O’zbekiston Respublikasi qishloq va suv xo’jaligi vazirligi

Samarqand qishloq xo’jalik instituti

Oliy matematika

(Hosila va differentsial)

Uslubiy qo’llanma

Samarqand -2010

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Uslubiy qo'llanma «Oliy matematika» kursining funktsiyaning hosila va differentsiali bo'limlarini o'z ichiga olgan bo'lib, oliy o'quv yurtlarining nomatematikaviy fakultetlari talabalariga, kasb-hunar kollejlari va akademik litsey talabalariga qo'llanma sifatida tavsiya qilinadi. Bundan o'rta umumta'lim maktablarining yuqori sinf o'quvchilari ham foydalanishlari mumkin.

Ushbu qo'llanma oliy matematika kursining o'quv rejalari asosida tuzilgan.

Sultonov J.S. Oliy matematika / Hosila va differentsial/.- Samarqand, 2010

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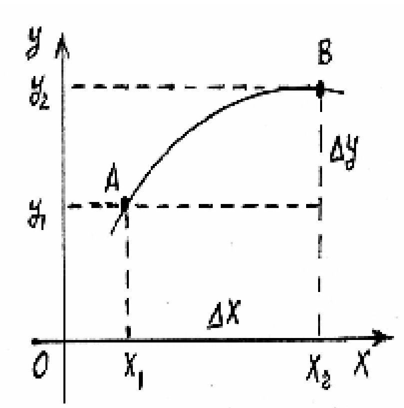
1§. Arnument va funktsiya orttirmasi

Faraz qilaylik, sonlar o'qida biror x_1 nuqta berilgan bo'lsin. Bu nuqta yangi x_2 nuqtaga ko'chirilsin. U holda, x_1 va x_2 nuqtalar orasida masofali farq hosil bo'ladi. Ana shu masofali farqning uzunligi $x_2 - x_1$ dan iborat bo'ladi. Bu farqni Δx bilan belgilasak,

$$x_2 - x_1 = \Delta x \qquad (1)$$

bo'ladi. Δx ni ***orttirma*** deb ataymiz.

Dekart koordinatalari sistemasida $y = f(x)$ funktsiyaning grafigi berilgan bo'lsin. Funktsiyaning ixtiyoriy A nuqtasining absissasi x_1 dan va ordinatasi y_1 dan iborat. Agar A nuqta grafik bo'ylab B nuqtagacha siljisa, B nuqtaning absissasi x_2, ordinatasi esa y_2 dan iborat bo'ladi.



U holda, absissadagi farq $x_2 - x_1 = \Delta x$, ordinatadagi farq esa $y_2 - y_1 = \Delta y$ dan iborat bo'ladi. Bundan quyidagicha mulohaza yuritish mumkin: funktsiyaning argumenti Δx orttirma olsa, funktsiya ham Δy orttirma oladi. Demak, $y = f(x)$ funktsiyaning orttirmasi argumentning Δx orttirmasi bilan aniqlanadi.

Ta'rif: $y = f(x)$ funktsiya uchun uning aniqlanish sohasidan olingan x_1 va x_2 argumentlari qiymatlarining ayirmasi ***argumentning orttirmasi*** deyiladi.

Ta'rif: $y = f(x)$ funktsiyaning qiymatlar sohasi (o'zgarish sohasi)dan olingan x_1 va x_2 argumentlariga mos $y_1 = f_1(x)$ va $y_2 = f_2(x)$ orasidagi ayirma ***funktsiyaning orttirmasi*** deyiladi.

Agar x argument Δx orttirma olgan bo'lsa, argumentning ortgan qiymati $x + \Delta x$, funktsiyaning unga mos qiymati esa quyidagidan iborat bo'ladi:

$$y+\Delta y=f(x+\Delta x).$$

$y=f(x)$ funktsiyaning orttirmasini topish uchun uning ortgan qiymatidan dastlabki qiymatini ayirish lozim, ya'ni

$$\begin{array}{r} y+\Delta y=f(x+\Delta x) \\ -\; y=f(x) \\ \hline \Delta y=f(x+\Delta x)-f(x) \end{array}$$

Misollar

1-misol. Agar $y=x+3$ funktsiya berilgan bo'lib, funktsiyaning argumenti o'z qiymatini $x_1=1$ dan $x_2=4{,}5$ ga o'zgartirsa, argument va funktsiya orttirmalarini toping.

Yechilishi: Dastlab x ning argument orttirmasi Δx ni topamiz:

$$y_1=f(x_1)=f(1)=1+3=4,$$

$$y_2=f(x_2)=f(4{,}5)=4{,}5+3=7{,}5.$$

U holda, funktsiya orttirmasi quyidagidan iborat bo'ladi:

$$\Delta y=y_2-y_2=f(x_2)-f(x_1)=7{,}5-4=3{,}5.$$

2-misol. $y=2x^2+x-6$ funktsiya berilgan bo'lsin. Agar x o'zgaruvchi o'z qiymatini $x_1=-2$ dan $x_2=0$ ga o'zgartirganligi ma'lum bo'lsa, Δx va Δy larni toping.

Yechilishi: Argument orttirmasi Δx ni topamiz:

$$\Delta x=x_2-x_1=0-(-2)=2.$$

Funktsiyaning $x_1=-2$ va $x_2=0$ ga mos qiymatlari $y_1=f(x_1)$ va $y_2=f(x_2)$ larni topamiz:

$$y_1=f(x_1)=f(-2)=2\cdot(-2)^2+(-2)-6=0;$$

$$y_2=f(x_2)=f(0)=2\cdot 0+0-6=-6.$$

Endi funktsiya orttirmasi Δy ni topamiz:

$$\Delta y=y_2-y_1=f(x_2)-f(x_1)=-6-0=-6.$$

Demak, argument orttirmasi $\Delta x = 2$, funktsiya orttirmasi esa $\Delta y = -6$ dan iborat ekan.

3-misol. $y = x^2 - 3x - 1$ funktsiya berilgan. $x = 4$ va $\Delta x = 0{,}5$ bo'lganda Δy ni toping.

Yechilishi: Berilgan funktsiyaning Δx ga mos ortgan qiymatini topamiz:

$$y + \Delta y = (x + \Delta x)^2 - 3(x + \Delta x) - 1 = x^2 + 2x \cdot \Delta x + (\Delta x)^2 - 3x - 3\Delta x - 1.$$

Endi funktsiya orttirmasi Δy ni topamiz:

$$\begin{array}{l} y + \Delta y = x^2 + 2x \cdot \Delta x + (\Delta x)^2 - 3x - 3\Delta x - 1, \\ ^{-}y \qquad\;\; = x^2 - 3x - 1 \\ \hline \Delta y = 2x \cdot \Delta x + (\Delta x)^2 - 3\Delta x = 2 \cdot 4 \cdot (0{,}5) + (0{,}5)^2 - 3 \cdot 0{,}5 = 4 + 0{,}25 - 1{,}5 = 2{,}75 \end{array}$$

Demak , $\Delta y = 2{,}75.$

Mustaqil yechish uchun mashqlar

№1. $y = 5x - 4$ funktsiya berilgan bo'lsa, argument o'z qiymatini $x_1 = -3$ dan $x_2 = -1$ ga o'zgartirsa, argument va funktsiya orttirmalarini toping.

№2. $y = \frac{1}{2}x + 3$ funktsiya berilgan. Argument o'z qiymatini $x_1 = 2$ dan $x_2 = 4$ ga o'zgartirgan bo'lsa, Δx va Δy larni toping.

№3. $y = x^3 - x^2 + x - 2$ funktsiya berilgan. Argumentning qiymati $[3; 4]$ oraliqda o'zgarsa, argument va funktsiya orttirmalarini toping.

№4. $f(x) = x^2$, $x_1 = 2$ va $x_2 = 1{,}9$ bo'lsa, Δx va Δy orttirmalarni toping.

№5. Qirrasi a ga teng bo'lgan kub berilgan. Agar qirra uzunligini hisoblashda Δx xatoga yo'l qo'yilgan bo'lsa, shu kubning hajmini hisoblashdagi ΔV xatoni toping.

№6. Agar $y = \frac{x^2}{2}$, $x_1 = 2$ va $\Delta x = 0{,}1$ ekanligi ma'lum bo'lsa, berilgan funktsiyaning x_1 nuqtadagi orttirmasini toping.

№7. $y=\frac{1}{2x}$ funktsiya berilgan. $x=2$ va $\Delta x=0{,}8$ bo'lganda Δy orttirmani toping.

№8. Agar $y=\sqrt[3]{x}$ funktsiya berilgan bo'lsa, $x=1$ va $\Delta x=0{,}2$ bo'lganda funktsiya orttirmasi Δy ni toping.

2§. Hosilaning fizikaviy ma'nosi

Ma'lumki, jismlarninng hamma harakatlari ichida eng soddasi to'g'ri chiziq bo'ylab tekis harakatidir. Bu shunday harakatki, unda jism o'z yo'nalishini o'zgartirmay, har qanday teng vaqtlar oralig'ida bir xil uzunlikdagi yo'llarni bosib o'tadi. Ba'zi oraliqlarda transport vositalari tekis va to'g'ri chiziqli harakat qiladi.

Harakatdagi jismning to'g'ri chiziq bo'ylab tekis harakatida vaqt birligida bosib o'tgan yo'li, shu ***harakatning tezligi*** deyiladi.

Odatda biz ko'pincha notekis harakatlarga duch kelamiz. Masalan, biror stansiyadan jo'nayotgan poyezd avval sekin-asta tezlasha boradi. U o'z harakatining ikkinchi minutida birinchi minutidagidan ko'p, uchinchi minutida esa ikkinchi minutidan ko'proq yo'l bosadi va hokazo. Ma'lum tezlikka erishib olgach, u har bir minutda bir xil uzunlikdagi yo'lni bosib o'tadi, tekis harakat qiladi. Poyezd belgilangan joyga yaqinlasha borgach, har qaysi minutda oldingisidan kamroq masofani o'tib, tezligini kamaytira boradi. Samolyot, paraxod, avtomobil, tramvay, trollebuslar haqida yuqoridagi fikrlarni aytish mumkin.

Har qanday notekis harakat qilayotgan jism turlicha, ammo miqdor jihatdan teng bo'lgan vaqt oraliqlarida turli uzunlikdagi masofalarni o'tishi mumkin. Shuning uchun notekis harakatni qandaydir vaqt oralig'ida o'tilgan yo'l uzunligi bilan to'la ifodalab bo'lmaydi, ammo notekis harakat ma'lum vaqt oralig'idagi o'rtacha tezlik bilan xarakterlanadi.

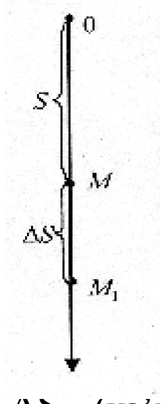
Tekis harakatda yo’lning har qanday qismidagi o’rtacha tezlik harakat tezligi bilan bir xil bo’ladi. O’rtacha tezlik odatda poyezdlarning harakati bilan xarakterlanadi. Masalan, Moskva-Sank Peterburg ekspress poyezdi 650 km.li yo’lni 6 soatda bosib o’tadi. Shunga ko’ra uning o’rtacha tezligi 108 km/soatga teng. Ammo bu poyezd har soatda 108 km masofani bosib o’tishini bildirmaydi. Masalan, harakatning poyezd tezlik olayotgan birinchi soatida u 80-90 km masofani bosadi. Undan keyingi soatda o’tilgan yo’l taxminan 130 km.ni tashkil etadi. Shundan keyin ekspress poyezdi katta shaharlardan birida to’xtaydi. Bu to’xtashga to’g’ri kelgan soatda ekspress 80-90 km dan ham kam masofani bosib o’tadi. Bulardan ko’rinib turibdiki, o’rtacha tezlik notekis harakatning to’la xarakteristikasi bo’la olmaydi.

Jism harakatining bir muncha to’laroq tavsifini ko’rib o’taylik.

Differentsial hisobga kirish sifatida tezlik haqidagi masalaga to’xtalamiz. Bu masala differentsial hisob asosiy tushunchasining vujudga kelishi bilan tarixiy bog’langan bo’lib, keyinroq bu tushuncha hosila nomini oladi.

Faraz qilaylik, biror moddiy nuqta t vaqt ichida S masofani bosib o’tgan bo’lsin. U holda, yo’l formulasi

$$S = \frac{gt^2}{2} \qquad (1)$$



ko’rinishda ifodalanishi ma’lum $(g = 9{,}81)$. Nuqta M vaziyatda bo’lganda harakatning berilgan t paytdagi v tezligini aniqlash talab qilinsin.

O’zgaruvchi t ga birorta Δt orttirma beramiz va nuqta M_1 vaziyatda bo’lgandagi $t + \Delta t$ paytni tekshiramiz.

Masofaning Δt vaqt oralig’ida qabul qilgan orttirmasini ΔS (ya’ni $MM_1 = \Delta S$) bilan belgilaymiz. (1) tenglikdagi t o’rniga $t + \Delta t$ ni qo’yib, yo’lning yangi qiymati uchun quyidagi ifodaga ega bo’lamiz.

$$S + \Delta S = \frac{g}{2}(t + \Delta t)^2$$

Bundan $\Delta S=\frac{g}{2}\left(t^2+2t\cdot\Delta t+\Delta t^2\right)-S.$ S ning o'rniga berilgan qiymatini qo'yamiz:

$$\Delta S=\frac{g}{2}t^2+\frac{g}{2}\cdot 2t\cdot\Delta t+\frac{g}{2}\cdot\Delta t^2-\frac{g}{2}t^2=\frac{g}{2}\left(2t\cdot\Delta t+\Delta t^2\right).$$

ΔS ni Δt ga bo'lib, moddiy nuqtaning MM_1 oraliqdagi o'rtacha tezligi topiladi:

$$v_{o'r}=\frac{\Delta S}{\Delta t}=gt+\frac{g}{2}\cdot\Delta t. \qquad (2)$$

(2) tezlik Δt ning o'zgarishi bilan birgalikda o'zgarishini hamda o'tgan Δt vaqt qancha kichik bo'lsa tushuvchi nuqtaning t paytdagi o'rtacha tezlik shuncha yaxshiroq tavsiflashi ko'rinib turibdi.

Nuqtaning t paytdagi v tezligi deb, Δt vaqt oralig'idagi $v_{o'r}$ o'rtacha tezlikning nolga intilgandagi limitiga aytiladi:

$$v=\lim_{\Delta t\to 0}\left(gt+\frac{g}{2}\cdot\Delta t\right)=gt. \qquad (3)$$

Umumiy holda ham v tezlik nuqtaning to'g'ri chiziqli harakatida shu singari hisoblanadi. Nuqtaning vaziyati, birorta boshlang'ich 0 nuqtadan hisoblangan S masofa bilan aniqlanadi hamda bu masofa ***o'tilgan yo'l*** deyiladi. t vaqt esa biror paytdan boshlab hisoblanadi va bu paytda nuqtaning 0 da bo'lishi shart emas. Ixtiyoriy payt uchun nuqtaning vaziyatini aniqlash imkonini beradigan harakat tenglamasi

$$S=f(t) \qquad (4)$$

ma'lum bo'lsa, ***harakat to'la berilgan*** deyiladi.

Berilgan t paytda v tezlikni aniqlash uchun yuqoridagidek, t_1 ga Δt orttirma berishga to'g'ri kelar edi, bunga S ning ΔS ga ortishi mos keladi. Δt oraliqdagi $v_{o'r}$ o'rtacha tezlikni

$$\frac{\Delta S}{\Delta t}$$

nisbat ifodalaydi. t paytdagi v oniy tezlik esa bu yerda limitga o'tish orqali topiladi, ya'ni:

$$v = \lim_{\Delta t \to 0} v_{o'r} = \lim_{\Delta t \to 0} \frac{\Delta S}{\Delta t}. \qquad (5)$$

Bu tenglik hosila deb qabul qilingan bo'lib, v ***tezlik o'tilgan*** S ***yo'lning vaqt bo'yicha hosilasi*** deb aytiladi. Uni quyidagicha yozish ham mumkin:

$$V = \frac{ds}{dt} \qquad (6)$$

1-misol. x argument $x_1 = 4$ dan $x_2 = 5$ gacha o'zgarganda $y = x^2 - 3x$ funktsiya o'zgarishining o'rtacha tezligini toping.

Yechilishi: Berilgan funktsiya argumentining orttirmasini topamiz:

$$\Delta x = x_2 - x_1 = 5 - 4 = 1.$$

Funktsiyaning x_1 va x_2 dagi qiymatlarini topamiz:

$$y_1 = 4^2 - 3 \cdot 4 = 4, \quad y_2 = 5^2 - 3 \cdot 5 = 10.$$

Funktsiya orttirmasini hisoblaymiz:

$$\Delta y = y_2 - y_1 = 10 - 4 = 6.$$

Berilgan funktsiya o'zgarishining o'rtacha tezligini topamiz:

$$v_{o'r} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{6}{1} = 6.$$

2-misol. Moddiy nuqtaning to'g'ri chiziqli harakati $S = 2t^2 - t + 3$ tenglama bilan berilgan bo'lsa, nuqtaning $t = 3$ paytdagi harakat tezligini toping (t-sekund, S -metrlarda).

Yechilishi: Moddiy nuqta harakatining o'rtacha tezligini topamiz:

$$S + \Delta S = 2(t + \Delta t)^2 - (t + \Delta t) + 3 = 2t^2 + 4t \cdot \Delta t + 2(\Delta t)^2 - t - \Delta t + 3;$$

$$\Delta S = 2t^2 + 4t \cdot \Delta t + 2(\Delta t)^2 - t - \Delta t + 3 - 2t^2 + t - 3 = 4t \cdot \Delta t + 2(\Delta t)^2 - \Delta t.$$

$$\frac{\Delta S}{\Delta t} = \frac{4t \cdot \Delta t + 2(\Delta t)^2 - \Delta t}{\Delta t} = 4t - 2\Delta t - 1.$$

Moddiy nuqta harakatining vaqtning t paytdagi haqiqiy tezligini topamiz:

$$v = \lim_{\Delta t \to 0} \frac{\Delta S}{\Delta t} = \lim_{\Delta t \to 0} (4t - 2\Delta t - 1) = 4t - 1.$$

Moddiy nuqta harakatining $t = 3$ sekund oxiridagi tezligini topamiz:

$$v_{t=5} = 4 \cdot 5 - 1 = 19 м / с .$$

3-misol. Moddiy nuqta $S = 5t^3 - 3t + 2$ qonun bo'yicha to'g'ri chiziqli harakat qiladi. Vaqtning $t = 5$ paytdagi tezligini toping.

Yechilishi: Nuqtaning t vaqtning istalgan paytidagi harakat tezligini topamiz, ya'ni:

$$v = \frac{ds}{dt} = (S)' = 15t^2 - 3.$$

Endi, moddiy nuqtaning $t = 5$ paytdagi harakat tezligini topamiz:

$$v_{t=5} = 15 \cdot 5^2 - 3 = 15 \cdot 25 - 3 = 375 - 3 = 372 m / s.$$

Demak, moddiy nuqtaning $t = 5$ paytdagi tezligi 372 *m/s* dan iborat ekan.

4-misol. Massasi 20 kg bo'lgan sharsimon jism $S = 8t^2 - 3t - 2$ qonun bo'yicha to'g'ri chiziqli harakat qiladi. Shu jismning harakat boshlagandan keyingi 7 s o'tgandagi kinetik energiyasi $\frac{mv^2}{2}$ ni toping.

Yechilishi: Sharsimon jismning vaqtning t paytidagi harakat tezligini topamiz:

$$v = \frac{ds}{dt} = (8t^2 - 3t - 2)' = 16t - 3.$$

Jismning $t = 7$ paytdagi tezligi quyidagidan iborat:

$$v_{t=7} = 16 \cdot 7 - 3 = 112 - 3 = 109 \text{ m/s}.$$

Endi $t = 7$ paytdagi jismning kinetik energiyasini topamiz:

$$\frac{mv^2}{2} = \frac{20 \cdot 109}{2} = 10 \cdot 109 = 1090 \, joul.$$

Demak, jismning $t = 7$ paytdagi kinetik energiyasi 1090 *joul*dan iborat ekan.

Mustaqil yechish uchun mashqlar

№9. Nuqta $S(t)=v_0t+\frac{at^2}{2}$ qonun bilan harakat qiladi. (S-o'yul, metr bilan; t - vaqt , minut bilan). Bu nuqtaning:

1) harakatning boshlang'ich paytidagi;
2) vaqtning t_0 paytdagi oniy tezligini toping.

№10. Nuqta $S(t)=t^2$ qonun bo'yicha harakat qilmoqda. Shu nuqtaning:

1) harakatning boshlang'ich paytidagi;
2) harakat boshlangandan 10 sekund o'tgandan keyingi;
3) $t=5$ minut paytdagi oniy tezliklarni toping.

№11. $S(t)=\sqrt{t}$ qonun bo'yicha harakatlanayotgan jismning t vaqtining istalgan minutdagi oniy tezligini toping.

№12. $y=6x+1$ qonun (tenglama) bilan harakatlanayotgan moddiy nuqtaning tezligini toping.

№13. Jismning harorati T ning t vaqtga bog'liq holda $T=2t^2+t-1$ qonun bo'yicha o'zgaradi. Bu jism vaqtning $t=6$ paytida qanday tezlik bilan qiziydi?

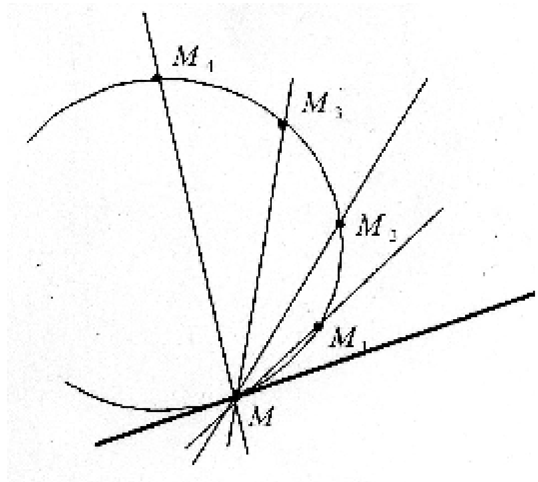
№14. $S=6t^2+5t+3$ qonun bo'yicha harakatlanayotgan jismning massasi 90 kg. Jismning harakat boshlangandan keyin 4s o'tgandagi kinetik energiyasini toping.

3§. Egri chiziq urinmasi

Maktab geometriya kursidan bizga faqatgina aylanaga urinma o'tkazish tushunchasi ma'lum bo'lgan. Aylanaga biror M nuqtadagi urinma deb, aylana bilan faqat bitta umumiy M nuqtaga ega bo'lgan to'g'ri chiziq tushunilgan edi. Bunday ta'rif har qanday egri chiziq uchun ham mos kelmaydi. Masalan, AB to'g'ri chiziq MNP egri chiziqqa, bu egri chiziq bilan ikkita M va P umumiy nuqtasi bo'lishiga qaramasdan M nuqtada urinadi, deyish mumkin.

CD to'g'ri chiziq MNP egri chiziq bilan faqat bitta umumiy N nuqtaga ega. Ammo buni MNP egri chiziqqa urinma deyish to'g'ri emas albatta.

Ixtiyoriy egri chiziqning M nuqtadagi urinmasini aniqlash uchun bu egri chiziqda yana bir M_1 nuqta olib, MM_1 kesuvchi o'tkazamiz. Agar M_1 nuqtani berilgan egri chiziq bo'yicha M nuqtaga cheksiz yaqinlashtirib siljitsak, bu holda, kesuvchi ketma-ket MM_1, MM_2, MM_3 va hokazo holatlarga ega bo'ladi.



MM_1 kesuvchining M_1 nuqtasi egri chiziq bo'ylab harakatlanib, M nuqtaga cheksiz yaqinlashgandagi limit holati egri chiziqqa M nuqtada o'tkazilgan ***urinma*** deb aytiladi.

Urinma tenglamasini topishga doir quyidagi misolni qaraymiz.

Misol. Quyidagi $y = 2x^2 - x + 3$ egri chiziqning absissasi $x = 1$ bo'lgan M nuqtasiga o'tkazilgan urinmaning tenglamasini tuzing.

Yechilishi: Egri chiziqning urinish nuqtasi koordinatalar (x, y)larni topamiz. Uning absissasi $x = 1$ ekanligi ma'lum. ordinatasini topamiz:

$$y = 2 \cdot 1^2 - 1 + 3 = 4$$

Demak, urinish nuqtasi $M(1;\, 4)$dan iborat. Endi $M(1;\, 4)$ nuqtadan o'tuvchi urinma tenglamasini tuzamiz. $x_1 = 1$ va $y_1 = 4$ bo'lganligi sababli, ularni berilgan nuqtadan o'tuvchi to'g'ri chiziq tenglamasi $y - y_1 = k(x - x_1)$ ga qo'yamiz. U holda, $y - 4 = k(x - 1)$ hosil bo'ladi. Urinmaning burchak koeffisiyenti k ni topamiz:

$$y + \Delta y = 2(x + \Delta x)^2 - (x + \Delta x) + 3 = 2x^2 + 4x \cdot \Delta x + 2(\Delta x)^2 - x - \Delta x + 3;$$

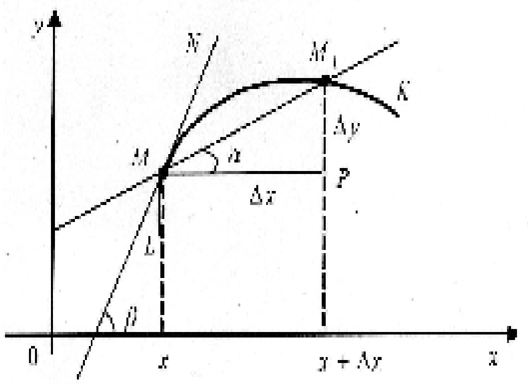
$$\Delta y = 2x^2 + 4x \cdot \Delta x + 2(\Delta x)^2 - x - \Delta x + 3 - 2x^2 + x - 3 = 4x \cdot \Delta x + 2(\Delta x)^2 + \Delta x;$$

$$y' = k = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \left(4x \cdot \Delta x + 2(\Delta x)^2 + \Delta x\right) = 0.$$

U holda, $y - 4 = k(x - 1)$ dan urinma tenglamasi $y = 4$ ekanligi topiladi.

