0 \Rightarrow x_{1,2}=-1\pm\sqrt{3}.$$

Berilgan tenglamani

$$(x^2+3x-2)^2+3\cdot(x^2+3x-2)-x-2=0 \quad (4)$$

ko'rinishida yozib olamiz.

(4) tenglamaning chap qismi to'rtinchi darajali ko'phad bo'lib, uning boshqa ildizlarini ham topishga harakat qilamiz.

Demak, $x_1=-1-\sqrt{3}$ va $x_2=-1+\sqrt{3}$ ildizlar (4) tenglamaning ildizi bo'ladi va tenglamaning chap qismi

$$(x-x_1)(x-x_2)=(x+1+\sqrt{3})(x+1-\sqrt{3})=(x+1)^2-3=x^2+2x-2$$

ga qoldiqsiz bo'linadi. Qavslarni ochamiz:

$$(x^2+3x-2)^2+3\cdot(x^2+3x-2)-x-2=0$$

$$(x^2+3x-2)(x^2+3x+1)-x-2=0$$

$$x^4+3x^3+x^2+3x^3+9x^2+3x-2x^2-6x-2-x-2=0$$

$$x^4+6x^3+8x^2-4x-4=0$$

$$\begin{array}{r|l}
x^4+6x^3+8x^2-4x-4 & x^2+2x-2 \\ \cline{2-2}
-\ x^4+2x^3-2x^2 \qquad\qquad & x^2+4x+2 \\ \cline{1-1}
4x^3+10x^2-4x-4 & \\
-\ 4x^3+8x^2-8x \qquad & \\ \cline{1-1}
2x^2+4x-4 & \\
-\ 2x^2+4x-4 & \\ \cline{1-1}
0 \qquad\quad &
\end{array}$$

Bundan $x^4+6x^3+8x^2-4x-4=(x^2+2x-2)(x^2+4x+2)$ bo‘lib, x^2+4x+2 kvadrat uchhadning ildizlari $x_{3,4}=-2\pm\sqrt{2}$ bo‘ladi.

Javob: $\{-1\pm\sqrt{3};-2\pm\sqrt{2}\}$.

Mustaqil yechish uchun mashqlar

10.1-misol. Tenglamalarni yeching.

1. $(x^2+2x-5)^2+2\cdot(x^2+2x-5)-5=x$

2. $(x^3+6)^3+6=x$

3. $\sqrt[3]{\sqrt[3]{x+24}+24}=x$

4. $(x^2-11x+11)^2-11\cdot(x^2-11x+11)+11=x$

5. $(x^2-2001x+2001)^2-2001\cdot(x^2-2001x+2001)+2001=x$.

11. Ko‘phadlar mavzusiga doir test topshiriqlarini yechish

1. Ko‘paytuvchilarga ajratish

1-misol. $(a+b+c+2)(a+b+c)-(1-a-b-c)^2+1$ ni ko‘paytuvchilarga ajrating.

$A)\ 4(a+b+c)^2$ $\quad B)\ 4(a+b+c)$

$C)\ (a+b+c+1)(a-b+c-1)$ $\quad D)(a+b+c+1)(a+b+c-1)$

Yechish:

$$(a+b+c+2)(a+b+c)-(1-a-b-c)^2+1=(a+b+c)^2+2(a+b+c)+$$
$$+1-(1-a-b-c)^2=(a+b+c+1)^2-(1-a-b-c)^2=$$
$$=(a+b+c+1-1+a+b+c)(a+b+c+1+1-a-b-c)=4(a+b+c)$$

Javob: $B)\ 4(a+b+c)$.

2-misol. $(3z-x)^3+(x-2y)^3-(3z-2y)^3$ ko'phadni ko'paytuvchilarga ajrating.

$A)\ 3(3z-x)(x-2y)(3z-2y)$ $\quad B)$ Ko'paytuvchilarga ajralmaydi

$C)\ -3(3z-2y)(3z-x)(x-2y)$ $\quad D)\ -6(3z-2y)(3z-x)(x-2y)$

Yechish: $a^3+b^3+c^3=(a+b+c)^3-3(a+b)(b+c)(a+c)$ ayniyatni qo'llaymiz.

$$(3z-x)^3+(x-2y)^3-(3z-2y)^3=(3z-x)^3+(x-2y)^3+(2y-3z)^3=$$
$$=(3z-x+x-2y+2y-3z)^3-3(3z-x+x-2y)(x-2y+2y-3z)\cdot$$
$$\cdot(3z-x+2y-3z)=-3(3z-2y)(x-3z)(2y-x)=-3(3z-2y)\cdot$$
$$\cdot[-(3z-x)][-(x-2y)]=-3(3z-2y)(3z-x)(x-2y).$$

Javob: $C)\ -3(3z-2y)(3z-x)(x-2y)$.

3-misol. $x(y^2-z^2)+y(z^2-x^2)+z(x^2-y^2)$ ni ko'paytuvchilarga ajrating.

$A)\ (2x-y)(3z-2x)$ $\quad B)\ (y-x-z)(x+y+z)(xy-z)$

$C)\ (x-1)(y-2)(z-3)(xyz-5)$ $\quad D)(x-y)(y-z)(z-x)$

Yechish: $z=y,\ y=x,\ z=x$ larda ko'phad nolga aylanadi. Bundan berilgan ko'phad $(x-y)(x-z)(y-z)$ ga qoldiqsiz bo'linishi kelib chiqadi.

$$x(y^2-z^2)+y(z^2-x^2)+z(x^2-y^2)=k(x-y)(x-z)(y-z)$$
$$xy^2-xz^2+yz^2-yx^2+zx^2-zy^2=k(x^2y-x^2z+xz^2-y^2x+$$
$$+zy^2-z^2y)\Rightarrow k=-1$$
$$x(y^2-z^2)+y(z^2-x^2)+z(x^2-y^2)=(x-y)(z-x)(y-z)$$

Javob: $D)(x-y)(y-z)(z-x)$.

4-misol. $(x+y+z)^3-x^3-y^3-z^3$ ni ko'paytuvchilarga ajrating.

$A)\ -3(x+y)(y+z)(x+z)$ $\quad B)\ -3(x-y)(y-z)(x-z)$

$C)\ 3(x-y)(y-z)(x-z)$ $\quad D)\ 3(x+y)(y+z)(x+z)$

Yechish:

$$(x+y+z)^3-x^3-y^3-z^3=(x+y)^3+3z(x+y)^2+3z^2(x+y)+z^3-$$
$$-x^3-y^3-z^3=x^3+y^3+3xy(x+y)+3z(x+y)^2+3z^2(x+y)-x^3-y^3=$$
$$=3xy(x+y)+3z(x+y)^2+3z^2(x+y)=3(x+y)(xy+z(x+y)+z^2)=$$
$$=3(x+y)\left[xy+xz+yz+z^2\right]=3(x+y)\left[x(y+z)+z(y+z)\right]=$$
$$=3(x+y)(y+z)(x+z).$$

Javob: $D)\ 3(x+y)(y+z)(x+z)$.

5-misol. x^6-a^6 ifodani ko'paytuvchilarga ajratganda nechta ratsional ko'paytuvchilardan iborat bo'ladi?

$A)\ 4$ $\quad B)\ 6$ $\quad C)\ 2$ $\quad D)5$

Yechish:

$$x^6-a^6=(x^3)^2-(a^3)^2=(x^3-a^3)(x^3+a^3)=$$
$$=(x-a)(x^2+ax+a^2)(x+a)(x^2-ax+a^2)$$

Javob: $A)\ 4$.

6-misol. $a(b^2-c^2)+b(c^2-a^2)+c(a^2-b^2)$ ni ko'paytuvchilarga ajrating.

$A)\ (a-b)(b-c)(c-a)$ $\quad B)\ (a-b)(b-c)(a-c)$

$C)\ (a-b)(b-c)(a+c)$ $\quad D)\ (a-b)(b+c)(c-a)$

Yechish:

$$a(b^2-c^2)+b(c^2-a^2)+c(a^2-b^2)=ab^2-ac^2+bc^2-a^2b+c(a^2-b^2)=$$
$$=(ab^2-a^2b)-(ac^2-bc^2)+c(a^2-b^2)=-ab(a-b)-c^2(a-b)+c(a-b)\cdot$$
$$\cdot(a+b)=(a-b)\left[-ab-c^2+c(a+b)\right]=(a-b)\left[-ab-c^2+ac+bc\right]=$$
$$=(a-b)\left[(-ab+bc)+(ac-c^2)\right]=(a-b)\left[-b(a-c)+c(a-c)\right]=$$
$$=(a-b)(a-c)(c-b)=(a-b)(b-c)(c-a).$$

Javob: $A)\ (a-b)(b-c)(c-a)$.

7-misol. m ning qanday qiymatlarida $x(x+3a)(x+b)\cdot$ $\cdot(x+3a+b)+\dfrac{9m^2}{16}$ ifoda to'la kvadrat bo'ladi?

$A)\ \pm\frac{ab}{2}$ $B)\ \pm 2ab$ $C)$ To'g'ri javob keltirilmagan $D)\ \frac{16}{9}a^2b^2$

Yechish: $x(x+3a)(x+b)(x+3a+b)+\frac{9m^2}{16}=x(x+3a+b)(x+3a)\cdot$

$$(x+b)+\frac{9m^2}{16}=(x^2+(3a+b)x)(x^2+(3a+b)x+3ab)+\frac{9m^2}{16}=$$

$$=t(t+3ab)+\frac{9m^2}{16}=t^2+3abt+\frac{9m^2}{16}=\left(t+\frac{3ab}{2}\right)^2-\frac{9a^2b^2}{4}+\frac{9m^2}{16}=$$

$$=\left(x^2+(3a+b)x+\frac{3ab}{2}\right)^2+\frac{9m^2}{16}-\frac{9a^2b^2}{4}.$$

Oxirgi ifoda to'liq kvadrat bo'lishi uchun $\frac{9m^2}{4}-\frac{9a^2b^2}{4}=0$ bo'lishi kerak. Bundan

$$\frac{9m^2}{16}-\frac{9a^2b^2}{4}=0\Rightarrow m^2=4a^2b^2\Rightarrow m=\pm 2ab.$$

Biz yuqorida $x^2+(3a+b)x=t$ belgilashdan foydalandik.

Javob: $B)\ \pm 2ab$.

8-misol. Agar $x=\frac{1+\sqrt{17}}{2}$ bo'lsa, $\frac{x^3-3x^2+8x-2}{x^2-x+1}$ kasrning qiymatini toping.

$A)\ \sqrt{17}+3$ $B)\ \sqrt{17}-1$ $C)\ \sqrt{17}$ $D)\ \sqrt{17}+2$

Yechish: $x^2-x+1=\left(x-\frac{1}{2}\right)^2+1-\frac{1}{4}=\left(\frac{1+\sqrt{17}}{2}-\frac{1}{2}\right)^2+\frac{3}{4}=\frac{17}{4}+\frac{3}{4}=5.$

$$\begin{array}{r|l} x^3-3x^2+8x-2 & x^2-x+1 \\ -\ \underline{x^3-x^2+x} & x-2 \\ -2x^2+7x-2 & \\ -\ \underline{-2x^2+2x-2} & \\ 5x & \end{array}$$

$$\frac{x^3-3x^2+8x-2}{x^2-x+1}=x-2+\frac{5x}{x^2-x+1}=x-2+\frac{5x}{5}=2x-2=2\cdot\frac{1+\sqrt{17}}{2}-2=\sqrt{17}-1.$$

Javob: $B)\ \sqrt{17}-1$.

2. Kasrlarni qisqartirish.

1-misol. $\dfrac{2a^4+a^3+4a^2+a+2}{2a^3-a^2+a-2}$ kasrni qisqartiring.

$A)\ \dfrac{a^2+2}{a+1}$ $B)\ \dfrac{a^2+1}{a+1}$ $C)\ \dfrac{a^2+2}{a-1}$ $D)\ \dfrac{a^2+1}{a-1}$

Yechish: $2a^3-a^2+a-2$ ko'phadni ratsional ildizlarini ozod hadning bo'luvchilari orasidan, ya'ni $\pm1;\pm2;\pm\dfrac{1}{2}$ lardan izlaymiz. Ko'rinib turibdiki, $a=1$ ko'phadning ildizi va berilgan ko'phad $a-1$ ga qoldiqsiz bo'linadi.

$$\begin{array}{r|l} 2a^3-a^2+a-2 & a-1 \\ \underline{2a^3-2a^2} & \overline{2a^2+a+2} \\ a^2+a-2 & \\ \underline{a^2+a} & \\ 2a-2 & \\ \underline{2a-2} & \\ 0 & \end{array}$$

$$2a^3-a^2+a-2=(a-1)(2a^2+a+2)$$

$2a^4+a^3+4a^2+a+2$ ko'phadni noma'lum koeffitsiyentlar usulida ko'paytuvchilarga ajratamiz. Bosh koeffitsiyent 2 ga teng.

$$2a^4+a^3+4a^2+a+2=(2a^2+xa+y)(a^2+za+t)$$

$$2a^4+a^3+4a^2+a+2=2a^4+2za^3+2ta^2+xa^3+xza^2+xta+ya^2+yza+yt$$

$$2a^4+a^3+4a^2+a+2=2a^4+(2z+x)a^3+(2t+xz+y)a^2+(xt+yz)a+yt$$

Mos koeffitsiyentlarni tenglaymiz:

$$\begin{cases} 2z+x=1 \\ 2t+xz+y=4 \\ xt+yz=1 \\ yt=2 \end{cases} \Rightarrow \begin{cases} x=1 \\ y=2 \\ z=0 \\ t=1 \end{cases}$$

Demak, $2a^4+a^3+4a^2+a+2=(2a^2+a+2)(a^2+1)$ o'rinli.

$$\frac{2a^4+a^3+4a^2+a+2}{2a^3-a^2+a-2}=\frac{(2a^2+a+2)(a^2+1)}{(a-1)(2a^2+a+2)}=\frac{a^2+1}{a-1}.$$

Javob: $D)\ \frac{a^2+1}{a-1}$.

2-misol. $\frac{x^4+1}{x^2-x\sqrt{2}+1}$ ni qisqartiring.

$A)\ x^2-1$ $B)\ x^2+1$ $C)\ x^2+x\sqrt{2}+1$ $D)\ x^2-x\sqrt{2}+1$

Yechish:

$$x^4+1=(x^2+1)^2-2x^2=(x^2+1)^2-(x\sqrt{2})^2=(x^2-x\sqrt{2}+1)(x^2+x\sqrt{2}+1).$$

$$\frac{x^4+1}{x^2-x\sqrt{2}+1}=\frac{(x^2-x\sqrt{2}+1)(x^2+x\sqrt{2}+1)}{x^2-x\sqrt{2}+1}=x^2+x\sqrt{2}+1.$$

Javob: $C)\ x^2+x\sqrt{2}+1$.

Mustaqil yechish uchun mashqlar

11.1-misol. Ko‘paytuvchilarga ajrating.

1. $(a+b+2)(a+b)-(a-b)^2+1$ ifodani ko‘paytuvchilarga ajrating.

$A)\ (a+b)(2a-1)$ $B)\ 2b(a+1)$

$C)\ (a+1)(2b-1)$ $D)\ (2b+1)(2a+1)$

2. $(a+b+c)^2+(a+b-c)^2+(a-b+c)^2+(b+c-a)^2$ ifodani soddalashtiring.

$A)\ 2(a^2+b^2+c^2)$ $B)\ 3(a^2+b^2+c^2)$

$C)\ 8(a^2+b^2+c^2)$ $D)\ 4(a^2+b^2+c^2)$

3. $(x-2y)^3-(3z-2y)^3-(x-3z)^3$ ko‘phadni ko‘paytuvchilarga ajrating.

$A)$ Ko'paytuvchilarga ajralmaydi $B)\ 3(x-3z)(x-2y)(3z-2y)$

$C)\ -3(x-2y)(3z-2y)(x-3z)$ $D)\ 6(x-2y)(3z-2y)(x-3z)$

4. $(2y-3z)^3-(x-3z)^3-(2y-x)^3$ ko‘phadni ko‘paytuvchilarga ajrating.

$A)\ 6(2y-x)(2y-3z)(x-3z)$ $B)\ -3(2y-3z)(x-3z)(2y-x)$

$C)\ 3(2y-x)(2y-3z)(x-3z)$ $D)$ Ko'paytuvchilarga ajralmaydi

5. $(a-b)^3+(b-c)^3+(c-a)^3$ ni ko'paytuvchilarga ajrating.

$A)\ 3(a-b)(b-c)(c-a)$ $B)\ 3(a-b)(b-c)(a-c)$

$C)\ -3(a-b)(b-c)(c-a)$ $A)\ 3(a-b)(b+c)(c+a)$

6. $ab(b-a)+bc(b+c)-ac(a+c)$ ni ko'paytuvchilarga ajrating.

$A)\ (b-c)(b-a)(a+c)$ $B)\ (b+c)(b-a)(a+c)$

$C)\ (b+c)(b-a)(a-c)$ $D)\ (b-c)(b-a)(a-c)$

7. $(ab+ac+bc)(a+b+c)-abc$ ni ko'paytuvchilarga ajrating.

$A)\ (a-b)(b+c)(a+c)$ $B)\ (a+b)(b-c)(a+c)$

$C)\ (a-b)(b-c)(a-c)$ $D)\ (a+b)(b+c)(a+c)$

8. m ning qanday qiymatlarida $x(x+a)(x+4b)(x+a+4b)+$ $+100m^2$ ifoda to'la kvadrat bo'ladi?

$A)\ \pm 5ab$ $B)$ To'g'ri javob keltirilmagan $C)\ \pm\frac{ab}{5}$ $D)\ \frac{a^2b^2}{100}$

9. m ning qanday qiymatlarida $x(x+5a)(x+2b)(x+5a+2b)+$ $+25m^2$ ifoda to'la kvadrat bo'ladi?

