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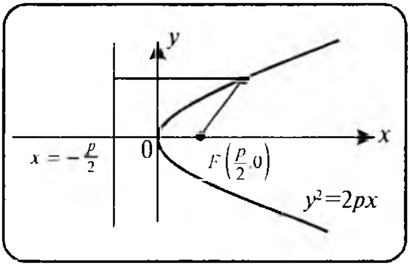
OLIY MATEMATIKADAN MISOL VA MASALALAR TO'PLAMI

Oliy texnika o'quv yurtlari talabalari uchun o'quv qo'llanma



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T a q r i z c h i l a r:

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Ushbu o‘quv qo‘llanmada oliy matematikaning chiziqli algebra, analiti' geometriya va matematik analiz kursining barcha bo‘limlariga doir qisq nazariy tushunchalar, formulalar va amaliy mashg‘ulotlar uchun misol v masalalar keltirilgan.

O‘quv qo‘llanma oliy texnika o‘quv yurtlari talabalari uchu mo‘ljallangan.

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So'zboshi

Komil inson g'oyasi azal-azaldan xalqimizning ezgu orzusi, uning ma'naviyatining uzviy bir qismi bo'lib kelgan. U sharq falsafasidan oziqlanib, yanada kengroq ma'no-mazmun kasb etib kelmoqda.

Erkin fuqarollk jamiyatini barkamol, ezgu g'oyalari, hayotiy e'tiqodi mustahkam bo'lgan insonlargina bunyod eta oladi. Shuning uchun yangilanayotgan jamiyatimizda sog'lom avlodni tarbiyalash, erkin fuqaro ma'naviyatini shakllantirish, ma'naviy-ma'rifiy ishlarni yuksak darajaga ko'tarish orqali barkamol insonlarni voyaga yetkazishga muhim e'tibor berilmoqda. Mamlakatimizda „Sog'lom avlod yili"dagi harakatning keng tus olgani, „Kadrlar tayyorlash Milliy dasturi" asosida ta'lim-tarbiya tizimining tubdan isloh etilayotgani ham ana shu ulug'vor maqsadni amalga oshirish yo'lidagi muhim qadamlardir.

Hozirgi davr yoshlari ruhiyatida chuqur va mustahkam bilimlarni shakllantirish, milliy istiqlol g'oyalariga sadoqatni, ona-Vatanga mehr-muhabbatni, bu yo'ldagi fidoyilikni tarbiyalashni davom ettirish oliy ta'limning asosiy vazifalaridandir.

„Ta'lim to'g'risida"gi Qonun va „Kadrlar tayyorlash Milliy dasturi"ning ikkinchi bosqichidagi va undan keyingi vazifalarni amalga oshirishda va yuqori malakali mutaxassislar tayyorlashda aniq fanlarga ehtiyoj kuchayib bormoqda, chunki asosiy muhandislik maxsus kurslari ana shu fanlar asosida qurilgan bo'ladi.

Oliy matematika fanidan har tomonlama chuqur bilim olish uchun faqat asosiy nazariy mavzularni o'zlashtirish kifoya qilmasdan, maxsus tanlangan misol va masalalarni yetarlicha yechish qobiliyatiga ham ega bo'lish zarur. Shuning uchun misol va masalalarni uzviy bog'liqlik asosida ma'lum tizim bilan tanlash va joylashtirish katta ahamiyatga ega.

E'tiboringizga havola qilinayotgan to'plam mavjud qo'llanmalar, ayniqsa V.P.Minorskiyning „Oliy matematikadan misol va masalalar to'plami" kitobi kabi texnik oliy o'quv yurtlari uchun mavjud namunaviy dastur asosida tuzilgan bo'lib, texnik oliy o'quv yurtlarining kunduzgi bo'lim talabalari uchun mo'ljallangan, undan, shuningdek, sirtqi bo'lim talabalari ham foydalanishlari mumkin.

Ushbu qo'llanmada chiziqli algebra, analitik geometriya va matematik analizdan misol va masalalar bo'limlar bo'yicha joylashtirilgan.

Har bir bo'lim boshlanishida misol va masalalarni yechish uchun kerak bo'ladigan ma'lumotlar va formulalar berilgan. Foydalanishga qulay bo'lishi uchun misol va masalalarning javoblari har bir bob oxirida joylashtirilgan.

To'plamning analitik geometriyaga tegishli boblardagi misol va masalalarni tanlashda marhum dotsent M.N. Nuritdinovning xizmatlarini alohida yodga olamiz.

Qo'llanmaga kiritilgan misol va masalalar Toshkent irrigatsiya va melioratsiya institutining „Oliy matematika" kafedrasida, auditoriyalarda tekshiruvdan o'tkazilgan. Darslik xato va kamchiliklardan xoli emasligini hisobga olib, ularni ko'rsatgan o'rtoqlarga mualliflar oldindan o'z minnatdorchiliklarini bildiradilar.

Mualliflar

I BOB | OLIY ALGEBRA

1- §. Determinantlar va matritsalar

1. Ikkinchi va uchinchi tartibli determinantlar

To'rtta sondan tuzilgan ushbu

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \qquad (1)$$

jadval *ikkinchi tartibli kvadratik matritsa* deyiladi.

$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$ simvol bilan belgilanib, qiymati $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{12} - a_{12}a_{21}$ ga teng bo'lgan songa *2- tartibli determinant* deyiladi. (2)

Elementlari 3×3 ta sondan iborat ushbu

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \qquad (3)$$

jadvalga *3- tartibli kvadratik matritsa* deyiladi.

Ushbu $$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} \qquad (4)$$

son *3- tartibli determinant* deyiladi.

(4) tenglikning o'ng tomonidagi 2- tartibli determinantlarning har biri berilgan 3- tartibli determinantning bitta satri va bitta ustununi o'chirishdan hosil bo'ladi va ular o'sha determinantning *minorlari* deyiladi.

(4) formula esa 3- tartibli determinantni *birinchi satri bo'yicha yoyish formulasi* deyiladi.

2. Determinantning xossalari

1°. *Determinantning satrlarini ustunlari bilan almashtirishdan uning qiymati o'zgarmaydi.*

2°. *Determinantning ikkita qatorini yoki ustunini o'zaro almashtirishdan uning qiymati o'zgarmaydi, ishorasi esa teskarisiga o'zgaradi.*

1° va 2° xossalardan ravshanki, determinantning istalgan qatorini birinchi satr o'rniga keltirish mumkin, shuning uchun uni istalgan qator elementlari bo'yicha yoyish mumkin.

3°. *Ikkita parallel qatori (ustuni) bir xil yoki proporsional bo'lgan determinant nolga teng.*

4°. *Biror qator elementlarining umumiy ko'paytuvchisini determinant belgisidan tashqariga chiqarish mumkin.*

5°. *Determinantning biror qatorining elementlariga unga parallel qator elementlarini, ixtiyoriy, bir xil songa ko'paytirib qo'shishdan determinant qiymati o'zgarmaydi. Masalan:*

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11}+ma_{13} & a_{12} & a_{13} \\ a_{21}+ma_{23} & a_{22} & a_{23} \\ a_{31}+ma_{33} & a_{32} & a_{33} \end{vmatrix}.$$

Bu xossaga asoslanib, 3- tartibli determinantning istalgan qatorida ikkita nol hosil qilish mumkin, buning natijasida determinantning o'sha qator elementlari bo'yicha yoyilmasi soddalashadi.

3. Uchlari $A(x_1; y_1)$, $B(x_2; y_2)$, $C(x_3; y_3)$ nuqtalarda bo'lgan uchburchak yuzi:

$$S = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}. \qquad (5)$$

Quyidagi determinantlarni hisoblang:

1. $\begin{vmatrix} 5 & 3 \\ 6 & 4 \end{vmatrix}$. **2.** $\begin{vmatrix} 8 & 5 \\ 3 & 2 \end{vmatrix}$. **3.** $\begin{vmatrix} 3 & 5 \\ 5 & 8 \end{vmatrix}$. **4.** $\begin{vmatrix} 3 & -2 \\ -4 & 5 \end{vmatrix}$.

5. $\begin{vmatrix} \sqrt{a} & -1 \\ a & \sqrt{a} \end{vmatrix}$. **6.** $\begin{vmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{vmatrix}$. **7.** $\begin{vmatrix} a+b & a-b \\ a-b & a+b \end{vmatrix}$.

8. $\begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix}$. **9.** $\begin{vmatrix} 2 & -3 & 1 \\ 6 & -6 & 2 \\ 2 & -1 & 2 \end{vmatrix}$. **10.** $\begin{vmatrix} 0 & x & 0 \\ x & 1 & x \\ 0 & x & 0 \end{vmatrix}$. **11.** $\begin{vmatrix} \sin\alpha & \sin\beta \\ \cos\alpha & \cos\beta \end{vmatrix}$.

Determinantlarni birinchi ustun elementlari bo'yicha yoyib hisoblang:

12. $\begin{vmatrix} 2 & 3 & 4 \\ 5 & -2 & 1 \\ 1 & 2 & 3 \end{vmatrix}$. **13.** $\begin{vmatrix} a & 1 & a \\ -1 & a & 1 \\ a & -1 & a \end{vmatrix}$. **14.** $\begin{vmatrix} 1 & 2 & -3 \\ 3 & 1 & -1 \\ 1 & 3 & 2 \end{vmatrix}$ **15.** $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$.

16. $\begin{vmatrix} 1 & b & 1 \\ 0 & b & 0 \\ b & 0 & -b \end{vmatrix}$. **17.** $\begin{vmatrix} -x & 1 & x \\ 0 & -x & -1 \\ x & 1 & -x \end{vmatrix}$. **18.** $\begin{vmatrix} a & -a & a \\ a & a & -a \\ a & -a & -a \end{vmatrix}$. **19.** $\begin{vmatrix} 1 & 2 & 5 \\ 3 & -4 & 7 \\ -3 & 12 & -10 \end{vmatrix}$.

20. $\begin{vmatrix} 12 & 6 & -4 \\ 6 & 4 & 4 \\ 3 & 2 & 8 \end{vmatrix}$. **21.** $\begin{vmatrix} x^2 & x & 1 \\ y^2 & y & 1 \\ z^2 & z & 1 \end{vmatrix}$. **22.** $\begin{vmatrix} 1+\cos\alpha & 1+\sin\alpha & 1 \\ 1-\sin\alpha & 1+\cos\alpha & 1 \\ 1 & 1 & 2 \end{vmatrix}$.

23. $\begin{vmatrix} 2\cos^2\frac{\alpha}{2} & \sin\alpha & 1 \\ 2\cos^2\frac{\beta}{2} & \sin\beta & 1 \\ 1 & 0 & 1 \end{vmatrix}$. **24.** $\begin{vmatrix} \sin a & \cos\alpha & 1 \\ \sin\beta & \cos\beta & 1 \\ \sin\gamma & \cos\gamma & 1 \end{vmatrix}$.

25. Uchlari $A(-2; 1)$, $B(2; -2)$ va $C(8; 6)$ nuqtalarda bo'lgan uchburchakning yuzini hisoblang.

26. $A(1;3)$, $B(2;4)$ va $C(3;5)$ nuqtalar bir to'g'ri chiziqda yotadimi?

27. Berilgan nuqtalardan o'tuvchi to'g'ri chiziq tenglamasini 3- tartibli determinant yordamida yozing:

1) $(x_1; y_1)$ va $(x_2; y_2)$; 2) $(2; 3)$ va $(-1; 5)$.

28. Berilgan tenglamalardan x ni toping va topilgan ildizlarni determinantga qo'yib tekshiring:

1) $\begin{vmatrix} x^2 & 3 & 2 \\ x & -1 & 1 \\ 0 & 1 & 1 \end{vmatrix} = 0;$ 2) $\begin{vmatrix} x^2 & 4 & 0 \\ x & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix} = 0;$

3) $\begin{vmatrix} x^2 & 3 & 2 \\ x & -1 & 1 \\ 0 & 1 & 4 \end{vmatrix} = 0;$ 4) $\begin{vmatrix} x^2 & 4 & 9 \\ x & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix} = 0.$

2- §. Chiziqli tenglamalar sistemasi

1°. Ikki noma'lumli chiziqli tenglamalar sistemasi

Agar $\Delta \neq 0$ bo'lsa, ushbu

$$\begin{cases} a_{11}x + a_{12}y = a_{13}, \\ a_{21}x + a_{22}y = a_{23} \end{cases} \quad (1)$$

sistema $x = \frac{\Delta_x}{\Delta}$; $y = \frac{\Delta_y}{\Delta}$ yechimga ega bo'ladi, bu yerda:

$$\Delta = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}, \quad \Delta_x = \begin{vmatrix} a_{13} & a_{12} \\ a_{23} & a_{22} \end{vmatrix}, \quad \Delta_y = \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix}. \quad (2)$$

2°. 3 noma'lumli 2 ta bir jinsli tenglamalar sistemasi

Ushbu

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = 0, \\ a_{21}x + a_{22}y + a_{23}z = 0 \end{cases} \quad (3)$$

tenglamalar sistemasi

$$x = k\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}; \quad y = -k\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix}; \quad z = k\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \quad (4)$$

yechimga ega, bu yerda k— ixtiyoriy son.