Demak, $M(1;\,4)$ nuqtadan o'tkazilgan urinma ordinata o'qidan 4 birlikda o'tib, absissalar o'qiga parallel bo'lgan to'g'ri chiziqdan iborat ekan.

4§. Hosilaning geometrik ma'nosi



Faraz qilaylik, KL egri chiziq $y = f(x)$ funktsiyaning grafigi bo'lsin. U holda, ikki nuqtani, ya'ni (x, y) koordinatali M nuqtani va $(x + \Delta x,\ y + \Delta y)$ koordinatali M_1 nuqtani belgilaymiz.

Absissa o'qiga parallel qilib MP to'g'ri chiziqni o'tkazamiz. MM_1P uchburchakda $MP = \Delta x$, $M_1P = \Delta y$. Shuning uchun $\frac{\Delta y}{\Delta x}$ nisbat MM_1 kesuvchining absissa o'qi bilan hosil qilgan a burchagining tangensiga teng. $x \to 0$ da M nuqta harakatsiz qoladi, M_1 nuqta esa egri chiziq bo'ylab M nuqtaga cheksiz yaqinlashadi. MM_1 kesuvchi bu vaqtda doim yo'nalishini o'zgartiradi. Shu bilan birga a burchak ham o'zgaradi. Bunda

$$\frac{\Delta y}{\Delta x} = tga\,. \qquad (1)$$

MM_1 vatar limit holatida absissalar o'qi bilan biror b burchak hosil qilib, MN urinma vaziyatini oladi. U holda

$$b = \lim_{\Delta x \to 0} a \qquad (2)$$

yoki

$$tgb = \lim_{\Delta x \to 0} tga \quad \text{bo'ladi.}$$

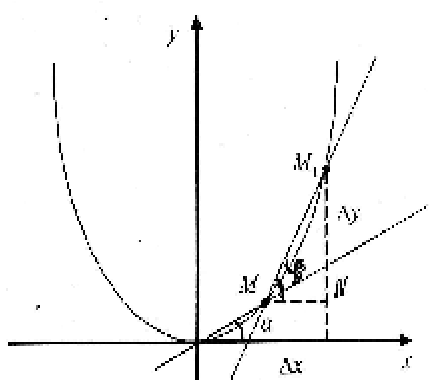
$tg\alpha = \dfrac{\Delta y}{\Delta x}$ ekanligini hisobga olsak, quyidagini hosil qilamiz.

$$tg\beta = \lim_{\Delta x \to 0} tg\alpha = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = y'. \qquad (3)$$

(3) tenglikni quyidagicha ta'riflash mumkin, ya'ni: $f(x)$ funktsiyaning x nuqtadagi hosilasi bu funktsiya grafigining x absissali nuqtasidagi urinma og'ish burchagining tangensiga teng.

5§. Hosilaning parabolaga tadbiqi

$y = ax^2$ parabolaga uning ixtiyoriy $M(x, y)$ nuqtasidan urinma o'tkazish masalasiga to'xtaylik. Urinmani $M(x, y)$ nuqtadan o'tkazish uchun urinmaning vaziyatini aniqlashda uning burchak koeffisiyentini (ya'ni $tg\alpha$ ni) bilish yetarlidir.



x absissaga Δx orttirma berib, egri chiziqning M nuqtasidan $x + \Delta x$ absissali hamda

$$y + \Delta y = a(x + \Delta x)^2$$

ordinatali M_1 nuqtasiga o'tamiz. MM_1 kesuvchining $tg\varphi$ burchak koeffisiyenti MNM_1 to'g'ri burchakli uchburchakdan topiladi. Undan MN katet absissaning Δx orttirmasiga teng. NM_1 katet esa ordinataning unga mos bo'lgan quyidagi orttirmasiga teng:

$$\Delta y = a(2x \cdot \Delta x + \Delta x^2).$$

Bundan:

$$tg\varphi = \frac{\Delta y}{\Delta x} = \frac{2a}{x} + a \cdot \Delta x.$$

Urinmaning burchak koefisiyentini topish uchun $\Delta x \to 0$ da limitga o'tish lozim, chunki bu limitga o'tish $MM_1 \to 0$ bilan teng kuchlidir. Bunda $j \to a$ hamda $tgj \to tga$. Shunday qilib, quyidagi natijaga kelamiz:

$$tga = \lim_{\Delta x \to 0} tgj = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} (2ax + a \cdot \Delta x) = 2ax = y'.$$

Bu tenglik ham hosilani ifodalaydi, ya'ni parabola tenglamasidan olingan hosila: $$y' = 2ax.$$

6§. *Hosila tushunchasi*

Ta'rif: $y = f(x)$ funktsiyaning x nuqtadagi ***hosilasi*** deb, funktsiyaning shu nuqtadagi orttirmasi $\Delta f(x)$ ning argument orttirmasi Δx ga nisbatining $\Delta x \to 0$ dagi limitiga aytiladi.

Demak, ta'rifga asosan

$$y' = f'(x) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}.$$

Berilgan $y = f(x)$ funktsiyadan hosila olish amali shu funktsiyani ***differentsiallash*** deyiladi.

Birorta oraliqning har bir nuqtasida hosilaga ega bo'lgan funktsiya shu oraliqda ***differentsiallanuvchi*** deyiladi.

$y = f(x)$ funktsiyaning hosilasi uchun quyidagi belgilashlar ishlatiladi:

$$y', y'_x, f', f'(x), \frac{dy}{dx}, \frac{df(x)}{dx}.$$

$y = f(x)$ funktsiyaning hosilasini hisoblash differesiallashning quyidagi bosqichlari bo'yicha bajariladi:

1^0. x argumentga Δx orttirma berib va funktsiya ifodasini x o'rniga orttirilgan qiymat $x + \Delta x$ ni qo'yib, funktsiyaning orttirilgan qiymati topiladi:

$$y + \Delta y = f(x + \Delta x).$$

2^0. Funktsiyaning orttirilgan qiymatidan uning boshlang'ich qiymatini ayirib, funktsiya orttirmasi topiladi:

$$\Delta y = f(x+\Delta x) - f(x).$$

3^0. Funktsiyaning orttirmasi Δy ni argument orttirmasi Δx ga bo'lib, quyidagi nisbat tuziladi:

$$\frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x}.$$

4^0. Bu nisbatning $\Delta x \to 0$ dagi limiti topiladi:

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}.$$

1-misol. $y = x - 9$ funktsiyaning hosilasini differentsiallash bosqichlari yordamida toping.

Yechilishi:

1^0. x argumentga va unga mos ravishda funktsiyaga orttirma beramiz:

$$y + \Delta y = x + \Delta x - 9.$$

2^0. Funktsiyaning orttirmasi topamiz:

$$\Delta y = x + \Delta x - 9 - y = x + \Delta x - 9 - x + 9 = \Delta x.$$

Demak, $\Delta y = \Delta x.$

3^0. Funktsiya orttirmasi Δy ning argument orttirmasi Δx ga bo'lgan nisbatini topamiz:

$$\frac{\Delta y}{\Delta x} = \frac{\Delta x}{\Delta x} = 1, \qquad \frac{\Delta y}{\Delta x} = 1.$$

4^0. $\Delta x \to 0$ da nisbatning limiti (hosila)ni topamiz:

$$y' = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} 1 = 1.$$

Demak, berilgan funktsiyaning hosilasi 1 dan iborat ekan.

2-misol. $y = 7x^2 - 3x + 2$ funktsiyaning $x = -2$ nuqtadagi xususiy hosilasini toping.

Yechilishi: Ushbu misol ham yuqoridagi misol singari 4 bosqichda hisoblanadi. Topilgan hosiladagi x o'rniga 2 qiymati qo'yiladi.

1^0. $y + \Delta y = 7(x+\Delta x)^2 - 3(x+\Delta x) + 2 = 7x^2 + 14x \cdot \Delta x + (\Delta x)^2 - 3x - 3\Delta x + 2.$

2^0. $\Delta y = 7x^2 - 14x\cdot\Delta x + (\Delta x)^2 - 3x - 3\Delta x + 2 - 7x^2 + 3x - 2 =$

$$= 14x\cdot\Delta x + (\Delta x)^2 - 3\Delta x.$$

3^0. $\dfrac{\Delta y}{\Delta x} = \dfrac{\Delta x(-14x + \Delta x - 3)}{\Delta x} = -14x + \Delta x - 3.$

4^0. $y' = \lim\limits_{\Delta x\to 0}\dfrac{\Delta y}{\Delta x} = \lim\limits_{\Delta x\to 0}(-14x + \Delta x - 3) = -14x - 3.$

$$y' = -14x - 3.$$

Endi x o'rniga 2 ni qo'yib, xususiy hosilaning qiymati topiladi:

$$y' = -14\cdot 2 - 3 = -28 - 3 = -31.$$

3-misol. $y = 4x^2 - 3x + 2$ funktsiya berilgan. $y'_{x=0}$ ni toping.

Yechilishi: Ushbu misolning ma'nosi yuqoridagi misolning ma'nosi bilan bir xil. Faqatgina funktsiyaning berilishi hamda nuqtaning qiymatlari farqli, xolos.

1^0. $y + \Delta y = 4(x + \Delta x)^2 - 3(x + \Delta x) + 2$

2^0. $\Delta y = 4(x + \Delta x)^2 - 3(x + \Delta x) + 2 - 4x^2 + 3x - 2 = 4x^2 + 8x\cdot\Delta x +$

$$+ 4(\Delta x)^2 - 3x - 3\Delta x + 2 - 4x^2 + 3x - 2 = 8x\cdot\Delta x + 4(\Delta x)^2 - 3\Delta x.$$

3^0. $\dfrac{\Delta y}{\Delta x} = \dfrac{\Delta x(8x + 4\Delta x - 3)}{\Delta x} = 8x + 4\Delta x - 3.$

4^0. $y' = \lim\limits_{\Delta x\to 0}\dfrac{\Delta y}{\Delta x} = \lim\limits_{\Delta x\to 0}(8x + 4\Delta x - 3) = 8x - 3.$

Endi $x = 0$ ni x ning o'rniga qo'yib hisoblaymiz:

$$y'_{x=0} = 8\cdot 0 - 3 = 0 - 3 = -3.$$

Mustaqil yechish uchun mashqlar

Funktsiyalarning hosilalarini differentsiallash bosqichlari asosida toping.

№15. Quyidagi funktsiyalarning hosilalarini toping:

a) $y = x - 4;$ b) $y = 2 - 3x;$

v) $y = \dfrac{1}{2}x - 3;$ g) $y = x + 5.$

№16. Funktsiyalarning hosilasini toping:

a) $y = 3x^2 + x - 2;$ b) $y = (x-1)^2;$

v) $y = x^2 - 6x + 1;$ g) $y = \frac{x-1}{2} + x^2;$

d) $y = 7x^2 + 3x - 12;$ ye) $y = 2x^2 - \frac{x+3}{4} - 5.$

№17. Agar $y = x - 3$ bo'lsa, $x = -1$da funktsiyaning hosilasini toping.

№18. $y = x^2 - 3x$ funktsiyaning:

a) 0; b)3; v)-1; g)-4.

nuqtalardagi hosilalarini hisoblang.

№19. $y = 4x^2 + x - 1$ funktsiyaning:

a) x; b) $x-1$; v) $x+1$; g) x^2.

nuqtalardagi hosilalarini hisoblang.

№20. $y = x^2$ funktsiyaning $x = 6$ nuqtadagi hosilasini toping.

№21. $y = 2x^2 - 8x$ funktsiya berilgan. Hosilaning $x = -4$ dagi xususiy qiymatini toping.

№22. $y = x^2 + 2x - 3$ funktsiya berilgan. $y'_{x=2}$ ni toping.

№23. $y = -2x^2 + 3x$ funktsiya berilgan. $y'_{x=0}$ ni toping.

№24. $y = 5\sqrt{x}$ funktsiya berilgan bo'lsa, $y'_{x=1}$ ni toping.

№25. $S = t^2$ funktsiya berilgan bo'lsa, $S'_{t=3}$ ni toping.

7§. Elementlar funktsiyalarning hosilalari haqidagi teoremalar

1-teorema. O'zgarmas sonning hosilasi 1 ga teng, ya'ni

$$f'(x) = c' = 0\,. \tag{1}$$

Isboti: Bu teorema yuqorida keltirilgan – differentsiallashning to'rt bosqichi asosida isbotlanadi, bu bosqichlarni ketma- ket qo'llaymiz.

Faraz qilaylik, $y = f(x) = c$ (c-o'zgarmas son) funktsiya berilgan bo'lsin.

1^0. $y + \Delta y = c$

2^0. $\Delta y = c - y = c - c = 0$

3^0. $\dfrac{\Delta y}{\Delta x} = \dfrac{0}{\Delta x} = 0$

4^0. $y' = \lim\limits_{\Delta x \to 0} \dfrac{\Delta y}{\Delta x} = \lim\limits_{\Delta x \to 0} 0 = 0.$

Demak, $y' = f'(x) = c' = 0$

2- teorema. Erkli o'zgaruvchining hosilasi 1 ga teng, ya'ni:

$$y' = f'(x) = x' = 1. \qquad (2)$$

Isboti:

1^0. $y + \Delta y = x + \Delta x$

2^0. $\Delta y = x + \Delta x - y = x + \Delta x - x = \Delta x, \quad \Delta y = \Delta x$

3^0. $\dfrac{\Delta y}{\Delta x} = \dfrac{\Delta x}{\Delta x} = 1, \quad \dfrac{\Delta y}{\Delta x} = 1$

4^0. $y' = \lim\limits_{\Delta x \to 0} \dfrac{\Delta y}{\Delta x} = \lim\limits_{\Delta x \to 0} 1 = 1, \quad y' = 1.$

Demak, $y' = f'(x) = x' = 1.$

3- teorema. Ikki funktsiya yig'indisining hosilasi shu funktsiyalar hosilalarining yig'indisiga teng, ya'ni

$$y' = u' + v'. \qquad (3)$$

Isboti: Faraz qilaylik, $u = f_1(x)$ va $v = f_2(x)$ bo'lsin. U holda $y = f(x) = u + v = f_1(x) + f(x_2).$

1^0. $u + \Delta u = f_1(x + \Delta x)$ va $v + \Delta v = f_2(x + \Delta x)$

2^0. $\Delta u = f_1(x + \Delta x) - f_1(x)$ va $\Delta v = f_2(x + \Delta x) - f_2(x)$

$$\Delta y = f_1(x + \Delta x) + f_2(x + \Delta x) - (f_1(x) + f_2(x)) =$$
$$= f_1(x + \Delta x) - f_1(x) + f_2(x + \Delta x) - f_2(x) = \Delta u + \Delta v$$

$3^0.\ \dfrac{\Delta y}{\Delta x}=\dfrac{\Delta u}{\Delta x}+\dfrac{\Delta v}{\Delta x}$

$4^0.\ y'=\lim\limits_{\Delta x\to 0}\dfrac{\Delta y}{\Delta x}=\lim\limits_{\Delta x\to 0}\dfrac{\Delta u}{\Delta x}+\lim\limits_{\Delta x\to 0}\dfrac{\Delta v}{\Delta x}=u'+v'.$

Demak, $y'=u'+v'.$

4-teorema. O'zgarmas c soni bilan biror funktsiya ko'paytmasining hosilasi shu son bilan funktsiya hosilasining ko'paytmasiga teng, ya'ni:

$$y'=(cu)'=cu'. \qquad (4)$$

Isboti: Faraz qilaylik, $u=f(x)$ bo'lsin. U holda $y=cu=cf(x)$ bo'ladi.

$1^0.\ y+\Delta y=cf(x+\Delta x)$

$2^0.\ \Delta y=cf(x+\Delta x)-cf(x)=c[f(x+\Delta x)-f(x)]=c\cdot\Delta u$

$3^0.\ \dfrac{\Delta y}{\Delta x}=\dfrac{c\cdot\Delta u}{\Delta x}$

$4^0.\ y'=\lim\limits_{\Delta x\to 0}\dfrac{\Delta y}{\Delta x}=\lim\limits_{\Delta x\to 0}\left(c\cdot\dfrac{\Delta u}{\Delta x}\right)=c\cdot\lim\limits_{\Delta x\to 0}\dfrac{\Delta u}{\Delta x}=cu'.$

Demak, $y'=(cu)'=cu'.$

5-teorema. Ikki funktsiya ko'paytmasining hosilasi birinchi funktsiya hosilasining ikkinchi funktsiyaning o'zi bilan, birinchi funktsiyaning ikkinchi funktsiya hosilasiga ko'paytmalarining yig'indisiga teng, ya'ni

$$y'=(u+v)'=u'v+uv'. \qquad (5)$$

Isboti: Faraz qilaylik, $u=f_1(x)$, $v=f_2(x)$ va $y=uv=f_1(x)\cdot f_2(x)$ hamda x o'zgaruvchining orttirmasi Δx va u, v, y larning orttirmalari mos ravishda $\Delta u, \Delta v, \Delta y$ lar bo'lsin. U holda

$1^0.\ y+\Delta y=(u+\Delta u)(v+\Delta v)$

$2^0.\ \Delta y=(u+\Delta u)\cdot(v+\Delta v)-uv=uv+(\Delta u)v+u(\Delta v)+(\Delta u)(\Delta v)-uv=$

$=(\Delta u)v+u(\Delta v)+\Delta u\cdot\Delta v$

$3^0.\ \dfrac{\Delta y}{\Delta x}=\dfrac{\Delta u}{\Delta x}v+u\dfrac{\Delta v}{\Delta x}\cdot\dfrac{\Delta v}{\Delta x}\cdot\Delta v$

$4^0.\ y' = \lim\limits_{\Delta x \to 0}\left(\frac{\Delta u}{\Delta x}v\right) + \lim\limits_{\Delta x \to 0}\left(u\frac{\Delta v}{\Delta x}\right) + \lim\limits_{\Delta x \to 0}\left(\frac{\Delta u}{\Delta x}\cdot \Delta v\right) =$

$$= u'v + uv' + \lim_{\Delta x \to 0}\Delta v = u'v + u'v + u'\cdot 0 = u'v + uv'.$$

Demak, $y' = (uv)' = u'v + uv'.$

6-teorema. Ikki funktsiya nisbatining hosilasi quyidagiga teng:

$$y' = \left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}. \tag{6}$$

Isbot: Faraz qilaylik, $y = \frac{u}{v}$ funktsiya berilgan bo'lsin. Bunda $u = f_1(x)$, $v = f_2(x)$ va $v \neq 0$. U holda

$1^0.\ y + \Delta y = \frac{u + \Delta u}{v + \Delta v}$

$2^0.\ \Delta y = \frac{u+\Delta u}{v+\Delta v} - \frac{u}{v} = \frac{(u+\Delta u) - u(v+\Delta v)}{v(v+\Delta v)} = \frac{uv\cdot\Delta u - uv - u\cdot\Delta v}{v(v+\Delta v)} =$

$$= \frac{v\cdot\Delta u - u\cdot\Delta v}{v(v+\Delta v)}$$

$3^0.\ \frac{\Delta y}{\Delta x} = \frac{\frac{\Delta u}{\Delta x}v - u\frac{\Delta v}{\Delta x}}{v\left(v + \frac{\Delta v}{\Delta x}\Delta x\right)}$

$4^0.\ y' = \lim\limits_{\Delta x \to 0}\left(\frac{\Delta u}{\Delta x}v\right) = \lim\limits_{\Delta x \to 0}\frac{\frac{\Delta u}{\Delta x} - u\frac{\Delta v}{\Delta x}}{v\left(v + \frac{\Delta v}{\Delta x}\Delta x\right)} = \frac{\lim\limits_{\Delta x \to 0}\left(\frac{\Delta u}{\Delta x}v\right) - \lim\limits_{\Delta x \to 0}\left(u\frac{\Delta v}{\Delta x}\right)}{v\left(v + \lim\limits_{\Delta x \to 0}\left(\frac{\Delta v}{\Delta x}\Delta x\right)\right)} =$

$$= \frac{u'v - uv'}{v(v - v'\cdot 0)} = \frac{u'v - uv'}{v^2}.$$

Demak, $y' = \left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}.$

7-teorem. $y = x^n$ (n-ixtiyoriy haqiqiy son) darajali funktsiyaning hosilasi quyidagiga teng:

$$y=\left(x^{n}\right)'=n\cdot x^{n-1}. \tag{7}$$

Isboti:

$1^0.$ $y+\Delta y=(x+\Delta x)^n$

$2^0.$ $\Delta y=(x+\Delta x)^n-x^n$

$3^0.$ $\dfrac{\Delta y}{\Delta x}=\dfrac{(x+\Delta x)^n-x^n}{\Delta x}=x^{n-1}\dfrac{\left(1+\dfrac{\Delta x}{x}\right)^n-1}{\dfrac{\Delta x}{x}}$

$4^0.$ $y'=\lim\limits_{\Delta x\to 0}\left(\dfrac{\Delta u}{\Delta x}\right)=\lim\limits_{\Delta x\to 0}\left[x^{n-1}\dfrac{\left(1+\dfrac{\Delta x}{x}\right)^n-1}{\dfrac{\Delta x}{x}}\right]=n\cdot x^{n-1},$ chunki

$$\lim_{\Delta x\to 0}\frac{\left(1+\dfrac{\Delta x}{x}\right)^n-1}{\dfrac{\Delta x}{x}}=n.$$

Demak, $y'=\left(x^n\right)'=n\cdot x^{n-1}.$

8- teorema. Ko’rsatkichli funktsiya $y=a^x$ $(a>0)$ning hosilasi $a^x\cdot\ln a$ ga teng, ya’ni:

$$y'=\left(a^x\right)'=a^x\ln a. \tag{8}$$

Isboti:

$1^0.$ $y+\Delta y=a^{x+\Delta x}$

$2^0.$ $\Delta y=a^{x+\Delta x}-a^x=a^x\cdot a^{\Delta x}-a^x=a^x\left(a^{\Delta x}-1\right)$

$3^0.$ $\dfrac{\Delta y}{\Delta x}=\dfrac{a^x\left(a^{\Delta x}-1\right)}{\Delta x}$

$4^0.$ $y'=\lim\limits_{\Delta x\to 0}\dfrac{\Delta y}{\Delta x}=\lim\limits_{\Delta x\to 0}\dfrac{a^x\left(a^{\Delta x}-1\right)}{\Delta x}=a^x\ln a,$ chunki $\lim\limits_{\Delta x\to 0}\dfrac{a^{\Delta x}-1}{\Delta x}=\ln a.$

Demak, $y'=\left(a^x\right)'=a^x\ln a.$

9-teorema. Logarifmik $y=\log_a x$ $(0<a\neq 1$ va $0<x<\infty)$ funktsiyaning hosilasi $\frac{1}{x}\log_a e$ ga teng, ya'ni:

$$y'=(\log_a x)'=\frac{1}{x}\log_a e. \tag{9}$$

Isboti:

$1^0.$ $y+\Delta y=\log_a(x+\Delta x)$

$2^0.$ $\Delta y=\log_a(x+\Delta x)-\log_a x$

$3^0.$ $\frac{\Delta y}{\Delta x}=\frac{\log_a(x+\Delta x)-\log_a x}{\Delta x}=\frac{1}{x}\cdot\frac{\log_a\left(1+\frac{\Delta x}{x}\right)}{\frac{\Delta x}{x}}$

$4^0.$ $y'=\lim\limits_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\frac{1}{x}\lim\limits_{\Delta x\to 0}\frac{\log_a\left(1+\frac{\Delta x}{x}\right)}{\frac{\Delta x}{x}}=\frac{1}{x}\log_a e.$

Demak, $y'=(\log_a x)'=\frac{1}{x}\log_a e.$

10-teorema. $y=\sqrt{x}$ funktsiyaning hosilasi $\frac{1}{2\sqrt{x}}$ ga teng, ya'ni:

$$y'=\left(\sqrt{x}\right)'=\frac{1}{2\sqrt{x}}. \tag{10}$$

Isboti:

$1^0.$ $y+\Delta y=\sqrt{x+\Delta x}$

$2^0.$ $\Delta y=\sqrt{x+\Delta x}-\sqrt{x}$

$3^0.$ $\frac{\Delta y}{\Delta x}=\frac{\sqrt{x+\Delta x}-\sqrt{x}}{\Delta x}$

$4^0.$ $y'=\lim\limits_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim\limits_{\Delta x\to 0}\frac{\sqrt{x+\Delta x}-\sqrt{x}}{\Delta x}=\lim\limits_{\Delta x\to 0}\frac{\left(\sqrt{x+\Delta x}-\sqrt{x}\right)\left(\sqrt{x+\Delta x}+\sqrt{x}\right)}{\Delta x\left(\sqrt{x+\Delta x}+\sqrt{x}\right)}=$

$$= \lim_{\Delta x \to 0} \frac{x + \Delta x - x}{\Delta x\left(\sqrt{x+\Delta x} + \sqrt{x}\right)} = \lim_{\Delta x \to 0} \frac{1}{\sqrt{x+\Delta x} + \sqrt{x}} = \frac{1}{\sqrt{x+0} + \sqrt{x}} = \frac{1}{2\sqrt{x}}.$$

Demak, $y' = \left(\sqrt{x}\right)' = \dfrac{1}{2\sqrt{x}}$.