$A)\ \pm ab$ $B)$ To'g'ri javob keltirilmagan $C)\ \frac{a^2b^2}{100}$ $D)\ \pm\frac{ab}{5}$

10. Agar $x^3-x+3=0$ bo'lsa, $(x^3-x+1)\cdot(x^3+3)$ ning qiymatini toping.

$A)\ 2x$ $B)\ 0$ $C)\ -4x$ $D)\ -2x$

11. Agar $x=\frac{1+\sqrt{17}}{2}$ bo'lsa, $\frac{x^3-2x^2+7x-1}{x^2-x+1}$ kasrning qiymatini toping.

$A)\ \sqrt{17}$ $B)\ \sqrt{17}+1$ $C)\ \sqrt{17}+2$ $D)\ \sqrt{17}+3$

12. Agar $a^3+a-2=0$ bo'lsa, $\frac{a^4+a^3+a^2+9}{a^3+a^2+a+6}$ ifodaning qiymatini toping.

$A)\ -2$ $B)\ 1$ $C)\ \frac{4}{3}$ $D)\ \frac{12}{11}$

11.2-misol. Kasrlarni qisqartiring.

1. Kasrni qisqartiring: $\frac{a^3c-2a^2c^2+ac^3-ab^2c}{(a^2+c^2-b^2)^2-4a^2c^2}$.

$A)\ \dfrac{ac}{(a+c)^2-b^2}$ $B)\ \dfrac{ac+b}{(a-c)^2-b^2}$ $C)\ \dfrac{ac(a+b-c)}{(a+c)^2+b^2}$ $D)\ \dfrac{ac}{(a+c)^2+b^2}$

2. $\dfrac{x^3+5x^2+3x-9}{x^3+x^2-5x+3}$ kasrni qisqartiring.

$A)\ \dfrac{x+3}{x-1}$ $B)\ \dfrac{x-3}{x+1}$ $C)\ \dfrac{x+3}{x+1}$ $D)\ \dfrac{x-3}{x-1}$

3. $\dfrac{32-6x^2+x^3}{x^2-8x+16}$ ni soddalashtiring.

$A)\ 1-x$ $B)\ x+2$ $C)\ 3x-2$ $D)\ \dfrac{x-1}{x+1}$

4. $\dfrac{p^3+4p^2+10p+12}{p^3-p^2+2p+16}\cdot\dfrac{p^3-3p^2+8p}{p^2+2p+6}$ ifodani soddalashtiring.

$A)\ \dfrac{p+1}{p-1}$ $B)\ p$ $C)\ -\dfrac{1}{p}$ $D)\ \dfrac{p+2}{p+1}$

5. $\dfrac{a^3-2a^2+5a+26}{a^3-5a^2+17a-13}$ kasrni qisqartiring.

$A)\ \dfrac{a+2}{a-2}$ $B)\ \dfrac{a-2}{a+2}$ $C)\ \dfrac{1-a}{a+2}$ $D)\ \dfrac{a+2}{a-1}$

6. $\dfrac{x^3+1}{x^4+x^2+1}$ kasrni qisqartiring.

$A)\ \dfrac{x+1}{x^2+x+1}$ $B)\ \dfrac{x-1}{x^2-x+1}$ $C)\ \dfrac{x+1}{x^2-x+1}$ $D)\ \dfrac{x-2}{x^2-x-1}$

7. $\dfrac{x^{33}-1}{x^{11}+x^{22}+x^{33}}$ ni qisqartiring.

$A)\ \dfrac{x^{11}-1}{x^{11}}$ $B)\ x^{11}+1$ $C)\ x^{11}-1$ $D)\ 1+\dfrac{1}{x^{11}}$

11.3-misol. Ko‘phadlarga oid misollarni yeching.

1. $P(x)=(x^2-3x+n)^3$ ko‘phadning koeffitsiyentlari yig‘indisi 64 ga teng bo‘lsa, n ni toping.

$A)\ 2$ $B)\ 6$ $C)\ 8$ $D)\ 4$

2. $x^4+mx^3+nx^2+8x+3$ ko‘phad $x-1$ ga qoldiqsiz bo‘linsa, $m+n$ ning qiymatini aniqlang.

$A)\ -13$ $B)\ -12$ $C)\ 13$ $D)\ 12$

3. $x^3 - ax^2 + 20x - 12 = 0$ tenglama ildizlaridan biri 2 ga teng. Uning qolgan ildizlari yig'indisini toping.

$A)\ 7 \quad B)\ 5 \quad C)\ -5 \quad D)\ 9$

4. $ax^3 + bx^2 + cx + d = (x+4)(x+5)(2x-3)$, $a+b+c+d=?$

$A)\ 24 \quad B)\ -30 \quad C)\ 30 \quad D)\ -18$

5. $3ax - 6x^2 - 8 + x^3$ ko'phad to'la kub bo'ladigan barcha a larni toping.

$A)\ 4 \quad B)\ 3 \quad C)\ -4 \quad D)\ -2$

6. $x^4 + 8x^3 + ax^2 + bx + 1$ ko'phad biror ko'phadning kvadrati bo'lsa, a va b koeffitsiyentlarning barcha qiymatlari yig'indisini toping.

$A)\ 27 \quad B)\ 26 \quad C)\ 48 \quad D)\ 32$

7. $P(x)$ va $Q(x)$ ko'phadlarni $x+2$ bo'lganda qoldiq mos ravishda 3 va -2 bo'lsa, $P(x+3)-(x+3)Q(x+3)$ ko'phadni $x+5$ ga bo'lgandagi qoldiqni toping.

$A)\ -2 \quad B)\ 1 \quad C)\ -1 \quad D)\ 0$

8. $\dfrac{P(x-4)}{Q(x-3)} = x^2 - 5x + 10$ munosabat berilgan. $P(x)$ ko'phadning ozod hadi 18 ga teng bo'lsa, $Q(x)$ ko'phadning koeffitsiyentlari yig'indisini toping.

$A)\ 2 \quad B)\ 4 \quad C)\ 5 \quad D)\ 3$

7-§. FUNKSIONAL TENGLAMALAR

1. Funksiya. Funksional bog'lanishlar

Dastlab funksiya tushunchasini ko'rib o'taylik.

Ta'rif. Agar X to'plamdan olingan har bir x elementga ($x \in X$) biror qoida yoki qonunga ko'ra Y to'plamdan bitta y element ($y \in Y$) mos qo'yilgan bo'lsa, u holda X to'plamda y *funksiya berilgan*(*aniqlangan*) deb ataladi va $f: x \to y$ yoki $y = f(x)$ kabi belgilanadi.

Bunda $x-$ *argument* yoki *erkli o'zgaruvchi*, $y-$ *erksiz o'zgaruvchi* yoki *funksiya* deb ataladi.

Ta'rif. O'zgaruvchi x ning $f(x)$ funksiya ma'noga ega bo'ladigan qiymatlari to'plami funksiyaning *aniqlanish sohasi* deyiladi va $D(f)$ harfi bilan belgilanadi.

Ta'rif. Funksiyaning qabul qiladigan qiymatlari to'plami *o'zgarish sohasi* (*qiymatlar to'plami*) deyiladi va $E(f)$ harfi bilan belgilanadi.

$$D(f) = X\ ,\ E(f) = Y = \{y: y = f(x),\ x \in X\}$$

Agar $x = x_0$ bo'lganda $y = f(x)$ funksiyaning qiymati y_0 bo'lsa, bu $y_0 = f(x_0)$ kabi belgilanadi.

1-misol. Agar $f(x) = \left(1 + \frac{1}{x}\right) \cdot (7 + 4x)$ bo'lsa, $f\left(-\frac{1}{2}\right)$ ni toping.

$A)\ 9 \qquad B)\ -3 \qquad C)\ -5 \qquad D)\ 15$

Yechish: Funksiyaning berilishidagi argument x lar o'rniga $-\frac{1}{2}$ ni qo'yib, funksiyaning shu nuqtadagi qiymatini topamiz:

$$f\left(-\frac{1}{2}\right) = \left(1 + \frac{1}{-1/2}\right)\left(7 + 4 \cdot \left(-\frac{1}{2}\right)\right) = (1-2)(7-2) = -5.$$

Javob: $C)\ -5$.

2-misol. Agar $f(x)=x^2-8x+8$ bo'lsa, $f(4-\sqrt{11})$ ni hisoblang.

$A)\ 2+\sqrt{11}$ $B)\ 5-\sqrt{11}$ $C)\ 2$ $D)\ 3$

Yechish: Funksiyani berilgan nuqtadagi qiymatini bevosita qo'yib topish ham mumkin. Lekin biz juda sodda usuldan foydalanib topsak, maqsadga tez erishamiz.

$x=4-\sqrt{11}\Rightarrow 4-x=\sqrt{11}\Rightarrow (4-x)^2=11\Rightarrow 16-8x+x^2=11\Rightarrow x^2-8x=-5.$

Bundan $f(4-\sqrt{11})=-5+8=3.$

Bu misolni to'liq kvadratga ajratish usuli bilan ham yechsa bo'ladi:

$f(x)=x^2-8x+8=(x-4)^2-8\Rightarrow f(4-\sqrt{11})=(4-\sqrt{11}-4)^2-8=11-8=3.$

Javob: $D)\ 3$.

3-misol. Agar $f(x)=\begin{cases}|x+1|, & x>-2\\ 3-4|x|, & x\le -2\end{cases}$ bo'lsa, $f(-1)-f(-\frac{9}{4})$ ni hisoblang.

$A)\ 9$ $B)\ 0$ $C)\ 6$ $D)\ 4$

Yechish: $f(-1)$ qiymat $f(x)$ funksiyaning $x=-1>-2$ nuqtadagi qiymati bo'lib, funksiya berilishining 1-sharti $f(x)=|x+1|,\ x>-2$ ga to'g'ri keladi va $f(-1)=|-1+1|=0$ ga teng.

$f(-\frac{9}{4})$ qiymat esa $f(x)$ funksiyaning $x=-\frac{9}{4}\le -2$ nuqtadagi qiymati bo'lib, funksiya berilishining 2-sharti $f(x)=3-4|x|$, $x\le -2$ ga to'g'ri keladi va $f(-\frac{9}{4})=3-4\cdot\left|-\frac{9}{4}\right|=3-9=-6$ ga teng. Natijada

$$f(-1)-f(-\frac{9}{4})=0-(-6)=6.$$

Javob: $C)\ 6$.

Funksiyalar kompozitsiyasi. Murakkab funksiya

Endi funksiyalarning kompozitsiyasi tushunchasini ko'rib o'tamiz.

X to'plamni Y to'plamga akslantiruvchi $y=g(x)$ funksiya va Y to'plamni Z to'plamga akslantiruvchi $F=f(y)$ funksiyalar berilgan bo'lsin.

Ta'rif. f va g funksiyalarning kompozitsiyasi yoki murakkab funksiya deb, X to'plamni Z to'plamga akslantiruvchi $F=f(g(x))$ funksiyaga aytiladi.

f va g funksiyalarning kompozitsiyasini $f\circ g$ ko'rinishida yozish qabul qilingan.

Masalan, $f\circ f$ kompozitsiya $y=f(f(x))$ funksiyani, $f\circ f\circ f$ esa $y=f(f(f(x)))$ funksiyani anglatadi.

4-misol. $\varphi(x)$ funksiya berilgan bo'lsa, $\varphi(\varphi(x))$ funksiyani aniqlang.

$$1)\ \varphi(x)=x^3 \qquad 2)\ \varphi(x)=\frac{3x+2}{2x+1} \qquad 3)\ \varphi(x)=\sqrt[5]{x}$$

Yechish: 1) $\varphi(\varphi(x))=(x^3)^3=x^9$;

$$2)\ \varphi(\varphi(x))=\frac{3\cdot\dfrac{3x+2}{2x+1}+2}{2\cdot\dfrac{3x+2}{2x+1}+1}=\frac{\dfrac{9x+6+4x+2}{2x+1}}{\dfrac{6x+4+2x+1}{2x+1}}=\frac{13x+6}{8x+5};$$

$$3)\ \varphi(\varphi(x))=\sqrt[5]{\sqrt[5]{x}}=\sqrt[25]{x}.$$

$f\circ f$ kompozitsiyani f^2 ko'rinishida yozish qabul qilingan. Xuddi shunday

$$f^3=f^2\circ f=(f\circ f)\circ f=f(f(f(x)))$$
$$f^4=f^3\circ f=((f\circ f)\circ f)\circ f=f(f(f(f(x))))\ \ldots$$

5-misol. $f(x)=\dfrac{x}{x-1}$ bo'lsa, $\underbrace{f(\ldots f(f(x))\ldots)}_{100\ marta}$ ni toping.

Yechish: $f^2 = f(f(x)) = \dfrac{\dfrac{x}{x-1}}{\dfrac{x}{x-1} - 1} = \dfrac{\dfrac{x}{x-1}}{\dfrac{1}{x-1}} = x$;

$$f^3 = f(f(f(x))) = \frac{x}{x-1};\ f^4 = f(f(f(f(x)))) = x;\ \ldots$$

Ko'rinib turibdiki, agar kompozitsiyalar soni juft bo'lsa, $f^{2n} = x$, $n \in N$ bo'ladi. Agar kompozitsiyalar soni toq bo'lsa, $f^{2n-1} = \dfrac{x}{x-1}$, $n \in N$ teng bo'lar ekan. Bundan

$$\underbrace{f(\ldots f(f(x))\ldots)}_{100\ marta} = x.$$

Javob: x.

6-misol. $f(x) = \begin{cases} 2x^2 + 1, |x| < 3 \\ 5x - 1, |x| \geq 3 \end{cases}$ funksiya berilgan. $f(x^2 + 7)$ ni toping.

$A)\ 5x^2 + 34$ $B)\ 2x^2 + 15$ $C)\ 2(x^2 + 7)^2 + 1$ $D)\ 5x^2 - 34$

Yechish: $f(x^2 + 7)$ funksiyaning argumenti uchun $x^2 + 7 \geq 7$, chunki $x^2 \geq 0$, $\forall x \in R$. Buni hisobga olsak, $f(x^2 + 7)$ funksiya uchun $f(x) = 5x - 1, |x| \geq 3$ shartdan foydalanamiz. Bu yerda $|x| \geq 3 \Leftrightarrow \left[\begin{array}{l} x \leq -3 \\ x \geq 3 \end{array}\right.$ ni e'tiborga oldik. Natijada

$$f(x^2 + 7) = 5(x^2 + 7) - 1 = 5x^2 + 34.$$

Javob: $A)\ 5x^2 + 34$.

7-misol. Agar $f(x) = x^2$ va $\varphi(x) = 2x - 1$ bo'lsa, x ning nechta qiymatida $f(\varphi(x)) = \varphi(f(x))$ bo'ladi?

$A)\ \varnothing$ $B)\ 1$ $C)\ 2$ $D)\ 3$

Yechish: Berilganlardan $f(\varphi(x)) = \varphi^2(x) = (2x - 1)^2$ ni va $\phi(f(x)) = 2f(x) - 1 = 2x^2 - 1$ larni topamiz. Natijada $f(\varphi(x)) = \varphi(f(x))$ dan

$$(2x - 1)^2 = 2x^2 - 1 \Rightarrow 4x^2 - 4x + 1 = 2x^2 - 1 \Rightarrow$$
$$\Rightarrow x^2 - 2x + 1 = 0 \Rightarrow (x - 1)^2 = 0 \Rightarrow x = 1.$$

Yuqoridagi shart faqat x ning bitta qiymatida bajariladi.

Javob: $B)$ 1.

8-misol. $f(x)=3x^2-5x+10$ va $g(f(x))=18x^2-30x+15$ bo'lsa, $g(x)=?$

$A)\ 6x+15$ $B)\ 3x+45$ $C)\ 3x+15$ $D)\ 6x-45$

Yechish: $g(f(x))=af(x)+b$ yoki $18x^2-30x+15=a(3x^2-$ $-5x+10)+b$, chunki ikkala funksiya ham kvadrat funksiyalar.

$$18x^2-30x+15=a(3x^2-5x+10)+b$$

$$18x^2-30x+15=3ax^2-5ax+10a+b$$

$$\begin{cases}3a=18\\-5a=-30\\10a+b=15\end{cases}\Rightarrow\begin{cases}a=6\\b=-45.\end{cases}$$

Demak, $g(f(x))=af(x)+b=6f(x)-45\Rightarrow g(x)=6x-45$.

Javob: $D)\ g(x)=6x-45$.

9-misol. $f(x)=\dfrac{1}{\sqrt[3]{1-x^3}}$; $\underbrace{f(\ldots f(f(19))\ldots)}_{2015\ \text{marta}}=?$

Yechish: x soni 0 va 1 dan farqli ixtiyoriy son bo'lsin. U holda

$$f(x)=\frac{1}{\sqrt[3]{1-x^3}};$$

$$f(f(x))=\frac{1}{\sqrt[3]{1-(f(x))^3}}=\frac{1}{\sqrt[3]{1-\dfrac{1}{1-x^3}}}=\sqrt[3]{\frac{x^3-1}{x^3}}=\sqrt[3]{1-\frac{1}{x^3}};$$

$$f(f(f(x)))=\frac{1}{\sqrt[3]{1-[f(f(x))]^3}}=\frac{1}{\sqrt[3]{1-\dfrac{x^3-1}{x^3}}}=x.$$

Bevosita tekshirib ishonch hosil qilish mumkinki, $f(\ldots f(f(x))\ldots)$ funksiyaning qiymatlari davri 3 ga teng qonuniyat bo'yicha takrorlanadi.

2015 ni 3 ga bo‘lgandagi qoldiq 2 teng bo‘lib,

$$\underbrace{f(\ldots f(f(19))\ldots)}_{2015 \text{ marta}} = f(f(19)) = \sqrt[3]{1 - \frac{1}{19^3}}$$

o‘rinli.