3°. 3 noma'lumli 3 ta chiziqli bir jinsli tenglamalar sistemasi

Ushbu

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = 0, \\ a_{21}x + a_{22}y + a_{23}z = 0, \\ a_{31}x + a_{32}y + a_{33}z = 0 \end{cases} \quad (5)$$

sistemaning determinanti

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 0$$ *bo'lsa, u noldan farqli yechimga ega bo'ladi, va aksincha.*

4°. Ikki noma'lumli uchta chiziqli tenglamalar sistemasi

Agar ushbu

$$\begin{cases} a_{11}x + a_{12}y = a_{13}, \\ a_{21}x + a_{22}y = a_{23}, \\ a_{31}x + a_{32}y = a_{33} \end{cases} \qquad (6)$$

sistemaning determinanti

$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 0$ *bo'lib, uning hech qaysi ikkita tenglamasi o'zaro zid bo'lmasa, (6) birgalikda bo'ladi.*

5°. Uch noma'lumli uchta chiziqli tenglamalar sistemasi

Agar ushbu

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = d_1, \\ a_{21}x + a_{22}y + a_{23}z = d_2, \\ a_{31}x + a_{32}y + a_{33}z = d_3 \end{cases} \qquad (7)$$

sistemaning determinanti

$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \neq 0$ bo'lsa,

(7) sistema yagona $x = \frac{\Delta_x}{\Delta}$, $y = \frac{\Delta_y}{\Delta}$, $z = \frac{\Delta_z}{\Delta}$ yechimga ega bo'ladi, bu yerda:

$$\Delta_x = \begin{vmatrix} d_1 & a_{12} & a_{13} \\ d_2 & a_{22} & a_{23} \\ d_3 & a_{32} & a_{33} \end{vmatrix}, \quad \Delta_y = \begin{vmatrix} a_{11} & d_1 & a_{13} \\ a_{21} & d_2 & a_{23} \\ a_{31} & d_3 & a_{33} \end{vmatrix}, \quad \Delta_z = \begin{vmatrix} a_{11} & a_{12} & d_1 \\ a_{21} & a_{22} & d_2 \\ a_{31} & a_{32} & d_3 \end{vmatrix}.$$

6°. Birgalikda bo'lmagan va aniqmas tenglamalar sistemasi

(7) tenglamalar sistemasining chap tomonlarini X_1, X_2, X_3 lar bilan belgilaylik, sistemaning determinanti $\Delta = 0$ *bo'lsin. U holda ikki hol bo'lishi mumkin.*

I. Δ determinantning ixtiyoriy ikki qatori elementlari proporsional. Masalan,

$m = \frac{a_{21}}{a_{11}} = \frac{a_{22}}{a_{12}} = \frac{a_{23}}{a_{13}}$. U holda $X_2 = mX_1$ va

1) agar $d_2 \neq md_1$ bo'lsa, sistema birgalikda emas (birinchi ikki tenglama bir- biriga zid);

2) agar $d_2 = md_1$ bo'lsa, sistema aniqmas (agar birinchi va uchinchi tenglamalar bir-biriga zid bo'lmasa).

II. Δ determinantda proporsional elementlarga ega bo'lgan satrlar yo'q. U nolga teng bo'lmagan shunday n va m sonlar mavjudki, $mX_1 + nX_2 = X_3$ va

1) agar $md_1 + nd_2 \neq d_3$ bo'lsa, sistema birgalikda emas;

2) agar $md_1 + nd_2 = d_3$ bo'lsa, sistema aniqmas.

m va n sonlarini mulohazalar yordami bilan yoki

$$a_{11}m + a_{12}n = a_{13}, \quad a_{21}m + a_{22}n = a_{23}, \quad a_{31}m + a_{32}n = a_{33}$$

tenglamalardan topish mumkin.

Tenglamalar sistemasini determinantlar yordamida yeching:

29. $\begin{cases} 2x + 3y = 1, \\ 3x + 5y = 4. \end{cases}$ **30.** $\begin{cases} 3x + y = 4, \\ 2x + 4y = 1. \end{cases}$ **31.** $\begin{cases} x + y = 1, \\ x - y = 2. \end{cases}$

32. $\begin{cases} 3x + 2y = 7, \\ 4x - 5y = 40. \end{cases}$ **33.** $\begin{cases} ax - 3y = 1, \\ ax + 2y = 6. \end{cases}$ **34.** $\begin{cases} mx - ny = (m - n)^2, \\ 2x - y = n \quad (m \neq 2n). \end{cases}$

35. Ushbu $3x \cdot ay = 1$, $6x + 4y = b$ tenglamalar sistemasi a va b larning qaysi qiymatlarida: 1) yagona yechimga ega;
2) yechim mavjud emas;
3) cheksiz ko'p yechimga ega ekanligini aniqlang.

36. Bir jinsli $13x + 2y = 0$, $5x + ay = 0$ tenglamalar sistemasi a ning qanday qiymatida noldan farqli yechimga ega ekanligini aniqlang.

Tenglamalar sistemasini yeching:

37. $\begin{cases} 2x - 3y + z - 2 = 0, \\ x + 5y - 4z + 5 = 0, \\ 4x + y - 3z + 4 = 0. \end{cases}$ **38.** $\begin{cases} 2x - 4y + 3z = 1, \\ x - 2y + 4z = 3, \\ 3x - y + 5z = 2. \end{cases}$ **39.** $\begin{cases} 2x + y + z = 0, \\ 3x - y + 2z = -3, \\ x + y - 3z = 4. \end{cases}$

40. $\begin{cases} x + 2y + 3z = 1, \\ 2x - y + 2z = 6, \\ x + y - 3z = 7. \end{cases}$ **41.** $\begin{cases} 2x - 5y + 2z = 0, \\ x + 4y - 3z = 0. \end{cases}$ **42.** $\begin{cases} 3x + 2y + 2z = 0, \\ 5x + 2y + 3z = 0. \end{cases}$

43. $\begin{cases} x-3y+5z=0, \\ 7x-9y-11z=0. \end{cases}$ **44.** $\begin{cases} x+2y+3z=0, \\ 2x-3y+4z=0, \\ 3x-y+7z=0. \end{cases}$ **45.** $\begin{cases} 3x+2y-z=0, \\ 2x-y+3z=0, \\ x+3y-4z=0. \end{cases}$

46. $\begin{cases} 3x+2y-z=0, \\ 2x-y+3z=0, \\ x+y-z=0. \end{cases}$ **47.** $\begin{cases} 3x-y+2z=0, \\ 2x+3y-5z=0, \\ x+y+z=0. \end{cases}$ **48.** $\begin{cases} 2x-y+3z=0, \\ x+2y-5z=0, \\ 3x+y-2z=0. \end{cases}$

49. $\begin{cases} x+2y+3z=4, \\ 2x+4y+6z=3, \\ 3x+y-z=1. \end{cases}$ **50.** $\begin{cases} x+2y+3z=4, \\ 2x+y-z=3, \\ 3x+3y+2z=7. \end{cases}$ **51.** $\begin{cases} x+2y+3z=4, \\ 2x+y-z=3, \\ 3x+3y+2z=10. \end{cases}$

52. $\begin{cases} 2x-y+z=2, \\ 3x+2y+2z=-2, \\ x-2y+z=1. \end{cases}$

Sistemalarni Gauss usuli bilan yeching:

53. $\begin{cases} x-y+3z=-4, \\ 2x+3y-2z=5, \\ 3x+5y+z=4. \end{cases}$ **54.** $\begin{cases} x-2y-3z=8, \\ 3x+y+z=3, \\ 4x+3y-2z=-1. \end{cases}$ **55.** $\begin{cases} 3x+y+2z=-4, \\ x-2y-z=-1, \\ 2x-3y-2z=0. \end{cases}$

56. $\begin{cases} x+y=4, \\ 2x+3y+z=7, \\ 2x+y+3z=-3. \end{cases}$ **57.** $\begin{cases} 2x+y+4z=20, \\ 2x-y-3z=3, \\ 3x+4y-5z=-8. \end{cases}$ **58.** $\begin{cases} x+2y+4z=31, \\ 5x+y+2z=29, \\ 3x-y+z=10. \end{cases}$

59. $\begin{cases} x-2y+3z=6, \\ 2x+3y-4z=16, \\ 3x-2y-5z=12. \end{cases}$ **60.** $\begin{cases} 2x+y-3z=3, \\ 3x+4y-5z=9, \\ 2y+7z=11. \end{cases}$

3- §. Chiziqli tenglamalar sistemasini teskari matritsalar yordamida yechish

n noma'lumli n ta chiziqli tenglamalar sistemasi berilgan bo'lsin:

$$\begin{cases} a_{11}x_1+a_{12}x_2+\cdots+a_{1n}x_n=b_1, \\ a_{21}x_1+a_{22}x_2+\cdots+a_{2n}x_n=b_2, \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ a_{n1}x_1+a_{n2}x_2+\cdots+a_{nn}x_n=b_n. \end{cases} \quad (8)$$

Quyidagi matritsalarni kiritamiz:

$$A=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix},\quad X=\begin{pmatrix} x_1 \\ x_2 \\ \cdots \\ x_n \end{pmatrix},\quad B=\begin{pmatrix} b_1 \\ b_2 \\ \cdots \\ b_n \end{pmatrix}, \tag{9}$$

bu yerda A xosmas matritsa. (8) sistemani matritsalar yordamida

$$A\times X=B \tag{10}$$

ко'rinishda yozamiz. Bu tenglama *matritsaviy tenglama* deyiladi. A xosmas matritsa bo'lgani sababli, unga teskari bo'lgan A^{-1} matritsa mavjud, shuning uchun (10) ni chap tomonidan A^{-1} ga ko'paytirib, $X=A^{-1}B$ yechim olinadi.

Bu yerda:

$$A^{-1}A=E.$$

61. Matritsalar ko'paytmasi AB va BA ni hisoblang:

1) $A=\begin{pmatrix} 3 & -2 \\ 5 & -4 \end{pmatrix},\quad B=\begin{pmatrix} 3 & 4 \\ 2 & 5 \end{pmatrix};$

2) $A=\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix},\quad B=\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix};$

3) $A=\begin{pmatrix} 5 & 8 & -4 \\ 6 & 9 & -5 \\ 4 & 7 & -3 \end{pmatrix},\quad B=\begin{pmatrix} 3 & 2 \\ 4 & 1 \\ 1 & 2 \end{pmatrix}.$

62. Berilgan matritsaga teskari matritsani toping:

1) $A=\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix};$ 2) $A=\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix};$ 3) $A=\begin{pmatrix} 2 & 5 & 7 \\ 6 & 3 & 4 \\ 5 & -2 & -3 \end{pmatrix};$

4) $A=\begin{pmatrix} 2 & 1 & -1 \\ 5 & 2 & 4 \\ 7 & 3 & 2 \end{pmatrix}.$

Tenglamalar sistemasini matritsalar yordamida yeching:

63. $\begin{cases} x+2y+z=4, \\ 3x-5y+3z=1, \\ 2x+7y-z=8. \end{cases}$ **64.** $\begin{cases} 2x-4y+9z=28, \\ 7x+3y-6z=-1, \\ 7x+9y-9z=5. \end{cases}$ **65.** $\begin{cases} 2x+y=5, \\ x+3z=16, \\ 5y-z=10. \end{cases}$

66. $\begin{cases} 7x + 2y + 3z = 15, \\ 5x - 3y + 2z = 15, \\ 10x - 11y + 5z = 36. \end{cases}$ 67. $\begin{cases} x + y - z = 36, \\ x + z - y = 13, \\ y + z - x = 7. \end{cases}$ 68. $\begin{cases} x + y + z = 36, \\ 2x - 3z = -17, \\ 6x - 5z = 7. \end{cases}$

69. $\begin{cases} 2x + y + 2z = 6, \\ x - 3y - z = -5, \\ 5x - 2y + z = -1. \end{cases}$ 70. $\begin{cases} x - 2y + 3z = 5, \\ 2x + 3y - z = -4, \\ 3x + y - 2z = -1. \end{cases}$

4- §. Kompleks sonlar

1°. Ta'riflar. *x va y haqiqiy sonlar, i esa qandaydir simvol ($\sqrt{-1} = i$) bo'lsa, $x + yi$ kompleks son deyiladi, bunda quyidagi shartlar qabul qilingan deb hisoblanadi:*

1) $x + 0i = x$; $0 + yi = yi$ *va* $1i = i$; $-1i = -i$,

2) *faqat* $x = x_1$; $y = y_1$ *bo'lgandagina* $x + yi = x_1 + iy_1$ *bo'ladi,*

3) $(x+yi) + (x_1+iy_1) = (x+x_1) + (y+y_1)i$,

4) $(x+yi)(x_1+iy_1) = (xx_1 - yy_1) + (xy_1+x_1y)i$,

1) *va* 4) *shartlardan i ning darajalari hosil bo'ladi:*

$$i^2=-1,\quad i^3=-i,\quad i^4=1,\quad i^5=i \text{ va hokazo.} \qquad (1)$$

$x + yi$ kompleks sonda $x = 0$, $y \neq 0$ bo'lsa, u *mavhum son* deyiladi, i son *mavhum birlik* deyiladi.