11-teorema. $y = e^x$ funktsiyaning hosilasi e^x ga teng, ya'ni:

$$y' = \left(e^x\right)' = e^x. \tag{11}$$

Isboti:

1^0. $y + \Delta y = e^{x+\Delta x}$

2^0. $\Delta y = e^{x+\Delta x} - e^x = e^x\left(e^{\Delta x} - 1\right)$

3^0. $\dfrac{\Delta y}{\Delta x} = \dfrac{e^x\left(e^{\Delta x} - 1\right)}{\Delta x}$

4^0. $y' = \lim\limits_{\Delta x \to 0} \dfrac{\Delta y}{\Delta x} = \lim\limits_{\Delta x \to 0} \dfrac{e^x\left(e^{\Delta x} - 1\right)}{\Delta x} = e^x \ln e = e^x \cdot 1 = e^x$

Demak, $y' = \left(e^x\right)' = e^x$.

12-teorema. $y = \sin x$ funktsiyaning hosilasi $\cos x$ ga teng, ya'ni:

$$y' = (\sin x)' = \cos x. \tag{12}$$

Isboti:

1^0. $y + \Delta y = \sin(x + \Delta x)$

2^0. $\Delta y = \sin(x + \Delta x) - \sin x = 2\sin\dfrac{\Delta x}{2}\cos\left(x + \dfrac{\Delta x}{2}\right)$

3^0. $\dfrac{\Delta y}{\Delta x} = \dfrac{2\sin\frac{\Delta x}{2}\cos\left(x + \frac{\Delta x}{2}\right)}{\Delta x} = \dfrac{\sin\frac{\Delta x}{2}}{\frac{\Delta x}{2}}\cos\left(x + \dfrac{\Delta x}{2}\right)$

4^0. $y' = \lim\limits_{\Delta x \to 0} \dfrac{\Delta y}{\Delta x} = \lim\limits_{\Delta x \to 0} \dfrac{\sin\frac{\Delta x}{2}}{\frac{\Delta x}{2}} \cdot \lim\limits_{\Delta x \to 0} \cos\left(x + \dfrac{\Delta x}{2}\right) = 1 \cdot \cos x = \cos x$

Demak, $y' = (\sin x)' = \cos x$.

13-teorema. $y=\cos x$ funktsiyaning hosilasi $-\sin x$ ga teng, ya'ni:

$$y'=(\cos x)'=-\sin x. \tag{13}$$

Isboti:

$1^0.$ $y+\Delta y=\cos(x+\Delta x)$

$2^0.$ $\Delta y=\cos(x+\Delta x)-\cos x=\cos x\cos\Delta x-\sin x\sin\Delta x-\cos x=$

$$=-\sin x\sin\Delta x-\cos x(1-\cos\Delta x)$$

$3^0.$ $\dfrac{\Delta y}{\Delta x}=\dfrac{-\sin x\sin\Delta x-\cos x(1-\cos\Delta x)}{\Delta x}=-\sin x\dfrac{\sin\Delta x}{\Delta x}-\cos\dfrac{1-\cos\Delta x}{\Delta x}$

$4^0.$ $y'=\lim\limits_{\Delta x\to 0}\dfrac{\Delta y}{\Delta x}=\lim\limits_{\Delta x\to 0}\left(-\sin x\dfrac{\sin\Delta x}{\Delta x}-\cos\dfrac{1-\cos\Delta x}{\Delta x}\right)=$

$$=\lim_{\Delta x\to 0}\left[-\sin x\frac{\sin\Delta x}{\Delta x}\right]-\lim_{\Delta x\to 0}\cos\frac{1-\cos\Delta x}{\Delta x}=-\sin x-0=-\sin x$$

Demak, $y'=(\cos x)'=-\sin x.$

14-teorema. $y=tgx$ funktsiyaning hosilasi $\dfrac{1}{\cos^2 x}$ yoki $\sec^2 x$ ga teng, ya'ni:

$$y'=(tgx)'=\frac{1}{\cos^2 x}=\sec^2 x. \tag{14}$$

Isboti:

$1^0.$ $y+\Delta y=tg(x+\Delta x)$

$2^0.$ $\Delta y=tg(x+\Delta x)-tgx$

$3^0.$ $\dfrac{\Delta y}{\Delta x}=\dfrac{tg(x+\Delta x)-tgx}{\Delta x}=\dfrac{\dfrac{\sin(x+\Delta x)}{\cos(x+\Delta x)}-\dfrac{\sin x}{\cos x}}{\Delta x}=$

$$=\frac{\sin(x+\Delta x)\cdot\cos x-\cos(x+\Delta x)\cdot\sin x}{\Delta x\cdot\cos x\cdot\cos(x+\Delta x)}=\frac{\sin\Delta x}{\Delta x}\cdot\frac{1}{\cos x\cdot\cos(x+\Delta x)}$$

$4^0.$ $y'=\lim\limits_{\Delta x\to 0}\dfrac{\Delta y}{\Delta x}=\lim\limits_{\Delta x\to 0}\left(\dfrac{\sin\Delta x}{\Delta x}\cdot\dfrac{1}{\cos x\cdot\cos(x+\Delta x)}\right)=$

$$= \lim_{\Delta x \to 0}\left(\frac{\sin \Delta x}{\Delta x}\right) \cdot \lim_{\Delta x \to 0} \frac{1}{\cos x \cdot \cos(x+\Delta x)} = 1 \cdot \frac{1}{\cos x \cdot \cos(x+0)} =$$

$$= \frac{1}{\cos x \cdot \cos x} = \frac{1}{\cos^2 x} = \sec^2 x$$

Demak, $y' = (tgx)' = \dfrac{1}{\cos^2 x} = \sec^2 x.$

15-teorema. $y = ctgx$ funktsiyaning hosilasi $-\dfrac{1}{\sin^2 x}$ yoki $-\cos ec^2 x$ ga teng, ya'ni:

$$y' = (ctgx)' = -\frac{1}{\sin^2 x} = -\cos ec^2 x. \qquad (15)$$

Isboti:

1^0. $y + \Delta y = ctg(x+\Delta x)$

2^0. $\Delta y = ctg(x+\Delta x) - ctgx$

3^0. $\dfrac{\Delta y}{\Delta x} = \dfrac{ctg(x+\Delta x) - ctgx}{\Delta x} = \dfrac{\dfrac{\cos(x+\Delta x)}{\sin(x+\Delta x)} - \dfrac{\cos x}{\sin x}}{\Delta x} =$

$$= \frac{\sin x \cdot \cos(x+\Delta x) - \cos x \cdot \sin(x+\Delta x)}{\Delta x \cdot \sin x \cdot \sin(x+\Delta x)} = \frac{\sin(x - x - \Delta x)}{\Delta x \cdot \sin x \cdot \sin(x+\Delta x)} =$$

$$= \frac{-\sin \Delta x}{\Delta x \cdot \sin x \cdot \sin(x+\Delta x)} = -\frac{\sin \Delta x}{\Delta x} \cdot \frac{1}{\sin x \cdot \sin(x+\Delta x)}$$

4^0. $y' = \lim\limits_{\Delta x \to 0} \dfrac{\Delta y}{\Delta x} = \lim\limits_{\Delta x \to 0} \dfrac{-\sin \Delta x}{\Delta x} \cdot \lim\limits_{\Delta x \to 0} \dfrac{1}{\sin x \cdot \sin(x+\Delta x)} =$

$$= -1 \cdot \frac{1}{\sin x \sin x} = -\frac{1}{\sin^2 x} = -\cos ec^2 x$$

Demak, $y' = (ctgx)' = -\dfrac{1}{\sin^2 x} = -\cos ec^2 x.$

8 §. *Murakkab funktsiya va uning hosilasi*

Faraz qilaylik, $y = f(u)$ va $u = j(x)$ funktsiyalar berilgan bo'lsin. U holda, $y = f(u)$ funktsiyaga x ning ***murakkab funktsiyasi*** deyiladi va bu funktsiya quyidagicha yoziladi:

$$y = f(j(x)). \qquad (1)$$

Teorema: Biror belgilangan $x = x_0$ nuqtada $u = j(x)$ funktsiya $j' = j'(x_0)$ hosilaga; $y = f(u)$ funktsiya esa unga mos ravishda $y_0 = j(x_0)$ nuqtada $y' = f'(u_0)$ hosilaga ega bo'lsin deb faraz qilaylik. U holda, $y = f(j(x))$ murakkab funktsiya ham x_0 nuqtada hosilaga ega bo'ladi, hamda bu hosila $f(u)$ va $j(x)$ funktsiyalar hosilalarining ko'paytmasiga tengdir, ya'ni:

$$y'_x = [f(j(x))]' = f'_u(j(x_0)) \cdot j'(x_0) \qquad (2)$$

yoki

$$y'_x = y'_u \cdot u'_x. \qquad (3)$$

Isboti: x ga ixtiyoriy Δx ortirma beramiz. $u = j(x)$ ning mos orttirmasi Δu, $y = f(u)$funktsiyaning orttirmasi Δy bo'lsin. Funktsiya orttirmasining $\Delta y = y'_x \cdot \Delta x + a \cdot \Delta x$ (a - miqdor Δx ga bog'liq bo'lib, u bilan birgalikda nolga intiladigan cheksiz kichik miqdor) formulasidan foydalanib, undagi x ni u bilan almashtirsak, quyidagi hosil bo'ladi:

$$\Delta y = y'_u \cdot \Delta u + a \cdot \Delta u. \qquad (4)$$

Bundan $$\frac{\Delta y}{\Delta x} = y'_x \cdot \frac{\Delta u}{\Delta x} + a \cdot \frac{\Delta u}{\Delta x}.$$

$\Delta x \to 0$ bo'lgan holda Δu va a lar ham nolga intiladi. U holda

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0}\left(y'_x \cdot \frac{\Delta u}{\Delta x}\right) + \lim_{\Delta x \to 0}\left(a \cdot \frac{\Delta u}{\Delta x}\right) = y'_x \cdot u'_x + 0 = y'_u \cdot u'_x. \qquad (5)$$

Demak, $y' = f'(u) = y'_u \cdot u'_x.$

1-misol. $y = \cos(a - bx)$, $y' = ?$

Yechish: Berilgan funktsiya murakkab funktsiya bo'lganligi uchun $y' = [f(j(x))]' = f'(j(x)) \cdot j'(x)$ formulani ko'llamiz:

$$y' = [\cos(a-bx)]' = -\sin(a-bx) \cdot (a-bx)' = -\sin(a-bx)(0-b) = b\sin(a-bx).$$

Demak, $y' = [\cos(a-bx)]' = b\sin(a-bx)$ ekan.

2-misol. $y = \dfrac{\sin^2 x}{\cos x}$, $y' = ?$

Yechish: Funktsiyaning hosilasini topish uchun avval

$$y' = \left[\frac{u(x)}{v(x)}\right]' = \frac{u'(x)v(x) - u(x)v'(x)}{v^2(x)}$$

formuladan, keyin esa $y' = \left(x^n\right)' = nx^{n-1}$, $y' = (\sin x)' = \cos x$ hamda $y' = (\cos x)' = -\sin x$ lardan foydalaniladi. Bu ishlarni quyidagicha amalga oshirish mumkin:

$$y' = \left[\frac{\sin^2 x}{\cos x}\right]' = \frac{\left(\sin^2 x\right)' \cdot \cos x - \sin^2 x \cdot (\cos x)'}{\cos^2 x} = \frac{2\sin x \cdot \cos x \cdot \cos x - \sin^2 x \cdot (-\sin x)}{\cos^2 x} =$$

$$= \frac{2\sin x \cdot \cos^2 x + \sin^3 x}{\cos^2 x} = \frac{\sin x(2\cos^2 x + \sin^2 x)}{\cos^2 x} = \frac{\sin x\left(\cos^2 x + \cos^2 x + \sin^2 x\right)}{\cos^2 x} =$$

$$= \frac{\sin x\left(1 + \cos^2 x\right)}{\cos^2 x} = \sin x\left(1 + \sec^2 x\right).$$

3-misol. $y = 3^{\sin x}$, $y' = ?$

Yechish: Funktsiyaning hosilasini topish uchun $u = \sin x$ deb almashtirish olamiz va $\left(a^u\right)' = a^u \ln a \cdot u'$ formulani qo'llaymiz, u holda:

$$y' = \left(3^{\sin x}\right)' = \left(3^u\right)' = 3^u \cdot \ln 3 \cdot u' = 3^{\sin x} \cdot \ln 3 \cdot \cos x.$$

4-misol. $y = ctg\sqrt{x}$, $y' = ?$

Yechish: Berilgan funktsiyadan hosila olish uchun quyidagi

$$(ctgu)' = -\frac{1}{\sin^2 u} \cdot u' \quad \text{hamda} \quad y' = \left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$$

formulalardan ketma-ket foydalanamiz:

$$y' = \left(ctg\sqrt{x}\right)' = -\frac{1}{\sin^2\sqrt{x}}\cdot\left(\sqrt{x}\right) = -\frac{1}{\sin^2\sqrt{x}}\cdot\frac{1}{2\sqrt{x}} = -\frac{1}{2\sqrt{x}\cdot\sin^2\sqrt{x}}.$$

5-misol. $y = \arccos\sqrt{2x},\ \ y' = ?$

Yechish: $(\arccos u)' = -\frac{1}{\sqrt{1-u^2}}\cdot u'$ va $\left(\sqrt{u}\right)' = \frac{1}{2\sqrt{u}}\cdot u'$ formulaga binoan quyidagini hosil qilamiz.

$$y' = \left(\arccos\sqrt{2x}\right)' = -\frac{1}{\sqrt{1-\left(\sqrt{2a}\right)^2}}\cdot\left(\sqrt{2x}\right)' = -\frac{1}{\sqrt{1-2x}}\cdot\frac{1}{2\sqrt{2x}}(2x)' =$$

$$= -\frac{1}{\sqrt{1-2x}}\cdot\frac{1}{2\sqrt{2x}}\cdot 2 = -\frac{1}{\sqrt{1-2x}}\cdot\frac{1}{2\sqrt{2x}} = -\frac{1}{\sqrt{2x(1-2x)}}.$$

Mustaqil yechish uchun mashqlar

Quyidagi murakkab funktsiyalarning hosilalarini toping:

№26. $y = \left(x^6+1\right)^2.$

№27. $y = (x-1)^2.$

№28. $y = \sin 3x.$

№29. $y = \cos 4x.$

№30. $y = \left(x^2-2x+1\right)^2.$

№31. $y = \left(x^2+3x+10\right)^2.$

№32. $y = \left(\frac{x}{x+1}\right)^3.$

№33. $y = ctg5x.$

№34. $y = \sin^2 x.$

№35. $y = \cos^2 x.$

№36. $y = tg^2 x.$

№37. $y = e^{10x}.$

№38. $y = e^{\sin x}.$

№39. $y = e^{\cos x}.$

№40. $y = \ln\sin x$

№41. $y = \ln\cos x.$

№42. $y = \sin\left(x^2+1\right) + \cos\left(x^2-1\right).$

№43. $y = \frac{x-1}{x^2}$ bo'lsa $f'(x)$ ni hisoblang.

№44. $y = \sqrt{1-x^2}.$

№45. $y = e^{-2x^3}.$

№46. $y=\left(x^2-3\right)^3$.   №47. $y=\left(\dfrac{1+x}{x^2-x}\right)^2$.

№48. $y=\left(\dfrac{x^2+x+1}{x^3-3x^2-5x}\right)^3$.

9§. Hosilani hisoblash qoidalari va hosilalar jadvali

Oldingi paragrafda bir nechta elementar funktsiyalar hosilalari haqidagi teoremalar keltirilib, ular isbotlangan edi. Shu teoremalarning barchasi differentsiallash qoidalaridan iborat. Ulardan eng asosiylari differentsiallash qoidalari sifatida alohida ajratib olinadi. Ajratilgan formulalarning hosilani hisoblash qoidalari deyilishiga sabab –hosilasini hisoblash talab qilinadigan har bir funktsiya hosila olish jarayonida shu qoidalarning biri bo’yicha yoyiladi. So’ngra esa ulardan hosila olinadi.

Quyidagi hosilani hisoblash qoidalari deb atalmish formulalarni keltiramiz. Ularning har biri teoremalardan iborat bo’lganligi sababli. Oldingi paragrafda isbotlandi.

1. Hosilani hisoblash qoidalari

I. $(cx)'=c$;

II. $(u\pm v\pm w)'=u'\pm v'\pm w'$;

III. $(uv)'=u'v+uv'$;

IV. $(uvw)'=u'vw+v'uw+w'uv$;

V. $\left(\dfrac{u}{v}\right)'=\dfrac{u'v-uv'}{v^2}$.

2. Hosilalar jadvali

1. $(c)'=0$.

2. $(x)'=1$.

3. $\left(x^n\right)' = nx^{n-1}$, $\qquad\left(u^n\right)' = nu^{n-1}\cdot u'$.

4. $\left(\sqrt{x}\right)' = \dfrac{1}{2\sqrt{x}}$, $\qquad\left(\sqrt{u}\right)' = \dfrac{1}{2\sqrt{u}}\cdot u'$.

5. $\left(\dfrac{1}{x}\right)' = -\dfrac{1}{x^2}$, $\qquad\left(\dfrac{1}{u}\right)' = -\dfrac{1}{u^2}\cdot u'$.

6. $(\sin x)' = \cos x$, $\qquad(\sin u)' = \cos u\cdot u'$.

7. $(\cos x)' = -\sin x$, $\qquad(\cos u)' = -\sin u\cdot u'$.

8. $(tgx)' = \dfrac{1}{\cos^2 x}$, $\qquad(tgu)' = \dfrac{1}{\cos^2 u}\cdot u'$.

9. $(ctgx)' = -\dfrac{1}{\sin^2 x}$, $\qquad(ctgu)' = -\dfrac{1}{\sin^2 u}\cdot u'$.

10. $\left(a^x\right)' = a^x\ln a$, $\qquad\left(a^u\right)' = a^u\ln a\cdot u'$.

11. $\left(e^x\right)' = e^x$, $\qquad\left(e^u\right)' = e^u\cdot u'$.

12. $(\ln x)' = \dfrac{1}{x}$, $\qquad(\ln u)' = \dfrac{1}{u}\cdot u'$.

13. $(\log_a x)' = \dfrac{1}{x\ln a}$, $\qquad(\log_a u)' = \dfrac{1}{u\ln a}\cdot u'$.

14. $(\arcsin x)' = \dfrac{1}{\sqrt{1-x^2}}$, $\qquad(\arcsin u)' = -\dfrac{1}{\sqrt{1-u^2}}\cdot u'$.

15. $(\arccos x)' = -\dfrac{1}{\sqrt{1-x^2}}$, $\qquad(\arccos u)' = -\dfrac{1}{\sqrt{1-u^2}}\cdot u'$.

16. $(arctgx)' = \dfrac{1}{1+x^2}$, $\qquad(arctgx)' = \dfrac{1}{1+u^2}\cdot u'$.

17. $(arcctgx)' = -\dfrac{1}{1+x^2}$, $\qquad(arcctgx)' = -\dfrac{1}{1+u^2}\cdot u'$.

10§. Teskari funktsiyaning hosilasi

Faraz qilaylik, $y=f(x)$ funktsiya berilgan bo’lsin. Agar $y=f(x)$ munosabatdan $x=j(y)$ munosabat kelib chiqsa, $j(y)$ funktsiya $f(x)$ funktsiyaga nisbatan ***teskari funktsiya*** deyiladi. Bunda y o’zgaruvchi miqdorning qabul qilishi mumkin bo’lgan har bir qiymatiga x o’zgaruvchi miqdorning to’la aniqlangan qiymati mos keladi, ya’ni teskari funktsiyaning aniqlanish sohasi to’g’ri funktsiyaning qiymatlari to’plami, to’g’ri funktsiyaning aniqlanish sohasi esa teskari funktsiyaning qiymatlari to’plamidan iborat bo’ladi.

Teorema. Agar $y = f(x)$ funktsiya biror oraliqda uzluksiz bo’lib, shu oraliqning x_0 nuqtasida $f'(x_0) \neq 0$ hosilaga ega bo’lsa, $x = j(y)$ teskari funktsiya ham $y_0 = f(x_0)$ nuqtada hosilaga ega bo’ladi, ya’ni:

$$j'(y_0) = \frac{1}{f'(x_0)}. \qquad (1)$$

Isboti: $x = j(y)$ teskari funktsiyaning y argumentiga $\Delta y \neq 0$ orttirma beramiz. $x = j(y)$ funktsiya ham $\Delta x \neq 0$ orttirma oladi. U holda, quyidagi o’rinli bo’ladi:

$$\frac{\Delta x}{\Delta y} = \frac{1}{\frac{\Delta y}{\Delta x}}. \qquad (2)$$

(2) tenglikning $\Delta y \to 0$ dagi limitini olamiz. Berilgan funktsiyaning uzluksiz bo’lganligi sababli, teskari funktsiya ham y_0 nuqtada uzluksiz bo’ladi. U holda, $\Delta y \to 0$ bo’lganligi sababli, $\Delta x \to 0$. Lekin $\Delta x \to 0$ da (2)ning o’ng tomonining limiti mavjud bo’lsa, u $\frac{1}{f'(x_0)}$ dan iborat bo’ladi. Shuningdek, tenglikning chap tomonining limiti ham mavjud. Uning limiti $j'(y_0)$ dir. U holda,

$$j'(y_0) = \frac{1}{f'(x_0)}$$

tenglik o’rinli bo’ladi. Teorema isbot bo’ldi.

1-misol. $y = x^{\frac{1}{3}}$ funktsiyaning (bunda $x > 0$) hosilasini toping.

Yechilishi: Berilgan funktsiya $x = y^3$ $(y > 0)$ funktsiya uchun teskari funktsiyadir. U holda, (1) ga asosan

$$\left(x^{\frac{1}{3}}\right)' = \frac{1}{\left(y^3\right)'} = \frac{1}{3y^2} = \frac{1}{3x^{\frac{2}{3}}} = \frac{1}{3}x^{-\frac{2}{3}}.$$

2-misol. $y = x^2$ funktsiyaning $x > 0$ dagi hosilasini toping.

Yechilishi: $y = x^2$ funktsiyaga teskari funktsiya $x = \sqrt{y}$ dan iborat. U holda,

$$\left(x^2\right)' = \frac{1}{\left(\sqrt{y}\right)'} = \frac{1}{2\sqrt{y}} = 2x.$$

3-misol. $y = \arcsin x$ funktsiyaning hosilasini toping. Bunda $x \in (-1;1)$.

Yechilishi: Berilgan funktsiyaga teskari funktsiya $x = \sin y$ dan iborat $\left(y \in \left(-\frac{p}{2}; \frac{p}{2}\right)\right)$. U holda, $(\arcsin x)' = \frac{1}{(\sin y)'} = \frac{1}{\cos y}$. $y \in \left(-\frac{p}{2}; \frac{p}{2}\right)$ ni hisobga olib, $\cos y = \sqrt{1 - \sin^2 y} = \sqrt{1 - x^2}$

ko'rinishda yozish mumkin. U holda,

$$(\arcsin x)' = \frac{1}{\sqrt{1 - x^2}} \quad \text{bo'ladi.}$$

Mustaqil yechish uchun mashqlar

№49. $y = \left(23 + 15x + x^3\right)^2$ funktsiyaning hosilasini toping.

№50. $y = \left(x^2 - 3\right)^3$ funktsiyaning hosilasini toping.

№51. $y = \left(\frac{1 + x}{x^2 - x}\right)^2$ funktsiyaning hosilasini toping.

№52. Quyidagi funktsiyaning hosilasini toping.

$$y = \frac{x^2 + x - 1}{\left(x^3 - 3x^2 - 5x\right)^2}$$

№53. $y = \arccos x$ funktsiyaning hosilasini toping.

№54. $y = arctgx$ funktsiyaning hosilasini toping.

№55. $y = arcctgx$ funktsiyaning hosilasini toping.

№56. $y = arctg\frac{1}{x}$ funktsiyaning hosilasini toping.

№57. Quyidagi funktsiyalarning hosilasini toping:

a) $y = \arcsin 2x;$

b) $y = arcctg2x.$

№58. $y = \arcsin x + arctgx + 2e^x$ ning hosilasini toping.

11§. Oshkormas va parametrik funktsiyalar hosilasi

Funktsiyaning formulaviy ifodasi funktsiyaga nisbatan yechilgan bo'lsa, bunday funktsiyaga ***oshkor funktsiya*** deyiladi, masalan,

$$y = f(x), \quad y = 5x^2 - 3x + 4, \ldots$$

Agar berilgan funktsiyaning formulaviy ifodasi funktsiyaga nisbatan yechilmagan bo'lsa, bunday funktsiyaga ***oshkormas funktsiya*** deyiladi, masalan,

$$x - y = 0, \quad x^3 - 2y + 5 = 0; \quad kx + y - b = 0.$$

$$f(x; y) = 0, \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0, \quad x^2 + y^2 = R^2.$$

Har qanday oshkormas funktsiyani har doim ham oshkor funktsiya shaklida ifodalash mumkin bo'lavermaydi. Masalan, $2x \cdot \sin xy + \arcsin(x - y) - 1 = 0$.

Oshkor funktsiyalardan hosila olish masalasi oldingi mavzularda ko'rib o'tildi. Oshkormas funktsiyalardan hosila olish tartibi quyidagicha, ya'ni, berilgan tenglikning har ikkala tomonidan hosila olinadi. Bunda y dan funktsiya sifatida hosila olinadi.