Javob: $\sqrt[3]{1-\frac{1}{19^3}}$.

10-misol. Agar $f(x)$ funksiyaning aniqlanish sohasiga tegishli barcha x lar uchun $f(x+4) = x \cdot f(x) + 7$ tenglik o‘rinli bo‘lsa, $f(8)$ ni toping.

$A)$ 14 $B)$ 42 $C)$ 21 $D)$ 35

Yechish: $x=0$ da $f(4) = 0 \cdot f(0) + 7 = 7$ o‘rinli.

$x = 4$ da $f(8) = 4 \cdot f(4) + 7 = 4 \cdot 7 + 7 = 35$.

Javob: $D)$ 35.

11-misol. Natural sonlarda aniqlangan $f(n)$ funksiya $f(n) = f(n-1) + 2^n$ va $f(1) = 1$ shartlarni qanoatlantiradi. $f(5)$ ni toping.

$A)$ 63 $B)$ 61 $C)$ 51 $D)$ 58

Yechish: Masala shartidan $f(1) = 1$. Natijada

$$f(2) = f(1) + 2^2 = 1 + 4 = 5;$$
$$f(3) = f(2) + 2^3 = 5 + 8 = 13;$$
$$f(4) = f(3) + 2^4 = 13 + 16 = 29;$$
$$f(5) = f(4) + 2^5 = 29 + 32 = 61.$$

Javob: $B)$ 61.

12-misol. x ning barcha haqiqiy qiymatlarida $f(x)$ funksiya $2f(x) + f(x^2 - 1) = 1$ tenglikni qanoatlantirsa, $f(-\sqrt{2})$ ni toping.

$A)\ -\sqrt{3}$ $B)\ \frac{1}{2}$ $C)\ \frac{1}{3}$ $D)$ 3

Yechish: $x = -\sqrt{2}$ da $2f(-\sqrt{2}) + f(1) = 1$ ni hosil qilamiz. $f(-\sqrt{2})$ ni topish uchun $f(1)$ ni topish yetarli. $2f(x) + f(x^2 - 1) = 1$

tenglikdan $x=0;-1;1$ da quyidagi tenglamalar sistemasini hosil qilamiz:

$$\begin{cases} 2f(0)+f(-1)=1 \\ 2f(-1)+f(0)=1 \Rightarrow f(0)=f(-1)=f(1)=\frac{1}{3}. \\ 2f(1)+f(0)=1 \end{cases}$$

Bundan $2f(-\sqrt{2})+f(1)=1 \Rightarrow 2f(-\sqrt{2})+\frac{1}{3}=1 \Rightarrow f(-\sqrt{2})=\frac{1}{3}$.

Javob: $C)\ \frac{1}{3}$.

13-misol. $f(x)$ funksiya barcha x lar uchun $f(x+1)=$ $=f(x)+2x+1$ tenglamani qanoatlantiradi. Agar $f(0)=0$ bo'lsa, $f(2015)$ ni toping.

Yechish: Tenglamani $f(x+1)-f(x)=2x+1$ ko'rinishida yozib olamiz. x ga ketma-ket ravishda $0,1,2,\dots,2014$ qiymatlarni beramiz va $f(0)=0$ ni e'tiborga olsak,

$$f(1)-f(0)=2\cdot 0+1$$
$$f(2)-f(1)=2\cdot 1+1$$
$$f(3)-f(2)=2\cdot 2+1$$
$$\dots\dots\dots\dots\dots\dots\dots\dots$$
$$f(2014)-f(2013)=2\cdot 2013+1$$
$$f(2015)-f(2014)=2\cdot 2014+1.$$

Hosil bo'lgan tengliklarni hadma-had qo'shsak,

$$f(2015)-f(0)=2(1+2+\dots+2013+2014)+1\cdot 2015=2\cdot\frac{1+2014}{2}\cdot$$
$$2014+2015=2015\cdot 2014+2015=2015^2=4060225,\ f(2015)=4060225.$$

Javob: 4060225.

Mustaqil yechish uchun mashqlar

1.1-misol. Quyidagilarni toping.

1. Agar $f(x)=\frac{x-1}{x+1}$ bo'lsa, $f\left(\frac{1}{x^4}\right)$ ni toping.

2. $f(x)=\sqrt{x^2-4}$ funksiya berilgan. Agar $a>1$ bo‘lsa, $f\left(a+\frac{1}{a}\right)$ ni toping.

3. $f(x)=\frac{1}{x}$ bo ‘lsa, $\underbrace{f(\ldots f(f(x))\ldots)}_{2016 \text{ marta}}=?$

4. Agar $f(x)=\frac{x+1}{1-x}$ bo‘lsa, quyidagilarni toping:

1) $f(f(x))$ 2) $f(f(f(x)))$ 3) $f(f(f(f(x))))$

5. Agar $\varphi(x)=\frac{x}{2}$ bo ‘lsa, $\underbrace{\varphi(\ldots\varphi(\varphi(x))\ldots)}_{n}=?$

6. Agar $f(x)=2x-5$ bo‘lsa, x ning qanday qiymatida $f(-x)=f(4x+1)$ tenglik bajariladi?

7. Agar $f(x)=x^3+x^2+1$ va $\varphi(x)=x-1$ bo‘lsa, $f(\varphi(x))=2-x$ tenglama nechta ildizga ega?

8. Agar $f(x)=x^2+12x+30$ bo‘lsa, $f(f(f(f(f(x)))))=0$ tenglamani yeching.

9. Agar $f(x)$ funksiya barcha x lar uchun aniqlangan va $2f(x)+f(1-x)=3x^2$ tenglikni qanoatlantirsa, $f(5)$ ni toping.

10. Agar $f(x)=1{,}5-x$ va $f(g(x))=\frac{1}{3+x}$ bo‘lsa, $g(x)$ ni toping.

11. $f(x)$ funksiya ixtiyoriy haqiqiy a va b lar uchun $f\left(\frac{a+2b}{3}\right)=\frac{f(a)+2f(b)}{3}$ tenglikni qanoatlantiradi. Agar $f(1)=1$ va $f(4)=7$ bo‘lsa, $f(1999)$ ning qiymatini toping.

12. $f(x)=ax^2+bx+c$ kvadrat uchhad berilgan va $f\left(\frac{a-b-c}{2a}\right)=f\left(\frac{c-a-b}{2a}\right)=0$ o‘rinli. $f(-1)\cdot f(1)=0$ ekanligini isbotlang.

1.2-misol. Test topshiriqlari.

1. Agar $f(x)=\left(x-\frac{1}{3}\right)\cdot\left(2x+\frac{1}{4}\right)$ bo‘lsa, $f(1)$ ni toping.

$A)\ 4{,}5$ $B)\ -4{,}5$ $C)\ 1{,}5$ $D)\ -1$

2. Agar $f(x)=(2x+3)\left(\frac{3}{x}-3\right)$ bo‘lsa, $f(-1)$ ni toping.

$A)\ 6$ $B)\ 0$ $C)\ -3$ $D)\ -6$

3. Agar $f(x)=x^2-4x+3$ bo‘lsa, $f(2+\sqrt{3})$ ni hisoblang.

$A)\ 5+\sqrt{3}$ $B)\ 2$ $C)\ 4-\sqrt{3}$ $D)\ 5$

4. Agar $f(a,b,c)=\frac{a}{b-c}$ bo‘lsa, $f(f(1,2,3),f(2,3,1),f(3,1,2))$ ni toping.

$A)\ 0$ $B)\ -\frac{1}{4}$ $C)\ -\frac{1}{2}$ $D)\ 1$

5. Agar $f(n)=\frac{1}{n(n+1)}$ bo‘lsa, $f(1)+f(2)+\cdots+f(2014)$ yig‘indini hisoblang.

$A)\ \frac{1}{2015}$ $B)\ \frac{1007}{2015}$ $C)\ \frac{1}{2014}$ $D)\ \frac{2014}{2015}$

6. Agar $f(x)=\frac{1}{x}-x$ bo‘lsa, $f(\frac{1}{a})+f(-\frac{1}{a})+a^2$ nimaga teng?

$A)\ 2a^2$ $B)\ a^2$ $C)\ 0$ $D)\ -a^2$

7. Agar $f(x)=x^2-5$ bo‘lsa, $f(a-1)-f(a+1)+2f(1-a^2)-2a^4+4a^2$ ni hisoblang.

$A)\ 4a+8$ $B)\ 4a-8$ $C)\ -4a-8$ $D)\ -4a+8$

8. Agar $f(x)=x^4-2x^2+1$ bo‘lsa, $f(1+a)-f(1-a)-4a^3+a^4$ nimaga teng?

$A)\ a^3+8$ $B)\ 4$ $C)\ 8a^3$ $D)\ a^4+4a^3$ $E)\ 4a^2+2a$

9. Agar $f(x)=\sqrt{x^3-1}$ bo‘lsa, $f(\sqrt[3]{x^2+1})$ ni toping.

$A)\ 0$ $B)\ -x$ $C)\ x$ $D)\ |x|$

10. $h(x)=|x|,\ g(x)=\frac{2x+3}{3x-1},\ f(x)=\sqrt{x+1}$ bo‘lsa, quyidagilardan qaysi biri to‘g‘ri?

$A)\ g(h(x))=\dfrac{2|x|+3}{3|x|+1}$ $B)\ h(f(x))=\sqrt{|x+1|}$

$C)\ f(h(x))=\sqrt{|x|+1}$ $D)\ f(g(x))=\dfrac{\sqrt{5x+1}}{\sqrt{3x-1}}$

11. $f(x)=\dfrac{x-3}{2}$ bo‘lsa, $f(4x-1)$ quyidagilardan qaysi biriga teng?

$A)\ 4f(x)+4$ $B)\ 4f(x)+1$ $C)\ f(x)+4$ $D)\ 4f(x)-1$

12. $f(x)=\begin{cases}2x-7, & x\le 2\\ 5-3x, & x>2\end{cases}$ bo‘lsa, $f(f(1))$ ni hisoblang.

$A)\ -17$ $B)\ 20$ $C)\ -3$ $D)\ -1$

13. $f(x)=\begin{cases}x^2-1, & x<0\\ 2x-1, & x\ge 0\end{cases}$ bo‘lsa, $f(f(-3))$ ni toping.

$A)\ 8$ $B)\ -7$ $C)\ 15$ $D)\ -16$

14. $f(x)=3x-5$ va $f(g(x))=\dfrac{2x+3}{x-5}$ bo ‘lsa, $g(3)$ ni hisoblang.

$A)\ \dfrac{1}{6}$ $B)\ \dfrac{1}{3}$ $C)\ 1$ $D)\ 3$

1.3-misol. Test topshiriqlari.

1. $f(x+2)=4\cdot f(x)+2;\ f(2)=4.\ f(6)=?$

$A)\ 74$ $B)\ 66$ $C)\ 72$ $D)\ 46$

2. $f(x+5)=x\cdot f(x)+4$ bo‘lsa, $f(10)$ ni toping.

$A)\ 24$ $B)\ 23$ $C)\ 30$ $D)\ 25$

3. Agar $(x-5)f(x)+2f(x+2)=3x+7$ bo‘lsa, $f(9)=?$

$A)\ 6$ $B)\ 4$ $C)\ 3$ $D)\ 22$

4. $f(x+1)+2\cdot f(x)=12$ va $f(2)=7$ bo‘lsa, $f(4)$ ning qiymatini toping.

$A)\ -2$ $B)\ 8$ $C)\ 12$ $D)\ 16$

5. Agar $(x-2)f(x-2)+f(2x)+f(x+2)=x+6$ bo‘lsa, $f(4)$ ni toping.

$A)\ 13$ $B)\ 2$ $C)\ 3$ $D)\ 4$

6. Agar $3f(x)=f(x+1)+f(x-1)$, $f(1)=3$, $f(2)=4$ bo'lsa, $f(5)$ ning nechta tub bo'luvchisi bor?

$A)\ 1$ $B)\ 3$ $C)\ 6$ $D)\ 2$

7. Agar $f(x)$ funksiya uchun $x\in(-\infty;\infty)$ da $f(x+3)=-\dfrac{1}{f(x+1)}$ tenglik bajarilsa, $\dfrac{f(4)}{f(0)}$ ni toping.

$A)\ 1$ $B)\ 3$ $C)\ 2$ $D)\ 4$

8. Agar $f(x)=3x^2-5$ bo'lsa, $f(x-2)=7$ tenglama ildizlari yig'indisini toping.

$A)\ -4$ $B)\ 4$ $C)\ 0$ $D)\ 2$

9. $f(x)=\dfrac{1}{x-4}+\dfrac{x}{4}+\dfrac{1}{4}$ funksiya uchun $f(a)=0$ bo'lsa, a ni toping.

$A)\ -5$ $B)\ 0$ $C)\ 0$ va -5 $D)\ 0$ va 3

2. Funksional tenglamalar

Ta'rif. Agar tenglamada noma'lum funksiya bo'lsa, bunday tenglamalarga funksional tenglamalar deyiladi.

Masalan, funksiyaning toqligi $f(-x)=-f(x)$, funksiyaning juftligi $f(-x)=f(x)$, funksiyaning davriyligi $f(x+T)=f(x)$ funksional tenglamalarga misol bo'la oladi.

2.1. O'zgaruvchilari erksiz funksional tenglamalarni yangi o'zgaruvchi kiritib yechish usuli

Bu turdagi funksional tenglamalarda noma'lum funksiya bitta o'zgaruvchiga bog'liq bo'lib, ozod o'zgaruvchularga ega bo'lmaydi. Usulning mohiyati, tenglamada qatnashgan erksiz o'zgaruvchi biror yangi o'zgaruvchi orqali belgilanadi va natijada soddaroq funksional tenglama hosil bo'ladi.

Test topshiriqlarini ko‘rib o‘taylik.

1-misol. Agar $f(x+1)=x^2-3x-3$ bo‘lsa, $f(x)$ ni toping.

$A)\ x^2-5x+6$ $\quad B)\ x^2-4$ $\quad C)\ x^2-5x+1$ $\quad D)\ x^2-3x-1$

Yechish: $x+1=t$ deb belgilash kiritamiz va $x=t-1$ ni topamiz. Funksiyaning dastlabki ko‘rinishiga qaytamiz:

$$f(t)=(t-1)^2-3(t-1)-3=t^2-5t+1.$$

Bundan $f(x)=x^2-5x+1$ ekanligi kelib chiqadi.

Javob: $C)\ x^2-5x+1$.

2-misol. Agar $1+2f(x-1)=2f(x)$ va $f(0)=0$ bo‘lsa, $f(2014)$ ni toping.

$A)\ 1007$ $\quad B)\ 1008$ $\quad C)\ 2014$ $\quad D)\ 2013$

Yechish: $1+2f(x-1)=2f(x)$ dan $f(x)=f(x-1)+\frac{1}{2}$ ni topamiz.

Qonuniyatni aniqlaymiz:

$$f(0)=0\ ;\ f(1)=f(0)+\frac{1}{2}=0+\frac{1}{2};$$

$$f(2)=f(1)+\frac{1}{2}=\frac{1}{2}+\frac{1}{2}=\frac{2}{2}=1$$

$$f(3)=f(2)+\frac{1}{2}=1+\frac{1}{2}=\frac{3}{2};$$

$$f(4)=f(3)+\frac{1}{2}=\frac{3}{2}+\frac{1}{2}=\frac{4}{2}=2;\ ...\ ;f(n)=\frac{n}{2}.$$

Demak, $n\in N$ uchun $f(n)=\frac{n}{2}$ o‘rinli.

Bundan $f(2014)=\frac{2014}{2}=1007$.

Javob: $A)1007$.

3-misol. Agar $f\left(\frac{ax-b}{bx-a}\right)=x^{50}+x^{49}+x^{48}+\cdots+x^2+x+1,\ (|a|\neq|b|)$ bo ‘lsa, $f(1)$ ni toping.

$A)\ 6$ $\quad B)\ 5$ $\quad C)\ 9$ $\quad D)\ 12$

Yechish: $f(1)$ ni topish uchun $\frac{ax-b}{bx-a}=1$ tenglikdan x ni topib olamiz:

$$\frac{ax-b}{bx-a}=1 \Rightarrow ax-b=bx-a \Rightarrow (a-b)x=-(a-b) \Rightarrow x=-1.$$

Bundan

$$f(1)=(-1)^{50}+(-1)^{49}+(-1)^{48}+\cdots+(-1)^2+(-1)+1=1.$$

Javob: $A)\ 1$.

4-misol. $f(x-2)=\frac{2x+1}{x+2}$ bo‘lsa, $f(f(3))$ ni toping.

Yechish: $x-2=t$ deb belgilash kiritamiz va $x=t+2$ ni topamiz. Funksiyaning dastlabki ko‘rinishiga qaytamiz: $f(t)=\frac{2(t+2)+1}{t+2+2}=\frac{2t+5}{t+4}$. Bundan $f(x)=\frac{2x+5}{x+4}$ ekanligi kelib chiqadi.

$$f(3)=\frac{2\cdot 3+5}{3+4}=\frac{11}{7} \Rightarrow f(f(3))=f\left(\frac{11}{7}\right)=\frac{\frac{22}{7}+5}{\frac{11}{7}+4}=\frac{57}{7}:\frac{39}{7}=\frac{57}{7}\cdot\frac{7}{39}=\frac{19}{13}=1\frac{6}{13}.$$

Javob: $\frac{19}{13}$.

Endi funksional tenglamalarni umumiy holda yechish usullarini ko‘rib o‘tamiz.

5-misol. x ning ixtiyoriy qiymatida $2f(x+2)+f(4-x)=$ $=2x+5$ tenglikni qanoatlantiruvchi $y=f(x)$ chiziqli funksiya mavjudmi?