2°. Kompleks sonlar ustida bajariladigan amallar. Kompleks sonlarni qo'shish, ayirish, ko'paytirish va darajaga ko'tarish amallari shu amallarni ko'p hadlilar ustida bajarish qoidalari asosida bajariladi, bunda i sonining darajalarini (1) formulalar bo'yicha almashtirish zarur.

Kompleks sonlarni bo'lish, kompleks sonlardan ildiz chiqarish amallari, mos ravishda, ko'paytirish va darajaga ko'tarish amallariga teskari amallar sifatida aniqlanadi.

3°. Kompleks sonning trigonometrik ko'rinishi. $x + yi$ kompleks son ikki haqiqiy $(x; y)$ son bilan aniqlanadi, shuning uchun ham u tekislikdagi $M(x; y)$ nuqta yoki uning $r = \overline{OM}$ radius-vektori bilan ifodalanadi. Bu vektorning uzunligi $r = \sqrt{x^2 + y^2}$ kompleks sonning *moduli*, vektor bilan Ox o'qi orasidagi φ burchak esa kompleks sonning *argumenti* deyiladi. $x = r\cos\varphi, y = r\sin\varphi$ bo'lgani uchun:

$$x + yi = r\,(\cos\varphi + i\sin\varphi). \qquad (2)$$

4°. Trigonometrik ko'rinishda berilgan kompleks sonlar ustida bajariladigan amallar:

$$\begin{aligned} &r(\cos\varphi + i\sin\varphi)\cdot r_1(\cos\varphi_1 + i\sin\varphi_1) = \\ &= r\,r_1\left[\cos(\varphi+\varphi_1) + i\sin(\varphi+\varphi_1)\right]. \end{aligned} \qquad (3)$$

$$\frac{r(\cos\varphi + i\sin\varphi)}{r_1(\cos\varphi_1 + i\sin\varphi_1)} = \frac{r}{r_1}\left[\cos(\varphi-\varphi_1) + i\sin(\varphi-\varphi_1)\right]. \qquad (4)$$

$$\left[r(\cos\varphi + i\sin\varphi)\right]^n = r^n(\cos n\varphi + i\sin n\varphi). \qquad (5)$$

$$\sqrt[n]{r(\cos\varphi + i\sin\varphi)} = \sqrt[n]{r}\left(\cos\frac{\varphi+2k\pi}{n}\right) + i\sin\left(\frac{\varphi+2k\pi}{n}\right), \qquad (6)$$

bunda $k = 0, 1, 2, \ldots, (n-1)$.

(5) va (6) formulalar *Muavr formulalari* deyiladi.

5°. Eyler formulasi:

$$e^{i\varphi} = \cos\varphi + i\sin\varphi.$$

6°. Kompleks sonning logarifmi quyidagicha yoziladi:

$$\ln z = \ln r + i\varphi_0 + i2k\pi.$$

φ ning $-\pi < \varphi < \pi$ tengliklarini qanoatlantiruvchi qiymati φ_0 bo'ladi. $\ln r + i\varphi_0$ ifoda logarifmning *bosh qiymati* deyiladi.

71—74- masalalarda ko'rsatilgan amallarni bajaring:

71. 1) $(2-3i)(-4+7i)$; 2) $(5-6i)(-10+8i)$;

3) $(\sqrt{3}+i)(\sqrt{2}-i\sqrt{3})$; 4) $(4+i\sqrt{5})(4-i\sqrt{5})$;

5) $(m+i\sqrt{n})(m-i\sqrt{n})$; 6) $(a+bi)(a-bi)$.

72. 1) $\frac{3i}{1-i}$; 2) $\frac{1+2i}{2-i}$; 3) $\frac{5+3i}{2+i}$; 4) $\frac{4-5i}{-2+7i}$.

73. 1) $\frac{1-\sqrt{3}i}{1+\sqrt{3}i}$; 2) $\frac{\sqrt{7}-i}{\sqrt{7}-2i}$; 3) $\frac{\sqrt{s}}{(s-2)i}$; 4) $\frac{m+i\sqrt{n}}{m-i\sqrt{n}}$; 5) $\frac{p+qi}{q-pi}$.

74. 1) $\frac{(3+4i)(-1+3i)}{6-8i}$; 2) $\frac{-4+6i}{(2+i)(3-2i)}$; 3) $\frac{(4-i)(1+2i)}{(-2+i)(1-3i)}$;

4) $\frac{(m+ni)(n+mi)}{n-mi}$.

Quyidagi kompleks sonlarni vektorlar bilan tasvirlang, ularning modullari va argumentlarini aniqlang hamda trigonometrik ko'rinishda yozing:

75. 1) $z = 3$; 2) $z = -2$; 3) $z = 3i$, $z = -2i$.

76. 1) $z = 2 - 2i$; 2) $z = 1 + i\sqrt{3}$; 3) $z = -\sqrt{3} - i$.

77. 1) $z = \sqrt{3} + i$; 2) $z = 3 + i\sqrt{3}$; 3) $z = -\sqrt{2} + i\sqrt{6}$.

78. 1) $z = -\sqrt{2} + i\sqrt{2}$; 2) $z = \sin\alpha + i(1 - \cos\alpha)$.

79. 75—78- masalalarda berilgan sonlarni ko'rsatkichli ($re^{i\varphi}$) shaklda tasvirlang ($-\pi < \varphi < \pi$).

80. 1) $x^2+25=0$; 2) $x^2-2x+5=0$; 3) $x^2+4x-13=0$ tenglamalarni yeching va ildizlarni tenglamaga qo'yib tekshiring.

81. Quyidagi shartlarni qanoatlantiruvchi z nuqtalarning sohalarini yasang:

1) $|z| \le 3$; 2) $|z| < 2$ va $\frac{\pi}{2} < \varphi < \pi$; 3) $2 < |z| < 4$ va $-\pi < \varphi < \frac{\pi}{2}$;

4) $|z| > 5$; 5) $|z - 4| \le 2$; 6) $|z + 2i| \ge 4$; 7) $|z - 3i| = 3$;

8) $|z + 1 - i| < 2$; 9) $|z - i| = |z - 1|$.

82. $|z_1 - z_2|$ ifoda z_1 va z_2 nuqtalar orasidagi masofa ekanligini ko'rsating.

83. $z_0 = -2 + 3i$ nuqta berilgan. $|z - z_0| < 1$ tengsizlikni qanoatlantiruvchi z nuqtalarning sohasini yasang.

84. Quyidagilarni Muavr formulasi bilan hisoblang:

1) $(1+i)^{10}$; 2) $(1-i\sqrt{3})^6$; 3) $(-1+i)^5$; 4) $(1 + \cos\frac{\pi}{4} + i\sin\frac{\pi}{4})^4$;

5) $(\sqrt{3} + i)^3$; 6) $(\cos 15^0 + i\sin 15^0)^8$; 7) $\left[\sqrt{2}(\cos 10^0 + i\sin 10^0)\right]^6$;

8) $\left[\frac{1}{\sqrt{2}}(\cos 57^0 + i\sin 57^0)\right]^{10}$.

85. $z = \sqrt[6]{1}$ ning barcha qiymatlarini toping va radiusi 1 ga teng doira yasab, topilgan qiymatlarni radius-vektorlar bilan tasvirlang.

86. 1) $\sqrt[3]{-1}$; 2) $\sqrt[6]{-1}$; 3) $\sqrt[3]{-2+2i}$ larni toping.

87. 1) $\sqrt{i}$; 2) $\sqrt[3]{-1+i}$; 3) $\sqrt[4]{-8+8i\sqrt{3}}$ larni toping.

88. 1) $x^3-8=0$; 2) $x^4+4=0$; 3) $x^3-8=0$; 4) $x^6+64=0$; 5) $x^4-81=0$ ikki hadli tenglamalarni yeching.

I bob javoblari

1. 2. **2.** 1. **3.** –1. **4.** 7.**5.** $2a$. **6.**1. **7.** $4ab$. **8.** 4. **9.**10. **10.** 0. **11.** $\sin(\alpha-\beta)$. **12.**-10. **13.** $4a$. **14.** –33. **15.** $3abc-a^3-b^3-c^3$. **16.** $-2b^3$. **17.** $-2x$. **18.** $-4a^3$. **19.** 144. **20.** 72. **21.** $(x-y)(y-z)(x-z)$. **22.** 1. **23.** $\sin(\beta-\alpha)$. **24.** $\sin(\beta-\gamma)+\sin(\gamma-\alpha)+\sin(\alpha-\beta)$. **25.** 25. **26.** $y=x+2$ to'g'ri chiziqda yotadi.

27. 1) $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$; 2) $\begin{vmatrix} x & y & 1 \\ 2 & 3 & 1 \\ -1 & 5 & 1 \end{vmatrix} = 0$; **28.** 1) $x_1=0$, $x_2=-\frac{1}{2}$; 2) $x_1=-6$, $x_2=2$; 3) $x_1=0$, $x_2=-2$; 4) $x_1=2$, $x_2=3$. **29.** $x=-7$, $y=5$. **30.** $x=\frac{3}{2}$, $y=-\frac{1}{2}$. **31.** $x=\frac{3}{2}$, $y=-\frac{1}{2}$. **32.** $x=5$, $y=-4$. **33.** $x=\frac{4}{a}$, $y=1$. **34.** $x=m$, $y=2m-n$. **37.** 5, 6, 10. **38.** –1, 0, 1. **39.** 0, 1, –1. **40.** 4, 0, –1. **41.** $7k$, $8k$, $13k$. **42.** $2k$, k, $-4k$. **43.** $78k$, $46k$, $12k$. **44.** $17k$, $2k$, $-7k$. **45.** $5k$, $-11k$, $-7k$. **46.** $x=y=z=0$. **47.** $x=y=z=0$. **48.** $-k$, $13k$, $5k$. **49.** Birgalikda emas. **50.** Aniqlanmagan: $x=\frac{2+5z}{3}$, $y=\frac{5-7z}{3}$. **51.** Birgalikda emas. **52.** 2, –1, –3. **53.** 0, 1, –1. **54.** 2, –3, 0. **55.**0, 2, –3. **56.** 2, 2, –3. **57.** 5, –2, 3. **58.** 3, 4, 5. **59.** 7, 2, 1. **60.** 2, 2, 1.

61. 1) $AB=\begin{pmatrix} 5 & 2 \\ 7 & 0 \end{pmatrix}$, $BA=\begin{pmatrix} 29 & -22 \\ 31 & -24 \end{pmatrix}$; 2) $AB=\begin{pmatrix} 1 & 3 \\ 3 & 7 \end{pmatrix}$, $BA=\begin{pmatrix} 4 & 6 \\ 3 & 4 \end{pmatrix}$;

3) $AB=\begin{pmatrix} 43 & 10 \\ 49 & 11 \\ 37 & 9 \end{pmatrix}$, BA ko'paytma ma'noga ega emas. **62.** 1) $A^{-1}=\begin{pmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix}$;

2) $A^{-1}=\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}=A$; 3) $A^{-1}=\begin{pmatrix} 1 & -1 & 1 \\ -38 & 41 & -34 \\ 27 & -29 & 24 \end{pmatrix}$;

4) $A^{-1}=\begin{pmatrix} -8 & -5 & 6 \\ 18 & 11 & -13 \\ 1 & 1 & -1 \end{pmatrix}$. **63.** 1, 1, 1. **64.** 2, 3, 4. **65.** 1, 3, 5. **66.** 2, –1, 1. **67.** $24\frac{1}{2}$, $21\frac{1}{2}$, 10. **68.** $13\frac{1}{4}$, $8\frac{1}{4}$, $14\frac{1}{2}$. **69.** –1, 0, 4. **70.** $\frac{1}{2}$, $-\frac{3}{2}$, $\frac{1}{2}$. **71.** 1) $13+26i$; 2) $-2+100i$; 3) $\sqrt{3}+\sqrt{6}+\left(\sqrt{2}-3\right)i$; 4) 21; 5) m^2+n; 6) a^2+b^2; **72.** 1) $-1{,}5+1{,}5i$; 2) i; 3) $2{,}6+0{,}2i$;

4) $-\frac{43}{53}-\frac{18}{53}i$; **73.** 1) $-\frac{1}{2}-\frac{\sqrt{3}}{2}i$; 2) $\frac{9}{11}+\frac{i\sqrt{7}}{11}$; 3) $\frac{\sqrt{s}}{2-s}i$;

4) $\frac{m^2-n}{m^2+n}+\frac{2m\sqrt{n}}{m^2+n}i$; 5) i. **74.** 1) $-1{,}3\ -0{,}9i$; 2) $-\frac{38}{65}+\frac{44}{65}i$;

3) $\frac{55}{8}-\frac{35}{8}i$; 4) $n+mi$; **77.** 1) $2\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$;

2) $2\sqrt{3}\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$; 3) $2\sqrt{2}\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)$.