1-misol. $8x^2 - 3y = 7, \quad y' = ?$

Yechilishi: $\left(8x^2 - 3y\right)' = (7)';$ $\qquad 16x - y \cdot y'_x = 0,$

$$y \cdot y'_x = 16x \quad \text{yoki} \quad y \cdot y' = 16x, \qquad y'_x = 16\frac{x}{y}.$$

2-misol. $e^x + \ln(x^2+1) + y^2 = 5. \quad y' = ?$

Yechilishi: $\left(e^x + \ln(x^2+1) + y^2\right)' = 5';$

$$e^x + \frac{2x}{x^2+1} + 2y \cdot y' = 0, \qquad 2y \cdot y' = -e^x - \frac{2x}{x^2+1}.$$

$$y' = -\frac{e^x}{2y} - \frac{2x}{2y(x^2+1)} = -\frac{1}{y}\left(\frac{e^x}{2} - \frac{x}{x^2+1}\right).$$

3-misol. $y^2 = 2px, \quad y'_x = ?$

Yechilishi: $2y \cdot y' = 2p,$ bundan $y' = \dfrac{p}{y}.$

Berilgan ifoda argument va funktsiyadan tashqari qatnashgan qo'shimcha o'zgaruvchi miqdorlarga ***parametrlar*** deyiladi. Masalan, $x^2 + y^2 = R^2$ aylananing parametrik tenglamasi $\begin{cases} x = R\cos a, \\ y = R\sin a \end{cases}$ ko'rinishida bo'ladi.

Parametrlar yordamida berilgan funktsiyalarga ***parametrik funktsiyalar*** deyiladi.

Parametrik funktsiyalarning hosilasi quyidagi formula yordamida topiladi:

$$\frac{dy}{dx} = \frac{dj(x)}{df(x)} = \frac{j'(x)}{f'(x)}. \tag{1}$$

Yuqoridagi tushunchalarni boshqacha talqin qilish ham mumkin, ya'ni:

$$x = j(t) \text{ va } y = f(t) \tag{2}$$

funktsiyalar berilgan bo'lsin. Bunda t-parametr. (2)dagi y murakkab funktsiya bo'lib, x ga bog'liqdir. (2) dagi birinchi tenglama t ga nisbatan yechilsa,

$$t = y(x). \tag{3}$$

hosil bo'ladi. Bundagi $y(x)$ funktsiya $j(t)$ ga teskari funktsiya bo'ladi. U holda, $y = f(t)$ va (3) dan quyidagini hosil qilamiz:

$$y = f\left(y\left(x\right)\right). \qquad (4)$$

Bu murakkab funktsiyadan y'_x ni topish og'irlik tug'dirmaydi.

x argumentga bog'liq bo'lgan y funktsiya $x = j\left(t\right)$ va $y = f\left(t\right)$ parametrik tenglamalari bilan berilgan bo'lib, $j\left(t\right)$ hamda $f\left(t\right)$ funktsiyalar differesiallanuvchi va $j'\left(t\right) \neq 0$ bo'lsa, bunday funktsiyaning hosilasi quyidagi formula yordamida topiladi:

$$y'_x = \frac{y'_t}{x'_t}. \qquad (5)$$

1-misol. Funktsiya $x = t^3$ va $y = t^4$ parametrik tenglamalar bilan berilgan bo'lsin. Funktsiyaning hosilasini toping.

Yechilishi: Berilganlardan t bo'yicha hosila olamiz:

$$x'_t = 3t^2 \quad \text{va} \quad y'_t = 4t^3.$$

(5) formulaga asosan:

$$y'_x = \frac{y'_t}{x'_t} = \frac{4t^3}{3t^2} = \frac{4}{3}t, \quad \text{ya'ni} \quad y'_x = \frac{4}{3}t.$$

2- misol. Parametrik tenglamalari $x = \sqrt{t}$, $y' = \frac{1}{2}t$ bilan berilgan funktsiyaning hosilasi y'_x ni toping.

Yechilishi: $\quad x'_t = \left(\sqrt{t}\right)' = \frac{1}{2\sqrt{t}}$ va $y'_t = \left(\frac{1}{2}t\right)' = \frac{1}{2}.$

Bundan, $y'_x = \frac{y'_t}{x'_t} = \frac{\frac{1}{2}}{\frac{1}{2\sqrt{t}}} = \sqrt{t}.$ Demak, $y'_t = \sqrt{t}.$

Mustaqil yechish uchun mashqlar

№59. Oshkor funktsiyalarning hosilasini toping:

a) $y=\frac{1}{x^2}$; b) $y=2\sqrt{x}+3\sqrt[3]{x^2}-\frac{1}{\sqrt[3]{x}}$;

v) $y=\frac{x^2}{2}+\frac{2}{x^2}$; g) $y=\frac{ax+b}{a+b}$;

d) $y=x^2(2x-1)$; ye) $y=\frac{2x}{1-x^2}$;

yo) $y=\frac{\sin x-\cos x}{\sin x+\cos x}$; j) $y=\sin^2 x$;

z) $y=\sin x^2$; i) $y=\sin^2 2x^2$.

№60. Quyidagi tenglamalar bilan berilgan differentsiallanuvchi oshkormas funktsiyalarning hosilalarini toping.

a) $x+y=1$; b) $x^2+y^2-xy=1$;

v) $x-y+xy=0$; g) $x^3+y^3-3xy=0$.

№61. Differentsiallanuvchi oshkormas funktsiyaning hosilasini toping:

$$y=x+\ln x$$

№62. Agar $\frac{x}{y}+2xy=3$ bo'lsin, $A(1;1)$ nuqtada y' ni toping.

№63. Quyidagi oshkormas funktsiyaning hosilasini toping:

$$3x^2y^3-5x=0.$$

№64. $x^5+5xy^3-y^5=0$ oshkormas funktsiyaning hosilasini toping.

№65. Parametrik tenglamalari bilan berilgan funktsiyaning hosilasi y'_x ni toping:

$$x=a\cos t \quad \text{va} \quad y=b\sin t.$$

№66. $x=wt$ va $y=te^{a\,t}$ tenglamalar bilan berilgan parametrik funktsiyaning hosilasini toping.

№67. $x=R\cos j$ va $y=R\sin j$ tenglamalar bilan berilgan funktsiyaning hosilasini toping.

№68. $t=0$ bo'lganda quyidagi parametrik tenglamalar bilan berilgan funktsiyaning hosilasini toping:

$$x=e^t\cos t \text{ va } y=e^t\sin t.$$

№69. Quyidagi funktsiyalarning hosilalarini toping:

a) $3y+5x^3=2;$ b) $y^2-5x+x^2=0.$

№70. Oshkormas funktsiyaning hosilasini toring:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$$

№71. $\frac{x^2}{27}+\frac{y^2}{24}=1$ ellipsga $A(-3;-4)$ nuqtada o'tkazilgan urinma va normalning tenglamasini tuzing.

12§. Hosilani hisoblashga doir mashqlar

Hosilani hisoblash qoidalari va jadvaldan foydalanib, funktsiyalarning hosilasini toping.

№ 1. $y=2x^4-x^3+5x-3, \quad y'=?$

Yechilishi: Bu funktsiyaning hosilasini topish uchun $y'=(u(x)\pm v(x))' \Leftrightarrow y'=u'(x)\pm v'(x)$ qoidadan foydalanamiz, ya'ni:

$$y'=\left(2x^4\right)'-\left(x^3\right)'+(5x)'-3.$$

Endi $y'=(cx)'=cx'$ ni hamda jadvaldagi $y'=c'=0$ hamda $y'=\left(x^n\right)'=n\cdot x^{n-1}$ formulalarni qo'llab, quyidagini hosil qilamiz:

$$y'=2\cdot 4x^3-3x^2+5-0=8x^3-3x^2+5.$$

Demak, berilgan funktsiyaning hosilasi

$y'=8x^3-3x^2+5$ funktsiyadan iborat ekan.

№ 2. $y=(x+1)x^2, \quad y'=?$

Yechilishi: Berilgan funktsiyaning hosilasini topish uchun uning shaklini o'zgartiramiz, ya'ni:

$$y=(x+1)x^2=x^3+x^2;$$

Ikki funktsiyaning yig'indisi haqidagi qoida hamda darajali funktsiya hosilasi formulalaridan foydalaniladi, ya'ni:

$$y'=\left(x^3+x^2\right)'=\left(x^3\right)'+\left(x^2\right)'=3x^2+2x.$$

№ 3. $y=x^2\sin x,\quad y'=?$

Yechilishi: Ushbu misolni yechish uchun hosilaning

$$y'=\left(u(x)\pm v(x)\right)'\Leftrightarrow y'=u'(x)\pm v'(x)$$

qoidasidan hamda jadvaldagi formulalardan $y'=\left(x^n\right)'=nx^{n-1}$ va $y'=(\sin x)'=\cos x$ lar qo'llaniladi. Bu holda hosilani hisoblash quyidagi ketma-ketlikda amalga oshiriladi:

$$y'=\left(x^2\sin x\right)'=\left(x^2\right)'\cdot\sin x+x^2\cdot(\sin x)'=2x\cdot\sin x+x^2\cdot\cos x=x(2\sin x+\cos x).$$

№ 4. $y=2\sqrt{x}\cdot e^x-x\cos x+\ln x,\qquad y'=?$

Yechilishi: Funktsiyaning hosilasini topish uchun tegishli qoidalardan hamda

$y'=\left(\sqrt{x}\right)'=\dfrac{1}{2\sqrt{x}}$, $y'=\left(e^x\right)'=e^x$, $y'=x'=1$, $y'=(\cos x)'=-\sin x$ va

$y'=(\ln x)'=\dfrac{1}{x}$ formulalardan foydalaniladi:

$$y'=\left(2\sqrt{x}\cdot e^x-x\cos x+\ln x\right)'=\left(2\sqrt{x}\cdot e^x\right)'-(x\cos x)'+(\ln x)'=$$

$$=2\cdot\frac{2}{2\sqrt{x}}\cdot e^x+2\sqrt{x}\cdot e^x-1\cdot\cos x-x(-\sin x)+\frac{1}{x}=\frac{e^x}{\sqrt{x}}+2\sqrt{x}\cdot e^x-\cos x+x\sin x=$$

$$=\frac{e^x+2xe^x-\sqrt{x}\cdot\cos x+\sqrt{x}\cdot\sin x}{\sqrt{x}}.$$

№5. $f(x)=\dfrac{e^x-1}{e^x-1}.\quad f'(-1)=?$

Yechilishi: Funktsiyadan hosila olish uchun $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$, $(u \pm v)' = u' \pm v'$, $(e^x)' = e^x$ va $x' = 1$ formulalarga asosan quyidagilarni hosil qilamiz:

$$f'(x) = \frac{(e^x+1)' \cdot (e^x-1) - (e^x+1) \cdot (e^x-1)'}{(e^x-1)^2} = \frac{e^x(e^x-1) - e^x(e^x+1)}{(e^x-1)^2} = -\frac{2e^x}{(e^x-1)^2}.$$

Endi hosil bo'lgan x o'zgaruvchi o'rniga -1 qiymatni qo'yamiz, ya'ni:

$$f'(x) = f'(-1) = -\frac{2 \cdot e^{-1}}{(e^{-1}-1)^2} = -\frac{2e}{(1-e)^2}.$$

13§. Differentsial tushunchasi

$y = f(x)$ funktsiya hosilasi $\lim\limits_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = y'_x$ va limitning ta'rifiga ko'ra $\frac{\Delta y}{\Delta x} - y' = a$ (bunda a - cheksiz kichik miqdori) ekanligi ma'lum.

Bundan $$\Delta y = y' \cdot \Delta x + a \cdot \Delta x. \tag{1}$$

(1)dan ko'rinadiki, funktsiya orttirmasi ikki qismdan iborat. Uning bosh qismi $y' \cdot \Delta x$, ***funktsiyaning differentsiali*** va $a \cdot \Delta x$ -cheksiz kichik miqdor deb nomlanadi.

Funktsiyaning differentsiali dy orqali belgilanadi:

$$dy = y' \cdot \Delta x. \tag{2}$$

Agar $y = x$ bo'lsa, $dx = x' \cdot \Delta x = \Delta x,$ ya'ni

$$dx = \Delta x. \tag{3}$$

(3)ni (2)ga qo'ysak, quyidagi hosil bo'ladi:

$$dy = y' \cdot dx. \tag{4}$$

Ta'rif: Funktsiyaning differentsiali uning hosilasini erkli o'zgaruvchining differentsialiga ko'paytirilganiga teng.

Yuqoridagilardan ko’rinadiki, funktsiyaning differentsial hosilani topishning o’zi bo’lib, faqatgina uni dx ga ko’paytirish lozim ekan.

Differentsialni hisoblash qoidalari va jadvali

1. $d[af(x)] = adf(x).$
2. $d[f(x) \pm g(x)] = df(x) \pm dg(x).$
3. $d[f(x) \cdot g(x)] = g(x)df(x) + f(x)dg(x).$
4. $d\dfrac{f(x)}{g(x)} = \dfrac{g(x)df(x) - f(x)dg(x)}{(g(x))^2}.$
5. $d(c) = 0.$
6. $d(x^n) = nx^{n-1}dx.$
7. $d(\sqrt{x}) = \dfrac{1}{2\sqrt{x}}dx.$
8. $d\left(\dfrac{1}{x}\right) = -\dfrac{1}{x^2}dx.$
9. $d(\ln x) = \dfrac{1}{x}dx.$
10. $d(e^x) = e^x dx.$
11. $d(\log_a x) = \dfrac{1}{x\ln a}, \quad (a > 0, a \neq 1).$
12. $d(a^x) = a^x \ln a dx, \quad (a > 0).$
13. $d(\sin x) = \cos x dx.$
14. $d(\cos x) = -\sin x dx.$
15. $d(tgx) = \dfrac{1}{\cos^2 x}dx.$
16. $d(ctgx) = -\dfrac{1}{\sin^2 x}dx.$
17. $d(\arcsin x) = \dfrac{1}{\sqrt{1-x^2}}dx.$

18. $d(\arccos x) = -\frac{1}{\sqrt{1-x^2}}dx.$

19. $d(arctgx) = \frac{1}{1+x^2}dx.$

20. $d(arcctgx) = -\frac{1}{1+x^2}dx.$

Misollar

Quyidagi funktsiyalarning differentsialini toping.

№ 1. $y = 8x+5, \quad dy = ?$

Yechilishi: $dy = d(8x+5) = (8x+5)'dx = (8\cdot 1+0)dx = 8dx.$

Demak, $dy = 8dx.$

№ 2. $y = \frac{x}{x+1}, \quad dy = ?$

Yechilishi:

$$dy = d\left(\frac{x}{x+1}\right) = \left(\frac{x}{x+1}\right)' dx = \frac{x'(x+1) - x(x+1)'}{(x+1)^2}dx =$$

$$= \frac{1\cdot(x+1) - x(1+0)}{(x+1)^2}dx = \frac{x+1-x}{(x+1)^2}dx = \frac{1}{(x+1)^2}dx.$$

Demak, $dy = \frac{1}{(x+1)^2}dx.$

№3. $y = x^3 + \cos 3x - 5, \quad dy = ?$

Yechilishi:

$$dy = (x^3 - \cos 3x - 5)' dx = (3x^2 + \sin 3x \cdot 3 - 0)dx = (3x^2 + 3\sin 3x)dx.$$

Demak, $dy = (3x^2 + 3\sin 3x)dx.$

№ 4. Agar $x = 3$ va $dx = 0{,}4$ bo'lsa $y = x^2$ funktsiya differentsialining qiymatini toping.

Yechilishi: (4) formula bo'yicha differentsiallaymiz va $x = 3$ hamda $dx = 0,4$ qiymatlarni qo'yamiz:

$$dy = d(x^2) = (x^2)' dx = 2xdx = 2 \cdot 3 \cdot 0,4 = 2,4.$$

№ 5. $y = \sin(1 - 4x^2), \quad dy = ?$

Yechilishi:

$$dy = [\sin(1-4x^2)]dx = \cos(1-4x^2)\cdot(1-4x^2)' dx = \cos(1-4x^2)(0-8x)dx =$$

$$= -8x \cdot \cos x(1 - 4x^2)dx.$$

Demak, $dy = -8x \cdot \cos x(1-4x^2)dx.$

№ 6. $y = x^3 \cdot e^{x^2}, \quad dy = ?$

Yechilishi: $dy = \left(x^3 \cdot e^{x^2}\right)' dx = \left(3x^2 e^{x^2} + x^3 e^{x^2} \cdot 2x\right)dx = x^2 e^{x^2}\left(3 + 2x^2\right)dx.$

Demak, $dy = x^2 e^{x^2}\left(3 + 2x^2\right)dx.$

Mustaqil yechish uchun mashqlar

№72. $y = x$ funktsiyaning differentsialini toping.

№73. $y = x^2 - 6x + 1$ funktsiyaning differentsialini toping.

№74. Quyidagi funktsiyalarning differentsialini toping:

a) $y = \sin x + \cos x;$ b) $y = \frac{1}{x} - x;$

v) $y = tgx - 7;$ g) $y = x^6 - 3\cos x + \ln x.$

№75. Agar $x = -1$ va $dx = 0,2$ bo'lsa, $y = x^2 - 3$ funktsiya differentsialining qiymatini toping.

№76. Agar $x = 90^0$ va $dx = 0,9$ bo'lsa, $y = 4\cos x$ funktsiya differentsialining qiymatini toping.

№77. Quyidagilar ma'lum bo'lsa, $y = x^2 - x - 1$ funktsiya differentsialining qiymatini toping:

a) $x = 0, \quad dx = 0,5.$

b) $x = -1{,}2, \qquad dx = 0{,}9.$

14§. Yuqori tartibli hosila va differentsiallar

Ta'rif: Berilgan $y = f(x)$ funktsiyaning hosilasi $y' = f'(x)$ dan olingan hosilaga ***ikkinchi tartibli hosila*** deyiladi va bunday belgilanadi:

$$(y')' = y'' = f''(x). \qquad (1)$$

Ta'rif: Berilgan $y = f(x)$ funktsiyaning differentsialidan olingan differentsialga ***ikkinchi tartibli differentsial*** deyiladi va quyidagicha belgilanadi:

$$d(dy) = d^2y = d[f'(x)dx] = [f'(x)dx]'dx = f''(x)(dx)^2 = f''(x)dx^2$$

Demak, $$d^2y = f''(x)dx^2. \qquad (2)$$

Uchinchi, to'rtinchi va yuqori tartibli differentsiallar ham xudi shunday ta'riflanadi.

Ta'rif: Agar $y = f(x)$ funktsiyaning $(n-1)$ tartibli hosilasi (yoki differentsiali) mavjud bo'lsa, undan olingan hosila (yoki differentsial)ga n-***tartibli hosila*** (yoki differentsial) deyiladi va bunday belgilanadi:

$$\left(y^{(n-1)}\right)' = y^{(n)} = f^{(n)}(x) \quad \text{yoki} \quad \frac{d''x}{dx^n} = \frac{d^n f(x)}{dx^n}. \qquad (3)$$

Hosila va differentsiallar orasidagi bog'lanishlarni quyidagicha yozish va o'qish mumkin:

$f'(x) = \dfrac{dy}{dx} -$ de igrek taqsim de iks;

$f''(x) = \dfrac{d^2y}{dx^2} -$ de kvadrat igrek taqsim de iks kvadrat;

$f''(x) = \dfrac{d^ny}{dx^n} -$ de en igrek taqsim de iks en.

Misollar

Quyidagi funktsiyalarning talab qilingan hosilasini toping.

№1. $y = 6x^3 - 5x^2 + 3x - 2, \quad y''' = ?$

Yechilishi: Berilgan funktsiyadan uchinchi tartibli hosila olish uchun undan ketma-ket uch marta hosila olish kerak. Bunda qoidalar va formulalardan bevosita foydalaniladi.

$$y' = \left(6x^3 - 5x^2 + 3x - 2\right)' = \left(6x^3\right)' - \left(5x^2\right)' + (3x)' - 2' =$$

$$= 6 \cdot 3x^2 - 5 \cdot 2x + 3 - 0 = 18x^2 - 10x + 3;$$

$$y'' = \left(18x^2 - 10x + 3\right)' = 18 \cdot 2x - 10 \cdot 1 + 0 = 36x - 10;$$

$$y''' = (36x - 10)' = 36 \cdot 1 - 0 = 36.$$

Demak, $y''' = 36.$

№2. $y = \sin x - \cos x,\ \ y^{IV} = ?$

Yechilishi: Tegishli qoida va formulalarni qo’llab, quyidagi natijalarga kelamiz:

$$y' = (\sin x - \cos x)' = (\sin x)' - (\cos x)' = \cos x + \sin x;$$

$$y'' = (\cos x + \sin x)' = (\cos x)' + (\sin x)' = -\sin x + \cos x;$$

$$y''' = (-\sin x + \cos x)' = (-\sin x)' + (\cos x)' = -\cos x - \sin x;$$

$$y^{IV} = (-\cos x - \sin x)' = (-\cos x)' - (\sin x)' = \sin x - \cos x.$$

Demak, $y^{IV} = (\sin x - \cos x)^{IV} = \sin x - \cos x$ ekan.

№3. $y = \left(x^3 + 1\right)(2x - 1),\quad y''' = ?$

Yechilishi: Funktsiyaning birinchi tartibli hosilasini topish uchun $y' = [u(x) \cdot v(x)]' = u'(x)v(x) - u(x)v'(x)$ formulani qo’llaymiz:

$$y' = \left[\left(x^3 - 1\right)(2x - 1)\right]' = \left(x^3 + 1\right)'(2x - 1) - \left(x^3 + 1\right)(2x - 1)' = \left(3x^3 + 0\right)(2x - 1) -$$

$$-\left(x^3 + 1\right)(2 - 0) = 3x^3(2x - 1) - 2\left(x^3 + 1\right) = 4x^4 - 3x^3 - 2x^3 - 2 = 6x^4 - 5x^3 - 2;$$

$$y'' = \left(6x^4 - 5x^3 - 2\right)' = 6 \cdot 4x^3 - 5 \cdot 3x^2 - 0 = 24x^3 - 10x^2;$$

$$y''' = \left(24x^3 - 10x^2\right)' = 24 \cdot 3x^2 - 10 \cdot 2x = 72x^2 - 20x.$$

Demak, $y'''' = 72x^2 - 20x.$

№4. $y = \ln\sqrt{x}, \quad y'' = ?$

Yechilishi: Funktsiyadan birinchi tartibli hosila olish uchun $y' = (\ln u)' = \frac{1}{u}\cdot u'$ va $y' = \left(\sqrt{u}\right)' = \frac{1}{2\sqrt{u}}\cdot u'$ formulalardan foydalaniladi, ya'ni:

$$y' = \left(\ln\sqrt{x}\right)' = \frac{1}{\sqrt{x}}\cdot\left(\sqrt{x}\right)' = \frac{1}{\sqrt{x}}\cdot\frac{1}{2\sqrt{x}} = \frac{1}{2x}.$$

Ikkinchi tartibli hosilani topish uchun $y' = \left(\frac{1}{x}\right)' = -\frac{1}{x^2}$ formula qo'llaniladi:

$$y'' = \left(\frac{1}{2x}\right)' = \frac{1}{2}\left(\frac{1}{x}\right)' = \frac{1}{2}\cdot\left(-\frac{1}{x^2}\right) = -\frac{1}{2x^2}.$$

Demak, $y'' = \left(\ln\sqrt{x}\right)^{n} = -\frac{1}{2x^2}.$

№5. $y = e^{\ln\frac{1}{x}}, \quad y'' = ?$

Yechilishi: Bu funktsiya murakkab funktsiya bo'lib, undan birinchi tartibli hosila olish uchun $y' = \left(e^{u}\right)' = e^{u}\cdot u'$ formuladan foydalanamiz:

$$y' = \left(e^{\ln\frac{1}{x}}\right) = e^{\ln\frac{1}{x}}\cdot\left(\ln\frac{1}{x}\right)' = e^{\ln\frac{1}{x}}\cdot\frac{1}{\frac{1}{x}}\cdot\left(\frac{1}{x}\right)^{2} = e^{\ln\frac{1}{x}}\cdot x\cdot\left(-\frac{1}{x^2}\right) = -\frac{1}{x}e^{\ln\frac{1}{x}};$$

Ikkinchi tartibli hosilani topish uchun yuqoridagi va quyidagi $y' = [u(x)\cdot v(x)]' = u'(x)v(x) - u(x)v'(x)$ formulalarni qo'llaymiz:

$$y'' = \left[-\frac{1}{x}e^{\ln\frac{1}{x}}\right]' = \left[-\left(\frac{1}{x}\right)'e^{\ln\frac{1}{x}} + \frac{1}{x}\left(e^{\ln\frac{1}{x}}\right)'\right] = -\left[-\frac{1}{x^2}e^{\ln\frac{1}{x}} + \frac{1}{x}e^{\ln\frac{1}{x}}\left(\ln\frac{1}{x}\right)'\right] =$$

$$= \frac{1}{x^2}e^{\ln\frac{1}{x}} - \frac{1}{x}e^{\ln\frac{1}{x}}\frac{1}{\frac{1}{x}}\left(\frac{1}{x}\right)' = \frac{1}{x^2}e^{\ln\frac{1}{x}} - \frac{1}{x}xe^{\ln\frac{1}{x}}\left(-\frac{1}{x^2}\right) = \frac{1}{x}e^{\ln\frac{1}{x}} + \frac{1}{x^2}e^{\ln\frac{1}{x}} = \frac{2}{x^2}e^{\ln\frac{1}{x}}.$$

Demak, $y'' = \left(e^{\ln\frac{1}{x}}\right)'' = \frac{2}{x^2} e^{\ln\frac{1}{x}}.$

№6. Quyidagi funktsiyaning uchinchi tartibli differentsialini toping, ya'ni:

$$y = 2x^5, \quad d^3y = ?$$

Yechilishi: Ma'lumki, $d^3y = \left(2x^5\right)^{III} dx^3$. Bundan ko'rinadiki, ketma- ket $2x^5$ ning uchinchi tartibli hosilasini topish kerak, ya'ni:

$$y' = \left(2x^5\right)' = 2 \cdot 5x^4 = 10x^4;$$

$$y'' = \left(10x^4\right)' = 10 \cdot 4x^3 = 40x^3;$$

$$y''' = \left(40x^3\right)' = 40 \cdot 3x^2 = 120x^2.$$

Bularni $d^3y = \left(2x^5\right)^{III} dx^2$ formulaga qo'yamiz:

$$d^3y = d\left(2x^5\right) = 120x^2 dx.$$

Demak, $d^3y = 120x^2 dx$ hosil bo'ladi.