Yechish: Chiziqli funksiya $f(x)=ax+b$ ko‘rinishga ega bo‘lib, yuqoridagi tenglikni qanoatlantirsin. Bundan

$$2(a(x+2)+b)+a(4-x)+b=2x+5$$

$$2ax+4a+2b+4a-ax+b=2x+5$$

$$ax+8a+3b=2x+5.$$

Oxirgi tenglik $\forall x\in R$ da bajarilishi uchun

$$\begin{cases} a=2 \\ 8a+3b=5 \end{cases} \Rightarrow \begin{cases} a=2 \\ b=-\dfrac{11}{3} \end{cases}$$

bo'ladi. Masala shartini $f(x)=2x-\frac{11}{3}$ funksiya qanoatlantiradi.

Berilgan funksional tenglamani yechimi faqat chiziqli funksiya bo'lishi shartmi? Boshqa turdagi funksiyalar bu tenglamani qanoatlantirishi mumkinmi? Bu savolga javob topish uchun berilgan funksional tenglamani umumiy holda yechaylik.

$2f(x+2)+f(4-x)=2x+5$ tenglamada x ni $x-2$ ga almashtiramiz:

$$2f(x)+f(6-x)=2x+1,\ \forall x\in R \quad (1)$$

(1) da x ni $6-x$ ga almashtiramiz:

$$2f(6-x)+f(x)=-2x+13,\ \forall x\in R \quad (2)$$

(1) va (2) tengliklardan $f(x)$ funksiyani topamiz.

$$\begin{cases} 2f(x)+f(6-x)=2x+1 \\ 2f(6-x)+f(x)=-2x+13 \end{cases} \left| \begin{matrix} \cdot 2 \\ \cdot 1 \end{matrix} \right. \Rightarrow 4f(x)-f(x)=$$

$$=4x+2+2x-13 \Rightarrow f(x)=2x-\frac{11}{3}.$$

Haqiqatan ham, $f(x)=2x-\frac{11}{3}$ funksiya tenglamani qanoatlantiradi.

Javob: $f(x)=2x-\frac{11}{3}$.

6-misol. Barcha $x\in R$ lar uchun $f(1-x)-f(2-x)=-2x+7$ tenglikni qanoatlantiruvchi $f(x)$ kvadrat funksiyani toping.

Yechish: Berilgan tenglikni qanoatlantiruvchi kvadrat uchhad $f(x)=ax^2+bx+c$ bo'lsin. U holda

$$a(1-x)^2+b(1-x)+c-[a(2-x)^2+b(2-x)+c]=-2x+7$$

$$a-2ax+ax^2+b-bx+c-4a+4ax-ax^2-2b+bx-c=-2x+7$$

$$2ax+(-3a-b)=-2x+7;$$

$$\begin{cases}2a=-2\\-3a-b=7\end{cases}\Rightarrow\begin{cases}a=-1\\b=-4\end{cases}$$

Demak, $f(x)=-x^2-4x+c, c\in R$ ko‘rinishda bo‘ladi.

Javob: $f(x)=-x^2-4x+c, c\in R$.

7-misol. $2f(1-x)+1=xf(x)$ tenglikni qanoatlantiruvchi $f(x)$ funksiyalarni toping.

Yechish: $2f(1-x)+1=xf(x)$ tenglikda x ni $1-x$ ga almashtiramiz. Natijada

$$2f(x)+1=(1-x)f(1-x)$$

ni hosil qilamiz. Bundan

$$\begin{cases}2f(1-x)+1=xf(x)\\2f(x)+1=(1-x)f(1-x)\end{cases}\Rightarrow\begin{cases}f(1-x)=\frac{1}{2}[xf(x)-1]\\2f(x)+1=(1-x)f(1-x)\end{cases}\Rightarrow$$

$$\Rightarrow 2f(x)+1=(1-x)\cdot\frac{1}{2}[xf(x)-1]\Rightarrow 4f(x)+2=(x-x^2)f(x)+x-1\Rightarrow$$

$$\Rightarrow(x^2-x+4)f(x)=x-3\Rightarrow f(x)=\frac{x-3}{x^2-x+4}.$$

Javob: $f(x)=\frac{x-3}{x^2-x+4}$.

8-misol. Barcha haqiqiy $x\neq 0$ lar uchun $f(x)+5x\cdot f\left(\frac{1}{x}\right)=3x^3$ shartni qanoatlantiruvchi $f(x)$ funksiyani toping.

Yechish: Berilgan tenglikda x ni $\frac{1}{x}$ ga almashtiramiz va

$$f\left(\frac{1}{x}\right)+\frac{5}{x}\cdot f(x)=\frac{3}{x^3}$$

ni hosil qilamiz.

Bundan

$$\begin{cases} f(x)+5x\cdot f\left(\frac{1}{x}\right)=3x^3 \\ f\left(\frac{1}{x}\right)+\frac{5}{x}\cdot f(x)=\frac{3}{x^3} \end{cases}\Bigg|\cdot 5x \Rightarrow \begin{cases} f(x)+5x\cdot f\left(\frac{1}{x}\right)=3x^3 \\ 5x\cdot f\left(\frac{1}{x}\right)+25f(x)=\frac{15}{x^2} \end{cases}\Rightarrow$$

$$\Rightarrow 25f(x)-f(x)=\frac{15}{x^2}-3x^3 \Rightarrow f(x)=\frac{5}{8x^2}-\frac{x^3}{8}.$$

Javob: $f(x)=\dfrac{5}{8x^2}-\dfrac{x^3}{8}$.

9-misol. $3f(x-1)+4f(1-x)=5x$ funksional tenglamani yeching.

Yechish: $x-1=t$ deb olamiz. U holda $x=t+1$ va $1-x=-t$ ni topamiz. Natijada

$$3f(t)+f4(-t)=5(t+1) \quad (1)$$

ni hosil qilamiz.

Bu tenglamada t ni $-t$ ga almashtiramiz va

$$3f(-t)+4f(t)=5(1-t) \quad (2)$$

ni hosil qilamiz. (1) va (2) dan $f(t)$ ni aniqlaymiz:

$$\begin{cases} 3f(t)+4f(-t)=5(t+1) \\ 4f(t)+3f(-t)=5(1-t) \end{cases}\begin{matrix}\cdot 3 \\ \cdot 4\end{matrix} \Rightarrow 16f(t)-9f(t)=20(1-t)-15(t+1)\Rightarrow$$

$$\Rightarrow 7f(t)=-35t+5 \Rightarrow f(t)=-5t+\frac{5}{7}.$$

Bundan $f(x)=-5x+\dfrac{5}{7}$ ga teng.

Javob: $f(x)=-5x+\dfrac{5}{7}$.

10-misol. $f\left(\dfrac{x+1}{x-2}\right)+2f\left(\dfrac{x-2}{x+1}\right)=x,\ (x\neq 0;\ x\neq \pm 1;\ x\neq 2),\ f(x)=?$

Yechish: $\dfrac{x+1}{x-2}=t \Rightarrow x+1=tx-2t \Rightarrow (1-t)x=-2t-1 \Rightarrow x=\dfrac{2t+1}{t-1}$ ni topamiz.

$$f(t)+2f\left(\frac{1}{t}\right)=\frac{2t+1}{t-1} \quad (1)$$

ni hosil qilamiz. (1) da t ni $\frac{1}{t}$ ga almashtiramiz:

$$f\left(\frac{1}{t}\right)+2f(t)=\frac{\frac{2}{t}+1}{\frac{1}{t}-1}=\frac{\frac{t+2}{t}}{\frac{1-t}{t}}=\frac{t+2}{1-t} \quad (2).$$

(1) va (2) ni birlashtirib, quyidagi sistemani yechamiz:

$$\begin{cases} f(t)+2f\left(\frac{1}{t}\right)=\frac{2t+1}{t-1} \\ 2f(t)+f\left(\frac{1}{t}\right)=\frac{t+2}{1-t} \end{cases} \Bigg| \cdot 2 \Rightarrow \begin{cases} f(t)+2f\left(\frac{1}{t}\right)=\frac{2t+1}{t-1} \\ 4f(t)+2f\left(\frac{1}{t}\right)=\frac{2(t+2)}{1-t} \end{cases} \Rightarrow$$

$$\Rightarrow 3f(t)=\frac{2(t+2)}{1-t}-\frac{2t+1}{t-1}=\frac{2(t+2)}{1-t}+\frac{2t+1}{1-t}=\frac{4t+5}{1-t}\Rightarrow$$

$$\Rightarrow f(t)=\frac{4t+5}{3(1-t)}\Rightarrow f(x)=\frac{4x+5}{3(1-x)}.$$

Javob: $f(x)=\dfrac{4x+5}{3(1-x)}$.

11-misol. Agar $f(x)$ va $g(x)$ funksiyalar

$$\begin{cases} f(2x+1)+g(x-1)=x \\ f(2x+1)-2x^2-2g(x-1)=0 \end{cases}$$

sistemani qanoatlantirsa, $4f(x)+g(x)\le 0$ tengsizlikni yeching.

Yechish: Berilgan sistemani quyidagicha yozib olamiz:

$$\begin{cases} f(2x+1)+g(x-1)=x \\ f(2x+1)-2g(x-1)=2x^2. \end{cases}$$

Sistemani $f(2x+1)$ va $g(x-1)$ ga nisbatan yechib,

$$\begin{cases} g(x-1)=\dfrac{x(1-2x)}{3} \\ f(2x+1)=\dfrac{2x(x+1)}{3} \end{cases}$$

ni topamiz.

1) $g(x-1)=\frac{x(1-2x)}{3}$ dan $g(x)$ ni aniqlaymiz. $x-1=t$ dan $x=t+1$. U holda

$$g(t)=\frac{(t+1)(1-2t-2)}{3}=-\frac{(t+1)(2t+1)}{3}\Rightarrow g(x)=-\frac{(x+1)(2x+1)}{3}.$$

2) $f(2x+1)=\frac{2x(x+1)}{3}$ dan $f(x)$ ni aniqlaymiz. $2x+1=y$ dan $x=\frac{y-1}{2}$. U holda

$$f(y)=\frac{2\cdot\frac{y-1}{2}\left(\frac{y-1}{2}+1\right)}{3}=\frac{(y-1)(y+1)}{6}=\frac{y^2-1}{6}\Rightarrow f(x)=\frac{x^2-1}{6}.$$

3) $f(x)=\frac{x^2-1}{6}$ va $g(x)=-\frac{(x+1)(2x+1)}{3}$ larni $4f(x)+g(x)\le 0$ ga qo'yamiz.

$$4\cdot\frac{x^2-1}{6}-\frac{(x+1)(2x+1)}{3}\le 0\Rightarrow 2(x-1)(x+1)-(x+1)(2x+1)\le 0\Rightarrow$$
$$\Rightarrow(x+1)(2x-2-2x-1)\le 0\Rightarrow x+1\ge 0\Rightarrow x\ge -1.$$

Javob: $x\ge -1$.

12-misol. Sistemani qanoatlantiruvchi $f(x)$ va $g(x)$ funksiyalarni toping:

$$\begin{cases} f(2x+2)+2g(4x+7)=x-1 \\ f(x-1)+g(2x+1)=2x. \end{cases}$$

Yechish: Sistemaning 1 – tenglamasidan $2x+2=t-1$ deb olib, $x=\frac{t-3}{2}$ ni va $4x+7=4\cdot\frac{t-3}{2}+7=2t+1$ ni hosil qilamiz. Topilganlarni sistemaning 1-tenglamasiga qo'yamiz.

$$f(t-1)+2g(2t+1)=\frac{t-3}{2}-1\Rightarrow f(t-1)+2g(2t+1)=\frac{t-5}{2}.$$

t ni x ga almashtiramiz: $f(x-1)+2g(2x+1)=\frac{x-5}{2}$.

Quyidagi sistemani hosil qilamiz va uni $f(x-1)$ va $g(2x+1)$ ga nisbatan yechamiz:

$$\begin{cases} f(x-1)+2g(2x+1)=\dfrac{x-5}{2} \\ f(x-1)+g(2x+1)=2x \end{cases} \Rightarrow \begin{cases} f(x-1)=\dfrac{7x+5}{2} \\ g(2x+1)=-\dfrac{3x+5}{2}. \end{cases}$$

1) $f(x-1)=\dfrac{7x+5}{2}$ dan $f(x)$ ni aniqlaymiz. $x-1=y$ deb olsak, $x=y+1$. Bundan

$$f(y)=\frac{7(y+1)+5}{2}=\frac{1}{2}(7y+12)\Rightarrow f(x)=\frac{1}{2}(7x+12).$$

2) $g(2x+1)=-\dfrac{3x+5}{2}$ dan $f(x)$ ni aniqlaymiz. $2x+1=z$ deb olsak, $x=\dfrac{z-1}{2}$.

Bundan

$$g(z)=-\frac{1}{2}\left(3\cdot\frac{z-1}{2}+5\right)=-\frac{1}{4}(3z+7)\Rightarrow g(x)=-\frac{1}{4}(3x+7).$$

Javob: $f(x)=\dfrac{1}{2}(7x+12)$; $g(x)=-\dfrac{1}{4}(3x+7)$.

Mustaqil yechish uchun mashqlar

2.1-misol. Test topshiriqlari.

1. Agar $f(x-1)=x^2+3x-8$ bo‘lsa, $f(x)$ ni aniqlang.

A) x^2+5x+2 *B*) x^2-6x+5 *C*) x^2+5x-4 *D*) x^2-x-2

2. Agar $f(x+1)=x^2-2x-3$ bo‘lsa, $f(x)$ ni aniqlang.

A) x^2-5x+1 *B*) x^2-3x-1 *C*) x^2-5x+6 *D*) x^2-4x

3. Agar $f(x+2)=x^3+6x^2+12x+8$ bo‘lsa, $f(\sqrt{3})$ ni toping.

A) $4\sqrt{3}$ *B*) $2\sqrt{3}$ *C*) $3\sqrt{3}$ *D*) 12

4. $x>0$ da $f(x^2+\sqrt{x})=x^4+(2x^2-1)\sqrt{x}-x^2+x$ tenglikni qanoatlantiruvchi $f(x)$ funksiyani toping.

$A)\ x^2+2x-1$ $B)\ x^2-x$ $C)\ x^2+x$ $D)\ x^2+2x+2$

5. $f\left(\frac{3x-1}{x+2}\right)=\frac{x+1}{x-1}$ bo'lsa, $f(x)$ ni toping.

$A)\ \frac{x+1}{x-1}$ $B)\ \frac{2x+1}{3-x}$ $C)\ \frac{3x-1}{x+2}$ $D)\ \frac{x+4}{3x-2}$

6. $f(2x-3)=\frac{x+3}{x-2}$ bo'lsa, $f(f(7))=?$

$A)\ 5$ $B)\ 2,(6)$ $C)\ 7$ $D)\ 3,(3)$

7. $f\left(\frac{3x-2}{2}\right)=x^2-x-1.\ f(1)-?$

$A)\ -\frac{5}{9}$ $B)\ -\frac{13}{9}$ $C)\ -\frac{7}{9}$ $D)\ -1$

8. $\frac{3}{f(2x)}-\frac{1}{x}=\frac{4}{f(2x)}-\frac{2}{3x}$ bo'lsa, $f(4)=?$

$A)\ 6$ $B)\ -6$ $C)\ 0,1(6)$ $D)\ -12$

9. $\frac{4}{f(4x)}-\frac{3}{2x}=\frac{3}{f(4x)}-\frac{4}{3x}$ bo'lsa, $f(6)=?$

$A)\ 0,(1)$ $B)\ 36$ $C)\ -36$ $D)\ 9$

10. $f(x+2)+f(x-1)=2(x^2+7)$ ekanligi ma'lum bo'lsa, $f(x)$ ko'phadni toping.

$A)\ f(x)=2x^2+1$ $B)\ f(x)=x^2-4$

$C)\ f(x)=x^2+3x+7$ $D)\ f(x)=x^2-x+5$

11. $P(x-3)+P(x+1)=2x^2-10x+16$ bo'lsa, $P(x)$ ni toping.

$A)\ x^2-x+3$ $B)\ x^2-3x$ $C)\ 2x^2-9$ $D)\ x^2-3x+1$

2.2-misol. Funksional tenglamalarni yeching.

1. Agar $f\left(\frac{x}{x+1}\right)=x^2$ bo'lsa, $f(x)$ ni toping.

2. Agar $f\left(1+\frac{1}{x}\right)=x^2-1,\ (x\neq 0,\ x\neq 1)$ bo'lsa, $f(x)$ ni toping.

3. $f\left(\frac{x+3}{2x-1}\right)=\frac{x+2}{x-2},\ \left(x\neq\frac{1}{2};\ x\neq 2;\ x\neq\frac{5}{3}\right)$ bo'lsa, $f(x)$ ni toping.

4. $f\left(\frac{3x-1}{x+2}\right)=\frac{x+1}{x-1}, (x\neq -2, x\neq \frac{2}{3}, x\neq 1)$ bo'lsa, $f(x)$ ni toping.

5. Quyidagi tenglamani qanoatlantiruvchi $f(x)$ funksiyani toping:

$$f(x)+(x-2)f(1)+3f(0)=x^3+2,\ x\in R.$$

6. Quyidagi tenglamani qanoatlantiruvchi $f(x)$ funksiyani toping:

$$f(x)+(1-x)f(0)+f(-1)=x^3-3,\ x\in R.$$

2.3-misol. Funksional tenglamalarni yeching.

1. $\forall x\in R$ uchun $2f(x)+f(1-x)=x^2$ tenglik bajarilsa, $f(x)$ ni toping.

2. $x>0$ da $5f(x)=3f\left(\frac{1}{x}\right)+\frac{1}{\sqrt{x}}$ tenglamani qanoatlantiruvchi $f(x)$ funksiyani toping.