78. 1) $2\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$; 2) $2\sin\frac{\alpha}{2}\left(\sin\frac{\alpha}{2}+i\sin\frac{\alpha}{2}\right)$; **84.** 1) $32i$;

2) 64; 3) $4(1-i)$; 4) $2(3+2\sqrt{2})i$; 5) $8i$; 6) $-\frac{1}{2}+\frac{i\sqrt{3}}{2}$;

7) $4+4\sqrt{3}i$; 8) $-\frac{\sqrt{3}}{64}-\frac{1}{64}i$; **85.** $\cos\frac{k\pi}{3}+i\sin\frac{k\pi}{3}$; $k=0, 1,\ldots,5$.

86. 1) $-i$, $\frac{i\pm\sqrt{3}}{2}$; 2) $\pm i$, $\frac{\pm\sqrt{3}\pm 1}{2}$; 3) $1+i$, $-1{,}36+0{,}365i$, $0{,}365-1{,}36i$; **87.** 1) $\pm\frac{1+i}{\sqrt{2}}$; 2) $\sqrt[6]{2}(\cos\varphi+i\sin\varphi), \varphi=45°, \varphi=165°$;

$\varphi=285°$; 3) $\pm2(\sqrt{3}+i)$, $\pm2\left(-1+i\sqrt{3}\right)$. **88.** 1) $-2, 1\pm i\sqrt{3}$; 2) $\pm1\pm i$;
3) $2, -1\pm i\sqrt{3}$; 4) $\pm 2i$, $\pm\sqrt{3}\pm i$; 5) $\pm 3, \pm 3i$.

II BOB TEKISLIKDAGI ANALITIK GEOMETRIYA

1- §. To‘g‘ri chiziq va tekislikdagi nuqtaning koordinatalari. Ikki nuqta orasidagi masofa

1°. O‘qdagi $A(x_1)$ va $B(x_2)$ nuqtalar orasidagi masofa:

$$d = |x_2 - x_1| = \sqrt{(x_2 - x_1)^2}. \quad (1)$$

2°. O‘qdagi yo‘naltirilgan AB kesmaning (algebraik) kattaligi:

$$AB = x_2 - x_1. \quad (2)$$

3°. Tekislikdagi $A(x_1;y_1)$ va $B(x_2;y_2)$ nuqtalar orasidagi masofa:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}. \quad (3)$$

4°. Tekislikdagi yo‘naltirilgan kesmaning boshi $A(x_1;y_1)$ va oxiri $B(x_2;y_2)$ nuqtalarda bo‘lgan $\overrightarrow{AB}$ vektorning koordinata o‘qlaridagi proyeksiyalari:

$$PR_x \overrightarrow{AB} = X = x_2 - x_1; \quad PR_y \overrightarrow{AB} = Y = y_2 - y_1. \quad (4)$$

89. Son o‘qida $A(-5)$, $B(+4)$ va $C(-2)$ nuqtalarni yasang va shu o‘qdagi AB, BC va AC kattaliklarni toping. $AB+BC=AC$ ekanligini tekshiring.

90. Berilgan har qaysi A va B nuqtalar orasidagi masofani toping:

1) $A(0)$, $B(-1)$; 2) $A(-2)$, $B(8)$; 3) $A(3)$, $B(-1)$; 4) $A(-7)$, $B(-5)$.

91. Quyidagi A va B nuqtalar orasidagi masofani aniqlang:

1) $A(5;-3)$ va $B(-3;1)$; 2) $A(4;2)$ va $B(7;-2)$;
3) $A(0;3)$ va $B(-2;3)$; 4) $A(k;1)$ va $B(k+q;0)$.

92. Quyidagi nuqtalarning har biri bilan koordinatalar boshi orasidagi masofani aniqlang:

1. $A(3; 4)$; 2. $B(5; -12)$; 3. $C(0; 8)$; 4. $D(a; b)$.

93. Uchlari $A(-4;2)$, $B(0;-1)$ va $C(3;3)$ nuqtalarda bo‘lgan uchburchak yasang va uning perimetrini hamda burchaklarini aniqlang.

94. Quyidagi har bir holda ABC uchburchak o‘tkir burchakli, o‘tmas burchakli yoki to‘g‘ri burchakli bo‘lishini aniqlang.

1) $A(1;4)$, $B(5;8)$, $C(3;2)$; 2) $A(2;1)$, $B(-2;5)$, $C(-1;3)$; 3) $A(1;-1)$, $B(-2;1)$, $C(1;2)$; 4) $A(-3;2)$, $B(0;-1)$, $C(-2;5)$.

Ko'rsatma. *Uchburchak katta tomonining kvadrati qolgan ikki tomoni kvadratlarining yig'indisidan kichik, katta yoki unga teng bo'lishiga qarab u o'tkir burchakli, o'tmas burchakli yoki to'g'ri burchakli bo'ladi.*

95. Ordinatalar o'qida $A(-5;1)$ va $B(3;2)$ nuqtalardan barobar uzoqlashgan nuqtani toping.

96. Uchlari $A(1;5)$, $B(2;2)$, $C(-3;-10)$ va $D(-3;-2)$ nuqtalarda bo'lgan to'rtburchak tomonlarining uzunliklarini aniqlang.

97. Ordinatasi 2 bo'lgan nuqta (10;3) va (8;10) nuqtalardan teng uzoqlikda turadi. Bu nuqtaning abssissasini toping.

98. Abssissalar o'qida shunday nuqta topingki, undan (5;12) nuqtagacha bo'lgan masofa 13 ga teng bo'lsin.

99. $A(2;2)$, $B(-5;1)$ va $C(3;-5)$ nuqtalardan teng uzoqlikda bo'lgan nuqtani toping.

100. Koordinata o'qlarida $K(-6;8)$ nuqtadan 10 birlik masofa uzoqlashgan nuqtalarni toping.

101. $A(-7;-3)$ nuqtadan markazi $C(5;-8)$ nuqtada va radiusi 5 ga teng bo'lgan aylanaga urinmalar o'tkazilgan. Ularning uzunliklarini toping.

102. $A(1;2)$, $B(9;2)$ va $C(2;-5)$ nuqtalardan barobar uzoqlashgan D nuqtani toping.

103. Koordinata o'qlaridan va $C(2;4)$ nuqtadan barobar uzoqlashgan nuqtani toping.

104. $A(-4;2)$ nuqtadan o'tib, abssissalar o'qining $B(2;0)$ nuqtasida urinuvchi aylananing markazini toping.

105. Koordinata o'qlarining har biriga urinuvchi va (2;–1) nuqtadan o'tuvchi aylananing markazi, radiusini toping.

106. Nuqta to'g'ri chiziqli harakat qilib, $M(5;5)$ va $N(1;3)$ nuqtalardan o'tdi. Ox o'qini kesib o'tgan nuqtasini toping.

2- §. Kesmani berilgan nisbatda bo'lish. Uchburchak va ko'pburchakning yuzi

1°. Kesmani berilgan nisbatda bo'lish. $A(x_1;y_1)$ va $B(x_2;y_2)$ nuqtalar berilgan. AB kesmani $AN:NB = \lambda$ nisbatda bo'luvchi $N(x;y)$ nuqtaning koordinatalari ushbu

$$x = \frac{x_1 + \lambda x_2}{1 + \lambda}; \quad y = \frac{y_1 + \lambda y_2}{1 + \lambda} \tag{1}$$

formulalar bilan aniqlanadi. Xususiy holda kesmani teng ikkiga, ya'ni $\lambda = 1 : 1 = 1$ nisbatda bo'linganda:

$$x = \frac{x_1 + x_2}{2}; \quad y = \frac{y_1 + y_2}{2}. \tag{2}$$

2°. Uchlari $A(x_1;y_1)$, $B(x_2;y_2)$, $C(x_3;y_3)$, ...,$F(x_n;y_n)$ nuqtalarda bo'lgan ko'phurchak yuzi:

$$S = \pm\frac{1}{2}\left\| \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} + \begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} + ... + \begin{vmatrix} x_n & y_n \\ x_1 & y_1 \end{vmatrix} \right\|. \tag{3}$$

107. $A(3;5)$ va $B(1;-4)$ nuqtalarni yasang. AB kesmani $AN:NB = 2:3$ nisbatda bo'luvchi $N(x;y)$ nuqtani toping.

108. $A(-2; 1)$ va $B(3; 6)$ nuqtalar berilgan. AB kesmani $AN:NB =-3:2$ nisbatda bo'luvchi $N(x;y)$ nuqtani toping.

109. Ox oqining $A(x_1)$ va $B(x_2)$ nuqtalariga m_1 va m_2 massalar joylashtirilgan. Bu sistemaning og'irlik markazini toping.

110. Uzunligi 40 sm va og'irligi 500 g bo'lgan bir jinsli sterjenning uchlariga og'irliklari 100 va 400 g li sharlar osilgan. Shu sistemaning og'irlik markazini aniqlang.

111. $A(-2; 4)$, $B(3; -1)$ va $C(2; 3)$ nuqtalarga, mos ravishda, 60, 40 va 100 g massalar qo'yilgan. Shu sistema massalarining og'irlik markazini aniqlang.

112. ABC uchburchakning $A(6; 2)$, $B(3; -2)$ uchlari va uning medianalari kesishgan nuqta $M(3;1)$ berilgan. C uchining koordinatalarini toping.

113. ABC uchburchak berilgan: $A(-1; 3)$, $B(2; 1)$; $C(7; -3)$. A burchak bissektrisasining uzunligini toping.

114. Uchlari $A(1;-1)$, $B(6; 4)$ va $C(2; 6)$ nuqtalarda bo'lgan uchburchakning og'irlik markazini toping.

Ko'rsatma. *Uchburchakning og'irlik markazi medianalarining kesishgan nuqtasida yotadi.*

115. Uchlari $A(2; 0)$, $B(5; 3)$ va $C(2; 6)$ nuqtalarda bo'lgan uchburchakning yuzini hisoblang.

116. $A(1;1)$, $B(-1; 7)$ va $C(0; 4)$ nuqtalarning bir to'g'ri chiziqda yotishini ko'rsating.

117. Uchlari $A(3;1)$, $B(4;6)$, $C(6;3)$ va $D(5;-2)$ nuqtalarda bo'lgan to'rtburchakning yuzini hisoblang.

118. $(-3;4)$ va $(2;2)$ nuqtalarnimg teng o'rtasi bilan $(-1;3)$ nuqta orasidagi masofani aniqlang.

119. Uchlarining koordinatalari (4;5), (–2;3) va (1;1) bo‘lgan uchburchakning har bir tomoni o‘rtasidagi nuqtalarning koordinatalarini toping.

120. $A(x;4)$ va $B(-6;y)$ nuqtalar orasidagi AB masofa $M(-1;1)$ nuqtada teng ikkiga bo‘lingan. A va B nuqtalarni aniqlang.

121. (–1;6) va (3;–2) nuqtalar orasidagi kesma to‘rtta teng bo‘lakka bo‘lingan. Bo‘linish nuqtalarining koordinatalarini toping.

122. Parallelogramm burchaklaridan uchtasining uchlari (4;–3), (6;4) va (–5;–2) nuqtalarda. Uning to‘rtinchi burchagi uchining koordinatalarini toping.

123. Burchaklari uchlari $A(-2;0)$, $B(0;-1)$, $C(2;0)$, $D(3;2)$ va $E(-1;3)$ nuqtalarda bo‘lgan beshburchakning yuzini hisoblang.

124. Bir jinsli to‘rtburchakli taxtaning uchlari $A(4;4)$, $B(5;7)$, $C(10;10)$ va $D(12;4)$ nuqtalarda joylashgan. Taxtaning og‘irlik markazini toping.

3- §. Chiziqning nuqtalarning geometrik o‘rni sifatidagi tenglamasi

Chiziqning tenglamasi deb x va y o‘zgaruvchilarga nisbatan tuzilgan shunday tenglamaga aytiladiki, uni shu chiziqda yotgan har qanday nuqtaning koordinatalari va faqat ulargina qanoatlantiradi.

Chiziqning tenglamasidagi x va y lar *o‘zgaruvchi koordinatalar* deb, harflar bilan belgilangan o‘zgarmaslar esa *parametrlar* deb ataladi.

Chiziqni bir xil (umumiy) xossaga ega bo‘lgan nuqtalarning geometrik o‘rni deb qarab, uning tenglamasini tuzish uchun:

1) chiziqning ixtiyoriy $M(x;y)$ nuqtasi olinadi;

2) barcha M nuqtalarning umumiy xossasi tenglik orqali yoziladi;

3) bu tenglikdagi kesmalar (shuningdek, burchaklar) $M(x;y)$ nuqta koordinatalari hamda masalaning shartida berilganlar orqali aniqlanadi.

125. Markazi koordinatalar boshida bo‘lib, radiusi R ga teng aylananing tenglamasi $x^2 + y^2 = R^2$ bo‘lishini ko‘rsating.

126. Markazi $C(3;4)$ nuqtada, radiusi $R = 5$ bo‘lgan aylana tenglamasini yozing. $A(-1;1)$, $B(2;3)$, O (0;0) va D (4;1) nuqtalar shu aylanada yotadimi?