№7. $y = 4x^3 - x^2 + \frac{1}{x}, \quad d^2y = ?$

Yechilishi: Funktsiyaning ikkinchi tartibli differentsialini topish uchun tegishli formulalarni qo'llaymiz. U holda:

$d^2y = \left(4x^3 - x^2 + \ln x\right)'' dx^2$ bo'lganligi uchun $4x^3 - x^2 + \ln x$ ning ketma – ket ikkinchi tartibli hosilasini topamiz:

$$\left(4x^3 - x^2 + \ln x\right)' = 12x^2 - 2x - \frac{1}{x};$$

$$\left(12x^2 - 2x - \frac{1}{x}\right)' = 24x - 2 - \left(-\frac{1}{x^2}\right) = 24x - 2 + \frac{1}{x^2} = \frac{24x^3 - 2x^2 + 1}{x^2}.$$

Hosil bo'lgan natijani formulaga qo'yamiz:

$$d^2y = \left(4x^3 - x^2 + \ln x\right)'' dx^2 = \frac{24x^3 - 2x^2 + 1}{x^2} dx^2.$$

Demak, $d^2y = \dfrac{24x^3 - 2x^2 + 1}{x^2}dx^2$ ekan.

Differentsiallanuvchi funktsiyalaning xossalari

Ta'rif: Berilgan nuqtada differentsialga ega bo'lgan uzluksiz funktsiyaga shu ***nuqtada differentsiallanuvchi funktsiya*** deyiladi.

Uzilishga ega bo'lgan funktsiya shu uzilish nuqtasida hosilaga ham, differentsialga ham ega bo'lmaydi.

Xossalari:

1^0. O'zgarmas sonning differentsiali 0 ga teng:

$$da = 0.$$

2^0. Erkli o'zgaruvchining differentsiali uning orttirmasiga teng:

$$dx = \Delta x.$$

3^0. Chiziqli funktsiyaning differentsiali uning orttirmasiga teng:

$$d(ax+b) = \Delta(ax+b) = a \cdot \Delta x.$$

4^0. Darajali x^n funktsiyaning differentsiali $nx^{n-1} \cdot \Delta x$ ga teng:

$$dx^n = nx^{n-1} \cdot \Delta x.$$

Asosiy xossalari:

1^0. O'zgarmas sonni differentsial belgisi oldiga chiqarish mumkin:

$$d[af(x)] = adf(x).$$

2^0. Funktsiyalar algebraik yig'indisining differentsiali ularning differentsiallari algebraik yig'indisiga teng:

$$d[f(x) \pm g(x)] = df(x) \pm dg(x).$$

3^0. Funktsiyaning differentsiali shu funktsiyaning hosilasi bilan argumenti ko'paytmasiga teng:

$$d[f(x)] = f'(x)dx.$$

Mustaqil yechish uchun mashqlar

Quyidagi funktsiyalarning ko'rsatilgan tartibdagi hosilalarini toping:

№ 78. $y = x - 6.$ y'' ni toping.

№ 79. $y = (x-2)^2.$ y''' ni toping.

№ 80. $y = x^5 + 3x^3 - x + 5.$ $y^{(IV)}$ ni toping.

№ 81. $y = \sin x.$ $y^{(IV)}$ ni toping.

№ 82. $y = \cos x.$ $y^{(IV)}$ ni toping.

№ 83. $y = (1+x)^{100}$ $y^{(IV)}$ ni toping.

№ 84. $y = (\ln x).$ $y^{(V)}$ ni toping.

№ 85. $y = \sin 3x - 2e^{4x} + tgx.$ y'' ni toping.

№ 86. $y = \sin x.$ $y^{(n)}$ ni toping.

Quyidagi funktsiyalarning ko'rsatilgan tartibdagi differentsiallarini toping:

№ 87. $y = x^2 - x + 9.$ $d^2y = ?$

№ 88. $y = 7x - \sqrt{x} + 3.$ $d^2y = ?$

№ 89. $y = \sin 2x + \ln x.$ $d^2y = ?$

№ 90. $y = x^4 - 3x^2 + 4.$ $d^3y = ?$

№ 91. $y = 4^{-x^2}.$ $d^2y = ?$

№ 92. $y = \sin^2 x.$ $d^3y = ?$

№ 93. $y = e^{-5x} + \cos x.$ a) $d^3y = ?$ b) $d^4y = ?$ v) $d^5y = ?$

15§. Funktsiya hosilasining tadbiqlari

1.**Funktsiyaning monotonlik oraliqlarini topish**

$y = f(x)$ funktsiya berilgan bo'lib, bu funktsiya $[a,b]$ kesmaning barcha nuqtalarida aniqlangan va uzluksiz bo'lsin.

Funktsiyaning monotonligini tekshirishda hosiladan foydalanamiz.

Agar berilgan $y = f(x)$ funktsiya biror oraliqning barcha nuqtalarida musbat hosilaga $(f'(x) > 0)$ ega bo'lsa, bu funktsiya shu oraliqda monoton o'suvchi; agar shu oraliqning barcha nuqtalarida manfiy
hosilaga $(f'(x) < 0)$ ega bo'lsa, funktsiya shu oraliqda kamayuvchi bo'ladi.

1-misol. $f(x) = 2x^3 - 9x^2 + 12x - 8$ funktsiyaning o'sish va kamayish oraliqlarini toping.

Yechilishi: Berilgan $f(x)$ funktsiya sonlar o'qining barcha nuqtalarida aniqlangan hamda hosilaga ega. Uning hosilasini topamiz:

$$f'(x) = \left(2x^3 - 9x^2 + 12x - 8\right)' = 6x^2 - 18x + 12\,;$$

Bundan $f'(x) = 6\left(x^2 - 3x + 2\right)$.

Hosil bo'lgan ifodani ko'paytuvchilarga ajratib, kritik nuqtalarni topamiz. Ular quyidagilardan iborat:

$$x_1 = 1 \text{ va } x_2 = 2.$$

U holda, $(-\infty; 1)$, $(1; 2)$ va $(2; +\infty)$ oraliqlar hosil bo'ladi. Bu oraliqlarda ifodaning ishoralarini aniqlaymiz. Bunda «intervallar usuli» dan foydalanish ma'qulroqdir.

$(-\infty; 1)$ oraliqdan ixtiyoriy nuqtani, masalan, $x = 0{,}5$ ni olaylik. Bu qiymatni $f'(x)$ ga qo'yamiz:

$$f'(0{,}5) = (0{,}5 - 1)(0{,}5 - 2) = 0{,}75 > 0.$$

Demak, berilgan funktsiya $(-\infty; 1)$ oraliqda o'suvchi ekan.

$(1; 2)$ oraliqda $x = 1{,}5$ nuqtani tekshiraylik:

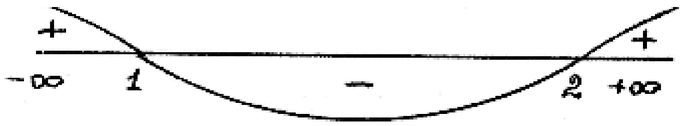
$$f'(1{,}5) = (1{,}5 - 1)(1{,}5 - 2) = -0{,}25 < 0.$$

Demak, funktsiya $(1; 2)$ oraliqda kamayuvchi ekan.

$(2; +\infty)$ oraliqda $x = 5$ nuqtani qaraylik. Bunda:

$$f'(5) = (5 - 1)(5 - 2) = 12 > 0.$$

Demak, berilgan funktsiya $(2;+\infty)$ oraliqda o'suvchi ekan. Shunday qilib quyidagi chizmani hosil qilamiz:



Yuqoridagi natijalarni bitta jadvalda mujassamlashtiramiz:

x	$(-\infty;1)$	1	$(1;2)$	2	$(2;+\infty)$
$f'(x)$	+	0	-	0	+
$f(x)$	↗	-3	↘	-4	↗

2-misol. $f(x)=(x-1)^3$ funktsiyaning monotonlik oraliqlarini toping.

Yechilishi: Berilgan funktsiyaning hosilasini topib, nolga tenglashtiramiz va uning kritik nuqtalarini topamiz:

$$f'(x)=3(x-1)^2$$

Bu ifoda faqatgina $x=1$da nolga aylanadi va u hamma vaqt musbatdir. U holda, $f'(x)$ funktsiya ishorasini o'zgartirmaydi.

Demak, berilgan funktsiya har doim o'suvchidir bqladi. Uning jadvali quyidagidan iborat:

x	$(-\infty;1)$	1	$(1;+\infty)$
$f'(x)$	+	0	+
$f(x)$	↗	0	↘

Mustaqil yechish uchun mashqlar

№ 94. $f(x)=2x^2-3x-6$ funktsiyaning monotonlik oraliqlarini toping.

№ 95. Quyidagi funktsiyalarning monotonlik oralig'ini toping:

a) $f(x)=3x^2+2x+1;$ b) $f(x)=2x^3-3x^2-12x-6.$

№ 96. $f(x)=\frac{2}{3}x^3-2x+1$ funktsiyaning monotonlik oraliqlarini toping.

№ 97. Quyidagi $f(x)=3x+\frac{3}{x}+5$ funktsiyaning monotonlik oraliqlarini toping.

№ 98. $f(x)=x^4-3x^2$ funktsiyaning monotonlik oraliqlarini toping.

№ 99. $f(x)=x\sqrt{4-x^2}$ funktsiyaning monotonlik oraliqlarni toping.

№ 100. $f(x)=\frac{x}{x^2+1}$ funktsiyaning monotonligini tekshiring.

№ 101. $f(x)=x^3+3x^2$ funktsiyaning monotonligini tekshiring.

№ 102. $y=\frac{x}{\ln x}$ funktsiyaning o'sish va kamayish oraliqlarini toping.

№ 103. $f(x)=\frac{1}{x^2+x+1}$ funktsiya kamayuvchi bo'lgan oraliqni toping.

№ 104. $f(x)=2x-\cos x$ funktsiya aniqlanish sohasining barcha nuqtalarida o'suvchi ekanligini isbotlang.

№ 105. $f(x)=\frac{\sin x}{\sin\left(x+\frac{p}{3}\right)}$ funktsiyaning $\left(0;\frac{p}{3}\right)$ oraliqda o'suvchi ekanligini isbotlang.

2. Funktsiyaning ekstremumlari. Ekstremum mavjudligining zaruriy va yetarli shartlari

1-ta'rif: $f(x)$ funktsiya aniqlanish sohasining funktsiyaning hosilasi nolga teng yoki mavjud bo'lmagan ichki nuqtalari funktsiyaning ***kritik nuqtalari*** deyiladi.

2-ta'rif: $f(x)$ funktsiyaning aniqlanish sohasidagi x_0 nuqta uchun (x_0-d; x_0+d) atrof mavjud bo'lib, barcha $x\neq x_0$ nuqtalar uchun

$$f(x)>f(x_0) \tag{1}$$

tenglik bajarilsa, x_0 nuqta $f(x)$ funktsiyaning ***minimum nuqtasi*** deyiladi.

3-ta'rif: $f(x)$ funktsiyaning aniqlanish sohasidagi x_0 nuqta uchun ($x_0 - d$; $x_0 + d$) atrof mavjud bo'lib, barcha $x \neq x_0$ nuqtalar uchun

$$f(x) < f(x_0) \qquad (2)$$

tenglik bajarilsa, x_0 nuqta $f(x)$ funktsiyaning ***maksimum nuqtasi*** deyiladi.

4-ta'rif: Maksimum va minimum nuqtalariga ***ekstremum nuqtalar,*** funktsiyaning shu nuqtalardagi qiymatlariga ***funktsiyaning ekstremumlari*** deyiladi.

Misol. $f(x) = (x-2)^2$ funktsiyani qaraylik. Bu funktsiya x ning 2 qiymatida nolga teng bo'ladi. x ning boshqa barcha qiymatlarida funktsiya musbatdir. Demak, x =2 bo'lganda funktsiya minimumga ega ekan.

Ferma teoremasi (estremum mavjudligining zaruriy sharti). $y = f(x)$ funktsiya uchun x_0 nuqta ekstremum nuqtasi bo'lsin. Agar x_0 nuqtada $f'(x_0) = 0$ hosila mavjud bo'lsa, u nolga teng bo'ladi, ya'ni

$$f'(x_0) = 0 .$$

Isboti: $f(x)$ funktsiya x_0 nuqtada ekstremumga ega bo'lganligi uchun ($x_0 - d$; $x_0 + d$) atrof mavjud bo'ladi. Bu atrofda $f(x_0)$ qiymat funktsiyaning boshqa qiymatlariga nisbatan eng katta yoki eng kichik bo'ladi. U holda, Ferma teoremasiga asosan berilgan funktsiyaning x_0 nuqtadagi hosilasi nolga teng bo'ladi, ya'ni

$$f'(x_0) = 0 .$$

Teorema isbot bo'ldi.

Teoremaning geometrik ma'nosi: ekstremum nuqtasida funktsiyaning grafigiga o'tkazilgan urinma absissalar o'qi OX ga parallel bo'ladi.

Teorema. (funktsiya ekstremum mavjudligining yetarli sharti). Agar x_0 kritik nuqtadan o'tishda $f'(x)$ hosila o'z ishorasini musbatdan manfiyga o’zgartirsa, x_0 - maksimum nuqta; agar x_0 kritik nuqtadan o’tishda $f'(x)$ hosila o’z ishorasini manfiydan musbatga o’zgartirsa, x_0 - minimum nuqtadir.

Isboti: Ishoraning plyusdan minusga o’zgargandagi holni qaraylik. Funktsiyaning $f'(x)$ hosilasi x_0 nuqtada o’z ishorasini (+)dan (-) ga o’zgartirsin va ($x \in (x_0 - d, x_0)$ bo’lsin.

Shartga asosan $x < x_0$ da hosila musbat. Bundan ko’rinadiki, $x < x_0$ da $f(x) < f(x_0)$, ya’ni funktsiya o’suvchidir. $x > x_0$ bo’lganda hosila manfiy. Bundan, $x > x_0$ da $f(x_0) > f(x)$ bo’ladi, ya’ni funktsiya kamayuvchidir. Demak, har doim kesmada $f(x_0) > f(x)$ dir. U holda $f(x_0)$ning funktsiyaning maksimal qiymatidan iborat ekanligi ko’rinadi. Teoremaning bir qismi isbot bo’ldi. Ikkinchi qismi ham xuddi shunday yo’l bilan isbot qilinadi.

1-misol. $f(x) = x^3 + 3x^2$ funktsiyaning ekstremum nuqtalarini toping.

Yechilishi: Berilgan $f(x)$ funktsiya x ning ixtiyoriy qiymatlarida aniqlangandir. Funktsiyaning hosilasini topamiz:

$$f'(x) = \left(x^3 + 3x^2\right)' = 3x(x+2).$$

Funktsiyaning hosilasini nolga tenglashtirib, kritik nuqtalarini topamiz:

$$3x(x+2) = 0, \quad x_1 = -2, \quad x_2 = 0.$$

Kritik nuqtalar yordamida sonlar o’qini oraliqlarga ajratami:

$$(-\infty; -2), \quad (-2; 0) \text{ va } (0; +\infty).$$

Shu oraliqlarda hosilaning ishoralarini aniqlaymiz:

$(-\infty; -2)$oraliqda, $x = -4$ bo’lganda $y'(-4) > 0;$

$(-2; 0)$ oraliqda, $x = -0{,}5$ bo’lganda $y'(-0{,}5) < 0;$

$(0; +\infty)$ oraliqda, $x = 2$ bo’lganda $y'(2) > 0.$

Demak, $x = -2$ kritik nuqtadan o’tganda hosila o’z ishorasini (+) dan (-) ga o’zgartiradi. U holda, funktsiya $x = -2$ nuqtada maksimumga ega bo’ladi:

$$y_{\max}(-2) = (-2)^3 + 3(-2)^2 = 4.$$

$x=0$ kritik nuqtadan o'tganda hosila o'z ishorasini (-)dan (+)ga o'zgartirib, $x=0$ nuqtada minimumga ega bo'ladi:

$$y_{\min}(0)=0.$$

Demak, ekstremum nuqtalar: $y_{\max}(-2)=4;\ \ y_{\min}(0)=0.$

2-misol. $y=\dfrac{2x^2-1}{x^4}$ funktsiyaning ekstremumlarini tekshiring.

Yechilishi: Berilgan funktsiya x ning $x\neq 0$ shartni qanoatlantiruvchi barcha qiymatlarida aniqlangan. Funktsiya hosilasini topamiz:

$$y'=\left(\frac{2x^2-1}{x^4}\right)'=\frac{4(1-x^2)}{x^5}.$$

Quyidagi tenglamani yechamiz: $\dfrac{4(1-x^2)}{x^5}=0.$ Bundan $1-x^2=0,\ \ x^2=1,$ $x_1=1$ va $x_2=-1.$

Demak, kritik nuqtalar $x_1=1$ va $x_2=-1$, hamda funktsiya va hosila aniqlangan $x=0$ nuqta topilib, $(-\infty;-1)$, $(-1;0)$, $(0;-1)$, $(1;+\infty)$ oraliqlarda funktsiya hosilasi tekshiriladi:

$$(-\infty;-1)\text{da},\quad x=-2,\quad y'(-2)=\frac{4(1-4)}{(-2)^5}>0,$$

$$(-1;0)\text{da},\quad x=-0{,}5,\quad y'(-0{,}5)=\frac{4(1-0{,}25)}{(-0{,}5)^5}<0,$$

$$(0;-1)\text{da},\quad x=0{,}5,\quad y'(0{,}5)=\frac{4(1-0{,}25)}{(0{,}5)^5}>0,$$

$$(1;+\infty)\text{da},\quad x=2,\quad y'(2)=\frac{4(1-4)}{2^5}<0.$$

Demak, $x=-1$ kritik nuqtada funktsiya maksimumga ega:

$$y_{\max}(-1)=\frac{2\cdot1-1}{(-1)^4}=1;$$

$x=1$ kritik nuqtada ham funktsiya maksimumga ega:

$$y_{max}(1) = \frac{2 \cdot 1 - 1}{1^4} = 1;$$

$x = 0$ nuqta ekstremum nuqtasi bo'laolmaydi, chunki bu nuqtada funktsiya va uning hosilasi aniqlanmagan.

Javob: $y_{max}(-1) = y_{max}(1) = 1.$

Mustaqil yechish uchun mashqlar

№ 106. Quyidagi a) $y = x^4 - 2x^2 + 5$; b) $y = x^2 e^x$ funktsiyalarning ekstremumlarini toping.

№ 107. $f(x) = x^3 - 3x$ funktsiyaning ekstremumlarini toping.

№108. $f(x) = \frac{1}{3}x^3 - 2x^2 + 3x - 1$ funktsiyaning ekstremal qiymatlarini toping.

№ 109. $f(x) = x^4 - 2$ funktsiyaning ekstremumlarini tekshiring.

№110. Quyidagi funktsiyalarning ekstremumlarini toping:

a) $f(x) = x^2 - 10x + 9;$

b) $f(x) = x^3 + x^2 - 8x + 1;$

v) $f(x) = x^3 + 9x - 1;$

g) $f(x) = x^4 - 14x^2 + 24x - 3;$

d) $f(x) = \frac{x}{\ln|x|}.$

№ 111. Funktsiyalarning maksimum va minimumlarini tekshiring:

a) $y = (x-1)^2(x-6)^3;$

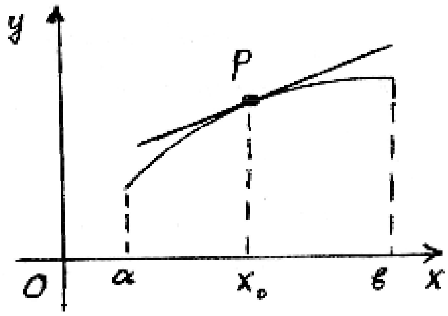
v) $y = \frac{x-1}{x^2+3};$

b) $y = \frac{x}{1+x^2};$

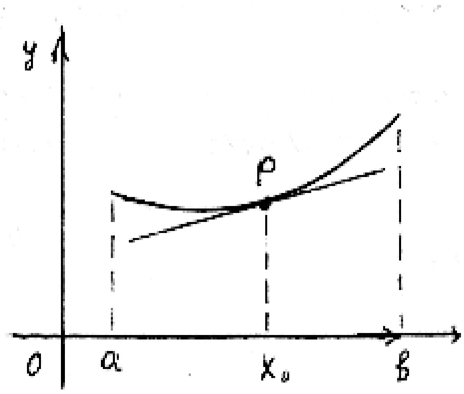
g) $y = \frac{\sqrt{x}}{x+1}.$

3. Funktsiyaning qavariq va botiqligi, bukilish nuqtasi

Ta'rif: Agar $y = f(x)$ differensiallanuvchi funktsiyaning grafigi unda yotgan ixtiyoriy P nuqtadan o'tuvchi urinmadan pastda yotgan bo'lsa, berilgan funktsiya grafigi (a, b) oraliqda ***qavariq*** deyiladi.



Ta'rif: Agar $y = f(x)$ differensiallanuvchi funktsiyaning grafigi unda yotgan ixtiyoriy P nuqtadan o'tuvchi urinmadan yuqorida yotgan bo'lsa, berilgan funktsiya grafigi (a, b) oraliqda ***botiq*** deyiladi.



Quyidagi teoremalar isbotsiz keltiriladi.

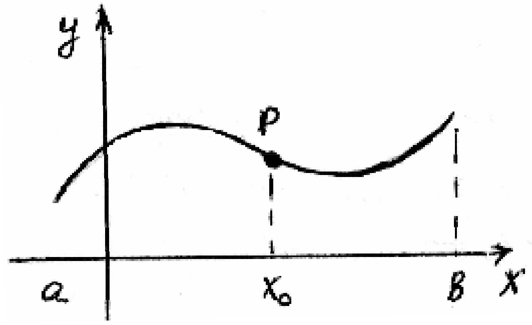
Teorema. (funktsiya grafigi qavariqligining yetarlilik sharti.)

1) Agarda, ikki martta differentsiallanuvchi $y = f(x)$ funktsiyaning ikkinchi tartibli $f''(x)$ hosilasi (a, b) oraliqda musbat bo'lsa, funktsiya grafigi shu oraliqda botiq bo'ladi.

2) Agarda, $y = f(x)$ funktsiyaning ikkinchi tartibli hosilasi (a, b) oraliqda manfiy bo'lsa, funktsiya grafigi shu oraliqda qavariq bo'ladi.

Ta'rif: Differentsiallanuvchi $y = f(x)$ funktsiya grafigining botiqlikdan qavariqlikka (yoki teskariga) o'tishdagi nuqtasiga ***bukilish nuqtasi*** deyiladi.

Teorema. Agarda $y = f(x)$ funktsiya uchun uning ikkinchi tartibli $f''(x)$ hosilasi x_0 nuqtada nolga aylansa va bu nuqtadan o'tishda o'z ishorasini teskariga almashtirsa, x_0 nuqta funktsiya grafigining bukilish nuqtasi bo'ladi.



Funktsiya grafigining qavariq oralig'ini topish quyidagi qoidalar asosida amalga oshiriladi:

1)Funktsiyaning ikkinchi tartibli hosilasi bo’yicha barcha kritik nuqtalari topiladi.

2) Kritik nuqtalar yordamida (a, b) oraliqning «oraliqchalar» da $f''(x)$ning ishoralari aniqlanadi.

3) (a, b) oraliq topilgan kritik nuqtalar yordamida «oraliqchalar»ga ajratiladi va $f''(x)$ning ulardagi ishoralari aniqlanadi.

1-misol. $f(x) = x^3$ funktsiya grafigining qavariqlik intervalini (oralig’ini) toping.

Yechilishi: Berilgan funktsiya $f'(x) = 3x^2$ va $f''(x) = 6x$ hosilalarga ega. Ikkinchi tartibli hosilasi bo’yicha bitta kritik nuqta, $x = 0$ mavjud. Bu nuqta sonlar o’qini ikki qismga, ya’ni $(-\infty; 0)$ va $(0; +\infty)$ga ajratadi.

Barcha $x > 0$ lar uchun $f''(x) > 0$ va barcha $x < 0$ lar uchun $f''(x) < 0$ bo’lganligi sababli, funktsiya grafigi $(0; +\infty)$ da botiq hamda $(-\infty; 0)$ da qavariqdir.

2- misol. $y = x^4 - 6x^3$ funktsiya grafigining qavariq, botiqlikligi hamda bukilish nuqtalarini tekshiring.

Yechilishi: Berilgan funktsiyaning birinchi va ikkinchi tartibli hosilalarini topamiz:

$$y' = 4x^3 - 12x, \quad y'' = 12x^2 - 12 = 12(x^2 - 1).$$

Ikkinchi tartibli hosila uchun kritik nuqtalarni topamiz:

$$12(x^2 - 1) = 0, \quad x^2 - 1 = 0, \quad x_1 = -1, \quad x_2 = 1.$$

Bu kritik nuqtalar sonlar o’qini uch qismga ajratadi:

$$(-\infty; -1), \ (-1; 1) \text{ va } (1; +\infty).$$

$(-\infty; -1)$ oraliqda $f''(x) > 0$, $(-1; 1)$ oraliqda $f''(x) < 0$, $(1; +\infty)$ oraliqda $f''(x) > 0$.