3. Barcha haqiqiy $x\neq 0$ lar uchun $f(x)+3x\cdot f\left(\frac{1}{x}\right)=2x^2$ shartni qanoatlantiruvchi $f(x)$ funksiyani toping.

4. $xf(x)+2f\left(-\frac{1}{x}\right)=3, x\neq 0$ bo'lsa, $f(x)$ ni toping.

5. $af(x-1)+bf(1-x)=cx$ funksional tenglamani yeching. Bu yerda a, b, c o'zgarmas sonlar bo'lib, $a^2\neq b^2$.

6. $f(x)+f\left(\frac{1}{1-x}\right)=x$ bo'lsa, $f(x)$ ni toping.

7. Barcha haqiqiy $x\neq 1$ lar uchun $(x-1)f\left(\frac{x+1}{x-1}\right)-f(x)=x$ tenglamani yeching.

8. Barcha haqiqiy $x\neq \frac{1}{2}$ lar uchun $f(x)+xf\left(\frac{x}{2x-1}\right)=2$ tenglamani yeching.

9. $f(x)+f\left(\frac{a^2}{a-x}\right)=x, a\neq 0$ tenglikni qanoatlantiruvchi $f(x)$ funksiyani toping.

10. $2f(3-x)+3f(x-1)=2x-1$ dan $f(x)$ funksiyani toping.

2.4-misol. Sistemani qanoatlantiruvchi $f(x)$ va $g(x)$ funksiyalarni toping.

1. $\begin{cases} f(2x+1)+2g(2x+1)=2x \\ f\left(\frac{x}{x-1}\right)+g\left(\frac{x}{x-1}\right)=x \end{cases}$

2. $\begin{cases} f(4x+3)+xg(6x+4)=2 \\ f(2x+1)+g(3x+1)=x+1 \end{cases}$

3. $\begin{cases} f(3x-2)+7g(x-5)=x+1 \\ f(x+1)-g\left(\frac{x}{3}-4\right)=3x \end{cases}$

2.2. O'zgaruvchilari erkli bo'lgan funksional tenglamalar

1-misol. $f(x)f(y)+1=f(x)+f(y)+xy$.

Yechish: Berilgan tenglamada $y=x$ almashtirishni bajaramiz:

$$f^2(x)+1=2f(x)+x^2 \Rightarrow f^2(x)-2f(x)+1=x^2 \Rightarrow [f(x)-1]^2=x^2 \Rightarrow$$
$$\Rightarrow f(x)-1=\pm x \Rightarrow f(x)=1\pm x.$$

Javob: $f(x)=1\pm x$.

2-misol. Agar $f(x-y)=f(x)+f(y)-2xy$ bo'lsa, $f(x)=?$

Yechish: $f(x-y)=f(x)+f(y)-2xy$ (1)

(1) tenglamada $y=-x$ almashtirishni bajaramiz:

$$f(2x)=f(x)+f(-x)+2x^2 \quad (2)$$

(1) da $y=2x$ almashtirishni bajaramiz:

$$f(-x)=f(x)+f(2x)-4x^2 \quad (3)$$

(2) va (3) dan

$$\begin{cases} f(2x)=f(x)+f(-x)+2x^2 \\ f(-x)=f(x)+f(2x)-4x^2 \end{cases} \Rightarrow \begin{cases} f(2x)=f(x)+f(-x)+2x^2 \\ f(2x)=-f(x)+f(-x)+4x^2 \end{cases} \Rightarrow$$

$$\Rightarrow f(x)+f(-x)+2x^2=-f(x)+f(-x)+4x^2 \Rightarrow f(x)=x^2.$$

Javob: $f(x)=x^2$.

3-misol. Agar $f(x^y)=y\cdot f(x)$, $x>0$ bo'lsa, $f(x)$ funksiyani toping.

Yechish: $x=e^z$ almashtirishni bajaramiz va $f(e^{zy})=yf(e^z)$ ni hosil qilamiz. Oxirgi tenglikdan $z=1$ da $f(e^y)=yf(e)$ yoki $f(e^y)=cy$, $f(e)=c$.

$e^y=x$ deb olsak, $y=\ln x$ o'rinli. Natijada $f(e^y)=cy$ dan $f(x)=c\ln x$.

Javob: $f(x)=c\ln x$, $x>0$.

4-misol. $f(x)$ funksiya barcha x larda haqiqiy qiymatlar qabul qiladi. Barcha x va y lar uchun $f(x+f(y))=2x+4y+3$ tenglikni qanoatlantiruvchi $f(x)$ funksiyalarni toping.

Yechish: $f(x+f(y))=2x+4y+3$ dan $f(f(x+f(y)))=f(2x+4y+3)$ o'rinli.

Masala shartidan

$$f(f(x+f(y)))=f(0+f(x+f(y)))=$$
$$=2\cdot 0+4(x+f(y))+3=4x+4f(y)+3.$$

Bundan $4x+4f(y)=f(2x+4y+3)$ ni hosil qilamiz.

x ni $2x+4y+3=y$ tenglikni qanoatlantiradigan qilib tanlaymiz va $2x=-3y-3$ yoki $x=-\frac{3y+3}{2}$. x ning ifodasini $4x+4f(y)=f(2x+4y+3)$ tenglikka qo'yamiz:

$$4\cdot\left(-\frac{3y+3}{2}\right)+4f(y)+3=f(y)\Rightarrow -6y-6+3f(y)+3=0\Rightarrow f(y)=2y+1.$$

Demak, $f(x)=2x+1$.

Javob: $f(x)=2x+1$.

5-misol. Faqat musbat sonlarda aniqlangan va ixtiyoriy x va y larda

$$f(x)\cdot f(y)=f(xy)+\frac{1}{x}+\frac{1}{y}$$

tenglikni qanoatlantiruvchi barcha $f(x)$ funksiyalarni toping.

Yechish: Agar berilgan munosabatda $x=y=1$ desak, u holda $f^2(1)-f(1)-2=0$ tenglama hosil bo'ladi. Bu yerda $f(1)=t$ belgilash kiritib $t^2-t-2=0$ tenglamaga kelamiz va uni yechib $f(1)=-1$ yoki $f(1)=2$ larni topamiz.

Endi, agar berilgan munosabatda $y=1$ desak, u holda

$$f(x)\cdot f(1)=f(x)+\frac{1}{x}+1$$

bo'ladi va bundan $f(x)=\frac{1}{f(1)-1}\left(\frac{1}{x}+1\right)$ kelib chiqadi. Shunday qilib, $f(x)=-\frac{1}{2}\left(\frac{1}{x}+1\right)$ yoki $f(x)=1+\frac{1}{x}$ javoblarga kelamiz. Tekshirib ko'rib, faqat ikkinchi funksiya berilgan tenglamani qanoatlantirishini aniqlaymiz.

Javob: $f(x)=1+\frac{1}{x}$.

6-misol. $f(x+y)-2f(x-y)+f(x)-2f(y)=y-2$ tenglikni qanoatlantiruvchi $f(x)$ funksiyani toping.

Yechish: $x=y=0$ da $f(0)=1$ bo'ladi. Funksional tenglamada $x=0$ bo'lsa,

$$f(y)-2f(-y)+f(0)-2f(y)=y-2\Rightarrow f(y)+2f(-y)=-y+3.$$

Oxirgi tenglikdan y ni $-y$ ga almashtiramiz:

$$f(-y)+2f(y)=y+3\Rightarrow 2f(y)+f(-y)=y+3.$$

Natijada

$$\begin{cases} f(y)+2f(-y)=-y+3 \\ 2f(y)+f(-y)=y+3 \end{cases}\Big|\cdot 2 \Rightarrow 4f(y)-f(y)=2y+6+y-3 \Rightarrow f(y)=y+1.$$

Oxirgi tenglikdan y ni x ga almashtiramiz va $f(x)=x+1$ ni topamiz.

Javob: $f(x)=x+1$.

7-misol. $f(x+y)+f(x-y)+f(x)+f(y)=3x^2+3y^2$ tenglikni qanoatlantiruvchi $f(x)$ funksiyani toping.

Yechish: $x=y=0$ da $f(0)=0$ bo'ladi. Funksional tenglamada $x=0$ bo'lsa,

$$f(y)+f(-y)+f(0)+f(y)=3y^2 \Rightarrow 2f(y)+f(-y)=3y^2 \quad (4)$$

(4) da y ni $-y$ ga almashtiramiz:

$$2f(-y)+f(y)=3y^2 \Rightarrow f(y)+2f(-y)=3y^2 \quad (5)$$

(4) va (5) dan

$$\begin{cases} 2f(y)+f(-y)=3y^2 \\ f(y)+2f(-y)=3y^2 \end{cases}\Big|\cdot 2 \Rightarrow 4f(y)-f(y)=3y^2 \Rightarrow f(y)=y^2.$$

$f(y)=y^2$ da y ni x ga almashtirsak, $f(x)=x^2$ bo'ladi.

Javob: $f(x)=x^2$.

Mustaqil yechish uchun mashqlar

2.5-misol. Funksional tenglamalarni yeching.

1. $f(x+y)+f(x-y)=6(x^2+y^2)$.
2. $f(x+y)+f(x-y)=4xy$ tenglikni qanoatlantiruvchi $f(x)$ funksiyani toping.
3. $2f(x+2y)+f(x)=f(x+y)(2e^y+e^{-y})$ funksional tenglamani yeching.
4. Barcha haqiqiy x va y larda $f(x+y)=x+yf(x)+(1-x)y$ tenglikni qanoatlantiruvchi $f(x)$ funksiyalarni toping.

5. Haqiqiy sonlarda berilgan va $f(xf(y))=f(xy)+x$ shartni qanoatlantiruvchi barcha $f(x)$ funksiyalarni toping.

6. $f(xy)=y^k f(x)$ tenglikni qanoatlantiruvchi $f(x)$ funksiyalarni toping.

7. $f(x+y)+2f(x-y)=3f(x)-y$ tenglikni qanoatlantiruvchi $f(x)$ funksiyani toping.

8. $f(x+y)+2f(x-y)+f(x)+2f(y)=4x+y$ tenglikni qanoatlantiruvchi $f(x)$ funksiyani toping.

9. $f(x+y)+f(x-y)-2f(x)(1+y)=2xy(3y-x^2)$ funksional tenglamani yeching.

10. $f(x+y)-f(x-y)+f(x)=4xy+x^2$ funksional tenglamani yeching.

2.3. Funksional tenglamalarni Koshi usulida yechish

Funksional tenglamalarni Koshi isbotlagan ba'zi turlarini ko'rib o'taylik:

$$f(x+y)=f(x)+f(y) \qquad (1)$$

$$f(x+y)=f(x)\cdot f(y) \qquad (2)$$

$$f(x\cdot y)=f(x)+f(y) \qquad (3)$$

$$f(x\cdot y)=f(x)\cdot f(y) \qquad (4)$$

Bu tenglamalarni qanoatlantiruvchi funksiyalar uzluksiz. Ko'rinib turibdiki, to'rttala tenglamaning barchasini $f(x)\equiv 0$ qanoatlantirishi aniq. Biz quyida yuqoridagi tenglamalarning $f(x)\equiv 0$ dan farqli bo'lgan barcha yechimlarini topamiz.

Funksiyaning uzluksizligi ta'rifini eslab o'taylik.

Ta'rif. $f(x)$ funksiya x_0 nuqtada uzluksiz deyiladi agar:

1) $x_0 \in D(f)$, ya'ni x_0 nuqta funksiyaning aniqlanish sohasiga tegishli;

2) $\lim\limits_{x \to x_0} f(x)$ mavjud va $\lim\limits_{x \to x_0} f(x) = f(x_0)$ bo'lsa.

1. $f(x+y) = f(x) + f(y)$ funksional tenglamaning uzluksiz yechimi $f(x) = kx$ ko'rinishida bo'ladi.

Isbot. Biz quyidagi tasdiqlarni isbotlaymiz:

1) $f(0) = 0$.

$f(x+y) = f(x) + f(y)$ tenglamada $x = y = 0$ bo'lganda $f(0) = 2f(0)$ bo'lib, bundan $f(0) = 0$ ekanligi kelib chiqadi.

2) $f(x)$ funksiya toq, ya'ni $f(-x) = -f(x)$. Haqiqatan ham,
$f(-x) = [f(-x) + f(x)] - f(x) = f(-x+x) - f(x) = f(0) - f(x) = -f(x)$.

Tenglamani yechimini $\forall x, y \in Q$ uchun isbotlaymiz.

Dastlab $x \in N$ uchun isbotlaymiz. y ni $x, 2x, 3x, \ldots$ ga teng qiymatlarida

$$y = x;\ f(2x) = f(x+x) = f(x) + f(x) = 2f(x);$$
$$y = 2x;\ f(3x) = f(x+2x) = f(x) + f(2x) = 3f(x);$$
$$y = 3x;\ f(4x) = f(x+3x) = f(x) + f(3x) = 4f(x);$$

....

Demak,

$$f(n \cdot x) = n \cdot f(x) \quad (5)$$

qonuniyatni topdik. Buni matematik induksiya usulida isbotlaymiz.

$n = 1$ da tenglikning to'g'riligi ravshan.

$n = k$ uchun $f(k \cdot x) = k \cdot f(x)$ to'g'ri deb faraz qilamiz.

$n = k+1$ uchun to'g'riligini isbotlaymiz:

$f((k+1) \cdot x) = f(x+kx) = f(x) + f(kx) = f(x) + kf(x) = (k+1)f(x)$.

Bundan $\forall n \in N$ uchun $f(n \cdot x) = n \cdot f(x)$ ekanligi kelib chiqadi.

(5) dan $x = 1$ da $f(n) = f(n \cdot 1) = n \cdot f(1)$ yoki $f(n) = k \cdot n$ (6)
bu yerda $k = f(1)$. Demak, $\forall x \in N$ uchun $f(x) = kx$.

Endi $x, y \in Q$ uchun isbotlaymiz. $r = \frac{m}{n}$ – musbat ratsional bo'lsin. U holda $f(m) = nf\left(\frac{m}{n}\right)$ yoki (6) tenglikni e'tiborga olsak, $f\left(\frac{m}{n}\right) = \frac{1}{n}f(m) = k \cdot \frac{m}{n}$ o'rinli. Demak, $f(x) = kx$ tenglik barcha $x \in Q^+$ lar uchun o'rinli.

$f(x)$ funksiyaning toqligidan $f\left(\frac{m}{n}\right) = -f\left(-\frac{m}{n}\right)$ va $f\left(-\frac{m}{n}\right) = k \cdot \left(-\frac{m}{n}\right)$.

Natijada $f(x) = kx$ tenglik barcha $x \in Q^-$ lar uchun o'rinli.

Demak, $f(x) = kx$ tenglikni barcha $x \in Q$ lar uchun bajarilishi kelib chiqadi.

Endi irratsional x lar uchun yuqoridagi tenglik o'rinli bo'lishini isbotlaymiz. x ixtiyoriy irratsional son bo'lsin. U holda x ga intiluvchi

$$r_1, r_2, \ldots, r_n, \ldots$$

ratsional sonlar ketma-ketligi mavjud bo'ladi. Yuqoridagi isbotdan

$$f(r_n) = k \cdot r_n \quad (n = 1, 2, 3, \ldots)$$

o'rinli. Bu yerda $n \to \infty$ da limitga o'tsak,

$$\lim_{n\to\infty} f(r_n) = \lim_{n\to\infty}(kr_n).$$

Tenglikning o'ng tomoni uchun $\lim_{n\to\infty}(kr_n) = kx$. Chap tomoni uchun $f(x)$ funksiyaning uzluksizligidan

$$\lim_{n\to\infty} f(r_n) = f\left(\lim_{n\to\infty} r_n\right) = f(x)$$

o'rinli. Bundan $\forall x \in R$ uchun $f(x) = kx$ ekanligi kelib chiqadi.

2. $f(x+y)=f(x)\cdot f(y)$ funksional tenglamani qanoatlantiruvchi uzluksiz funksiyalar $f(x)=a^x$ $(a>0,\ a\neq 1)$ ko'rinishida bo'ladi.

Bu yerda $f(x)\equiv 0$ dan tashqari barcha yechimlar nazarda tutiladi.

Isbot. $f(x)$ barcha haqiqiy x larda aniqlangan va uzluksiz. $f(x)\equiv 0$ yechimlarni chiqarib tashlaymiz. U holda biror $x=x_0$ uchun $f(x_0)\neq 0$.

(2) ga $y=x_0-x$ ni qo'yamiz va $f(x)\cdot f(x_0-x)=$ $=f(x_0)\neq 0$ bo'ladi. Bu $f(x)$ funksiya x ning ixtiyoriy qiymatida noldan farqliligini bildiradi.

(2) da x va y ni $\frac{x}{2}$ ga almashtiramiz va

$$f(x)=\left[f\left(\frac{x}{2}\right)\right]^2$$

ni hosil qilamiz. Oxirgi tenglikdan x ning barcha qiymatlarida $f(x)>0$ ekanligi kelib chiqadi. (2) tenglikning ikkala tomonini e asosga ko'ra logarifmlaymiz:

$$\ln f(x+y)=\ln f(x)+\ln f(y).$$

Bundan $\varphi(x)=\ln f(x)$ belgilash kiritsak, $\phi(x+y)=$ $=\phi(x)+\phi(y)$ Koshining (1) tenglamasiga kelamiz. $\varphi(x)$ funksiya uzluksiz, chunki uzluksiz funksiyalar kompozitsiyasi ham uzluksiz. Natijada

$$\varphi(x)=\ln f(x)=kx \Rightarrow f(x)=e^{kx}=(e^k)^x=a^x.$$

Bu yerda $k=const$, $a=e^k$.

3. $f(xy)=f(x)+f(y)$ funksional tenglamaning barcha musbat x va y lar uchun aniqlangan uzluksiz yechimi $f(x)=\log_a x$ $(a>0,\ a\neq 1)$ ko'rinishida bo'ladi.