127. $A(3;2)$ va $B(-1;4)$ nuqtalardan teng uzoqlikda harakat qiluvchi $M(x;y)$ nuqta trayektoriyasining tenglamasini yozing.

$C(-1;1)$, $D(1;-3)$, $E(0;-1)$ va $F(2;2)$ nuqtalar o‘sha chiziqda yotadimi?

128. $B(0;4)$ nuqtaga nisbatan $A(0;12)$ nuqtadan uch marta uzoqroqda harakat qiluvchi $M(x;y)$ nuqta trayektoriyasining tenglamasini yozing.

129. $B(-4;4)$ nuqtaga nisbatan $A(-1;1)$ nuqtadan ikki marta yaqinroqda harakat qiluvchi $M(x;y)$ nuqta trayektoriyasining tenglamasini yozing.

130. Har bir nuqtasidan $F(2;0)$ va $F(-2;0)$ nuqtalargacha bo‘lgan masofalarining yig‘indisi $2\sqrt{5}$ ga teng nuqtalarning geometrik o‘rnining tenglamasini yozing va u bo‘yicha chiziq yasang.

131. Ushbu: $y=2x+5$; 2) $y=7-2x$; 3) $y=2x$; 4) $y=4$; 5) $y=4-x^2$; 6) $2x-y+1=0$; 7) $y=\sqrt{9-x^2}$; 8) $y=\frac{1}{1+x^2}$; 9) $y=\frac{1}{x}$; 10) $y=x^2-1$ tenglamalarga mos keladigan chiziqlarni (nuqtalar bo‘yicha) yasang.

132. Tenglamalari $y^2-x^2=5$ va $2x+y-1=0$ bo‘lgan L_1 va L_2 chiziqlarning kesishish nuqtasini toping.

133. Quyidagi chiziqlarning kesishish nuqtalarini toping:

1) $2x-y+1=0$ va $2x+y-5=0$; 2) $x^2+y^2=25$ va $x=3$;

3) $y=\frac{1}{x+1}$ va $x+y-1=0$; 4) $y=\frac{6}{x}$ va $x^2+y^2=13$.

134. Koordinatalar boshidan va $A(-2;-3)$ nuqtadan barobar uzoqlashgan nuqtalar geometrik o‘rnining tenglamasini tuzing.

135. Koordinatalar boshidan va $A(1;3)$ nuqtadan barobar uzoqlashgan nuqtalar geometrik o‘rnining tenglamasini tuzing.

136. $A(3;0)$ nuqtadan va koordinatalar o‘qidan barobar uzoqlashgan nuqtalar geometrik o‘rnimg tenglamasini tuzing.

137. Markazi $C(a;b)$ nuqtada, radiusi r bo‘lgan aylana tenglamasini tuzing.

138. Berilgan $A(1;-2)$ va $B(-1;2)$ nuqtalargacha masofalarning kvadratlari yig‘indisi 20 ga teng o‘zgarmas kattahk bo‘lgan nuqtalar geometrik o‘rnining tenglamasini toping.

139. Berilgan $A(0;4)$ va $B(-1;2)$ nuqtalargacha masofalarning ayirmasi 1 ga teng o‘zgarmas kattalik bo‘lgan nuqtalar geometrik o‘rnining tenglamasini yozing.

140. Agar M nuqta harakatlanayotgan har bir momentda undan $A(2\sqrt{3};2)$ nuqtagacha bo‘lgan masofa $B(\sqrt{3};-1)$ nuqtagacha masofadan ikki marta ortiq bo‘lsa, M nuqta trayektoriyasining tenglamasini keltirib chiqaring.

4- §. To‘g‘ri chiziqning burchak koeffitsiyentli tenglamasi, kesmalar bo‘yicha tenglamasi, umumiy tenglamasi

1°. To‘g‘ri chiziqning burchak koeffitsiyentli tenglamasi

$$y = kx+b \qquad (1)$$

ko‘rinishga ega, bu yerda: k – parametr to‘g‘ri chiziqning Ox o‘qqa og‘ish burchagi α ning tangensiga teng bo‘lib ($k=\mathrm{tg}\alpha$), to‘g‘ri chiziqning *burchak koeffitsiyenti, b* parametr *boshlang‘ich ordinata* yoki Oy o‘qidan ajratgan kesma kattaligi deyiladi.

2°. To‘g‘ri chiziqning umumiy tenglamasi:

$$Ax+By+C=0. \qquad (2)$$

Xususiy hollar:

1) $C=0$ bo‘lsa, $y=-\frac{A}{B}x$ bo‘lib, to‘g‘ri chiziq koordinatalar boshidan o‘tadi;

2) $B=0$ bo‘lsa, $x=-\frac{C}{A}=a$ bo‘lib, to‘g‘ri chiziq Oy o‘qqa parallel bo‘ladi;

3) $A=0$ bo‘lsa, $y=-\frac{C}{B}=b$ bo‘lib, to‘g‘ri chiziq Ox o‘qqa parallel bo‘ladi;

4) $B=C=0$ bo‘lsa, $Ax=0$ yoki $x=0$ bo‘lib, to‘g‘ri chiziq Oy o‘qdan iborat bo‘ladi;

5) $A=C=0$ bo‘lsa, $By=0$ yoki $y=0$ bo‘lib, to‘g‘ri chiziq Ox o‘qdan iborat bo‘ladi.

3°. To‘g‘ri chiziqning o‘qlardan ajratgan kesmalar bo‘yicha tenglamasi:

$$\frac{x}{a}+\frac{y}{b}=1. \qquad (3)$$

Bu yerda a va b –to‘g‘ri chiziqning o‘qlardan ajratgan (kesgan) kesmalarining kattaliklari.

141. Oy o‘qdan $b=5$ kesma ajratib, Ox o‘q bilan 1) 45°; 2) 135° burchak tashkil qiluvchi to‘g‘ri chiziqlarni yasang. O‘sha to‘g‘ri chiziqlarning tenglamalarini yozing.

142. Oy o‘qdan b=–4 kesma ajratib, Ox o‘q bilan 1) 60°; 2) 120° burchak tashkil qiluvchi to‘g‘ri chiziqlarni yasang. Bu to‘g‘ri chiziqlarning tenglamalarini yozing.

143. Koordinatalar boshidan o‘tib, Ox o‘qi bilan 1) $45°$; 2) $60°$; 3) $90°$; 4) $120°$; 5) $135°$ burchak tashkil qiluvchi to‘g‘ri chiziqlar tenglamalarini yozing.

144. Koordinatalar boshidan va $(-2;-3)$ nuqtadan o‘tuvchi to‘g‘ri chiziqni yasang va uning tenglamasini yozing.

145. $A(-2;3)$ nuqtadan o‘tuvchi to‘g‘ri chiziq Ox o‘qi bilan $135°$ li burchak tashkil etadi. Bu to‘g‘ri chiziq tenglamasini toping.

146. 1) $2x-3y=6$; 2) $2x+3y=0$; 3) $y=-3$;

4) $\frac{x}{4}+\frac{y}{3}=1$; 5) $x-4y-7=0$; 6) $y+5=0$;

7) $5x-2y=0$; 8) $10x+5y+12=0$

to‘g‘ri chiziqlarning har qaysisi uchun k va b parametrlarni aniqlang.

147. Quyidagi tenglamalar bilan ifodalangan to‘g‘ri chiziqlardan qaysilari Ox o‘qining musbat yo‘nalishi bilan o‘tkir burchak va qaysilari o‘tmas burchak tashkil qiladi:

1) $y=-x+5$; 2) $y=\frac{4}{5}x-1$; 3) $y=x$;

4) $3x-5y+1=0$; 5) $y=-x$; 6) $3x=4y$;

7) $x+y+4=0$; 8) $2x+3y=0$?

148. Quyidagi to‘g‘ri chiziqlardan qaysilari koordinatalar boshidan o‘tadi? Ikkala koordinata o‘qini kesadi? Ox o‘qiga parallel? Oy o‘qiga parallel:

1) $y+3=0$; 2) $x+2y=0$; 3) $2x-5=0$;

4) $x+y=2$; 5) $3y+x=0$; 6) $6y+7=0$;

7) $2x-y-1=0$; 8) $x=y$?

149. Kvadratning uchlaridan biri koordinatalar boshi bilan, qarama- qarshi uchi esa $A(5;-5)$ nuqta bilan ustma-ust tushadi. Kvadrat tomonlarining tenglamalarini yozing.

150. Kvadrat uchlaridan biri koordinatalar boshida, diagonallari kesishgan nuqta $S(-1;1)$ nuqtada joylashgan. Kvadrat tomonlarining tenglamalarini tuzing.

151. To‘g‘ri to‘rtburchakning ikkita tomoni koordinata o‘qlarida yotadi, uchlaridan biri $(-2;-3)$ koordinataga ega. To‘g‘ri to‘rtburchak tomonlarining tenglamalarini tuzing.

152. 1) $2x-3y=6$; 2) $3x-2y+4=0$; 3) $2x-y+3=0$; 4) $5x+2y-8=0$; 5) $3x+8y+16=0$ to'g'ri chiziqlarning tenglamalarini o'qlardan ajratgan kesmalariga nisbatan yozing.

153. $O(0;0)$ va $A(-3;0)$ nuqtalar berilgan. Bir tomoni OA kesmadan iborat bo'lgan va diagonallari $B(0;2)$ nuqtada kesishuvchi parallelogramm tomonlarining va diagonallarining tenglamalarini yozing.

154. $A(4;3)$ nuqtadan o'tuvchi va koordinatalar burchagidan yuzi 3kv birlikka teng uchburchak kesuvchi to'g'ri chiziq tenglamasini yozing.

155. Abssissalar o'qining musbat yarmidan 2 birlikka teng, ordinatalar o'qining manfiy yarmidan 5 birlikka teng bo'lgan kesma ajratadigan to'g'ri chiziqning tenglamasini tuzing ($5x-2y-10=0$).

156. Rombning diagonallari 8 va 3 birllikka teng. Agar rombning katta diagonalini Ox o'qi uchun, kichigini Oy o'qi uchun qabul qilsak, romb tomonlarining tenglamalarini yozing.

($3x+8y-12=0$, $3x-8y+12=0$, $3x+8y+12=0$, $3x-8y-12=0$).

157. Koordinata o'qlari bilan $2x-5y+20=0$ to'g'ri chiziq orasida joylashgan uchburchak yuzini toping (20 kvadrat birlik).

158. $A(1;2)$ nuqta orqali koordinatalar o'qining musbat yarim o'qlaridan teng kesmalar ajratadigan to'g'ri chiziq tenglamasini tuzing ($x+y-3=0$).

159. $(2; 3)$ nuqtadan o'tuvchi va koordinata burchagidan yuzi 12 kv birlikka teng bo'lgan uchburchak ajratuvchi to'g'ri chiziq tenglamasini tuzing ($3x+2y-12=0$).

160. Asoslari 10 va 6 bo'lgan teng yonli trapetsiyaning o'tkir burchagi 60°, trapetsiyaning katta asosi Ox o'qi, uning simmetriya o'qini Oy o'qi deb olib, uning tomonlari tenglamasini toping.

$$\left(y = 0;\ y = 2\sqrt{3};\ y = \sqrt{3}x + 5\sqrt{3};\ y = -\sqrt{3}x + 5\sqrt{3}\right).$$

161. Tomoni a ga teng bo'lgan kvadratning diagonallari koordinata o'qlaridan iborat. Kvadrat tomonlarining tenglamalarini tuzing.

$$\left(y = \pm x \pm \frac{a\sqrt{2}}{2}\right).$$

162. $y = \frac{2}{3}x - 4$ to'g'ri chiziq bo'ylab yo'nalgan nur Ox o'qiga yetib, qaytadi. Nurning Ox o'qi bilan uchrashgan nuqtasini va qaytgan nur tenglamasini toping.

$$\left(x_1=6;\ y_1=0;\quad y = -\frac{2}{3}x + 4\right).$$

5- §. Ikki to‘g‘ri chiziq orasidagi burchak. Berilgan nuqtadan o‘tuvchi to‘g‘ri chiziqlar dastasining tenglamasi. Berilgan ikki nuqtadan o‘tuvchi to‘g‘ri chiziqning tenglamasi. Ikki to‘g‘ri chiziqning kesishish nuqtasi

1°. $y = k_1x + b_1$ to‘g‘ri chiziqdan $y = k_2x + b_2$ to‘g‘ri chiziqqacha soat strelkasiga qarshi yo‘nalishda hisoblanuvchi α burchak

$$\mathrm{tg}\alpha = \frac{k_2 - k_1}{1 + k_1k_2} \qquad (1)$$

formula bilan aniqlanadi.

$A_1x+B_1y+C_1=0$ va $A_2x+B_2y+C_2=0$ tenglamalar bilan berilgan to‘g‘ri chiziqlar uchun (1) ko‘rinishga ega formula quyidagicha ifodalanadi:

$$\mathrm{tg}\alpha = \frac{A_1B_2 - A_2B_1}{A_1A_2 + B_1B_2}.$$

Ikkita to‘g‘ri chiziqning:

Parallellik sharti: $k_1=k_2$ yoki $\frac{A_1}{A_2} = \frac{B_1}{B_2}$.