Demak, $x = -1$ dan o’tishda $f''(x)$ o’z ishorasini (+) dan (-) ga o’zgartiradi, ya’ni $x = -1$ da bukilish nuqtasi mavjuddir. $x = 1$ dan o’tishda

$f''(x)$ o’z ishorasini (-) dan (+) ga o’zgartiradi, ya’ni $x=1$da bukilish nuqtasi mavjud.

Mustaqil yechish uchun mashqlar

№ 112. $y=-x^2+6x-8$ funktsiyaning qavariq va botiqligini tekshiring.

№ 113. $y=tgx$ funktsiyaning qavariq va botiqligini tekshiring.

№ 114. $y=xe^{-x}$ funktsiya grafigining qavariq va botiqlik oraliqlarini toping.

№115. Quyidagi funktsiya grafiklarining qavariqlik va botiqlik oraliqlarini toping:

a) $f(x)=x^3-6x^2+12x+4;$

b) $f(x)=(x+1)^4;$

v) $f(x)=x^4-6x^2+4;$

g) $f(x)=x^4+8x^2+16.$

№ 116. $f(x)=x^3-3x^2+2x+1$ funktsiya grafigining qavariqlik va botiqligini tekshiring.

№ 117. Funktsiya grafigining qavariq va botiqligini tekshiring:

a) $f(x)=-x^4-2x^3+12x^2+15x-6;$

b) $f(x)=3x^5-10x^4-30x^3+12x+7;$

v) $f(x)=\dfrac{x-5}{x+7};$

№ 118. Quyidagi funktsiya grafiklarining bukilish nuqtalarini toping:

a) $f(x)=6x^2-x^3;$

b) $f(x)=x^4-8x^3+18x^2-4x+31;$

v) $f(x)=\dfrac{x}{1+x^2};$

g) $f(x)=x^4-8x^3+24x^2;$

d) $f(x)=5+\sqrt[3]{x-4}$;

ye) $f(x)=\ln(x^2+4)$;

yo) $f(x)=(x-1)\sqrt[7]{(x-1)^6}$.

4. Funktsiyaning kesmadagi eng katta va eng kichik qiymatlari

$[a,b]$ oraliqda aniqlangan va uzluksiz hamda shu kesmaning barcha nuqtalarida differentsiallanuvchi $y=f(x)$ funktsiya berilgan bo'lsin. Bu funktsiya o'zining eng katta va eng kichik qiymatlarini yoki kritik nuqtalarida yoki kesmaning chetki nuqtalarida qabul qiladi.

Agar berilgan $y=f(x)$ funktsiya $[a,b]$ kesmada monoton bo'lsa, u holda, bu funktsiya o'zining eng katta va eng kichik qiymatlariga shu kesmaning chetki nuqtalarida erishadi, ya'ni:

- agar $y=f(x)$ funktsiya o'suvchi bo'lsa, $f(a)$-eng kichik qiymat va $f(b)$ -eng katta qiymati bo'ladi.

- agar $f(x)$ funktsiya kamayuvchi bo'lsa, u holda, $f(a)$- eng katta qiymat va $f(b)$-eng kichik qiymati bo'ladi.

Agar $y=f(x)$ funktsiya monoton bo'lmasa, u holda, berilgan funktsiya $[a,b]$ kesmadagi o'zining eng katta qiymatiga shu kesmadagi maksimum nuqtalardan birida yoki kesmaning chetki nuqtalaridan birida erishadi. Shuningdek, funktsiya $[a,b]$ kesmadagi eng kichik qiymatiga shu kesmadagi minimum nuqtalardan birida yoki shu kesmaning chetki nuqtalaridan birida erishadi.

Agar $y=f(x)$ funktsiyaning $[a,b]$ oraliqdagi eng katta va eng kichik qiymatlarini topish lozim bo'lsa, ularni topish quyidagi sxema bo'yicha amalga oshiriladi:

1) Berilgan $y=f(x)$ funktsiyaning $[a,b]$ oraliqning barcha nuqtalarida aniqlanganligiga ishonch hosil qilinib, $f'(x)$ hosila topiladi.

2) $y' = f'(x) = 0$ tenglama yechiladi hamda kritik nuqtalar topiladi. Topilgan nuqtalar ichida $[a, b]$ kesmada yotmaydiganlarini tashlaymiz, ya'ni qaramaymiz.

3) $y = f(x)$ funktsiyaning qolgan kritik nuqtalardagi va $[a, b]$ kesmaning chetki $x = a$ va $x = b$ nuqtalardagi qiymatlari hisoblanadi.

4) Topilgan qiymatlar o'zaro taqqoslanib, ular ichida eng kattasi va eng kichigi ajratib olinadi. Bu qiymatlar funktsiyaning $[a, b]$ kesmadagi eng katta va eng kichik qiymatlaridan iborat bo'ladi.

1-misol. $f(x) = x^3 - 2x^2$ funktsiyaning $[1; 3]$ kesmadagi eng katta va eng kichik qiymatlarini toping.

Yechilishi: 1. Berilgan funktsiya algebraik funktsiya bo'lib, u $[1; 3]$ kesmaning barcha nuqtalarida aniqlangan. Funktsiyaning hosilasini topamiz:

$$f'(x) = \left(x^3 - 2x^2\right)' = 3x^2 - 6x.$$

2. $3x^2 - 6x = 0$ tenglama yechiladi:

$$x(3x - 6) = 0, \quad x_1 = 0, \quad x_2 = 2.$$

Berilgan intervalga $x_1 = 0$ nuqta tegishli emas. Shuning uchun funktsiyaning kritik nuqtasi $x = 2$ dan iborat bo'ladi.

3. Kesmaning chetki $x = 1$ va $x = 3$ hamda $x = 2$ nuqtalarda funktsiyaning qiymatlarini hisoblaymiz:

$$f(1) = -2, \quad f(3) = 0 \text{ va } f(2) = -4.$$

4. Bu qiymatlardan eng kattasi 0, eng kichigi esa -4 dan iborat.

Demak, $f(x) = x^3 - 3x^2$ funktsiyaning $[1; 3]$ kesmadagi eng katta qiymati 0 va eng kichik qiymati -4 ga teng ekan.

2- misol. $f(x) = 2 \cdot 3^{3x} - 4 \cdot 3^{2x} - 2 \cdot 3^x$ funktsiyaning $[-1; 1]$ kesmadagi eng katta va eng kichik qiymatlarini toping.

Yechilishi: 1.Funktsiya berilgan oraliqning barcha nuqtalarida aniqlangan. Berilgan funktsiyaning hosilasini $\left(a^u\right)' = a^u \ln a \cdot u'$ formula yordamida topamiz:

$$f'(x)=2\cdot3^{3x}\cdot\ln3\cdot3-4\cdot3^{2x}\ln3\cdot2+2\cdot3^{x}\ln3=$$

$$2\cdot3^{x}\cdot\ln3\left(3\cdot3^{2x}-4\cdot3^{x}-1\right).$$

2. $f'(x)=0$ tenglamani yechib, kritik nuqtalarni topamiz.

$2\cdot3^{x}\cdot\ln3\neq0$ bo'lganligi sababli,

$$3\cdot3^{2x}-4\cdot3^{x}+1=0$$

tenglamani yechamiz:

$$3^{x}=t,\ t>0\Rightarrow 3t^{2}-4t+1=0.$$

$$\frac{D}{4}=4-3=1;\quad t=\frac{2\pm1}{3};\quad t_1=\frac{1}{3},\quad t_2=1.$$

Endi, eski o'zgaruvchi x ga qaytiladi:

a) $3^{x}=\frac{1}{3}\Rightarrow 3^{x}=3^{-1}\Rightarrow x_1=-1,$ b) $3^{x}=1\Rightarrow 3^{x}=3^{0}\Rightarrow x_2=0.$

3. Berilgan funktsiyaning $x=-1$, $x=0$, $x=1$ nuqtalardagi qiymatlarini topamiz:

$$f(-1)=2\cdot3^{-3}-4\cdot3^{-2}+2\cdot3^{-1}=\frac{2}{27}-\frac{4}{9}+\frac{2}{3}=\frac{8}{27},$$

$$f(0)=2\cdot3^{0}-4\cdot3^{0}+2\cdot3^{0}=2-4+2=0,$$

$$f(1)=2\cdot3^{3}-4\cdot3^{2}+2\cdot3=54-36+6=24.$$

4. Berilgan funktsiyaning $[-1;1]$ kesmadagi eng katta qiymati $f(1)=24$ va eng kichik qiymati $f(0)=0$ dan iborat ekan, ya'ni:

$$\underset{[-1;1]}{f_{\max}}(1)=24,\quad \underset{[-1;1]}{f_{\min}}(0)=0.$$

3-misol. $f(x)=\frac{3}{2}x+\sin x$ funktsiyaning $[a;b]$ kesmadagi eng katta va eng kichik qiymatlarini toping.

Yechilishi: Berilgan funktsiyaning hosilasini topamiz:

$$f'(x)=\frac{3}{2}+\cos x.$$

$f'(x)$ funktsiya x ning barcha qiymatlarida musbat. Shuning uchun bu funktsiya sonlar to’g’ri chizig’ining hamma yerida monoton o’sadi. Bunday holda, $[a; b]$ ning chap oxirida ($x = a$ da) bu funktsiya eng kichik $\frac{3}{2}a + \sin a$ qiymatni, o’ng oxirida ($x = b$ da) eng katta $\frac{3}{2}b + \sin b$ qiymatni qabul qiladi.

Mustaqil yechish uchun mashqlar

№ 119. $f(x) = x^3 - 3x$ funktsiyaning quyidagi kesmalardagi eng kichik va eng katta qiymatlarini toping:

a) $[-0{,}5;\ 0{,}5]$, b) $[-1{,}5;\ 2]$.

№ 120. $f(x) = \sin x - \sqrt{3}\cos x$ funktsiyaning quyidagi kesmalardagi eng kichik va eng katta qiymatlarini toping:

a) $[-p;\ 0]$, b) $\left[0;\ \frac{p}{2}\right]$.

№121. $f(x) = \frac{1}{2}x - \sin x$ funktsiyaning:

a) $\left[0;\ \frac{p}{2}\right]$, b) $\left[-\frac{p}{4};\ \frac{p}{4}\right]$, v) $\left[-\frac{p}{2};\ \frac{p}{2}\right]$ oraliqdagi eng kichik va eng katta qiymatlarini toping.

№122. $f(x) = 5 + 4\cos x - \sin^2 x$ funktsiyaning $\left[-\frac{p}{2};\ \frac{p}{2}\right]$ oraliqdagi eng katta va eng kichik qiymatlarini toping.

№123. 20 sonini shunday ikkita qo’shiluvchiga ajratish kerakki, ularning ko’paytmasi eng katta bo’lsin.

№124. Asosi va balandligi yig’indisi k ga teng bo’lgan barcha uchburchaklar ichida eng katta yuzaga ega bo’lgan uchburchak topilsin.

№125. $f(x) = x + 2\sqrt{x}$ funktsiyaning $[0;\ 4]$ kesmadagi eng katta va eng kichik qiymatlarini toping.

№126. $f(x)=\sin x\cdot\cos^2\frac{x}{2}$ funktsiyaning $[0;\ p]$ kesmadagi eng katta va eng kichik qiymatlarini toping.

№127. $f(x)=tgx+ctgx$ funktsiyaning $\left[\frac{p}{6};\ \frac{p}{3}\right]$ kesmadagi eng katta va eng kichik qiymatlarini toping.

№128. $f(x)=e^{2x-1}+2e^{1-2x}+7x-3$ funktsiyaning $[0{,}14;\ 1]$ kesmadagi eng katta va eng kichik qiymatlarini toping.

№129. Ikki sonning ayirmasi 18 ga teng. Bu sonlarning ko'paytmasi eng kichik bo'lishi uchun bu sonlar qanday bo'lishi kerak?

№130. Teng yonli uchburchakning yon tomoni 2 dm. ga teng. Eng katta yuzali uchburchakning asosi qanday bo'lishi kerak?

5. Funktsiyaning asimptotalari

Ta'rif: Agar $x\to+\infty$ bo'lganda quyidagi

$$\lim_{x\to+\infty}\left(f(x)-kx-b\right)=0 \qquad (1)$$

tenglik o'rinli bo'lsa, $y=kx+b$ to'g'ri chiziqqa $f(x)$ funktsiyaning ***og'ma asimptotasi*** deyiladi.

(1) tenglikdan $f(x)-kx-b=a(x),$ (2)

bunda $\lim\limits_{x\to\infty}a(x)=0.$ U holda, (2) dan

$$k=\frac{f(x)}{x}-\frac{b+a(x)}{x},\qquad b=f(x)-kx-a(x).$$

Bundan,

$$k=\lim_{x\to+\infty}\frac{f(x)}{x}\quad \text{va} \qquad (3)$$

$$b=\lim_{x\to+\infty}\left(f(x)-kx\right). \qquad (4)$$

(3) va (4) formulalar yordamida $x\to+\infty$ da $y=kx+b$ asimptotaning burchak koeffisiyenti k hamda uning boshlang'ich ordinatasi b ni hisoblab, topish

mumkin. $f(x)$ funktsiya grafigining $x \to +\infty$ dagi asimptotasi yuqoridagidek aniqlanadi.

Agar $k=0$ bo'lsa, asimptotaning tenglamasi

$$y = b \qquad (5)$$

dan iborat bo'ladi.

Ta'rif: (5) tenglama bilan aniqlanadigan asimptotaga ***gorizontal asimptota*** deyiladi.

Ta'rif: Agar $\lim\limits_{x\to a-0} f(x) = \infty$ yoki $\lim\limits_{x\to a+0} f(x) = \infty$ tengliklar o'rinli bo'lsa,

$$x = a \qquad (6)$$

to'g'ri chiziqqa ***vertikal asimptota*** deyiladi.

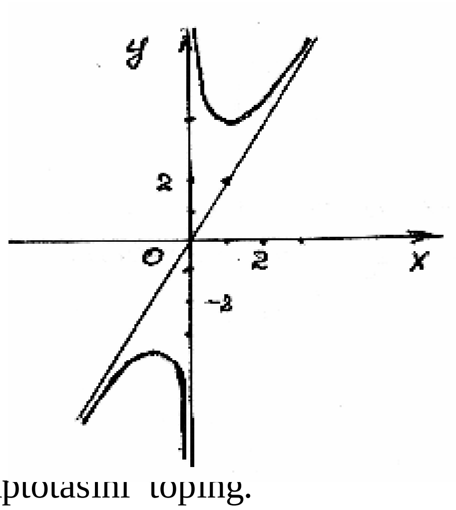
1-misol. $f(x) = \dfrac{2x^2-1}{x}$ egri chiziqning asimptotasini toping.

Yechilishi: (3) va (4) formulalar yordamida asimptotaning burchak koeffisiyenti k va boshlang'ich ordinatasi b ni topamiz:

$$k = \lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \frac{2x^2-1}{x^2} = 2,$$

$$b = \lim_{x\to+\infty}\left(f(x) - kx\right) = \lim_{x\to+\infty}\left(\frac{2x^2-1}{x^2} - 2x\right) = \lim_{x\to+\infty} \frac{2x^2-1-2x^2}{x} = \lim_{x\to+\infty}\left(-\frac{1}{x}\right) = 0.$$

Demak, $y = 2x$ to'g'ri chiziq $f(x)$ funktsiyaning $x \to +\infty$ dagi og'ma asimptotasi ekan. Shu to'g'ri chiziq $x \to -\infty$ da ham asimptota bo'ladi. Asimptotasi $x = 0$ to'g'ri chiziq, ya'ni OY o'qidan iborat.



2-misol. $f(x) = \dfrac{x^2+1}{x-2}$ egri chiziqning asimptotasini toping.

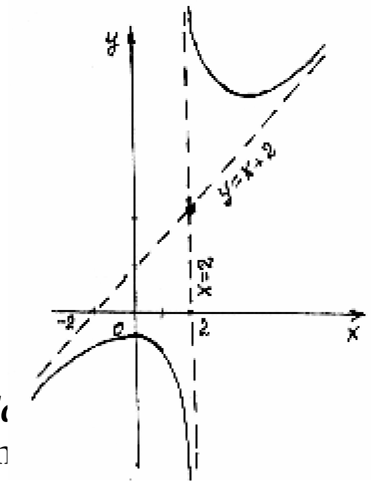
Yechilish: $\lim\limits_{x\to 2+0} \dfrac{x^2+1}{x-2} = +\infty, \quad \lim\limits_{x\to 2-0} \dfrac{x^2+1}{x-2} = -\infty.$

U holda, $x=2$ egri chiziqning vertikal asimptotasi bo'ladi. Endi k va b ni topamiz:

$$k=\lim_{x\to\pm\infty}\frac{x^2+1}{x(x-2)}=1,$$

$$b=\lim_{x\to\pm\infty}\left(\frac{x^2+1}{x-2}-x\right)=\lim_{x\to\pm\infty}\frac{x^2+1-x^2+2x}{x-2}=\lim_{x\to\pm\infty}\frac{2x+1}{x-2}=2.$$

$k=1$, $b=2$ bo'lganligi uchun asiptotaning tenglamasi $y=x+2$ bo'lib, u $x\to\pm\infty$ da og'ma asimptotadan iborat bo'ladi.



Demak, berilgan $f(x)$ funktsiyaning og'ma asimptotasi $y=x+2$ va vertikal asimptotasi $x=2$ dir.

Mustaqil yechish uchun mashql

Quyidagi berilgan funktsiyalarning asimptotalarin

№ 131 $f(x)=\dfrac{1}{x-2}.$

№ 132 $f(x)=\dfrac{x^3}{x-1}.$

№ 133 $f(x)=\dfrac{1}{1-x^2}.$

№ 134 $f(x)=\sqrt{x^2-1}.$

№ 135 $f(x)=\dfrac{x^2}{x-1}.$

№ 136 $f(x)=\dfrac{5}{x^2-25}.$

№ 137 $f(x)=\dfrac{x^2+6x-5}{x}.$

№ 138 $f(x)=\dfrac{x^2-5x+4}{x-4}.$

№ 139 $f(x)=\dfrac{3x^2+1}{x}.$

№ 140 $f(x)=\dfrac{4}{x^2-4}.$

6. Funktsiyani hosila yordamida tekshirib grafigini chizish sxemasi

Hosiladan foydalanish differentsiallanuvchi funktsiyalarni tekshirishni va ularning grafiklarini yasashni ancha yengillashtiradi. Hosila yordamida funktsiyalarning o'sish va kamayish oraliqlarini (intervallarini), kritik nuqtalarini hamda ekstremumlarini aniqlash mumkin. Bu esa tekshirilayotgan funktsiyalarning grafiklarini aniqroq yasash imkoniyatini beradi.

$y=f(x)$ funktsiya berilgan bo'lib, uning grafigini yasash talab qilinsin. Buning uchun funktsiyani uning hosilasi yordamida quyidagi reja asosida tekshiriladi va grafigi chiziladi:

1. Berilgan funktsiyaning aniqlanish sohasi topiladi.

2. $y=f(x)$ funktsiyaning juft yoki toqligi, davriyligi tekshiriladi. Bunda shuni hisobga olish kerakki, agar $f(-x)=f(x)$ bo'lsa, berilgan funktsiya juft bo'ladi va uning grafigi OY ordinatalar o'qiga nisbatan simmetrik bo'ladi. Agar $f(-x)=-f(x)$ bo'lsa, funktsiya toq bo'lib, uning grafigi koordinatalar boshiga nisbatan simmetrik bo'ladi.

Agar $f(x+T)=f(x)$ tenglik T o'zgarmas son bo'lganda bajarilsa, ***funktsiya davriy*** deyiladi. Davriy funktsiyaning grafigi T davr osha takrorlanadi.

3. Berilgan funktsiyaning $f'(x)$ hosilasi topiladi. $f'(x)=0$ tenglama yechilib, funktsiyaning kritik nuqtalari aniqlanadi.

4. Berilgan $y=f(x)$ funktsiyaning monotonlik oraliqlari va ekstremumlari topiladi. Monotonlik oraliqlarini topishda ikkita quyidagi shartdan foydalaniladi: 1) ularning chegaralari kritik nuqtalar va funktsiya aniqlangan nuqtalardan iborat; 2) agar qaralayotgan oraliqda hosila musbat (ya'ni $f'(x)>0$) bo'lsa, funktsiya bu oraliqda o'sadi; agar qaralayotgan oraliqda funktsiyaning hosilasi manfiy (ya'ni $f'(x)<0$) bo'lsa, funktsiya kamayuvchi bo'ladi.

Agar $x=x_0$ kritik nuqtadan o'tganda funktsiyaning hosilasi o'z ishorasini musbatdan manfiyga o'zgartirsa, $x=x_0$ kritik nuqta maksimumga ega bo'ladi. Agar funktsiyaning ishorasi manfiydan musbatga o'zgarsa, funktsiya $x=x_0$ nuqtada minimumga ega bo'ladi.

5. Berilgan funktsiya grafigining koordinata o'qlari bilan kesishish nuqtalari topiladi. Bunda $f(x)=0$ deb olinib, funktsiya grafigining absissalar o'qi bilan, $x=0$ deb, grafikning ordinatalar o'qi OY bilan kesishish nuqtalari topiladi. Bunda chegaralari grafikning OX o'qi bilan kesishish nuqtalari va funktsiya aniqlanmagan nuqtalar bo'lgan ishora o'zgarmaslilik oraliqlarini topish mumkin. Bu nuqtalarni sonlar o'qida belgilab, shu oraliqlarning har birida funktsiyaning ishorasi aniqlanadi.

6. Yuqoridagi olingan ma'lumotlar asosida jadval tuzib, funktsiya grafigi chiziladi.

Misol. $f(x)=x^3-12x$ funktsiyani hosila yordamida tekshiring va grafigini chizing.

Yechilishi: 1. Berilgan funktsiya algebraik funktsiya bo'lib, uning aniqlanish sohasi barcha haqiqiy sonlar to'plamidan iborat.

2. Berilgan funktsiya toq, chunki grafigi koordinatalar boshiga nisbatan simmetrikdir, u davriy emas.

$$f(-x)=(-x)^3-12(-x)=-(x^3-12x).$$

3. Funktsiyaning hosilasini topamiz:

$$f'(x)=(x^3-12x)'=(x^3)'-(12x)'=3(x^2-4).$$

$x^2-4=0$ tenglamani yechib, kritik nuqtalar topiladi:

$$x^2=4, \quad x_1=-2, \quad x_2=2.$$

4. Sonlar o'qida $x_1=-2$ va $x_2=2$ nuqtalarni belgilaymiz va $(-\infty;-2)$, $(-2;2)$, $(2;+\infty)$ oraliqlarda hosilaning ishorasini aniqlaymiz. Bu oraliqlarning har birida funktsiyaning o'zgarishini aniqlaymiz:

$(-\infty;-2)$da: $x=-3$ bo'lsa, $f'(-3)=3(9-4)>0$ - funktsiya o'sadi;

$(-2;2)$da: $x=0$ bo'lsa, $f'(0)=3(0-4)<0$ -funktsiya kamayadi.

$(2;+\infty)$da: $x=3$ bo'lsa, $f'(3)=3(9-4)>0$ - funktsiya o'sadi.

$x=-2$ nuqtadan o’tishda hosila o’z ishorasini musbatdan manfiyga o’zgartiradi, $x=2$ nuqtadan o’tganda esa manfiydan musbatga o’zgartiradi. Shuning uchun $x=-2$ nuqtada funktsiya maksimumga erishadi, $x=2$ da esa minimumga erishadi:

$$f_{\max}(-2)=(-2)^3-12(-2)16, \qquad f_{\min}(x)=2^2-12\cdot 2=-16.$$

5. Funktsiya grafigining koordinata o’qlari bilan kesishish nuqtalarini topamiz.

$x=0$ bo’lsin, u holda, $y=0$ bo’ladi, ya’ni grafik koordinatalar boshidan o’tadi va OY o’qi bilan boshqa umumiy nuqtalarga ega bo’lmaydi.

$y=0$ bo’lsin, u holda, funktsiya grafigining OX o’q bilan kesishish nuqtalari quyidagicha topiladi:

$x^3-12x=0,\quad x(x^2-12)=0$. Bundan, $x_1=0$ va $x_{2,3}=\pm 2\sqrt{3}$.

Sonlar o’qida $x=-2\sqrt{3}$, $x=0$ va $x=2\sqrt{3}$ nuqtalarni belgilab olamiz va $(-\infty;\,2\sqrt{3})$, $(2\sqrt{3};\,0)$, $(0;\,2\sqrt{3})$ hamda $(2\sqrt{3};\,+\infty)$ oraliqlarning har birida funktsiyaning ishorasini tekshirib, funktsiyaning ishora o’zgarmaslilik oraliqlarini topamiz:

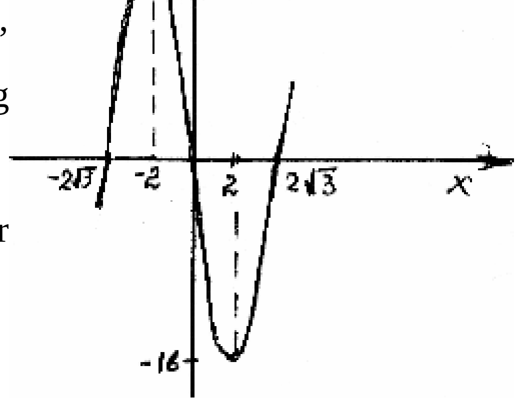
$(-\infty;\,2\sqrt{3})$da $x=-4$ bo’lsin, u holda, $f(-4)=-4(16-12)<0$, ya’ni funktsiyaning grafigi OX o’qidan pastda joylashgan.

$(-2\sqrt{3};\,0)$ da $x=-1$ bo’lsin, u holda, $f(-1)=-1(1-12)>0$, ya’ni grafik OX o’qidan yuqorida joylashgan.

$(0;\,2\sqrt{3})$da $x=1$ bo’lsin, u holda, $f(1)=1(1-12)<0$, ya’ni grafik OX o’qidan pastda joylashgan.