Isbot. $u\in(-\infty;\ \infty)$ aniqlangan $x=e^u$ va $\varphi(u)=f(e^u)$ funksiyalarni kiritamiz, chunki $x>0$. Bundan

$$u = \ln x \text{ va } f(x) = \varphi(\ln x).$$

U holda $\varphi(x)$ funksiya (1) tenglamani qanoatlantiradi, ya'ni

$$\varphi(u+v) = f(e^{u+v}) = f(e^u \cdot e^v) = f(e^u) + f(e^v) = \varphi(u) + \varphi(v).$$

Bundan $\varphi(u) = ku$ va $f(x) = \varphi(\ln x) = k \ln x$ hosil qilamiz. Natijada

$$f(x) = \log_a x \ (a > 0, a \neq 1), \ a = e^{1/k}.$$

4. $f(x \cdot y) = f(x) \cdot f(y), \ x > 0, \ y > 0$ funksional tenglamani $f(x) = x^p$ ko'rinishidagi uzluksiz funksiyalar qanoatlantiradi.

Isbot. $x > 0$ dan $x = e^u$ va $\varphi(u) = f(e^u)$ funksiyalarni kiritamiz.

Bundan $u = \ln x$ va $f(x) = \varphi(\ln x)$. U holda $\varphi(x)$ funksiya (3) tenglamani qanoatlantiradi, ya'ni

$$\varphi(u+v) = f(e^{u+v}) = f(e^u \cdot e^v) = f(e^u) \cdot f(e^v) = \varphi(u) \cdot \varphi(v).$$

Oxirgi tenglikdan $\varphi(x)$ funksiya Koshining (3) tenglamasini qanoatlantirishi kelib chiqadi va $\varphi(u) = a^u, u > 0$ bo'ladi. Bundan

$$f(x) = \varphi(\ln x) = a^{\ln x} = x^{\ln a} = x^p, \ p = \ln a.$$

Koshi tenglamalarini qo'llanilishiga doir misollar ko'rib o'taylik.

1-misol. $f\left(\frac{x+y}{2}\right) = \frac{f(x)+f(y)}{2}$ tenglikni qanoatlantiruvchi $f(x)$ funksiyani toping.

Yechish: Bu tenglama *Yensen* tenglamasi deb ataladi.

$$f\left(\frac{x+y}{2}\right) = \frac{f(x)+f(y)}{2} \quad \text{(A)}$$

x ni $x+y$ va y ni 0 ga almashtiramiz:

$$f\left(\frac{x+y}{2}\right) = \frac{f(x+y)+f(0)}{2} \quad \text{(B)}$$

(A) va (B) ning o'ng qismlarini tenglashtiramiz.

$$\frac{f(x+y)+f(0)}{2}=\frac{f(x)+f(y)}{2} \text{ dan}$$

$$f(x+y)=f(x)+f(y)-b,\ b=f(0).$$

Oxirgi tenglamada $\varphi(x)=f(x)-b$ almashtirish bajarsak, $\varphi(x+y)=\varphi(x)+\varphi(y)$ Koshi tenglamasi hosil bo‘ladi va $\varphi(x)=kx$ o‘rinli. Bundan $f(x)=\varphi(x)+b=kx+b$ ni topamiz.

Javob: $f(x)=kx+b$.

2-misol. $f(x+y)=f(x)+f(y)+2xy$ funksional tenglamaning uzluksiz yechimini toping.

Yechish: $g(x)=f(x)-x^2$ yordamchi funksiyani kiritamiz. Bundan $f(x)=g(x)+x^2$ ni topib, tenglamaning dastlabki ko‘rinishiga qo‘yamiz. Natijada

$$g(x+y)+(x+y)^2=g(x)+x^2+g(y)+y^2+2xy$$
$$g(x+y)+(x+y)^2=g(x)+g(y)+(x+y)^2$$
$$g(x+y)=g(x)+g(y)$$

Oxirgi tenglama Koshining (1) tenglamasini ifodalaydi va $g(x)=kx$ ni topamiz. Berilgan tenglamaning yechimi

$$f(x)=g(x)+x^2=x^2+kx.$$

Javob: $f(x)=x^2+kx$.

3-misol. $f\left(\frac{x+y}{2}\right)=\sqrt{f(x)\cdot f(y)}$ tenglamani yeching.

Yechish: $f\left(\frac{x+y}{2}\right)=\sqrt{f(x)\cdot f(y)}$ (C)

(C) da x ni $x+y$ va y ni 0 ga almashtiramiz:

$$f\left(\frac{x+y}{2}\right)=\sqrt{f(x+y)\cdot f(0)} \quad \text{(D)}$$

(C) va (D) ni o‘ng tomonlarini tenglashtiramiz:

$$\sqrt{f(x+y)\cdot f(0)}=\sqrt{f(x)\cdot f(y)}$$
$$f(x+y)\cdot f(0)=f(x)\cdot f(y)$$
$$f(x+y)=\frac{f(x)}{C}\cdot f(y),\ C=f(0)$$

$\varphi(x)=\frac{f(x)}{C}$ funksiyani kiritsak, oxirgi tenglikdan $\phi(x+y)=$ $=\phi(x)\cdot\phi(y)$ Koshi tenglamasini hosil qilamiz. Bu tenglamaning yechimi $\varphi(x)=a^x$.

Belgilashga qaytamiz:

$$\varphi(x)=\frac{f(x)}{C}\Rightarrow f(x)=C\varphi(x)=Ca^x.$$

Javob: $f(x)=Ca^x$.

Mustaqil yechish uchun mashqlar

2.6-misol. Funksional tenglamalarni yeching.

1. Barcha x, y ratsional sonlar uchun $f(x+y)=f(x)+f(y)$ va $f(10)=-\pi$ bo'lsa, $f\left(-\frac{2}{7}\right)$ ni toping.

2. Barcha x, y ratsional sonlar uchun $f(x-y)=f(x)-f(y)$ va $f(6)=-\sqrt{3}$ bo'lsa, $f\left(-\frac{5}{4}\right)$ ni toping.

3. Ixtiyoriy musbat x, y sonlarda $f(x)$ funksiya $f(x\cdot y)=$ $=f(x)+f(y)$ tenglikni qanoatlantiradi. Agar $f\left(\frac{1}{2013}\right)=1$ bo'lsa, $f(2013)$ ni toping.

2.4. Uzluksiz funksiyalar sinfida o'zgaruvchilari erksiz funksional tenglamalarni yechish

Yuqorida biz funksiyalarning uzluksizligi shartlarini ko'rib o'tdik. Endi o'zgaruvchilari erksiz funksional tenglamalarni uzluksizlik sharti asosida yechishni o'rganamiz.

1-misol. Ushbu $f(f(x))=f(x)+x$ shartni qanoatlantiruvchi $y=f(x)$ uzluksiz funksiyani toping.

Yechish: Bu funksiyaning xossalarini ko'rib chiqamiz:

1) $f(0)=0$ bo'ladi.

Haqiqatan, agar $f(0)=c$ deb olsak, u holda

$$f(c)=f(f(0))=f(0)+0=f(0)=c$$

bo'ladi. Bundan va funksiyaning berilishidan

$$f(c)=f(f(c))=f(c)+c$$

tenglikka, ya'ni $c=0$ ga kelamiz.

2) agar $x_1 \neq x_2$ bo'lsa, $f(x_1)\neq f(x_2)$ bo'ladi.

Haqiqatan, agar $f(x_1)=f(x_2)$ bo'lsa, u holda $f(f(x_1))=$ $=f(f(x_2))$ bo'ladi. Bundan esa $f(x_1)+x_1=f(x_2)+x_2$, ya'ni $x_1=x_2$ kelib chiqadi.

Bu xossalardan $f(x)$ funksiya $f(x)=ax$ ko'rinishda bo'lishi kerakligi xulosa qilinadi. U holda $f(f(x))=a^2x$ bo'lib, masalada berilgan bog'lanish quyidagi ko'rinishga keladi:

$$a^2x=ax+x$$

Endi $a^2-a-1=0$ tenglamani yechib, $a_{1,2}=\frac{1\pm\sqrt{5}}{2}$ ildizlarini topamiz va $y=\frac{1-\sqrt{5}}{2}x$ va $y=\frac{1+\sqrt{5}}{2}x$ funksiyalar masala yechimi bo'ladi.

Javob: $y=\frac{1\pm\sqrt{5}}{2}x$.

2-misol. Haqiqiy sonlar to'plamida berilgan va 0 nuqtada uzluksiz $f(x)$ funksiya ixtiyoriy $x\in R$ da $2f(2x)=f(x)+x$ tenglikni qanoatlantiradi. Barcha $f(x)$ funksiyalarni toping.

Yechish: $f(x)$ funksiya yuqoridagi shartlarni qanoatlantirsin. Berilgan tenglamani $f(2x)=\frac{1}{2}f(x)+\frac{x}{2}$ ko'rinishida yozib olamiz. x ga ketma-ket

$$\frac{x}{2},\frac{x}{4},\frac{x}{8},\dots,\frac{x}{2^n},\dots$$

qiymatlarni bersak,

$$f(x)=\frac{1}{2}f\left(\frac{x}{2}\right)+\frac{x}{4};$$

$$f\left(\frac{x}{2}\right)=\frac{1}{2}f\left(\frac{x}{4}\right)+\frac{x}{8}\Rightarrow f(x)=\frac{1}{2}\left(\frac{1}{2}f\left(\frac{x}{4}\right)+\frac{x}{8}\right)+\frac{x}{4}=\frac{1}{4}f\left(\frac{x}{4}\right)+\frac{x}{4}+\frac{x}{16};$$

$$=f\left(\frac{x}{4}\right)=\frac{1}{2}f\left(\frac{x}{8}\right)+\frac{x}{16}\Rightarrow f(x)=\frac{1}{4}\left(\frac{1}{2}f\left(\frac{x}{8}\right)+\frac{x}{16}\right)+\frac{x}{4}+\frac{x}{16}=$$

$$=\frac{1}{8}f\left(\frac{x}{8}\right)+\frac{x}{4}+\frac{x}{16}+\frac{x}{64};\dots;f(x)=\frac{1}{2^n}f\left(\frac{x}{2^n}\right)+\frac{x}{4}+\frac{x}{16}+\frac{x}{64}+\dots+\frac{x}{4^n}.$$

$\frac{x}{4}+\frac{x}{16}+\frac{x}{64}+\dots+\frac{x}{4^n}$ yig'indiga geometrik progressiyaning dastlabki n ta hadi yig'indisi formulasini qo'llaymiz:

$$\frac{x}{4}+\frac{x}{16}+\frac{x}{64}+\dots+\frac{x}{4^n}=\frac{x}{4}\left(1+\frac{1}{4}+\dots+\frac{1}{4^{n-1}}\right)=\frac{x}{4}\cdot\frac{1-\frac{1}{4^n}}{1-\frac{1}{4}}=\frac{x}{3}\left(1-\frac{1}{4^n}\right).$$

U holda $f(x)=\frac{1}{2^n}f\left(\frac{x}{2^n}\right)+\frac{x}{3}\left(1-\frac{1}{4^n}\right)$ bo'ladi. Bundan $n\to\infty$ limitga o'tib, $f(x)$ funksiyani uzluksizligini e'tiborga olsak,

$$f(x)=\lim_{n\to\infty}\left(\frac{1}{2^n}\right)\lim_{n\to\infty}f\left(\frac{x}{2^n}\right)+\frac{x}{3}\lim_{n\to\infty}\left(1-\frac{1}{4^n}\right)=0+\frac{x}{3}(1-0)=\frac{x}{3}.$$

Bevosita tekshirish orqali $f(x)=\frac{x}{3}$ tenglamaning yechimi ekanligini ko'ramiz.

Javob: $f(x)=\frac{x}{3}$.

3-misol. $f(x)$ funksiya R da aniqlangan va $x=0$ nuqta atrofida chegaralangan. $\forall x \in R$ uchun $f(x)-\frac{1}{2}f\left(\frac{x}{2}\right)=x-x^2$ funksional tenglamani qanoatlantiruvchi $f(x)$ funksiyani toping.

Yechish: **1-usul.** $x=0$ da $f(0)=0$ ekanligi ravshan. Har safar tenglikning ikkala tomonini 2 ga bo'lib, x ga ketma-ket

$$\frac{x}{2}, \frac{x}{4}, \frac{x}{8}, \dots, \frac{x}{2^n}, \dots$$

qiymatlarni beramiz.

$$f(x)-\frac{1}{2}f\left(\frac{x}{2}\right)=x-x^2;$$

$$\frac{1}{2}f\left(\frac{x}{2}\right)-\frac{1}{4}f\left(\frac{x}{4}\right)=\frac{1}{2}\left[\frac{x}{2}-\left(\frac{x}{2}\right)^2\right]=\frac{x}{4}-\frac{x^2}{8};$$

$$\frac{1}{4}f\left(\frac{x}{4}\right)-\frac{1}{8}f\left(\frac{x}{8}\right)=\frac{1}{4}\left[\frac{x}{4}-\left(\frac{x}{4}\right)^2\right]=\frac{x}{4^2}-\frac{x^2}{8^2};$$

$$\frac{1}{8}f\left(\frac{x}{8}\right)-\frac{1}{16}f\left(\frac{x}{16}\right)=\frac{1}{8}\left[\frac{x}{8}-\left(\frac{x}{8}\right)^2\right]=\frac{x}{4^3}-\frac{x^2}{8^3};$$

....

$$\frac{1}{2^n}f\left(\frac{x}{2^n}\right)-\frac{1}{2^{n+1}}f\left(\frac{x}{2^{n+1}}\right)=\frac{x}{4^n}-\frac{x^2}{8^n}$$

Hosil bo'lgan tengliklarni hadma – had qo'shsak,

$$f(x)-\frac{1}{2^{n+1}}f\left(\frac{x}{2^{n-1}}\right)=x\left(1+\frac{1}{4}+\frac{1}{4^2}+\dots+\frac{1}{4^n}\right)-x^2\left(1+\frac{1}{8}+\frac{1}{8^2}+\dots+\frac{1}{8^n}\right)=$$

$$=\frac{4x}{3}\left(1-\frac{1}{4^{n+1}}\right)-\frac{8x^2}{7}\left(1-\frac{1}{8^{n+1}}\right).$$

Natijada $f(x)=\frac{1}{2^{n+1}}f\left(\frac{x}{2^{n+1}}\right)+\frac{4x}{3}\left(1-\frac{1}{4^{n+1}}\right)-\frac{8x^2}{7}\left(1-\frac{1}{8^{n+1}}\right)$.

$f(x)$ funksiyani uzluksizligini va $f(0)=0$ ekanligini e'tiborga olib, $n\to\infty$ da limitga o'tsak, $f(x)=\frac{4}{3}x-\frac{8}{7}x^2$ bo'ladi.

2-usul. Tenglamaning o‘ng tomoni kvadrat uchhaddan iborat. Bundan funksional tenglamaning yechimini $f(x)=ax^2+bx+c$ ko‘rinishida izlaymiz.

$$ax^2+bx+c-\frac{1}{2}\left[a\cdot\left(\frac{x}{2}\right)^2+b\cdot\frac{x}{2}+c\right]=x-x^2,\quad \frac{7a}{8}x^2+\frac{3b}{4}x+\frac{c}{2}=$$

$$=-x^2+x+0\Rightarrow a=-\frac{8}{7};\ b=\frac{4}{3};\ c=0\Rightarrow f(x)=\frac{4}{3}x-\frac{8}{7}x^2$$

Javob: $f(x)=\frac{4}{3}x-\frac{8}{7}x^2$.

3-misol. $f(x)$ funksiya R da uzluksiz va $f(x)+f\left(\frac{2}{3}x\right)=x$ tenglikni qanoatlantiradi. $f(x)$ funksiyani toping.

Yechish: $f(0)=0$ ekanligi ravshan. x ga ketma-ket

$$\frac{2}{3}x,\ \frac{4}{9}x,\ \frac{8}{27}x,\ \ldots,\ \left(\frac{2}{3}\right)^n x,\ \ldots$$

qiymatlarni bersak,

$$f(x)+f\left(\frac{2}{3}x\right)=x;$$

$$f\left(\frac{2}{3}x\right)+f\left(\frac{4}{9}x\right)=\frac{2}{3}x;$$

$$f\left(\frac{4}{9}x\right)+f\left(\frac{8}{27}x\right)=\frac{4}{9}x;$$

$$f\left(\frac{8}{27}x\right)+f\left(\frac{8}{27}x\right)=\frac{8}{27}x$$

....

$$f\left(\left(\frac{2}{3}\right)^n x\right)+f\left(\left(\frac{2}{3}\right)^{n+1} x\right)=\left(\frac{2}{3}\right)^n x$$

Juft o‘rinda turgan tengliklarning yig‘indisidan toq o‘rinda turgan tengliklar yig‘indisini chap va o‘ng tomonlarini mos ravishda ayiramiz. Natijada

$$f(x)-f\left(\left(\frac{2}{3}\right)^{n+1}x\right)=x\left(1-\frac{2}{3}+\frac{4}{9}-\frac{8}{27}+\cdots\right)=x\frac{1-\left(-\frac{2}{3}\right)^n}{1-\left(-\frac{2}{3}\right)}=\frac{3}{5}x\left(1-\left(-\frac{2}{3}\right)^n\right)$$

$$f(x)-f\left(\left(\frac{2}{3}\right)^{n+1}x\right)=\frac{3}{5}x\left(1-\left(-\frac{2}{3}\right)^{n}\right).$$

Oxirgi tenglikdan limitga o'tsak va $f(0)=0$ ekanligini hisobga olsak, $f(x)=\frac{3}{5}x$ bo'ladi.