Ustma-ust tushishlik sharti: $\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}$.

Perpendikularlik sharti: $k_1 = \frac{1}{k_2}$ yoki $A_1A_2+B_1B_2= 0$.

2°. Berilgan $A(x_1;y_1)$ nuqtadan o‘tuvchi to‘g‘ri chiziqlar dastasining tenglamasi quyidagicha yoziladi:

$$y - y_1 = k(x - x_1). \qquad (2)$$

3°. Berilgan $A(x_1; y_1)$ va $B(x_2;y_2)$ nuqtalardan o‘tuvchi to‘g‘ri chiziq tenglamasi quyidagicha yoziladi:

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}. \qquad (3)$$

4°. Parallel bo‘lmagan ikki $A_1x+B_1y+C_1=0$ va $A_2x+B_2y+C_2=0$ to‘g‘ri chiziqning kesishish nuqtasini topish uchun ularning tenglamalarini birgalikda yechiladi:

$$x = \frac{\begin{vmatrix} -C_1 & B_1 \\ -C_2 & B_2 \end{vmatrix}}{\begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} A_1 & -C_1 \\ A_2 & -C_2 \end{vmatrix}}{\begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix}}. \qquad (4)$$

163. Quyidagi to‘g‘ri chiziqlar orasidagi burchakni aniqlang:

1) $\begin{cases} y = 2x - 3, \\ y = \frac{1}{2}x + 1; \end{cases}$ 2) $\begin{cases} y = 3x, \\ y = -2x + 5; \end{cases}$ 3) $\begin{cases} y = 4x - 7, \\ y = -\frac{1}{4}x + 2; \end{cases}$

4) $\begin{cases} y = 5x - 3, \\ y = 5x + 8; \end{cases}$ 5) $\begin{cases} y = \sqrt{3}x - 5, \\ y = -\sqrt{3}x + 1; \end{cases}$ 6) $\begin{cases} y = 7x - 2, \\ y = x - \sqrt{2}; \end{cases}$

7) $\begin{cases} 5x - y + 7 = 0, \\ 2x - 3y + 1 = 0; \end{cases}$ 8) $\begin{cases} 2x + y = 0, \\ y = 3x - 4; \end{cases}$ 9) $\begin{cases} 3x + 2y = 0, \\ 6x + 4y + 9 = 0; \end{cases}$

10) $\begin{cases} 3x - 4y = 0, \\ 8x + 6y = 11; \end{cases}$ 11) $\begin{cases} \frac{x}{a} + \frac{y}{b} = 1, \\ \frac{x}{b} + \frac{y}{a} = 1. \end{cases}$

164. Quyidagi to‘g‘ri chiziqlarning ustma-ust tushishligi, parallelligi yoki kesishishini (bunda kesishish nuqtasini topish kerak bo‘ladi) tekshiring:

1) $x-y+3=0$, $2x-2y-7=0$; 2) $2x-y+4=0$, $4x-2y+9=0$;
3) $x+3y-1=0$, $2x+6y-2=0$; 4) $5x-y+1=0$, $10x-3y+2=0$;
5) $3x+2y-4=0$, $5x+6y-12=0$; 6) $2x-3y=0$, $6x-9y=0$;
7) $y-5=0$, $3y+15=0$; 8) $4x-1=0$, $8y+2=0$;
9) $2x+3=0$, $2x-1=0$; 10) $4x-y+1=0$, $2x+3y-17=0$;
11) $5x+3=0$, $10x+7y+2=0$.

165. $A(2;3)$ nuqtadan o‘tuvchi to‘g‘ri chiziqlar dastasining tenglamasini yozing. Shu dastadan Ox o‘qi bilan: 1) 45°; 2) 60°; 3) 135°; 4) 0° burchak tashkil etuvchi to‘g‘ri chiziqlarni tanlab oling va yasang.

166. $A(-2;5)$ nuqta va $2x-y=0$ to‘g‘ri chiziqni yasang. A nuqtadan o‘tuvchi to‘g‘ri chiziqlar dastasining tenglamasini yozing va o‘sha dastadan berilgan to‘g‘ri chiziqqa: 1) parallel; 2) perpendikular bo‘lgan to‘g‘ri chiziqlarni tanlab oling.

167. $2x-5y-10=0$ to‘g‘ri chiziqning koordinata o‘qlari bilan kesishgan nuqtalaridan bu to‘g‘ri chiziqqa perpendikular chiqarilgan. Ularning tenglamalarini yozing.

168. Berilgan ikki nuqtadan o‘tgan to‘g‘ri chiziqning tenglamasini yozing:

1) $A(-1;3)$ va $B(4;-2)$; 2) $(-1;+4)$ va $(-3;2)$; 3) $(0;6)$ va $(3;6)$; 4) $(0;0)$ va $(2;3)$; 5) $(-1;1)$ va $(0;1)$.

169. $A(-1;6)$ va $B(9;-8)$ nuqtalar berilgan. AB kesmaning o‘rtasidan $2x-3y+5=0$ to‘g‘ri chiziqqa parallel bo‘lgan to‘g‘ri chiziq tenglamasini tuzing.

170. Uchburchak uchlarining koordinatalari berilgan: $A(-2;3)$, $B(4;-2)$ va $C(1;5)$. Har qaysi uchidan qarshi tomonga parallel bo‘lib o‘tgan to‘g‘ri chiziqlarning tenglamalarini tuzing.

171. Uchlari $A(-2;0)$, $B(2;6)$ va $C(4;2)$ nuqtalarda bo‘lgan uchburchakning BD balandligi va BE medianasi o‘tkazilgan. AC tomon, BE medianasi va BD balandlikning tenglamalarini tuzing.

172. To‘rtburchakning uchlari berilgan: $A(-4;-2)$, $B(-3;1)$, $C(4;3)$ va $D(5;-3)$. Bu to‘rtburchak tomonlarining o‘rtalari parallelogrammning uchlari ekanligini ko‘rsating.

173. Parallelogrammning $x-y+1=0$ va $2x+3y-6=0$ tomonlarini hamda uning uchlaridan biri $C(7;1)$ ni bilgan holda qolgan ikkita tomonning tenglamalarini tuzing.

174. Uchburchakning uchlari $A(0;0)$, $B(1;2)$ va $C(-2;3)$ nuqtalarda. Uning istalgan ikki tomonining o‘rtasidan o‘tgan to‘g‘ri chiziq qolgan tomoniga parallel ekanligini isbot qiling.

175. $(0;2)$ nuqtadan o‘tib, $y-x-5=0$ to‘g‘ri chiziqqa perpendikular bo‘lgan to‘g‘ri chiziqning tenglamasini tuzing.

176. $(5;-3)$ nuqtadan o‘tib, $4x+3y+15=0$ to‘g‘ri chiziqqa perpendikular bo‘lgan to‘g‘ri chiziqning tenglamasini tuzing.

177. Uchburchakning uchlari $A(2;5)$, $B(2;-3)$ va $C(4;-1)$ nuqtalarda. Bu uchburchak balandliklarining tenglamalarini tuzing.

178. Uchburchakning uchlari $A(-2;3)$, $B(0;0)$ va $C(3;5)$ nuqtalarda. Uning har bir tomonining o‘rtasiga perpendikular bo‘lgan to‘g‘ri chiziqlarning tenglamalarini tuzing.

179. Tomonlari $x+y=4$, $3x-y=0$, $x-3y-8=0$ tenglamalar bilan berilgan uchburchak yasang, uning burchaklari va yuzini toping.

180. Uchlari $A(-4;2)$, $B(2;-5)$ va $C(5;0)$ nuqtalarda bo‘lgan uchburchak medianalarining kesishgan nuqtasini va balandliklarining kesishgan nuqtasini toping.

181. $A(-2;-2)$, $B(-3;1)$, $C(7;7)$ va $D(3;1)$ nuqtalar trapetsiyaning uchlari ekanligini tekshiring. Bu trapetsiyaning o‘rta chizig‘i va diagonallarining tenglamalarini tuzing ($BC||DA$; $3x-5y+5=0$; $x-y=0$; $y-1=0$).

182. Uchburchakning $A(3;5)$, $B(6;1)$ uchlari va uning medianalarining kesishgan nuqtasi $N(4;0)$ bo‘yicha uning tomonlari tenglamalarini tuzing ($4x+3y-27=0(AB)$; $x=3(AC)$; $7x-3y-39=0$ (BC)).

183. Ikkita: $A(-3;1)$ va $B(3;-7)$ nuqtalar berilgan. Ordinata o'qida shunday N nuqta topingki, AN va BN to'g'ri chiziqlar bir-biriga perpendikular bo'lsin ($N_1(0;2)$ va $N_2(0;-8)$).

184. $M(4;-3)$ nuqtadan o'tib, koordinata o'qlari bilan, yuzasi 3 kv birlikka teng bo'lgan uchburchak hosil qiluvchi to'g'ri chiziq tenglamasini tuzing.

$$\left(\frac{x}{2}+\frac{y}{3}=1 \quad \text{va} \quad \frac{x}{4}+\frac{y}{\frac{3}{2}}=-1\right).$$

6- §. To'g'ri chiziqning normal tenglamasi. Nuqtadan to'g'ri chiziqqacha bo'lgan masofa. Bissektrisalarning tenglamalari. Berilgan ikki to'g'ri chiziqning kesishish nuqtasidan o'tuvchi to'g'ri chiziqlar dastasining tenglamasi

1°. To'g'ri chiziqning normal tenglamasi quyidagicha yoziladi:

$$x\cos\beta + y\sin\beta - p = 0. \qquad (1)$$

Bunda p — koordinatalar boshidan to'g'ri chiziqqa tushirilgan perpendikular (normal) uzunligi, β esa o'sha perpendikularning Ox o'qqa og'ish burchagi.

To'g'ri chiziqning $Ax+By+C=0$ umumiy tenglamasini normal ko'rinishga keltirish uchun uning barcha hadlarini $M=\pm\frac{1}{\sqrt{A^2+B^2}}$ normallovchi ko'paytuvchiga ko'paytirish kerak. M ning ishorasi tenglamadagi ozod had C ning ishorasiga teskari qilib olinadi.

2°. $(x_0;y_0)$ nuqtadan to'g'ri chiziqqacha bo'lgan d masofa

$$d=|x_0\cos\beta + y_0\sin\beta - p| \qquad (2)$$

yoki

$$d=\frac{|Ax_0+By_0+C|}{\sqrt{A^2+B^2}} \qquad (2')$$

formula bilan hisoblanadi.

3°. $Ax+By+C=0$ va $A_1x+B_1y+C_1=0$ to'g'ri chiziqlar orasidagi burchaklar bissektrisalarining tenglamalari:

$$\frac{Ax+By+C}{\sqrt{A^2+B^2}}=\pm\frac{A_1x+B_1y+C_1}{\sqrt{A_1^2+B_1^2}}. \qquad (3)$$

4°. Berilgan ikki to‘g‘ri chiziqning kesishish nuqtasidan o‘tuvchi to‘g‘ri chiziqlar dastasining tenglamasi:

$$\alpha(Ax+By+C)+\beta(A_1x+B_1y+C_1)=0. \qquad (4)$$

185. 1) $4x-3y+10=0$; 2) $5x+12y-39=0$; 3) $6x+8y-15=0$; 4) $x-2y+3=0$; 5) $y-x\sqrt{3}=4$; 6) $x\cos 10°+y\sin 10°+4=0$ to‘g‘ri chiziqlarning tenglamalarini normal ko‘rinishga keltiring.

186. $A(4;3)$; $B(2;1)$ va $C(1;0)$ nuqtalardan $3x+4y-10=0$ to‘g‘ri chiziqqacha bo‘lgan masofalarni toping va to‘g‘ri chiziqni yasang.

187. Normal uzunligi $p=2$ va uning Ox o‘qqa og‘ish burchagi β: 1) 45°; 2) 135°; 4) 315° bo‘lgan to‘g‘ri chiziqlarni yasang.

188. Berilgan nuqtadan berilgan to‘g‘ri chiziqqacha bo‘lgan masofani hisoblang: 1) $P_1(4;-2)$, $8x-15y-11=0$; 2) $P_2(2;7)$, $12x+5y-7=0$; 3) $P_3(-3;5)$, $9x-12y+2=0$; 4) $P_4(-3;2)$, $4x-7y+26=0$; 5) $P_5(8;5)$, $3x-4y-15=0$.

189. Koordinatalar boshidan $12x-5y+39=0$ to‘g‘ri chiziqqacha bo‘lgan masofani toping.

190. 1) $3x-4y+10=0$ va $6x-8y+15=0$; 2) $2x-3y-6=0$ va $4x-6y-25=0$ to‘g‘ri chiziqlar o‘zaro parallel ekanligini ko‘rsating va ularning orasidagi masofani aniqlang.

191. $y=kx+5$ to‘g‘ri chiziq koordinatalar boshidan $d=\sqrt{5}$ masofa uzoqlilkda bo‘lsa, k ni toping.