$(2\sqrt{3};\,+\infty)$da $x=4$ bo’lsin, u holda, $f(4)=4(16-12)>0$, ya’ni funktsiyaning grafigi OX o’qidan yuqorida joylashgan.

6. Funktsiya grafigini topilganlar asosida chizamiz.



Mustaqil yechish uchun mashqlar

Quyidagi funktsiyalarni hosila yordamida tekshiring va grafiklarini yasang.

№141. $y = x^4 - 2x - 3.$

№142. $y = \frac{1}{10}\left(x^4 - 13x^2 + 36\right).$

№143. $y = x^3 - 9x.$

№144. $y = x^3 - 3x^2 + 2.$

№ 145. $y = \frac{x-1}{x^2 - 3x - 4}.$

№ 146. $y = x\left(3 - x^2\right).$

№ 147. $y = \frac{x^2}{2} + \frac{1}{x}.$

№ 148. $y = \frac{x}{x^2 + 1}.$

№ 149. $y = \frac{x^2 + 4}{x}.$

№ 150. $y = \frac{1}{x^2 + 2x}.$

№151. $y = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x.$

№ 152. $y = \frac{x^2}{x^3 - 1}.$

№ 153. $y = \frac{1}{4}x^4 - \frac{1}{2}x^2.$

№ 154. $y = -\frac{1}{3}x^3 + \frac{1}{2}x^2.$

№ 155. $y = x^3 - 6x^2 + 9x.$

№ 156. $y = \frac{1}{2}x + \sin x.$

№ 157. $y = \sqrt{3}\cos x - 2x.$

№ 159. $y = \sin^2 x + 2\cos x + \frac{1}{4}.$

16§. O’rta qiymatlar haqidagi teoremalar

a) ***Ferma teoremasi***. $y = f(x)$ funktsiya biror X oraliqda aniqlangan va oraliqning ichki c nuqtasida eng katta (eng kichik) qiymatga ega bo’lsin. Agar bu nuqtada chekli $f'(c)$ hosila mavjud bo’lsa, u holda, $f'(c) = 0$ bo’lishi zarur.

Isboti. $f(x)$ funktsiya c nuqtada eng katta qiymatga ega bo’lmin, deb faraz qilaylik. U holda, X oraliqdagi barcha x lar uchun quyidagi o’rinlidir:

$$f(x) \le f(c). \qquad (1)$$

Hosilaning ta'rifiga asosan: $f'(c)=\lim\limits_{x\to c}\dfrac{f(x)-f(c)}{x-c}.$ (2)

$x>c$ bo'lsa, $\dfrac{f(x)-f(c)}{x-c}\le 0.$ (3)

Agar $x\to c\ne 0$ da limitga o'tilsa, $f'(c)\le 0.$ (4)

Agar $x<c$ bo'lsa, u holda $\dfrac{f(x)-f(c)}{x-c}\ge 0.$ (5)

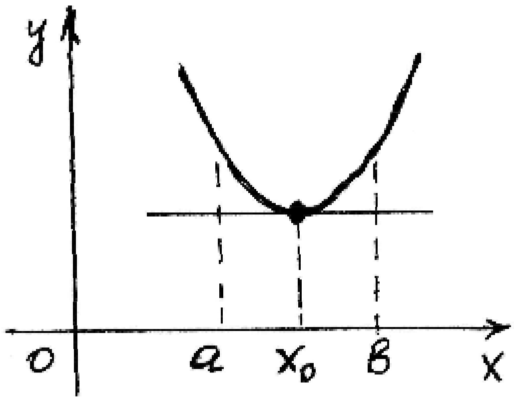
(5)dan $x\to c-0$ bo'lganda limitga o'tilsa, quyidagi hosil bo'ladi:

$$f'(c)\ge 0. \qquad (6)$$

(4) va (6) munosabatlarni taqqoslab, quyidagi kutilgan natijaga ega bo'lamiz:

$$f'(c)=0. \qquad (7)$$

Ferma teoremasining geometrik ma'nosi shundan iboratki, $f(x)$ differentsiallanuvchi funktsiya x_0 nuqtada eng katta yoki eng kichik qiymatga ega bo'lsa, u holda, $f(x)$ funktsiya grafigining (x_0; $f(x_0)$) nuqtasiga urinuvchi to'g'ri chiziq absissalar o'qiga parallel bo'ladi.

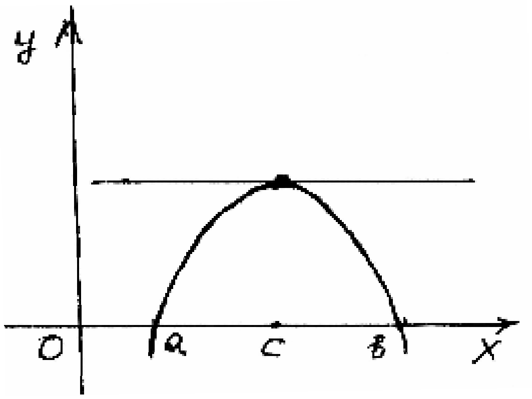


b) ***Rolli teoremasi***. Agar $y=f(x)$ funktsiya $[a;b]$ yopiq oraliqda aniqlangan va uzluksiz, oraliq ichki nuqtalarining barchasida hosilaga ega hamda oraliqning chetlarida funktsiya teng $f(a)=f(b)$qiymatlarni qabul qilsa, $[a;b]$ning hyech bo'lmaganda bitta shunday c ichki nuqtasi topiladiki, bu nuqtada $y'=f'(c)=0$ bo'ladi.

Isboti: Berilgan funktsiya $[a;b]$ da uzluksiz bo'lganligi uchun bu oraliqda uning katta M va eng kichik m qiymatlari mavjud.

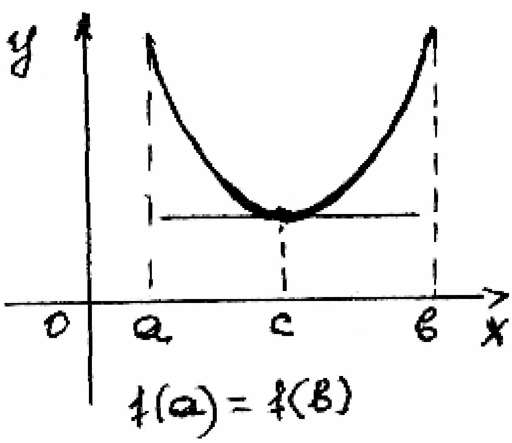
Agar $M=m$ bo'lsa $y=f(x)$ funktsiya har qanday nuqtada o'zgarmas $f(x)=M$ qiymatga ega bo'lib, har qanday nuqtada $f'(x)=0$ bo'ladi. $M=m$ hol uchun teorema isbot bo'ldi.

Agar $M > m$ bo’lsa, funktsiya ikkala qiymatga ham erishishi ma’lum, ammo $f(a) = f(b)$ bo’lganligi uchun ularning ikkalasiga ham oraliqning chetlarida erishishi mumkin emas, ulardan hyech bo’lmaganda bittasiga, a va b orasidagi c nuqtada erishiladi. Bunday holda, Ferma teoremasiga asosan bu nuqtada $f'(c)$ hosila nolga aylanadi, ya’ni $f'(c) = 0$.



Demak, teorema isbotlandi.

Bu teoremaning geometrik ma’nosi shundan iboratki, berilgan $f(x)$ funktsiya $[a; b]$ kesmada uzluksiz, shu kesma ichida differentsiallanuvchi hamda kesma uchlarida bir xil qiymat qabul qilsa, shu oraliqda birorta $(c, f(c))$ nuqta topiladiki, bu nuqtaga urinuvchi to’g’ri chiziq OX o’qqa parallel bo’ladi.



v) Lagranj teoremasi. $y = f(x)$ funktsiya $[a; b]$ oraliqda aniqlangan va uzluksiz hamda oraliqning ichki nuqtalarida hosilaga ega (differentsiallanuvchi) bo’lsa, u holda, bu kesmaning hyech bo’lmaganda bitta shunday ichki c $(a < c < b)$ nuqtasi mavjud bo’ladiki, unda

$$\frac{f(b) - f(a)}{b - a} = f'(c), \tag{8}$$

ya’ni kesuvchining burchak keeffisiyenti urinmaning burchak koeffisiyentiga teng bo’ladi.

Isboti: Berilgan $[a; b]$ oraliqda qo’yidagi tenglik bilan aniqlanuvchi yordamchi funktsiya kritamiz:

$$F(x) = f(x) - f(a) - \frac{f(b) - f(a)}{b - a}(x - a). \tag{9}$$

Bu funktsiya $[a;b]$ da uzluksiz, chunki u $f(x)$ uzluksiz funktsiya bilan chiziqli funktsiyaning ayirmasidan iborat hamda $(a;b)$ oraliqda quyidagi chekli hosilaga ega:

$$F'(x)=f'(c)-\frac{f(b)-f(a)}{b-a}. \qquad (10)$$

va bevosita qo'yib ko'rish bilan

$$F(a)=F(b)=0 \qquad (11)$$

ekanligiga, ya'ni $F(x)$ funktsiya oraliqning chetlarida teng qiymatlarga ega bo'lganligi uchun ham Roll teoremasining hamma shartlarini qanoatlantiradi.

Demak, $F(x)$ funktsiyaga Roll teoremasini qo'llash va $(a;b)$ da $F'(c)=0$ bo'ladigan c nuqtaning mavjudligini tasdiqlash mumkin.

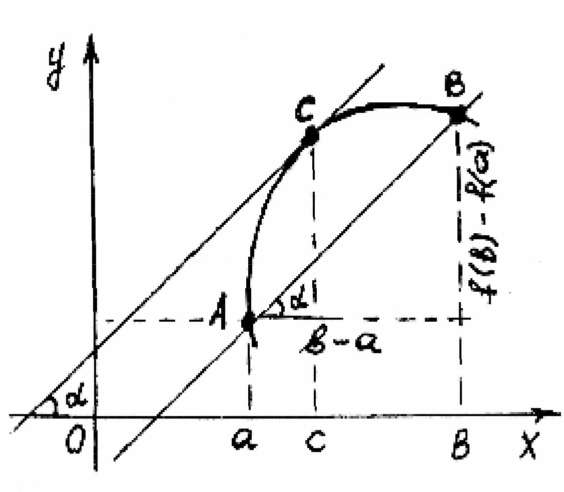
Xulosa qilib, quyidagini yozish mumkin:

$$f'(c)-\frac{f(b)-f(a)}{b-a}=0.$$

Bundan esa
$$\frac{f(b)-f(a)}{b-a}=f'(c). \qquad (12)$$

Teorema isbot bo'ldi.

Logranj teoremasining geometrik ma'nosi: $y=f(x)$ funktsiya grafigining $A(a, f(a))$ va $B(b, f(b))$ nuqtalaridan o'tuvchi to'g'ri chiziqning burchak koeffisiyenti $\frac{f(b)-f(a)}{b-a}$ dan iborat. $C(c, f(c))$ nuqtaga urinuvchi urinmaning burchak koeffisiyenti esa $f'(c)$ dan iborat. Teorema mazmuniga



asosan c nuqtaga urinuvchi to'g'ri chiziq AB kesuvchiga parallel bo'ladi.

g) Koshi teoremasi (O'rta qiymat haqidagi umumlashgan teorema)

Teorema. $f(t)$ va $j(t)$ funktsiyalar $(a;b)$ oraliqda differentsiallanuvchi hamda $f'(t)$ va $j'(t)$ hosilalarga ega bo'lib, shu oraliqning biror joyida ikkalasi bir vaqtda 0(nol)ga aylanmasin. Funktsiyalardan biri masalan, $j(t)$ ning shu oraliq chegaralaridagi qiymatlari o'zaro teng bo'lmasin, ya'ni $j(a)\neq j(b)$. U holda, berilgan funktsiyalarning $f(b)-f(a)$ va $j(b)-j(a)$ orttirmalari nisbati shu funktsiyalarning $(a;b)$ oraliqda yotuvchi $t=t$ nuqtadagi hosilalarining nisbati kabi bo'ladi:

$$\frac{f(b)-f(a)}{j(b)-j(a)}=\frac{f'(t)}{j'(t)}.$$

17§. Lopital qoidast

Ma'lumki, $\frac{0}{0}, \frac{\infty}{\infty}, 0\cdot\infty, \infty-\infty, 1^x, 0^0, \infty^0$ kabi aniqmasliklar mavjud. Bunday aniqmasliklarni ochishda elementar usullardan foydalanilgan edi. Endi esa hosila tushunchasidan foydalanamiz.

Teorema(Lopital qoidasi). Faraz qilaylik, $f(x)$va $g(x)$ funktsiyalar $x=a$ nuqta atrofida mavjud va differentsiallanuvchi bo'lib,

$$\lim_{x\to a} f(x)=f(a)=0 \text{ va } \lim_{x\to a} g(a)=0.$$

yoki

$$\lim_{x\to a} f(x)=f(a)=\infty \text{ va } \lim_{x\to a} g(a)=\infty$$

hamda $g'(a)\neq 0$ bo'lsa, u holda, $\lim_{x\to a}\frac{f(x)}{g(x)}=\frac{f'(a)}{g'(a)}$ bo'ladi.

Isboti. 1) $\lim_{x\to a} f(x)=0$ va $\lim_{x\to a} g(x)=0$ bo'lgan hol uchun isbot qilamiz.

$f(a)=g(a)=0$ bo'lganligi uchun $\frac{f(x)}{g(x)}=\frac{f(x)-f(a)}{g(x)-g(a)}=\frac{\frac{f(x)-f(a)}{x-a}}{\frac{g(x)-g(a)}{x-a}}$.

Tenglikning ikkala tomonidan $x\to a$ da limit olsak:

$$\lim_{x\to a}=\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{\dfrac{f(x)-f(a)}{x-a}}{\dfrac{g(x)-g(a)}{x-a}}=\frac{\lim\limits_{x\to a}\dfrac{f(x)-f(a)}{x-a}}{\lim\limits_{x\to a}\dfrac{g(x)-g(a)}{x-a}}=\frac{f'(a)}{g'(a)}=\lim_{x\to a}=\frac{f(x)}{g(x)}.$$

Demak, $$\lim_{x\to a}=\frac{f(x)}{g(x)}=\lim_{x\to a}=\frac{f'(x)}{g'(x)}=\frac{f'(a)}{g'(a)}.$$

2) $\lim\limits_{x\to a} f(x)=\infty$ va $\lim\limits_{x\to a} g(x)=\infty$ hol uchun ham teorema xuddi shu usul bilan isbot qilinadi.

Boshqa ko’rinishdagi aniqmasliklarni ochish uchun u aniqmasliklar avval $\frac{0}{0}$ yoki $\frac{\infty}{\infty}$ ko’rinishlaridan biriga keltiriladi, keyin esa yuqoridagi usul bilan aniqmaslik ochiladi.

Misollar

№1. $\lim\limits_{x\to 2}\frac{x-2}{x^2-4}=\frac{2-2}{4-4}=\frac{0}{0}.$

Yechish: $\lim\limits_{x\to 2}\frac{x-2}{x^2-4}=\lim\limits_{x\to 2}\frac{(x-2)'}{(x^2-4)'}=\lim\limits_{x\to 2}\frac{1-0}{2x-0}=\lim\limits_{x\to 2}\frac{1}{2x}=\frac{1}{2\cdot 2}=\frac{1}{4}=0{,}25.$

№2. $\lim\limits_{x\to 0}\frac{3x^2-5x}{3x^3+5x}=\frac{0}{0}.$

Yechish: $\lim\limits_{x\to 0}\frac{3x^2-5x}{3x^3+5x}=\lim\limits_{x\to 0}\frac{(3x^2-5x)'}{(3x^3+5x)'}=\lim\limits_{x\to 0}\frac{6x-5}{9x^2+5}=\frac{6\cdot 0-5}{9\cdot 0+5}=\frac{-5}{5}=-1.$

№3. $\lim\limits_{x\to 3}\frac{x^2-5x+6}{3x^3-9x}=\frac{3^2-5\cdot 3+6}{3\cdot 3^2-9\cdot 3}=\frac{0}{0}.$

Yechish:

$$\lim_{x\to 3}\frac{x^2-5x+6}{3x^2-9x}=\lim_{x\to 3}\frac{(x^2-5x+6)'}{(3x^2-9x)'}=\lim_{x\to 3}\frac{2x-5}{6x-9}=\frac{2\cdot 3-5}{6\cdot 3-9}=\frac{6-5}{18-9}=\frac{1}{9}.$$

№4. $\lim\limits_{x\to 5}\frac{x^2-7x+10}{x^2-9x+20}=\frac{25-35+10}{25-45+20}=\frac{0}{0}.$

Yechish: $\lim\limits_{x\to 5}\dfrac{\left(x^2-7x+10\right)'}{\left(x^2-9x+20\right)'}=\lim\limits_{x\to 5}\dfrac{2x-7}{2x-9}=\dfrac{2\cdot 5-7}{2\cdot 5-9}=\dfrac{10-7}{10-9}=3.$

№5. $\lim\limits_{n\to\infty}\dfrac{3n+1}{7n-1}=\dfrac{\infty}{\infty}.$

Yechish: $\lim\limits_{n\to\infty}\dfrac{(3n+1)'}{(7n-1)'}=\lim\limits_{n\to\infty}\dfrac{3}{7}=\dfrac{3}{7}.$

№6. $\lim\limits_{n\to\infty}\dfrac{2n^2+3n-4}{3n^2-4n+1}=\dfrac{\infty}{\infty}.$

Yechish: $\lim\limits_{n\to\infty}\dfrac{\left(2n^2+3n-4\right)'}{\left(3n^2-4n+1\right)'}=\lim\limits_{n\to\infty}\dfrac{4n-3}{6n-4}=\lim\limits_{n\to\infty}\dfrac{(4n-3)'}{(6n-4)'}=\dfrac{4}{6}=\dfrac{2}{3}.$

№7. $\lim\limits_{x\to\infty}\dfrac{x^6+x^5}{x^4+x^3}=\dfrac{\infty}{\infty}.$

Yechish:

$$\lim_{x\to\infty}\frac{\left(x^6+x^5\right)'}{\left(x^4+x^3\right)'}=\lim_{x\to\infty}\frac{6x^5+5x^4}{4x^3+3x^2}=\lim_{x\to\infty}\frac{\left(6x^5+5x^4\right)'}{\left(4x^3+3x^2\right)'}=\lim_{x\to\infty}\frac{30x^4+20x^3}{12x^2+6x}=$$

$$=\lim_{x\to\infty}\frac{\left(30x^4+20x^3\right)'}{\left(12x^2+6x\right)'}=\lim_{x\to\infty}\frac{120x^3+60x^2}{24x+6}=\lim_{x\to\infty}\frac{\left(120x^3+60x^2\right)'}{(24x+6)'}=$$

$$=\lim_{x\to\infty}\frac{360x^2+120x}{24}=\frac{\infty}{24}=\infty.$$

Mustaqil yechish uchun mashqlar

Lopital qoidasidan foydalanib hisoblang

№160. a) $\lim\limits_{x\to 2}\dfrac{x^2-4}{x-2}$; b) $\lim\limits_{x\to 2}\dfrac{x^3-8}{x-2}$; v) $\lim\limits_{x\to 0}\dfrac{1-\cos x}{x^2}$;

g) $\lim\limits_{x\to 0}\dfrac{tgx-x}{x-\sin x}$; d) $\lim\limits_{x\to 0}\dfrac{e^x-1}{\sin x}$; ye) $\lim\limits_{x\to -\frac{p}{2}}\dfrac{\cos^2 x}{1+\sin^3 x}$.

№ 161. a) $\lim\limits_{x\to p}\dfrac{\sin^2 x}{1+\cos x}$; b) $\lim\limits_{x\to \frac{p}{4}}\dfrac{\sin x-\cos x}{tgx-1}$; v) $\lim\limits_{x\to 0}\dfrac{\sin 4x}{3x}$.

№ 162. a) $\lim\limits_{x\to 0}\dfrac{x-\sin x}{x^3}$; b) $\lim\limits_{x\to 0}\dfrac{e^x-1}{x}$; v) $\lim\limits_{x\to 1}\dfrac{x^2+\ln x-1}{e^x-e}$;

g) $\lim\limits_{x\to 1}\dfrac{x^3-3x^2+2}{x^3-4x^2+3}$; d) $\lim\limits_{x\to 0}\dfrac{e^x-e^{-x}}{\ln(1+x)}$; ye) $\lim\limits_{x\to 0}\dfrac{e^x-e^{\sin x}}{x-\sin x}$.

№ 163. a) $\lim\limits_{x\to 0}\dfrac{\sin x-x}{x^3}$; b) $\lim\limits_{x\to 0}\dfrac{2-\left(e^x+e^{-x}\right)\cdot\cos x}{x^4}$; v) $\lim\limits_{x\to -3}\dfrac{x^3+27}{x+3}$;

g) $\lim\limits_{x\to 0}\dfrac{x+x^2-\sin x}{\left(x+x^2\right)\sin x}$; d) $\lim\limits_{x\to 0}\dfrac{e^{2x}-1}{\ln(1+2x)}$; ye) $\lim\limits_{x\to 0}\dfrac{\ln(\cos ax)}{\ln(\cos bx)}$.

№ 164. a) $\lim\limits_{x\to 0}\dfrac{2^x-1}{\sin x}$; b) $\lim\limits_{x\to 0}\dfrac{\arcsin 3x}{2x}$; v) $\lim\limits_{x\to \frac{p}{2}}\dfrac{\frac{p}{2}-x}{ctgx}$.

№ 165. a) $\lim\limits_{x\to \infty}\dfrac{2x^3+3x^2+1}{x^2+4}$; b) $\lim\limits_{x\to \infty}\dfrac{\log x}{x^n}$; v) $\lim\limits_{x\to \infty}\dfrac{x^n}{e^x}$;

g) $\lim\limits_{x\to \infty}\dfrac{\ln x}{x}$; d) $\lim\limits_{x\to \infty}\dfrac{e^x}{x^2}$; ye) $\lim\limits_{x\to \infty}\dfrac{a^x}{x}$.

№ 166. a) $\lim\limits_{x\to 0}(x\ln x)$; b) $\lim\limits_{x\to 0}\left(ctgx-\dfrac{1}{x}\right)$; v) $\lim\limits_{x\to \infty}\left(x-\sqrt{x^2+1}\right)$.

18§. Funktsiya differentsialining taqribiy hisoblashda qo'llanilishi

Biror x kattalikni taqribiy hisoblashda Δx xatoga yo'l qo'yiladi. Bu esa y miqdor uchun Δy xatoni vujudga keltiradi.

Agar funktsiya argumentining hisoblangan taqribiy qiymatini x

deb qaraladigan bo'lsa, x ni o'lchashda yo'l qo'yilgan absolyut xato Δx bo'ladi. U holda, o'lchanadigan kattalikning haqiqiy qiymati $x+\Delta x$ dan iborat bo'ladi. Natijada, $f(x)$ funktsiyaning taqribiy qiymatini x, $f(x+\Delta x)$ funktsiyaning haqiqiy qiymatini esa $x+\Delta x$ aniqlaydi. U holda, funktsiyaning absolyut xatosi quyidagi tenglik orqali ifodalanadi:

$$|\Delta y| = |f(x+\Delta x) - f(x)|. \qquad (1)$$

Agar Δx ning nolga yaqin, ya'ni kichik qiymatlarida Δy ni dy differentsial bilan almashtirilsa

$$\Delta y = f(x+\Delta x) - f(x) \approx f'(x)dx = dy$$

hosil bo'ladi.

Ixtyoriy y kattalik $y = f(x)$ tenglamadan topilsin deb hisoblaylik. U holda, x kattalikni topishda yul qo'yiladigan Δx absolyut xato y kattalikning Δy absolyut xatosini hosil qiladi.

$\Delta y \approx dy$ bo'lsa, r nisbiy xato uchun quyidagi ifoda o'rinli bo'ladi:

$$r = \left|\frac{dy}{y}\right|. \qquad (2)$$

(2) ifodani quyidagicha ta'riflash mumkin: qaralayotgan kattalik absolyut xatosining (dy) shu kattalik taqribiy qiymati y ga nisbatining absolyut qiymati ***nisbiy xato*** deb aytiladi va r prosentlarda ifodalanadi.

1-misol. Radiusi $r = 125$ sm bo'lgan doiraning yuzini hisoblashda qilingan nisbiy xatolarni taqqoslang, bunda:

a) absolyut xato doira yuzining orttirmasiga teng bo'lsin;

b) absolyut xato doira yuzining differentsialiga teng bo'lsin.

Yechilishi:

a) Qaralayotgan doira yuzining orttirmasi ΔS ni va doira yuzi $S = pR^2$ ni hisoblaymiz. Doiraning radiusini hisoblashdagi xato $|\pm 0{,}5\, см|$ dan oshmaydi deb hisoblaymiz. U holda,

$$\Delta S = p\left(R+\Delta R\right)^2 - pR^2 = p[2R\cdot\Delta R + \left(\Delta R\right)^2] = p\left(2\cdot 125\cdot 0{,}5 + 0{,}25\right) = 125{,}25p .$$

$$\frac{\Delta S}{S} = \frac{125{,}25p}{p\cdot 125^2} = 0{,}008016 \approx 0{,}8\%.$$

b) Doiraning yuzini hisoblashdagi dS - differentsialni va $\frac{dS}{S}$ nisbiy xatoni topamiz:

$$dS = 2p\cdot R\cdot\Delta R = 2p\cdot 125\cdot 0{,}5 = 125p ,$$

$$\frac{dS}{S} = \frac{2p\cdot R\cdot\Delta R}{p\cdot R^2} = y\frac{dR}{R}.$$

Qaralayotgan doiraning yuzini hisoblashdagi nisbiy xato radiusni hisoblashdagi nisbiy xatoning ikkilanganiga teng:

$$\frac{dS}{S} = 2\frac{dR}{R} = 2\cdot\frac{0{,}5}{125} = 0{,}8\%.$$

Demak, b) hola hisoblash ancha sodda va hisoblash aniqligiga zarar yetkazilmasdan bajarilgan.