Javob: $f(x)=\frac{3}{5}x$.

Mustaqil yechish uchun mashqlar

2.7-misol. Funksional tenglamalarni yeching.

1. $f(x)$ funksiya R da uzluksiz va $f(x)=f\left(\frac{x}{2}\right)$. $f(x)$ ni toping.

2. $f(x)$ funksiya R da uzluksiz va $f(x)=\frac{1}{3}f\left(\frac{x}{2}\right)+x$ tenglikni qanoatlantiradi. $f(x)$ funksiyani toping.

3. $f(x)$ funksiya R da aniqlangan va $x=0$ nuqta atrofida chegaralangan. $\forall x\in R$ uchun $f(x)-\frac{1}{4}f\left(\frac{x}{3}\right)=x^2$ funksional tenglamani qanoatlantiruvchi $f(x)$ funksiyani toping.

JAVOBLAR

1-§. CHIZIQLI TENGLAMALAR

1.

1.1-misol. 1. 376263 **2.** 45 **3.** 11 **4.** 6051 **5.** 6849 **6.** 2 **7.** 10 **8.** 18 **9.** 30

1.2-misol. 1.A **2.**C **3.**B

2.

2.1-misol. 1. 0,5 **2.** −2 **3.** 3 **4.** Cheksiz ko‘p yechimga ega. **5.** Cheksiz ko‘p yechimga ega. **6.** Cheksiz ko‘p yechimga ega. **7.** 6,3 **8.** 0,5 **9.** 1 **10.** 1 **11.** 319 **12.** 37 **13.** 6 **14.**Ø **15.**Ø **16.** −7

2.2-misol. 1.C 2.D 3.C 4.B 5.C 6.C 7.B 8.B 9.B 10.E 11.C 12.B 13.B 14.D 15.C 16.B

3.1-misol. 1.A 2.A 3.C 4.C 5.A 6.A 7. D **8.C 9.E 10.A 11.D 12.A 13.B**

2-§. KVADRAT TENGLAMALAR

1.

1.1-misol. 1. $\{\pm 7\}$ **2.** $\{\pm 11\}$ **3.** $\{\pm 2\sqrt{2}\}$ **4.** $\{\varnothing\}$ **5.** $\{\varnothing\}$ **6.** $\left\{\pm\frac{3}{4}\right\}$ **7.** $\left\{\pm\frac{3}{2}\right\}$ **8.** $\{\pm\sqrt{41}\}$ **9.** $\{0\}$ **10.** $\{-4; 12\}$ **11.** $\{-23; 7\}$ **12.** $\left\{-\frac{1}{4}; 1\frac{1}{4}\right\}$.

2.

2.1-misol. 1. $\left\{-1; -\frac{1}{2}\right\}$ **2.** $\left\{\frac{1}{2}; 3\right\}$ **3.** $\left\{\frac{3}{4}; 2\right\}$ **4.** $\left\{-\frac{1}{2}; 4\right\}$

5. $\left\{-1; \frac{1}{3}\right\}$ **6.** $\left\{-\frac{1}{6}; 1\right\}$ **7.** $\{-3; -1\}$ **8.** $\{1; 7\}$

9. $\left\{-\frac{1}{4}\right\}$ **10.** $\left\{\frac{3}{5}\right\}$ **11.** $\{-5\}$ **12.** $\left\{\frac{1}{3}\right\}$ **13.** $\left\{-\frac{1}{2}\right\}$

14. $\{\varnothing\}$ **15.** $\{\varnothing\}$ **16.** $\{\varnothing\}$ **17.** $\{\varnothing\}$ **18.** $\left\{1 \pm \frac{1}{2}\sqrt{5}\right\}$

19. $\left\{\frac{1 \pm \sqrt{17}}{6}\right\}$ **20.** $\left\{\frac{-2 \pm \sqrt{7}}{3}\right\}$

2.2-misol. 1. $\{-8; 9\}$ **2.** $\{-8; 7\}$ **3.** $\left\{1 \pm \frac{1}{2}\sqrt{10}\right\}$ **4.** $\left\{\frac{1}{5}; 1\right\}$

5. $\left\{-\frac{3}{2}; -\frac{2}{3}\right\}$ **6.** $\{-11; 7\}$ **7.** $\{-3; 0{,}6\}$

2.3-misol. 1.B 2.B 3.E 4.C 5.A 6.D 7.C 8.D

3.

3.1-misol. 1. $\{0\}$ **2.** $\{-4; 4\}$ **3.** $\{-3\sqrt{3}; 3\sqrt{3}\}$ **4.** $\{\varnothing\}$

5. $\{-2; 2\}$ **6.** $\left\{-\frac{5}{4}; \frac{5}{4}\right\}$ **7.** $\{-8; 8\}$ **8.** $\left\{-\frac{7}{4}; \frac{7}{4}\right\}$

9. $\{-20; 20\}$ **10.** $\left\{-\frac{4}{3}; \frac{4}{3}\right\}$

3.2-misol. 1. $\{0; 7\}$ **2.** $\{0; -10\}$ **3.** $\left\{0; \frac{9}{4}\right\}$ **4.** $\left\{0; \frac{1}{9}\right\}$

5. $\left\{0; \frac{3}{4}\right\}$ **6.** $\left\{0; \frac{25}{16}\right\}$ **7.** $\{0; 10\}$ **8.** $\{0; 0{,}01\}$.

4.

4.1-misol. **1.** $2\frac{17}{26}$ **2.** $17\frac{1}{4}$ **3.** $59\frac{1}{8}$ **4.** $213\frac{1}{16}$

5. $\frac{11\sqrt{17}}{4}$, agar $x_1 > x_2$ bo'lsa; $-\frac{11\sqrt{7}}{4}$, agar $x_1 < x_2$ bo'lsa.

6. $\frac{95\sqrt{17}}{8}$, agar $x_1 > x_2$ bo'lsa; $-\frac{95\sqrt{17}}{8}$, agar $x_1 < x_2$ bo'lsa.

7. $\frac{759\sqrt{17}}{16}$, agar $x_1 > x_2$ bo'lsa; $-\frac{759\sqrt{17}}{16}$, agar $x_1 < x_2$ bo'lsa.

4.2-misol. **1.D** **2.B** **3.C** **4.D**

5.

5.1-misol. **1.** $\{-5;1\}$ **2.** $\{-1;7\}$ **3.** $\{-10;4\}$ **4.** $\{-1;9\}$
5. $\{9\}$ **6.** $\{-11\}$ **7.** $\{\varnothing\}$ **8.** $\{\varnothing\}$.

5.2-misol. **1.** $\{\sqrt{3} \pm 2\}$ **2.** $\{\sqrt{5} \pm 2\}$ **3.** $\{-2\sqrt{2};\sqrt{2}\}$
4. $\{2(\sqrt{7} \pm \sqrt{6})\}$ **5.** $\{1+\sqrt{2} \pm \sqrt{3}\}$

6.

6.2-misol. **1.C** **2.D** **3.A** **4.C** **5.E** **6.B** **7.B** **8.A** **9.D**
10.D **11. B** **12.C** **13.D** **14.B** **15.B** **16.A**

7.

7.1-misol. **1.** $x^2 - x - 12 = 0$ **2.** $x^2 - 11x + 30 = 0$
3. $x^2 + 15x + 56 = 0$ **4.** $x^2 - 3x - 18 = 0$ **5.** $x^2 - 5x + 6 = 0$
6. $x^2 - 2x - 80 = 0$

7.2-misol. **1.A** **2.C** **3.D** **4.D** **5.B** **6.E** **7.A** **8.D**

8.

8.1-misol. **1.** $(x+4)(2x-1)$ **2.** $(x+1)(6x+1)$ **3.** Ko'paytuvchilarga ajralmaydi. **4.** $(4x+1)^2$ **5.** $(x-1)(x+5)$

6. $(x-3)(x+8)$ **7.** $(x-6)(x+7)$ **8.** $(x+2)^2$ **9.** Ko‘paytuvchilarga ajralmaydi. **10.** $(x+4)(x+7)$ **11.** $(x-7)(x+8)$ **12.** $(x+7)(x-8)$.

8.2-misol. **1.** $\frac{1}{x+7}$ **2.** $\frac{x-2}{2x-1}$ **3.** $\frac{x-9}{x+8}$ **4.** $-\frac{x-9}{x-5}$ **5.** $\frac{2x+1}{3x-2}$ **6.** $\frac{x+7}{x(x+3)}$

8.3-misol. **1.**C **2.**B **3.**D **4.**B **5.**A **6.**D **7.**E **8.**E **9.**B **10.**A **11.**A **12.**D **13.**D **14.**C

3-§. KVADRAT TENGLAMAGA KELTIRILADIGAN TENGLAMALAR

1.

1.1-misol. **1.** $\left\{\pm\frac{\sqrt{2}}{2};\pm\sqrt{2}\right\}$ **2.** $\left\{\pm\frac{\sqrt{2}}{2};\pm 3\right\}$ **3.** $\{\pm 1;\pm 7\}$ **4.** $\{\pm 3;\pm 4\}$ **5.** $\{\pm 2;\pm 5\}$ **6.** $\{\pm 1;\pm 2\}$ **7.** $\{\pm\sqrt{2};\pm 3\}$ **8.** $\{\pm 2\}$ **9.** $\{\pm 2\}$ **10.** $\{\varnothing\}$

1.2-misol. **1.**B **2.**E **3.**C **4.**B **5.**D **6.**B **7.**D **8.**B

2.

2.1-misol. **1.** $\{\pm 1\}$ **2.** $\left\{-1;\frac{-3\pm\sqrt{5}}{2}\right\}$ **3.** $\{\pm 1; 4\}$ **4.** $\{1;2;3\}$ **5.** $\{2;3\}$ **6.** $\{-2;1\}$ **7.** $\{-2;1\}$ **8.** $\{1\}$ **9.** $\left\{-\frac{3}{5};\frac{3\pm\sqrt{109}}{10}\right\}$ **10.** $\left\{-\frac{5}{3};\frac{2}{3}\right\}$ **11.** $\{-\sqrt{2}\}$ **12.** $\left\{-\frac{2}{3};-\frac{1}{2};3\right\}$

15. $\left\{2;\ 4;\ \frac{5\pm\sqrt{5}}{2}\right\}$

2.2-misol. 1. $\{-1\}$ **2.** $\{\pm1\}$ **3.** $\{\pm1; 2; 4\}$ **4.** $\left\{\pm\frac{1}{2}\right\}$ **5.** $\left\{\pm\frac{\sqrt{3}}{3}; -\frac{1}{2}; 2\right\}$

6. $\{1\}$ **7.** $\{\pm\sqrt{2}; \sqrt[3]{3}\}$ **8.** $\{-\sqrt{2}\pm1; \sqrt{2}\pm1; 2\}$ **9.** $\{-2{,}5; 1\}$

3.

3.1-misol. 1.B 2.D 3.A 4.B 5.B 6.B 7.A 8.A 9.E 10.C 11.B 12.D

4.

4.1-misol. 1. $\{-2; 1\}$ **2.** $\{1; \sqrt[3]{2}\}$ **3.** $\{\pm1\}$ **4.** $\left\{\pm\frac{\sqrt{2}}{2}; \pm1\right\}$.

4.2-misol. 1. $\{-4; 2\}$ **2.** $\left\{0; -1; \frac{-1\pm\sqrt{5}}{2}\right\}$ **3.** $\{-4; 2; 3; 9\}$

4. $\{1; 2; 3; 4\}$ **5.** $\{\pm1; -4; -6\}$ **6.** $\{-1; -2; -3; -4\}$

7. $\{3\pm2\sqrt{5}; 3\}$ **8.** $\{-1; -2\}$ **9.** $\left\{\frac{1}{2}\left(-7\pm\frac{\sqrt{3}}{3}\right); \frac{1}{2}(-7\pm\sqrt{2})\right\}$.

4.3-misol. 1. $\left\{-\frac{5}{4}; -\frac{1}{2}\right\}$ **2.** $\left\{\frac{-5\pm\sqrt{3}}{6}\right\}$ **3.** $\left\{\frac{-2\pm\sqrt{6}}{4}\right\}$

4. $\left\{-\frac{3}{2}; -\frac{5}{6}\right\}$

4.4-misol. 1. $\left\{\frac{1\pm\sqrt{5}}{2}; \frac{2\pm\sqrt{2}}{2}; 1; 2\right\}$ **2.** $\left\{a; b; \frac{a+b}{2}\right\}$ **3.** $\{2; 3\}$

4. $\left\{\frac{5}{2}\right\}$ **5.** $\left\{\frac{5}{3}; 15\right\}$ **6.** $\{-1\}$ **7.** $\left\{\frac{1\pm\sqrt{5}}{2}; \frac{1\pm\sqrt{57}}{4}; -\frac{4}{3}; 2\right\}$.

4.5-misol. 1.A 2.A 3.A 4.C 5.E 6.D 7.D 8.B

5.

5.1- misol. 1. $\{-6;1\}$ **2.** $\{-4;1\}$ **3.** $\{-3;2\}$

4. $\{-6;-2;-4\pm\sqrt{6}\}$ **5.** $\left\{\frac{-5\pm\sqrt{5+4\sqrt{2}}}{2}\right\}$ **6.** $\left\{\frac{-1\pm\sqrt{13}}{2}\right\}$

7. $\left\{\frac{1}{6};1;\frac{7\pm\sqrt{601}}{12}\right\}$ **8.** $\left\{-1\frac{5}{6};-\frac{1}{2};-\frac{1}{3};1\right\}$

9. $\left\{-25;\frac{12\pm\sqrt{70-6\sqrt{21}}}{2};\frac{12\pm\sqrt{70+6\sqrt{21}}}{2}\right\}$.

5.2-misol. 1.D **2.**C

6.

6.1-misol. 1. $\{-2;-1\}$ **2.** $\left\{\frac{-1\pm\sqrt{5}}{2};1\pm\sqrt{2}\right\}$.

6.2-misol. 1.A **2.**D

7.

7.1-misol. 1. $\left\{\frac{-35\pm\sqrt{265}}{4};-8;-\frac{15}{2}\right\}$ **2.** $\{5\pm\sqrt{17}\}$ **3.** $\{-4;5;-5\pm3\sqrt{5}\}$

8.

8.1-misol. 1. $\{2;3\}$ **2.** $\{-3;-5\}$ **3.** $\{1;3\}$ **4.** $\{-2\pm\sqrt{3\sqrt{2}-3}\}$
5. $\{a;b\}$

8.2-misol. 1. $\{-1;0;1\}$ **2.** $\{2;4\}$ **3.** $\{2;4\}$ **4.** $\{-2\}$

9.

9.1-misol. 1. $\left\{\frac{-\sqrt{2}\pm\sqrt{8\sqrt{2}-2}}{2}\right\}$ **2.** $\{1\pm\sqrt{3}\}$ **3.** $\left\{\frac{-1\pm\sqrt{13}}{2}\right\}$

4. $\left\{\sqrt{2};\frac{1\pm\sqrt{1+4\sqrt{2}}}{2}\right\}$. Ko'rsatma: $\sqrt{2}$ ga nisbatan kvadrat tenglama ko'rinishiga keltiring. **5.** $\left\{\frac{1}{\sqrt[3]{2}-1}\right\}$ **6.** $\left\{-1;-\frac{1}{3};\frac{1}{6};2\right\}$

7. $\{-4;-2\}$ **8.** $\left\{-\frac{3+\sqrt[3]{6}}{2}\right\}$ **9.** $\{4\}$ **10.** $\{-1;0\}$ **11.** $\left\{\sqrt[3]{\frac{3}{2}}\right\}$.

9.2-misol. 1. $\{25\}$ **2.** $\{55\}$ **3.** $\{1\}$ **4.** $\{3\}$ **5.** $\{7\}$

4-§. QAYTMA TENGLAMALAR

1.

1.1-misol. 1. $\left\{-1;\frac{1}{3};3\right\}$ **2.** $\{-1;3\pm2\sqrt{2}\}$ **3.** $\{-1\}$ **4.** $\{\pm1\}$

5. $\left\{-1;\frac{1}{2};2\right\}$ **6.** $\{-1\}$

1.2-misol. 1. $\left\{\frac{-4\pm\sqrt{7}}{3};\frac{1}{2};2\right\}$ **2.** $\left\{\frac{-3\pm\sqrt{5}}{2};\frac{1}{2};2\right\}$

3. $\left\{\frac{3\pm\sqrt{5}}{2}\right\}$ **4.** $\left\{-3;-\frac{1}{2};\frac{1}{3};2\right\}$ **5.** $\{1\}$ **6.** $\left\{1;\frac{-11\pm\sqrt{85}}{6}\right\}$

7. $\{1+\sqrt{3}\pm\sqrt{3+2\sqrt{3}}\}$ **8.** $\left\{2;-\frac{1}{2};-1\pm\sqrt{2}\right\}$

1.3-misol. 1. $\left\{\pm1;-2;-\frac{1}{2}\right\}$ **2.** $\{-1\}$ **3.** $\left\{\pm1;\frac{-3\pm\sqrt{5}}{2}\right\}$

4. $\left\{-1;\frac{-1+\sqrt{101}\pm\sqrt{38-2\sqrt{101}}}{8};\frac{-1-\sqrt{101}\pm\sqrt{38+2\sqrt{101}}}{8}\right\}$

5. $\left\{\pm2;\pm\frac{1}{2}\right\}$

1.4-misol. 1. $\{2\pm\sqrt{3};3\pm2\sqrt{2}\}$ **2.** $\{1\}$

3. $\left\{1;\frac{5+\sqrt{37}\pm\sqrt{46+10\sqrt{37}}}{4}\right\}$ **4.** $\left\{\frac{1}{2};2\right\}$

2.