192. $4x-3y=0$ to‘g‘ri chiziqdan 4 birlik uzoqlikdagi nuqtalar geometrik o‘rnining tenglamalarini yozing.

193. $5x+Ay-15=0$ tenglamadagi A koeffitsiyenti qanday bo‘lganida u to‘g‘ri chiziqning ordinatalar o‘qidan kesgan kesmasi 2 ga teng bo‘ladi?

194. $8x-15y=0$ to‘g‘ri chiziqqa parallel bo‘lib, $A(4;-2)$ nuqtadan 4 birlik uzoqlikdagi to‘g‘ri chiziqning tenglamasmi yozing.

195. 1) $2x+3y=12$ va $3x+2y=12$; 2) $3x+4y=12$ va $y=0$; 3) $2x-9y+18=0$ va $6x+7y-21=0$ to‘g‘ri chiziqlar orasidagi burchaklar bissektrisalarining tenglamalarini yozing.

196. $M(x;y)$ nuqta $y = 4-2x$ to‘g‘ri chiziqqa nisbatan $y = 2x-4$ to‘g‘ri chiziqdan 3 marta uzoqda harakat qiladi. O‘sha nuqta trayektoriyasining tenglamasini yozing.

197. Uchlari $A\left(\frac{9}{5};\ \frac{2}{5}\right)$, $B(0;4)$ va $C(-3;-2)$ nuqtalarda bo‘lgan uchburchakka ichki chizilgan doira markazining koordinatalarini toping.

198. $N(1;2)$ nuqtadan o‘tuvchi va $A(3;3)$, $B(5;2)$ nuqtalardan bir xil uzoqlikda bo‘lgan to‘g‘ri chiziq tenglamasini toping.

199. Ikkita: $A(2;-3)$ va $B(5;-1)$ nuqtalar berilgan. A nuqtadan 6 birlik va B nuqtadan 4 birlik uzoqda bo‘lgan to‘g‘ri chiziq tenglamasini toping.

200. $P(6;-2)$ nuqtadan 4 birlik masofada yotgan va $y = \frac{8}{15}x + 1$ to‘g‘ri chiziqqa parallel bo‘lgan to‘g‘ri chiziq tenglamasini toping.

201. $x+7y-6=0$ va $5x-5y+1=0$ to‘g‘ri chiziqlar orasidagi burchak bissektrisalari tenglamalarini yozing.

202. $A(3;5)$ va $B(5;2)$ nuqtalardan teng uzoqlikda bo‘lib, $M(1;2)$ nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasini yozing.

Aralash masalalar

203. $(-1;2)$ nuqtadan o‘tib, abssissalar o‘qining musbat yonalishi bilan tashkil qilgan burchagining sinusi 0,8 ga teng bo‘lgan to‘g‘ri chiziq tenglamasini tuzing.

204. $P(0;1)$ nuqtadan o‘tib, $x-3y+10=0$, $2x+y-8=0$ to‘g‘ri chiziqlarning orasidagi kesmasi P nuqtada teng ikkiga bo‘linuvchi to‘g‘ri chiziqni o‘tkazing.

205. $3x-2y+1=0$ va $x+3y-7=0$ to‘g‘ri chiziqlarning kesishish nuqtasidan ularning birinchisiga perpendikular to‘g‘ri chiziq o‘tkazilgan. Hosil qilingan to‘g‘ri chiziqdan koordinatalar boshigacha bo‘lgan masofa qancha?

206. Tenglamasi $4x+3y+1=0$ bo‘lgan to‘g‘ri chiziq berilgan. Bu to‘g‘ri chiziqqa parallel bo‘lgan shunday to‘g‘ri chiziq topilsinki, u bilan berilgan to‘g‘ri chiziq orasidagi masofa 3 birlikka teng bo‘lsin.

207. (2;7) nuqtadan shunday to‘g‘ri chiziq o‘tkazilganki, u koordinata o‘qlari bilan yuzi 64 kv birlikka teng bo‘lgan uchburchak tashkil qiladi. Bu chiziqning tenglamasini tuzing.

208. Tenglamalari $3x-4y+6=0$, $x-2=0$ va $y=2x-1$ bo‘lgan to‘g‘ri chiziqlarning bir nuqtadan o‘tishini isbot qiling.

209. Uchburchakning uchlari berilgan: $A(-8;1)$, $B(1;-2)$ va $C(6;3)$. Uchburchakka tashqi chizilgan aylananing markazi va radiusini toping.

210. Uchburchakning uchlari $A(-2;-1)$, $B(1;3)$ va $C(-2;3)$ nuqtalarda. Uchburchakka ichki chizilgan aylananing markazi va radiusini toping.

211. $2x+5y+3=0$ va $3x-4y-7=0$ to‘g‘ri chiziqlarning kesishgan nuqtasidan shunday to‘g‘ri chiziq o‘tkazingki, uning bilan $y=4x+3$ to‘g‘ri chiziqning orasidagi burchak 45° bo‘lsin.

212. Teng yonli uchburchakda yon tomonlarining tenglamalari $3x-y+6=0$ va $x+3y-2=0$. Uchburchakning asosi $(1;-2)$ nuqtadan o‘tadi. Asosining tenglamasini tuzing.

213. Uchburchakning tomonlaridan ikkitasining tenglamalari $y+1=0$, $x+1=0$ va medianalari kesishgan nuqta $(-1;0)$ berilgan. Uchinchi tomon tenglamasini tuzing.

214. Uchburchakning tomonlaridan ikkitasining tenglamalari $3x+2y+6=0$, $x+y-3=0$ va balandliklarining kesishgan nuqtasi $(0;0)$ berilgan. Uchinchi tomon tenglamasini tuzing.

215. Uchburchakning $A(0;-4)$, $B(3;0)$ va $C(0;6)$ uchlari berilgan. C uchidan A burchakning bissektrisasigacha bo‘lgan masofani toping.

216. Parallelogrammning AB va BC tomonlari, mos ravishda, $2x-y+5=0$ va $x-y+4=0$ tenglamalar bilan berilgan,diagonallari $M(1;4)$ nuqtada kesishadi. Uning balandliklarining uzunliklarini toping.

217. Romb ikki tomonining tenglamalari $x+2y=4$ va $x+2y=10$ hamda diagonallaridan birining tenglamasi $y=x+2$ ma’lum bo‘lsa, romb uchlarining koordinatalarini toping.

218. $2x+y-6=0$ to‘g‘ri chiziq va unda ordinatalari $y_A=6$ va $y_B=-2$ bo‘lgan ikki A va B nuqta berilgan. AOB uchburchakning AD balandligining tenglamasini yozing, uning uzunligi va DAB burchakni toping.

219. Uchburchakning bitta uchi $A(3;-4)$ va ikkita balandliklarining tenglamalari: $7x-2y-1=0$ va $2x-7y-6=0$ ga ko‘ra uchburchak tomonlarining tenglamalarini tuzing.

220. Uchburchakning bitta uchi $A(-4;2)$ va ikkita medianalarining tenglamalari $3x-2y+2=0$ va $3x+5y-12=0$ ga ko‘ra uchburchak tomonlarining tenglamalarini tuzing.

II bob javoblari

89. $AB = 9$, $BC = -6$, $AC = 3$, $9 - 6 = 3$. **90**. 1) 1; 2) 10; 3) 4; 4) 2. **91**. 1) $4\sqrt{5}$; 2) 5; 3)2; 4) $\sqrt{q^2 + l^2}$. **92**. 1) 5; 2) 13; 3) 8; 4) $\sqrt{a^2 + b^2}$; **93**. $5(2+\sqrt{2})$, 90^0, 45^0. **94**. 1) to'g'ri burchakli; 2) o'tmas burchakli; 3) o'tkir burchakli; 4) to'g'ri burchakli uchburchak. **95**. $M(0;-6{,}5)$. **96**. $AB = \sqrt{10}$; $BC = 13$; $CD = \frac{1}{8}$; $AD = 11\sqrt{65}$. **97.** $x = -6{,}75$. **98**. (0;0) va (10;0). **99**. (–1; –2). **100**. (0;0), (–12;0), (0;16). **101.** 12. **102**. $D(5;-1)$. **103**. (10;10), (2;2). **104.** (2; 10). **105.** $M_1(1;-1)$, $r_1 = 1$ va $M_2(5;-5)$. $r_2 = 5$. **106.** (–5; 0). **107**. $N\left(\frac{11}{5};\frac{7}{5}\right)$. **108.** $N(13;16)$. **109.** $x = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$. **110.** 100g og'irlikdagi shar markazidan 26 sm uzoqlikda. **111**. (1;2,5). **112**. (0;3). **113.** $\frac{14}{3}\sqrt{2}$. **114**. (3;3). **115.** 9 kv bir. **117**. 13 kv bir. **118.** 0,5. **119.** (1;4), $\left(-\frac{1}{2};2\right)$, $\left(2\frac{1}{2};3\right)$. **120**. (4;4), (–6;2). **121.** (0;4), (2;0). **122**. (–3;5) yo (15; 3), yo (–7;–9). **123**. 12,5 kv bir. **124**. (8,2; 6,2). **126**. $x^2+y^2-6x-8y=0$, A va O aylanada yotadi. **127**. $2x-y+1=0$, D va E chizqda yotadi. **129**. $x^2+y^2=8$. **130.** $\frac{x^2}{5}+y^2=1$. **132.** $\left(-\frac{2}{3};-\frac{7}{3}\right)$. **133.** 1) (1; 3); 2) (3; 4), (3; -4); 3) (0; 1); 4) (2; 3), (3; 2), (–2; –3), (–3; –2). **134.** $4x+6y+13=0$. **135.** $x+3y-5=0$. **136**. $y^2 = 6x - 9$. **137**. $(x-a)^2+(y-b)^2=r^2$. **138**. $x^2+y^2=5$. **139**. $x+2y-5=0$. **140.** $x^2+y^2=8$. **141.** $y=x+5$, $y=-x+5$. **142.** $y=x\sqrt{3}-4$, $y = -x\sqrt{3-4}$. **144**. $y=1{,}5x$. **145.** $x+y-1=0$. **146.** 1) $k = \frac{2}{3}$, $b=-2$; 2) $k = -\frac{2}{3}$, $b=0$; 3) $k=0$, $b=-3$; 4) $k = -\frac{3}{4}$, $b=3$; 5) $k = \frac{1}{4}$, $b = -\frac{7}{4}$; 6) $k=0$, $b = -\frac{5}{3}$; 7) $k = \frac{5}{2}$, $b=0$; 8) $k=-2$, $b = -\frac{12}{5}$. **147**. 1) o'tmas; 2) o'tkir; 3) o'tkir; 4) o'tkir; 5) o'tmas; 6) o'tkir; 7) o'tmas; 8) o'tmas. **148.** 2,5 va 8 – chiziqlar koordinata boshidan o'tadi; 4 va 7— ikkala koordinata o'qlarini kesadi; 1 va 6—Ox o'qiga parallel; 3—Oy o'qiga parallel. **149.** $x=0$, $y=0$, $x-5=0$, $y+5=0$. **150**. $x=0$, $y=0$, $x+2=0$, $y-2=0$. **151.** $x=0$, $y=0$, $x+2=0$, $y+3=0$. **152.** 1) $\frac{x}{3}+\frac{y}{-2}=1$; 2) $\frac{x}{-\frac{4}{3}}+\frac{y}{2}=1$; 3) $\frac{x}{3}+\frac{y}{3}=1$;