ΔS orttirmani dS differentsial bilan almashtirishdagi yaqinlashishning nisbiy xatosi quyidagidan iborat bo'ladi:

$$\Delta S - ds = 125{,}25p - 125p = 0{,}25p ,$$

$$\frac{\Delta S - ds}{dS} = \frac{0{,}25\cdot p}{125\cdot p} = 0{,}002 = 0{,}2\%.$$

Demak, yaqinlashishning nisbiy xatosi $0{,}2\%$ ekan.

Differentsial yordamida funktsiya orttirmasini topish uchun berilgan $y = f\left(x\right)$ funktsiyaning $\Delta y = f\left(x+\Delta x\right) - f\left(x\right)$ orttirmasi topiladi. Funktsiyaning differentsiali esa $\Delta y = f'\left(x\right)dx$ dan iborat. U holda, x argumentning yetarlicha kichik orttirmasida

$$\Delta y \approx dy$$

deb olish mumkin bo'ladi. Demak, funktsiyaning orttirmasi uning differentsialiga taqriban teng bo'ladi.

2- misol. $y=4x^2+2x-1$ da $x=2$ va $\Delta x = dx = 0{,}001$ bo'lganda Δy ni toping.

Yechilishi:

$$\Delta y \approx dy = d\left(4x^2+2x-1\right)=(8x+2)dx=(8\cdot 2+2)\cdot 0{,}001=18\cdot 0{,}001=0{,}018.$$

Δy ning haqiqiy qiymati quyidagidan iborat bo'ladi:

$$\Delta y = 4\cdot 2{,}001^2+2\cdot 2{,}001-1-\left(4\cdot 2^2+2\cdot 2-1\right)=16{,}016004+$$
$$+3{,}002-19=19{,}018004-19=0{,}018004.$$

Funktsiyaning taqribiy son qiymatini hisoblash uchun berilgan $y=f(x)$ funktsiyaning orttirmasi va differentsiali topiladi:

$$\Delta y = f(x+\Delta x)-f(x),$$
$$dy = f'(x)dx.$$

$\Delta y = dy$ almashtirish bajarilsa, $f'(x)dx = f(x+\Delta x)-f(x).$ (3)

Bundan, $f(x+\Delta x)=f(x)+f'(x)\Delta x.$ (4)

3-misol. $y=5x^3-2x+3$ funktsiyaning $x=2{,}01$ bo'lgandagi taqribiy qiymatini toping.

Yechilishi:Bunda $f'(x)dx = f(x+\Delta x)-f(x)$ va $f(x+\Delta x)=f'(x)\Delta x+f(x)$ ekanligini hisobga olamiz va quyidagilarni topamiz:

$x=2{,}01$ dan $x=2$ va $\Delta x=0{,}01$ deb, $f(x)$ hamda $f'(x)\Delta x$ larning son qiymatlarini aniqlaymiz:

$$f(x)=f(2)=5\cdot 2^3-2\cdot 2+3=39;$$

$$f'(x)\Delta x = f'(2)\cdot 0{,}01=\left(5x^3-2x+3\right)'\cdot\Delta x=\left(15x^2-2\right)\cdot\Delta x=$$
$$=\left(15\cdot 2^2-2\right)\cdot 0{,}01=0{,}58\,.$$

Yuqorida keltirilgan ikkinchi formuladan:

$$f(2{,}01)=39+0{,}58=39{,}58.$$

Funktsiyaning aniq qiymatini topamiz:

$$f(2{,}01)=5\cdot(2{,}01)^2-2\cdot 2{,}1+3=39{,}583005.$$

Yaqinlashish xatosi $\frac{39{,}583005-39{,}58}{39{,}58}\approx 0{,}008\%$, ya'ni juda kichik ekan.

4- misol. $y=3x^2-7$ funktsiyaning $x=2$ va $\Delta x=0{,}001$ bo'lgandagi orttirmasining taqribiy qiymatini toping.

Yechilishi: Berilgan funktsiyaning ikkala tomonini differentsiallaymiz. U holda,

$$dy=6xdx$$

bo'ladi. Bundan, $dy=6\cdot 2\cdot 0{,}001=0{,}012$ bo'ladi.

Orttirmaning haqiqiy qiymatini topamiz:

$$|\Delta y|=f(x+\Delta x)-f(x)=3(x+\Delta x)^2-7-\left(3x^2-7\right)=$$

$$=3x^2+6x\cdot\Delta x+3(\Delta x)^2-7-3x^2+7=6x\cdot\Delta x+3(\Delta x)^2-6\cdot 2\cdot 0{,}001+$$

$$+3\cdot 0{,}000001=0{,}012003.$$

Absolyut xatoni topamiz: $|\Delta y-dy|=|0{,}01003-0{,}012|=0{,}000003.$

Nisbiy xato quyidagicha bo'ladi:

$$\left|\frac{\Delta y-dy}{dy}\right|=\frac{0{,}000003}{0{,}012}=0{,}00025=0{,}025\%.$$

Darajali funktsiyaning taqribiy qiymatini hisoblash quyidagi tartibda olib beriladi: argumentga orttirma beriladi: $x+\Delta x$. U holda $f(x+\Delta x)=(x+\Delta x)^n$ hosil bo'ladi. (4) dan foydalanib, uning taqribiy qiymatini hisoblaymiz:

$$f(x+\Delta x)\approx f(x)+f'(x)\Delta x.$$

U holda, quyidagilarga ega bo'linadi:

$$f(x+\Delta x)=(x+\Delta x)^n;\quad f(x)=x^n;\quad f'(x)\Delta x=n\cdot x^{n-1}\Delta x.$$

Bulardan esa: $(x+\Delta x)^n\approx x^n+n\cdot x^{n-1}\cdot\Delta x.$ (5)

(5) tenglik darajali funktsiyalarning taqribiy qiymatini hisoblash formulasi bo'lib, undagi n-ixtiyoriy son bo'lishi mumkin.

5-misol. $(4{,}012)^2$ ifodaning taqribiy qiymatlarini toping.

Yechilishi: Misolni yechish uchun $f(x+\Delta x)\approx f(x)+f'(x)\Delta x$ formuladan foydalanib, $(x+\Delta x)^n \approx x^n + n\cdot x^{n-1}\cdot\Delta x$ ekanligini aniqlaymiz hamda uni qo'llaymiz. Berilganlarga ko'ra $x=4,\ \ \Delta x=0{,}012$ larni hisobga olib, quyidagilarni hosil qilamiz:

$$(4{,}012)^2=(4+0{,}012)^2=4^2-2\cdot4\cdot0{,}012=16{,}096\approx16{,}1.$$

Yaqinlashishning nisbiy xatosi:

$\dfrac{16{,}1-16{,}096}{16{,}1}\approx0{,}025\%$ dan iborat. Bu esa juda kichik bo'lganligi sababli, javob o'rnida $16{,}1$ ni qabul qilish mumkin.

Differentsialdan foydalanib, ildizlarni taqribiy hisoblash mumkin. Masalan, $f(x)=\sqrt{x}$ funktsiyaning taqribiy qiymatini topish talab qilinsin. Uni topish uchun x argumentga Δx orttirma beriladi. U holda, (4)ga asosan $f(x+\Delta x)=\sqrt[n]{x+\Delta x}$ funktsiyaning taqribiy qiymatini hisoblaymiz:

$$f(x+\Delta x)=\sqrt[n]{x+\Delta x},\qquad f(x)=\sqrt[n]{x},\quad f'(x)\cdot\Delta x=\frac{\Delta x}{n\sqrt[n]{x^{n-1}}}.$$

Bulardan esa quyidagi formula hosil bo'ladi:

$$\sqrt[n]{x+\Delta x}\approx\sqrt[n]{x}+\frac{\Delta x}{n\sqrt[n]{x^{n-1}}}. \qquad (6)$$

(6)- ildizni taqribiy hisoblash formulasidir.

6-misol. $\sqrt{0{,}96}$ ildizning qiymatini taqribiy hisoblang.

Yechilishi: Berilgan ildizni quyidagi ko'rinishda yozamiz: $\sqrt{0{,}96}=\sqrt{1-0{,}04}$. Bunda $x=1$ va $\Delta x=-0{,}04$. (6) formulaga asosan:

$$\sqrt{0{,}96}=\sqrt{1-0{,}04}\approx1-\frac{0{,}04}{2}=0{,}98.$$

Mustaqil yechish uchun mashqlar

№ 167. $r=50$ sm, $\Delta r=0{,}01$ bo'lsa, aylana uzunligini hisoblashdagi nisbiy xatoni toping.

№ 168. $x=2$ va $\Delta x=0{,}01$ bo'lsa, $y=x^3$ formula bilan berilgan kattalikni hisoblashdagi nisbiy xatoni toping.

№ 169. $x=2$ va $\Delta x=0{,}001$ bo'lganda $y=2x^3+5$ funktsiyaning taqribiy qiymatini toping.

№ 170. $y=3x^2+5x+1$ funktsiyaning $x=3$ va $\Delta x=0{,}001$ bo'lgandagi orttirmasi qiymatini toping.

№ 171. Quyidagi funktsiyalarning orttirmalari qiymatini toping:

a) $y=x^3+x-1;$ $x=2$ va $\Delta x=0{,}01$ da;

b) $y=\ln x;$ $x=10$ va $\Delta x=0{,}01$ da.

№ 172. Radiusi R bo'lgan shar qizdirilganda uning radiusi ΔR ga uzayishi ma'lum bo'lsa, sharning hajmi qanchaga oshadi?

№ 173. Qirrasi 10 sm bo'lgan kub qizdirilganda qirrasining uzayishi $0{,}02$ sm bo'lsin. U holda kubning hajmi qanchaga ortgan bo'ladi?

№ 174. Quyidagi funktsiyalarning taqribiy qiymatlarini argumentning keltirilgan qiymatlari bo'yicha hisoblang:

a) $f(x)=2x^2-x+1,$ $x=2{,}01$ da;

b) $f(x)=x^2+3x+1,$ $x=3{,}02$ da;

v) $f(x)=\frac{1}{3}x^3+\frac{1}{2}x^2-2x+4,$ $x=1{,}1$ da.

№ 175. Quyidagilarning taqribiy qiymatlarini toping:

a) $(9{,}6)^2;$ b) $(1{,}012)^3;$ v) $(9{,}95)^3;$ g) $(1{,}005)^{10};$ d) $(0{,}975)^4.$

№ 176. $5{,}013^3$ ning taqribiy qiymatini toping.

№ 177. Quyidagilarning taqribiy qiymatlarini toping:

a) $(0{,}9756)^4;$ b) $(3{,}025)^4.$

№ 178. Quyidagi ildizlarning taqribiy qiymatlarini toping:

a) $\sqrt[3]{1{,}006};$ b) $\sqrt{24{,}84};$ v) $\sqrt{99{,}5};$ g) $\sqrt[10]{1{,}03};$

d) $\sqrt[3]{1{,}012};$ ye) $\sqrt{25{,}16};$ yo) $\sqrt{101}.$

Javob va ko'rsatmalar

№ 4. $\Delta x = -0{,}1$; $\Delta y = -0{,}39$. № 5. $\Delta v = 3a^2 \cdot \Delta x + 3a(\Delta x)^2 + (\Delta x)^3$. № 6. $\Delta y = 0{,}205$. № 7. $\Delta y = 3x^2 \cdot \Delta x + 3x(\Delta x)^2 + (\Delta x)^2 = 2{,}791$. № 8. $\Delta y = \sqrt[3]{x+\Delta x} - \sqrt[3]{x} = 0{,}06$. №9. 1) v_0; 2) $v_0 + at_0$. № 10. 1) 0 m/minut; 2) $\frac{1}{12}$ m/minut; 3) 75 m/minut. № 11. $\frac{1}{2\sqrt{t}}$. № 12. $V = 6$. № 13. 25 sek. № 14. $\frac{mv^2}{2}$ joul. № 15. a)1; b) -3; w) $\frac{1}{2}$; g)1. № 16. a) $6x$; b) $2x-2$; v) $2x-6$; g) $\frac{1}{2}+2x$; d) $14x+3$; e) $2x-\frac{1}{4}$. № 17. -4. № 18. a) -3; b) 3; v) -5; g) -11. № 19. a) $2x-3$; b) $2x-5$; v) $2x-1$; g) $2x^2-3$. № 20. 12. № 21. -24. № 22. 6. № 23. 3. № 24. 2,5. № 25. 6. № 26. $12(x^6+1)$. № 27. $2(x-1)$. № 28. $3\cos 3x$. № 29. $-4\sin 4x$. № 30. $2(x^2-2x+1)(2x-2)$. № 31. $2(x^2+3x+10)(2x+3)$. № 32. $\frac{3x^2}{(x+1)^4}$. № 33. $-\frac{5}{\sin^2 5x}$. № 34. $\sin 2x$. № 35. $-\sin 2x$. № 36. $2tgx \cdot \frac{1}{\cos^2 x} = \frac{2\sin x}{\cos^3 x}$. № 37. $10e^{10x}$. № 38. $\cos e^{\sin x}$. № 39. $-\sin \cdot e^{\cos x}$. № 40. $ctgx$. № 41. $-tgx$. № 42. $2x(\cos(x^2+1) - \sin(x^2-1))$. № 43. $-\frac{2}{x^2}+\frac{3}{x^4}$. № 44. $-\frac{x}{\sqrt{1-x^2}}$. № 45. $-6x^2e^{-2x}$. № 46. $6x(x^2-3)^2$. № 47. $\frac{2(1+x)(1-2x-x^2)}{(x^2-x)^3}$. № 48. $\frac{3(x^2+x+1)(5+6x-5x^2-2x^3-x^4)}{(x^3-3x^2-5x)^4}$. № 49. $6(x^2+5)(x^3+15x+23)$. № 50. $6x(x^2-3)^2$. № 51. $\frac{2(1+x)(1-2x-x^3)}{x^2-x}$. № 52. $\frac{3(x^2+x+1)(5+6x-5x^2-2x^3-x^4)}{(x^3-3x^2-5x)^4}$. № 53. $-\frac{1}{\sqrt{1-x^2}}$. № 54. $\frac{1}{1+x^2}$. № 55. $-\frac{1}{1+x^2}$. № 56. $\frac{1}{1+x^2}$. № 57. a) $\frac{2}{\sqrt{1-4x^2}}$; b) $-\frac{2}{1+4x^2}$. № 58. $\frac{1}{\sqrt{1-x^2}}+\frac{1}{1+x^2}+2e^x$. № 59. a) $y^1 = -\frac{2}{x^2}$; b) $y^1 = \frac{1}{\sqrt{x}}+\frac{2}{\sqrt[3]{x}}+\frac{1}{3x\sqrt[3]{x}}$; v) $y^1 = x-\frac{4}{x^2}$; g) $y^1 = \frac{a}{a+b}$; d) $y^1 = 6x^2-2x$; e) $y^1 = \frac{2(1+x^2)}{(1-x^2)^2}$; yo) $y^1 = \frac{2}{(\sin x+\cos x)^2}$; j) $y^1 = 2\sin x$; z) $y^1 = 2x\cdot\cos x$; i) $y^1 = 4x\cdot\sin 4x$. № 60. v) $y^1 = \frac{2x-y}{x-2y}$; g) $y^1 = \frac{x^2-y}{x-y^2}$. № 61. $y^1 = \frac{y}{y-1}$. № 62. $y^1 - 3$. № 63. $y^1 = \frac{5-6xy^3}{9x^2y^2}$. № 64. $y^1 = \frac{x^4-y^3}{3xy^2-y^4}$. № 65. $y^1{}_x = -\frac{b}{a}ctgt$. № 66. $y_x^1 = \frac{1+at}{w}\cdot e^{at}$. № 68. $y_x^1 = -1$. № 69. a) $y^1 = -5x^2$; b) $y^1 = \frac{5-2x}{2y}$. № 70. $y^1 = -\frac{b^2x}{a^2y}$. № 71. urunma tenglamasi: $2x+3y+18=0$. normal tenglamasi: $3x-2y+1=0$. № 72. $dy = dx$. № 73. $dy = (2x-6)dx$. № 74. a) $dy = (\cos x - \sin x)dx$; b) $dy = \left(-\frac{1}{x^2}-1\right)dx$; v) $dy = \frac{1}{\cos^2 x}dx$; g) $dy = \left(6x^5+3\sin x+\frac{1}{x}\right)dx$. № 75. $dy = -0{,}4$. № 76. $dy = -3{,}6$. № 77. a) $dy = -0{,}5$.

b) $dy=-3{,}36$. № 78. 0. № 79. 0. № 80. 60. № 81. $-\cos x$. № 82. $\sin x$. № 83. $9700200(1+x)^{97}$. № 84. $-\dfrac{6}{x^4}$. № 85. $3\cos 3x-8e^{4x}+\dfrac{1}{\cos^2 x}$. № 86. $\sin\left(x+n\dfrac{p}{2}\right)$. № 87. $2dx^2$. № 88. $\dfrac{1}{4x\sqrt{x}}dx^2$. № 89. $-\left(4\sin 2x+\dfrac{1}{x^2}\right)dx^2$. № 90. $24xdx^3$. № 91. $4^{-x^2}\cdot 2\ln 4\left(2x^2\cdot\ln 4-1\right)dx^2$. № 92. $-4\sin 2xdx^3$. № 93. a) $\left(25\cdot e^{-5x}-\cos x\right)dx^3$; b) $\left(-125e^{-5x}+\sin x\right)dx^4$; v) $\left(625e^{-5x}+\cos x\right)dx^5$. № 94. $(-\infty;0{,}75)$ va $(0{,}75;+\infty)$. № 95. a) $\left(-\infty;-\dfrac{1}{3}\right)$ va $\left(-\dfrac{1}{3};+\infty\right)$; b) $(-\infty;-1)$ va $(2;+\infty)$. № 96. $(-\infty;-1)$; $(-1;1)$; $(1;+\infty)$. № 97. $(-\infty;-1)$ va $(1;+\infty)$ da o'suvchi; $(-1;0)$ va $(0;1)$ da kamayuvchi. № 98. $(-\infty;0)$ va $(2;+\infty)$ oraliqlarida o'sadi; $(0;2)$ da kamayadi. № 99. $(-2;-\sqrt{2})$ va $(\sqrt{2};2)$ da kamayadi; $(-\sqrt{2};\sqrt{2})$ da o'sadi. № 100. $(-1;1)$ da o'sadi; $(-\infty;-1)$ va $(1;+\infty)$ da kamayadi. № 101. $(-\infty;-2)$ va $(0,+\infty)$ da o'sadi; $(-2;0)$ da kamayadi. № 102. $(e;+\infty)$ da o'sadi; $(0;1)$ va $(1;e)$ da kamayadi. № 103. $\left(-\dfrac{1}{2};+\infty\right)$. № 106. a) $y_{\max}(0)=5$, $y_{\min}(1)=4$; b) $y_{\max}(-2)=\dfrac{4}{e^2}$, $y_{\min}(0)=0$. № 107. $y_{\max}=f(-1)=2$, $y_{\min}=f(1)=-2$. № 108. $y_{\max}=f(1)=-2\dfrac{2}{3}$, $y_{\min}=f(3)=-4$. № 109. $y_{\min}f(0)=-2$. № 110. a) $y_{\min}=f(5)=-16$; b) $y_{\max}=f(-2)=13$, $y_{\min}f\left(\dfrac{4}{3}\right)=-5\dfrac{14}{27}$; v)yekstremumlari mavjud yemas; g) $y_{\min}=f(-3)=-120$, $y_{\max}=f(1)=8$, $y_{\min}=f(2)=5$; d) $y_{\max}=f(-e)=-e$, $y_{\min}=f(e)=e$. № 111. a) $Y_{\max}(1)=Y_{\max}(6)=0$, $Y_{\max}(3)=-108$; b) $Y_{\max}(1)=\dfrac{1}{2}$, $Y_{\min}=(-1)=-\dfrac{1}{2}$; v) $Y_{\max}(3)=\dfrac{1}{6}$, $Y_{\min}(-1)=-\dfrac{1}{2}$; g) $Y_{\max}(1)=\dfrac{1}{2}$. № 112. Sonlar o'qining barcha nuqtalarida qavariq. №113. $-\dfrac{p}{2}<x<0$ 1 da qavariq; $0<x<\dfrac{p}{2}$ da botiq; $x=0$-bukilish nuqtasi. №114. $(-\infty,2)$ da qavariq; $(2,+\infty)$ da botiq. № 115. a) $(-\infty,2)$ da qavariq; $(2,+\infty)$ da botiq; b) R da botiq; v) $(-\infty,-1)$ va $(1,+\infty)$ da botiq; $(-1;1)$ da qavariq; g) R da botiq. № 116. $(-\infty;1)$ da qavariq; $(1;+\infty)$ da botiq. № 117. a) $(-\infty;-2)$ va $(1;+\infty)$ da qavariq; $(-2;1)$ da botiq; b) $(-\infty;-1)$ va $(0;3)$ da qavariq; $(-1;0)$ va $(3;+\infty)$ da botiq; v) $(-7;+\infty)$ da qavariq; $(-\infty;-7)$ da botiq. № 118. a) $(2;16)$; b) $(1;38)$ va $(3;46)$; v) $\left(-\sqrt{3};-\sqrt{\dfrac{3}{4}}\right)$; $(0;0)$ va $\left(\sqrt{3};\sqrt{\dfrac{3}{4}}\right)$; g) bukilish nuqtalari mavjud emas; d) $(4;5)$; ye) $(-2;3\ln 2)$; $(2;3\ln 2)$; f) $(1;0)$. № 119. a) $-1{,}375$, $1{,}375$; b) $-2;2$. № 120. a) $-2;\sqrt{3}$; b) $-\sqrt{3};1$. № 121. a) $\dfrac{p}{6}-\dfrac{\sqrt{3}}{2},0$; b) $\dfrac{p}{8}-\dfrac{\sqrt{2}}{2}$, $-\dfrac{p}{8}+\dfrac{\sqrt{2}}{2}$; v) $\dfrac{p}{6}-\dfrac{\sqrt{3}}{2}$, $-\dfrac{p}{6}+\dfrac{\sqrt{3}}{2}$. № 122. $9;4$. № 123. $10;10$. № 124. $a=\dfrac{k}{2}$, $h=\dfrac{k}{2}$. № 125. $8;0$. № 126. $\dfrac{3\sqrt{3}}{8};0$. № 127. $\dfrac{4\sqrt{3}}{3};2$. № 128. $e+\dfrac{2}{e}+4$; $\dfrac{1}{2}(10-7\ln 2)$. №129. 9 va -9. № 130. $2\sqrt{2}$ dm. № 131/ $x=2$; $y=0$. № 132. $x=1$ -vertikal asimptota. №133. $x=1$, $x=-1$, $y=0$. № 134. $x\to-\infty$ da $y=-x$, $x\to+\infty$ da $y=x$. № 135. $x=1$, $y=x+1$. № 136. $x=-5$, $x=5$, $y=0$. № 137. $x=-1$; $y=x-3$. № 138. $x=4$; $y=x-1$.

№139. $x=0$ vertikal; $y=3x$ og'ma asimptotalar. № 140. $x=2$; $x=-2$ vertikal asimptotalar. № 160 a) 4; b) 12; v) $\frac{1}{2}$. Ko'rsatma: kasrning surati va maxrajidan ketma – ket ikki marta hosila olinadi; g) -1; d)1; ye) $\frac{2}{3}$. № 161. a) 2; b) $\frac{\sqrt{2}}{2}$; v) $\frac{4}{3}$. № 162 a) $\frac{1}{6}$; b) 1; v) $\frac{3}{e}$; g) $\frac{3}{5}$; d) 2; ye) 1. № 163 a) $-\frac{1}{6}$; b) $\frac{1}{3}$; v) 27; g) 1; d) 1; ye) $\frac{a}{b}$. № 164. a) $\ln 2$; b) $\frac{3}{2}$; v)1. № 165 a) ∞; b) 0; v) 0; g) 0; d) ∞; ye) ∞. № 166. a) O. ko'rsatma: Ifodani $x\ln x=\frac{\ln x}{\frac{1}{x}}$ ko'rinishga keltirib, $\frac{\infty}{\infty}$ kabi aniqmaslikka keltiriladi. b) O. Ko'rsatma: $ctgx=\frac{\cos x}{\sin x}$ kabi ifodalab, $\lim\limits_{x\to 0}\frac{x\cos x-\sin x}{x\sin x}$ ga keltiriladi va $\frac{0}{0}$ shakldagi anikmaslik xosil bo'ladi. v) O. Ko'rsatma: Ifodani o'zining qo'shmasiga ko'paytirilib, bulinadi. Natijada; $\lim\limits_{x\to\infty}\frac{-1}{x+\sqrt{x^2+1}}$ ko'rinishga keladi. № 167. $r\approx 1\%$. № 168. $r\approx 1{,}5\%$. № 169. $0{,}05\%$.

№ 170. $0{,}023$. № 171. a) $0{,}13$; b) $0{,}001$. № 172. $dv=4pR^2\cdot dR$. № 173. $6\,sm^3$. № 174. a) $7{,}07$; b) $19{,}18$; v) $2{,}83$. № 175. a) $82{,}08$; b) $1{,}036$; v) 985; g) $1{,}05$; d) $0{,}9$. № 176. $125{,}975$. № 177 a) $0{,}9$; b) $83{,}7$. № 178 a) $\approx 1{,}002$; b) $\approx 4{,}984$; v) $\approx 9{,}975$; g) $\approx 1{,}003$; d) $\approx 1{,}004$; ye) $\approx 5{,}016$; yo) $\approx 10{,}05$.

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