2.1-misol. 1. $\left\{-1; -\frac{1}{3}; \frac{3\pm\sqrt{3}}{3}\right\}$ **2.** $\left\{-\frac{3}{2}; 2; \frac{9\pm\sqrt{129}}{4}\right\}$

3. $\left\{-2; \frac{3}{2}; \frac{-1\pm\sqrt{13}}{2}\right\}$ **4.** $\left\{-\frac{3}{4}; 4; \frac{-21\pm\sqrt{633}}{8}\right\}$

5. $\left\{-\frac{2}{3}; 1; \frac{-1\pm\sqrt{97}}{12}\right\}$ **6.** $\{-2; 1; 4\}$ **7.** $\{-3; -1\}$

2.2-misol. 1.B **2.**D **3.**A

5-§. KASR-RATSIONAL TENGLAMALAR

1.

1.1-misol. 1. $\{-20; 40\}$ **2.** $\{-4\}$ **3.** $\{9\}$ **4.** $\{2\}$ **5.** $\left\{-\frac{4}{3}\right\}$ **6.** $\{3\}$

7. $\{2; 4\}$ **8.** $\{20\}$ **9.** $\{1\}$ **10.** $\{\pm\sqrt{2}\}$ **11.** $\left\{1; \frac{1\pm\sqrt{33}}{4}\right\}$

12. $\{6\}$ **13.** $\{9\}$ **14.** $\{\pm\sqrt{3}\}$

1.2-misol. 1. $\{0; 2\}$ **2.** $\left\{0; 5; \frac{38}{11}\right\}$ **3.** $\{-4; 2\}$ **4.** $\{-5; 5\}$

1.3-misol. 1. $\{11\}$ **2.** $\{1\}$ **3.** $\{3\}$ **4.** $\left\{\frac{1}{10}\right\}$ **5.** 37.

2.

2.1-misol. 1. $\left\{\frac{5(9\pm\sqrt{69})}{9}\right\}$ **2.** $\{\pm 1\}$ **3.** $\{-1\}$ **4.** $\{-1\}$

5. $\left\{1; 4; \frac{7\pm\sqrt{33}}{4}\right\}$ **6.** $\{-3; -2\}$ **7.** $\{-1; 0\}$ **8.** $\{2\pm\sqrt{\sqrt{3}-1}\}$

9. $\left\{0; -3; \frac{-3\pm\sqrt{73}}{2}\right\}$ **10.** $\left\{-5; 1; \frac{-4\pm\sqrt{10}}{2}\right\}$

2.2-misol. **1.** $\left\{\frac{1}{2}; 2\right\}$ **2.** $\left\{\frac{1}{7}; 7\right\}$ **3.** $\left\{2; \frac{1}{2}; \frac{-9\pm\sqrt{65}}{4}\right\}$

4. $\left\{\frac{1}{2}; 2\right\}$ **5.** $\{-2; 6; 3\pm\sqrt{21}\}$ **6.** $\{-1; 3\}$ **7.** $\{-2; 1\}$

8. $\left\{\frac{1}{8}(5\pm\sqrt{33})\right\}$ **9.** $\left\{\frac{1}{3}(2\pm\sqrt{7})\right\}$ **10.** $\{1\}$ **11.** $\{-3\pm\sqrt{15}\}$

3.

3.1-misol. **1.** $\{1,2; 2,4\}$ **2.** $\left\{-2\pm\sqrt{\frac{15-\sqrt{145}}{10}}; -2\pm\sqrt{\frac{15+\sqrt{145}}{10}}\right\}$

3. $\{0; \pm 5\sqrt{2}\}$ **4.** $\left\{-\frac{33}{4}; 6\right\}$ **5.** $\{\pm\sqrt{2}; \pm\sqrt{3}\}$

3.2-misol. **1.** $\left\{-\frac{5}{4}; 5\right\}$ **2.** $\{0; 7\pm 2\sqrt{3}\}$ **3.** $\left\{\frac{1}{2}\left(-1\pm\sqrt{\frac{69}{5}}\right)\right\}$

4. $\{-5\}$ **5.** $\left\{\pm\frac{1}{2}; \pm 4\right\}$ **6.** $\left\{0; -3\pm\frac{2}{3}\sqrt{3}\right\}$

3.3-misol. **1.** $\left\{0; \frac{-5\pm\sqrt{3}}{2}\right\}$ **2.** $\left\{0; \frac{-5\pm\sqrt{3}}{2}\right\}$

3.4-misol. **1.** $\left\{\frac{-5\pm\sqrt{13}}{2}\right\}$ **2.** $\left\{\frac{-11\pm\sqrt{97}}{6}\right\}$ **3.** $\left\{\frac{7\pm\sqrt{31}}{6}; \frac{7\pm\sqrt{41}}{4}\right\}$

4. $\{3\pm\sqrt{2}\}$

3.5-misol. **1.** $\{5\pm\sqrt{10}; 10\pm\sqrt{85}\}$ **2.** $\{7\pm\sqrt{34}\}$.

4.

4.1-misol. **1.** $\left\{\frac{1\pm\sqrt{21}}{2}\right\}$ **2.** $\{1\pm\sqrt{19}\}$ **3.** $\{-1; 2\}$ **4.** $\left\{-\frac{5}{7}; 1\right\}$

5. $\left\{-\frac{2}{3}; 2\right\}$ **6.** $\{-1\pm\sqrt{3}; 2\}$ **7.** $\left\{\frac{\sqrt{2}-1\pm\sqrt{2\sqrt{2}-1}}{2}\right\}$

4.2-misol. 1. $\left\{\pm\frac{3\sqrt{11}}{11};\pm\frac{\sqrt{5}}{2}\right\}$. **2.** $\left\{\pm 3;\pm\frac{\sqrt{65}}{13}\right\}$

5.

5.1-misol. 1. $\{0;3;\pm\sqrt{3}\}$ **2.** $\left\{\pm\frac{\sqrt{35}}{7}\right\}$ **3.** $\{-1;1\}$ **4.** $\{-1;-2\pm\sqrt{3}\}$

5. $\{-1\pm\sqrt{7}\}$ **6.** $\{-4;-2\}$ **7.** $\left\{\frac{1\pm\sqrt{5}}{2}\right\}$ **8.** $\left\{-3\pm\sqrt{3};\frac{1\pm\sqrt{5}}{2}\right\}$

9. $\{2\}$.

5.2-misol. 1. $\left\{\frac{2}{3};3\right\}$ **2.** $\left\{\frac{4}{5};3\right\}$.

6.

6.1-misol. 1. $\{2\}$ **2.** $\{\varnothing\}$ **3.** $\{-3;1\}$ **4.** $\left\{0;19;9\frac{10}{19}\right\}$

5. $\{1;2\}$ **6.** $\{0;-1\}$ **7.** $\{\pm 5\}$.

7.

7.1-misol. 1.C 2.C 3.A 4.D 5.C 6.B 7.E 8.A 9.A 10.E 11.B 12.A 13.A 14.D 15.B 16.D 17.B 18.A 19.D 20.C 21.A 22.B

6-§. KO'PHADLAR

1.

1.2-misol. 1.B 2.C 3.A 4.B 5.A 6.B

3.

3.3-misol. $\frac{5}{x(x+20)}$.

3.5-misol. 1.B **2.**E **3.**D **4.**D

4.

4.1-misol. $a=-2$

4.2-misol. $a=24$

4.3-misol. 1.C **2.**A **3.**E. **4.**A

5.

5.1-misol. 1. $2x+1$. **2.** $-2x+2$. **3.** $a=3; b=-4$.

4. $p_1=-7, q_1=-1;\ p_2=-12, q_2=-2$. **6.** -4 va -10.

5.2-misol. 1.C. **2.**B **3.**A **4.**A **5.**C **6.**C **7.**B **8.**A **9.**C

5.3-misol. 1. $\{-1; 1; -2\}$ **2.** $\{-3; 2\}$ **3.** $\{-5; -3; 2\}$

4. $\{-4; -3; 1; 2\}$ **5.** $\{-3; -1; 2\}$

5.4-misol. 1. $\left\{\frac{2}{3}\right\}$ **2.** $\left\{-1; \frac{1}{2}\right\}$ **3.** $\left\{-2; -\frac{2}{3}; -\frac{1}{2}; 2\right\}$

4. $\{-1; \frac{1}{2}; 2\pm\sqrt{3}\}$

6.

6.1-misol. 1. $q(x)=5x^2-x+20;\ r=96$

2. $q(x)=4x^2-9x-9;\ r=0$

3. $q(x)=x^3+x^2+10x+30;\ r=136$

4. $q(x)=x^4+3x^2-x-1;\ r=-7$

5. $q(x)=x^5-3x^4+5x^3-14x^2+40x-120;\ r=365$.

6.2-misol. 1. $P(2)=0; P(3)=0; P(-3)=-120$ **2.** $P(4)=39$

3. $P(-4)=84$ **4.** $P(-2)=-8$ **5.** $P(3)=0$ **6.** $P(2)=67$

7. $P(4)=28559$.

7.

7.1-misol. **1.** $(x-1)(x+2)^2$ **2.** $(x^2-x\sqrt{2}+1)(x^2+x\sqrt{2}+1)$

3. $(x^2-4x+8)(x^2+4x+8)$ **4.** $(x^2-x\sqrt{3}+1)(x^2+x\sqrt{3}+1)$

5. $(x-2)(x-1)(x+1)(x+2)$ **6.** $(x^2-x+3)(x^2+x+3)$

7. $(x^2-x+1)(x^2+x+1)(x^4+x^2+1)$ **8.** $(x^2+x+1)(x^6-x^5+x^3-x^2+1)$

9. $(x^4-x^2+2)(x^4+x^2+2)$ **10.** $(x-1)(x+2)(x^2+x+5)$

11. $\left(x+\frac{3+\sqrt{5}}{2}\right)\left(x+\frac{3-\sqrt{5}}{2}\right)(x^2+3x+3)$

12. $12\left(x+\frac{11-\sqrt{217}}{24}\right)\left(x+\frac{11+\sqrt{217}}{24}\right)(12x^2+11x+3)$

13. $(x^2+x+1)(x^5-x^4+x^3-x+1)$

14. $(x-1)(x+1)(x^2+1)(x^2-x\sqrt{2}+1)(x^2+x\sqrt{2}+1)$

15. $(x^2-x+1)(x^3+x^2-1)$.

8.

8.1-misol. **1.** $(x+1)(x-2)(2x+1)$ **2.** $(x-3)(x-2)^3$

8.2-misol. $(x^2+2x+3)(x^2-4x+5)$.

8.3-misol. **1.** $a=310$ va $b=6889$ **2.** $a=\frac{7}{8}$ va $b=\frac{49}{64}$.

8.4-misol. $a_1=b_1=c_1=0$; $a_2=-3\sqrt{3}, b_2=9, c_2=-\sqrt{3}$; $a_3=3\sqrt{3}$, $b_3=9, c_3=\sqrt{3}$.

9.

9.1-misol. **1.** $x^2-6x+4=0$ **2.** $x^4-10x^2+1=0$ **3.** $x^3-5=0$

4. $x^3-6x^2+12x-5=0$ **5.** $(x+1)^4-2=0$.

9.2-misol. $x_{1,2}=3\pm\sqrt{2}, x_3=-1, x_4=2$.

9.3-misol. $x_{1,2}=1\pm\sqrt{2}$; $x_3=0$; $x_{4,5}=\pm1$.

9.3-misol. $x_3=-\frac{q}{ab}$.

9.5-misol. 1. yo‘q **2.** ha.

9.6-misol. Ko‘rsatma: $n+1$ ta $x=c_1, c_2, \ldots, c_{n+1}$ nuqtalarda ma’lum qiymatlarni qabul qiluvchi darajasi n dan oshmaydigan ko‘phadni

$$P(x)=b_0+b_1(x-c_1)+b_2(x-c_1)(x-c_2)+\cdots+b_n(x-c_1)(x-c_2)\ldots(x-c_{n+1})$$

ko‘rinishida qidirish maqsadga muvofiq bo‘ladi.

$c_1=0,\ c_2=1,\ c_3=2$ ni hisobga olsak, izlangan ko‘phad

$$P(x)=b_0+b_1(x-c_1)+b_2(x-c_1)(x-c_2)+b_3(x-c_1)(x-c_2)(x-c_3)$$

ko‘rinishida bo‘ladi. Bundan

$P(x)=b_0+b_1(x-0)+b_2(x-0)(x-1)+b_3(x-0)(x-1)(x-2)$ yoki

$P(x)=b_0+b_1x+b_2x(x-1)+b_3x(x-1)(x-2)$. Masala shartidan

$P(0)=b_0+b_1\cdot 0+b_2\cdot 0\cdot(0-1)+b_3\cdot 0\cdot(0-1)(0-2)=1\Rightarrow b_0=1$;

$P(1)=b_0+b_1+0+0=2\Rightarrow b_0+b_1=2\Rightarrow 1+b_1=2\Rightarrow b_1=1$;

$P(2)=b_0+2b_1+2b_2=6\Rightarrow 1+2+2b_2=6\Rightarrow b_2=1{,}5$.

Natijada $P(x)=1+x+1{,}5x(x-1)=1{,}5x^2-0{,}5x+1$.

1. $P(x)=1{,}5x^2-0{,}5x+1$ **2.** $P(x)=x^3+2x$.

10.

10.1-misol. 1. $\left\{\frac{-1\pm\sqrt{21}}{2}; \frac{-3\pm\sqrt{17}}{2}\right\}$ **2.** $\{-2\}$ **3.** $\{3\}$

4. $\{1; 11; 5\pm 2\sqrt{6}\}$ **5.** $\{1; 2001; 1000\pm 3\sqrt{111111}\}$.

11.

11.1-misol. 1.D **2.**D **3.**B **4.**C **5.**A **6.**B **7.**D **8.**C **9.**A **10.**D **11.**A **12.**C

11.2-misol. 1.A **2.**A **3.**B **4.**B **5.**D **6.**A **7.**A

11.3-misol. 1.B **2.**B **3.**A **4.**B **5.**A **6.**D **7.**C **8.**D

7-§. FUNKSIONAL TENGLAMALAR

1.

1.1-misol. **1.** $\frac{1-x^2}{1+x^2}$ **2.** $a-\frac{1}{a}$ **3.** x **4.** 1) $-\frac{1}{x}$ 2) $\frac{x-1}{x+1}$ 3) x

5. $\frac{x}{2^n}$ **6.** $x=-0{,}2$ **7.** 1 ta **8.** $x=-6\pm\sqrt[32]{6}$ **9.** $f(5)=34$

10. $g(x)=\frac{3x+7}{2x+6}$ **11.** $f(1999)=3997$.

1.2-misol. **1.C 2.D 3.B 4.B 5.D 6.B 7.C 8.D 9.D 10.C 11.A 12.A 13.C 14.A**

1.3-misol. **1.A 2.A 3.C 4.D 5.D 6.B 7.A 8.B 9.D**

2.

2.1-misol. **1.C 2.D 3.C 4.B 5.D 6.C 7.A 8.B 9.D 10.D 11.B**

2.2-misol. **1.** $f(x)=\left(\frac{x}{1-x}\right)^2,\ x\neq 1$ **2.** $f(x)=\frac{2x-2x^2}{(x-1)^2},\ x\neq 1$

3. $f(x)=\frac{5x+1}{5-3x}$ **4.** $f(x)=\frac{x+4}{3x-2},\ x\neq\frac{2}{3},\ x\neq 3$

5. $f(x)=x^3-x+1$ **6.** $f(x)=x^3-x-1$.

2.3-misol. **1.** $f(x)=\frac{1}{3}(x^2+2x-1)$ **2.** $f(x)=\frac{5+3x}{16\sqrt{x}}$

3. $f(x)=\frac{3}{4x}-\frac{x^2}{4}$ **4.** $f(x)=\frac{6x+3}{5x}$ **5.** $f(x)=\frac{c}{a-b}x+\frac{c}{a+b}$

6. $f(x)=\frac{1}{2}\left(x+1-\frac{1}{x}-\frac{1}{1-x}\right)$ **7.** $f(x)=2x+1$

8. $f(x)=\frac{4x-2}{x-1},\ x\neq\frac{1}{2},\ x\neq 1$ **9.** $f(x)=\frac{x^3-a^2x+a^3}{2x(x-a)}$

10. $f(x)=2x-1{,}4$

2.4-misol. **1.** $f(x)=\dfrac{x^2-4x+1}{1-x},\ g(x)=\dfrac{x^2-3x+1}{x-1},\ x\neq 1.$

2. $f(x)=\dfrac{x^2-2x-19}{2(x-7)},\ x\neq 7;\ g(x)=\dfrac{2(x-4)}{10-x},\ x\neq 10$

3. $f(x)=\dfrac{1}{12}(32x-29),\ g(x)=-\dfrac{1}{4}(4x+15).$

2.5-misol. **1.** $f(x)=3x^2$ **2.** $f(x)=x^2+C,\ C=f(0)$

3. $f(x)=Ce^x,\ C=f(0)$ **4.**. $f(x)=x$ **5.** $f(x)=c+\dfrac{x}{c}$

6. $f(x)=ax^k$ **7.** $f(x)=x+a$ **8.** $f(x)=x$ **9.** $f(x)=x^3$

10. $f(x)=x^2$

2.6-misol. **1.** $f\left(-\dfrac{2}{7}\right)=\dfrac{\pi}{35}$ **2.** $f\left(-\dfrac{5}{4}\right)=\dfrac{5\sqrt{3}}{24}$ **3.** $f(2013)=-1$

2.7-misol. **1.** $f(x)=C=const$ **2.** $f(x)=\dfrac{6}{5}x$ **3.** $f(x)=\dfrac{36}{35}x^2$

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MUNDARIJA

4-§. QAYTMA TENGLAMALAR

5-§. KASR-RATSIONAL TENGLAMALAR

6-§. KO'PHADLAR

7-§. FUNKSIONAL TENGLAMALAR