4) $\frac{x}{8}+\frac{y}{4}=1$; 5) $\frac{x}{16}+\frac{y}{-2}=1$. **153**. $y=0$; $4x-3y=0$; $y=4$; $4x-3y+12=0$; $x=0$; $2x-3y+6=0$. **154.** $\frac{x}{2}-\frac{y}{3}=1$ yoki $-\frac{x}{4}+\frac{y}{3}=1$. **155.** $5x-2y-10=0$. **156.** $3x+8y-12=0$, $3x-8y+12=0$. $3x+8y+12=0$, $3x-8y-12=0$. **157.** 20kv birlik. **158.** $x+y-3=0$. **159.** $3x+2y-12=0$. **160.** $y=0$; $y=2\sqrt{3}$; $y=\sqrt{3}x+5\sqrt{3}$; $y=-\sqrt{3}x+5\sqrt{3}$. **161.** $y=\pm x+\frac{a\sqrt{2}}{2}$. **162.** $x_1=6$, $y_1=0$; $y=-\frac{2}{3}x+4$. **163.** 1) arctg $\frac{3}{4}$; 2) $\frac{\pi}{4}$; 3) $\frac{\pi}{2}$; 4) 0; 5) $\frac{\pi}{3}$; 6) arctg $\left(-\frac{3}{4}\right)$; 7) $\frac{\pi}{4}$; 8) $\frac{\pi}{4}$; 9) 0; 10) $\frac{\pi}{2}$; 11) arctg $\frac{a^2-b^2}{2ab}$. **164**. 1) parallel; 2) parallel; 3) ustma-ust tushadi; 4) $\left(-\frac{1}{4};0\right)$; 5) (0;2); 6) ustma-ust tushadi; 7) parallel; 8) $\left(\frac{1}{4}-\frac{1}{4}\right)$; 9) parallel; 10) (1;5); 11) $\left(-\frac{3}{5};\frac{4}{7}\right)$. **167.** $5x+2y+4=0$; $5x+2y=25$. **168.** 1) $x+y-2=0$; 2) $y-3x-7=0$; 3) $y-6=0$; 4) $y=\frac{3}{2}x$; 5) $y-1=0$. **169**. $2x-3y-12=0$. **170**. $6y=5x-35$; $3y=2x-14$; $3y=-7x-5$. **171.** $x-2y+2=0$; $5x-y=4$; $3x-y=0$. **173**. $x-y-6=0$; $2x+3y-17=0$. **175.** $x+y-2=0$. **176.** $3x-4y-27=0$. **177.** $x-3y-1=0$, $y+1=0$, $x+y-7=0$. **178.** $4x-6y+13=0$, $3x+5y-17=0$, $10x+4y-21=0$. **179**. $\mathrm{tg}A=\frac{4}{3}$; $\mathrm{tg}B=\mathrm{tg}C=2$; $S=16$. **180.** $(1;-1)$, $\left(\frac{8}{3};-2\right)$. **181.** $BC\parallel DA$; $3x-5y+5=0$; $x-y=0$; $y-1=0$. **182.** $4x+3y-27=0$ (AB); $x=3(AC)$; $7x-3y-39=0$ (BC). **184.** $\frac{x}{2}+\frac{y}{3}=1$ va $\frac{x}{4}+\frac{y}{3/2}=-1$. **185.** 1) $-\frac{4}{5}x+\frac{3}{5}y-2=0$; 2) $\frac{3}{15}x+\frac{12}{15}y-3=0$; 3) $0{,}6x+0{,}8y-1{,}5=0$; 4) $\frac{x-2y+3}{-\sqrt{5}}=0$; 5) $\frac{y}{2}-\frac{\sqrt{3}}{2}x-2=0$; 6) $-x\cos10^0-y\sin10^0-4=0$ yoki $x\cos190^0+y\sin190^0-4=0$; **188**. 1) 3; 2) 4; 3) $5\frac{2}{3}$; 4) 0; 5) 2,2; **189**. 3. **190**. 1) 3,5; 2) $\frac{\sqrt{13}}{2}$. **191**. $k=\pm2$. **192.** $4x-3y-20=0$. **193.** 7,5. **195.** 1) $x-y=0$ va $x+y-\frac{4}{8}=0$; 2) $3x-y=12$ va $x+3y=4$; 3) $8x-2y-3=0$ va $4x+16y-39=0$. **196**. $x+y=2$ yoki $4x+y-8=0$. **197**. $M(0;1)$. **198**.

$x+2y-5=0$ va $x-6y+11=0$. **199.** $y-3=0$ va $12x-5y-117=0$. **200.** $y=\frac{8}{15}x-\frac{2}{3}$. **201.** $4x-12y+7=0$ va $6x+2y-5=0$. **202.** $x+2y-5=0$ va $x-6y+11=0$. **203.** $4x-3y+10=0$ yoki $4x+y-2=0$. **204.** $x+4y-4=0$. K o ' r s a t m a . $P(0;1)$ nuqtadan o'tuvchi to'g'ri chiziq tenglamasi $y=kx+1$ bo'ladi. Bu to'g'ri chiziq bilan berilgan chiziqlarni kesishgan nuqtalari $x_1=\frac{7}{3k-1}$ va $x_2=\frac{7}{2+k}$ larni topamiz. Ularning yarim yig'indisini 0 ga tenglab, k ni topamiz. **205.** $\frac{8\sqrt{3}}{13}$. **206.** $4x+3y+1\pm15=0$. **207.** $\frac{7}{16}x+\frac{1}{56}y=1$ yoki $\frac{x}{16}+\frac{y}{8}=1$. **211.** $x+4y+3=0$. **212.** $x-2y-5=0$. **213.** $x-y+3=0$. **214.** $25y-20x+6=0$. **215.** $\sqrt{10}$. **216.** $h_1=h_2=\frac{6}{\sqrt{5}}$. **217.** (0;2), (4;0), (2;4), (–2;6). **218** $y=x+6$, $\frac{12}{\sqrt{15}}$; $\angle DAB\approx53^0$. **219.** $2x+7y+22=0$; $7x+2y-13=0$; $x-y+2=0$. **220.** $2x+y-8=0$; $x-3y+10=0$; $x+4y-4=0$.

III BOB

IKKINCHI TARTIBLI EGRI CHIZIQLARNING ELEMENTAR XOSSALARI

1- §. Aylana

Tekislikda markaz deb ataluvchi nuqtadan baravar uzoqlikda yotgan nuqtalarning geometrik o'rni *aylana* deb ataladi.

Markazi $C(a;b)$ nuqtada va radiusi R bo'lgan aylana tenglamasi quyidagicha yoziladi:

$$(x-a)^2+(y-b)^2=R^2. \qquad (1)$$

Agar (1) tenglamadagi qavslarni ochsak, u holda tenglama

$$x^2+y^2+mx+ny+p=0 \qquad (2)$$

ko'rinishga keladi, bu yerda $m=-2a$, $n=-2b$, $p=a^2+b^2-R^2$.

(2) tenglamadan (1) tenglamaga qaytadan o'tish uchun (2) tenglamaning chap tomonida to'la kvadratni ajratish kerak, ya'ni

$$\left(x+\frac{m}{2}\right)^2+\left(y+\frac{n}{2}\right)^2=\frac{m^2}{4}+\frac{n^2}{n}-p. \qquad (3)$$

221. Markazi 1) $C(1;2)$ nuqtada va radiusi $R=3$; 2) $C(0;-3)$, $R=5$; 3) $C(-4;3)$, $R=5$ bo'lgan aylana tenglamasini yozing.

222. $A(-4;6)$ nuqta berilgan. Diametri OA kesmadan iborat aylana tenglamasini yozing.

223. Aylana diametrlaridan birining uchlari $M_1(2;-7)$ va $M_2(-4;-3)$ nuqtalarda yotishi ma'lum. Aylana tenglamasini yozing.

224. Diametri $12x+5y+60=0$ to'g'ri chiziqning koordinata o'qlari orasidagi kesmasidan iborat bo'lgan aylananing tenglamasini tuzing.

225. Quyidagi aylanalarning radiuslari va markazlarining koordinatalarini aniqlang:

a) $x^2+y^2+2x-6y+5=0$; *b*) $x^2+y^2+4x-5=0$;

d) $x^2-5+y^2+4y=0$; *e*) $4x^2-5x+4y^2-8=0$.

226. $x^2+y^2-5x-7y+6=0$ aylananing koordinata o'qlari bilan kesishgan nuqtalarini aniqlang.

227. Aylana Ox o'qqa koordinata boshida urinadi va Oy o'qini $(0;10)$ nuqtada kesib o'tadi. Aylana tenglamasini yozing.

228. Markazi Ox o‘qda va $A(6;\ 4\sqrt{2})$ hamda $B(0;\ -2\sqrt{5})$ nuqtalardan o‘tuvchi aylananing tenglamasini tuzing.

229. $A(3;\ -1)$ va $B(-4;\ -8)$ nuqtalardan o‘tuvchi aylananing radiusi $R=13$ ga teng. Aylana tenglamasini yozing.

230. $A(-3;0)$ va $B(3;6)$ nuqtalar berilgan. Diametri AB kesmadan iborat aylana tenglamasini yozing.

231. 1) $x^2+y^2-6x+4y-23=0$; 2) $x^2+y^2+5x-7y+2{,}5=0$;

3) $x^2+y^2+7y=0$ aylanalarning markazlari va radiuslarini toping. Aylanalarni yasang.

232. Koordinata o‘qlariga urinadigan aylana $M(-2;\ -4)$ nuqtadan o‘tadi. Uning tenglamasini yozing.

233. Koordinata o‘qlariga urinuvchi va radiusi $\sqrt{5}$ ga teng bo‘lib IV chorakda yotuvchi aylana tenglamasini yozing.

234. Koordinata o‘qlariga urinuvchi aylana markazi $2x-y+3=0$ to‘g‘ri chiziqda yotadi. Aylana tenglamasini yozing.

235. Berilgan $A(-1;3)$, $B(-2;-4)$ va $C(6;2)$ nuqtalardan o‘tuvchi aylana tenglamasini yozing.

236. Uchburchakning uchlari quyidagi koordinatalarga ega: $A(-2;9)$, $B(-4;9)$ va $C(5;8)$. Uchburchakka tashqi chizilgan aylana tenglamasini yozing.

237. $x^2+y^2=25$ aylananing $2x-y+1=0$ to‘g‘ri chiziqqa parallel bo‘lgan urinmalarini toping.

238. $(1;1)$ nuqtadan o‘tib $7x+y-3=0$ va $x+7y-3=0$ to‘g‘ri chiziqlarga urinuvchi aylana tenglamasini yozing.

239. $A(1;\ -2)$, $B(0;\ -1)$ va $C(-3;0)$ nuqtalardan o‘tuvchi aylanaga koordinatalar boshidan o‘tkazilgan urinmalar tenglamalarini yozing.

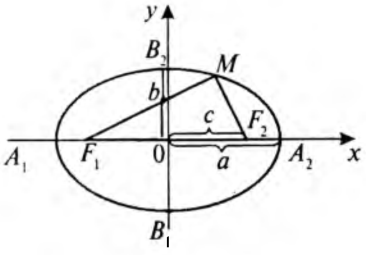
240. $x^2+y^2-4x+6y-5=0$ aylananing Ox o‘q bilan kesishgan nuqtalariga o‘tkazilgan radiuslari o‘rtasidagi burchaklarini toping.

2- §. Ellips

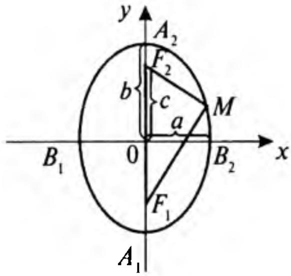
Ellips *deb har bir nuqtasidan berilgan* F_1 *va* F_2 *nuqtalargacha (fokuslargacha) masofalarning yig‘indisi* F_1F_2 *dan katta o‘zgarmasga teng nuqtalarning geometrik o‘rniga aytiladi.*

Ellipsning *kanonik* (eng sodda) tenglamasi:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1. \qquad (1)$$



1- chizma.



2- chizma.

(1) tenglama bilan berilgan ellips koordinata o'qlariga nisbatan simmetrikdir (1,2- chizmalar). a va b parametrlar ellipsning *yarim o'qlari* (a — katta yarim o'q, b — kichik yarim o'q) deyiladi. Agar $a>b$ bo'lsa, F_1 va F_2 fokuslar Ox o'qda bo'lib, markazdan $c=\sqrt{a^2-b^2}$ masofada bo'ladi. $\frac{c}{a}=\varepsilon<1$ nisbat ellipsning *ekssentrisiteti* deyiladi.

Ellipsning $M(x;y)$ nuqtasidan fokuslarigacha bo'lgan masofalar (fokal radius-vektorlar) $r=a-\varepsilon x$, $r_1=a+\varepsilon x$ formulalar bilan aniqlanadi.

Agar $a<b$ bo'lsa, fokuslar Oy o'qida bo'lib, $c=\sqrt{b^2-a^2}$, $\frac{c}{b}=\varepsilon$, $r=b\pm\varepsilon y$ bo'ladi.

$x=\pm\frac{a}{\varepsilon}$ — ellipsning *direktrisalari* tenglamalari deyiladi.

241. Quyidagi ellipslarning uchlari koordinatalarini, yarim o'qlarini, fokuslarini va ekssentrisitetini toping:

1) $16x^2+25y^2=400$; 2) $4x^2+9y^2=36$;

3) $16x^2+9y^2=144$; 4) $25x^2+9y^2=900$.

242. $\frac{x^2}{9}+\frac{y^2}{16}=1$ ellipsga nisbatan $A(-3;0)$, $B(0;-5)$, $C(2;3)$, $D\left(-\frac{3}{4};\sqrt{15}\right)$, $E(2;-2)$ va $P\left(3;\frac{4}{3}\right)$ nuqtalar qanday joylashgan?

243. Ellipsning katta yarim o'qi $a=4$ va ellipsda yotuvchi $M\left(-2;\frac{3\sqrt{3}}{2}\right)$ nuqta ma'lum. Ellipsning eng sodda tenglamasini tuzing va M nuqtadan ellips fokuslarigacha bo'lgan masofani toping.

244. Kichik yarim o'qi 24 ga teng bo'lgan va fokuslardan biri $(-5;0)$ koordinatalarga ega bo'lgan ellipsning eng sodda tenglamasini tuzing.

245. Yarim o'qlarining yig'indisi 36 ga, Oy o'qida yotuvchi fokuslari orasidagi masofa 48 ga teng bo'lgan ellipsning eng sodda tenglamasini tuzing.

246. Agar ellipsning fokuslaridan biri (6;0) nuqtada bo'lsa va ekssentrisiteti $\varepsilon = \frac{2}{3}$ ga teng bo'lsa, uning eng sodda tenglamasini tuzing.

247. Ellipsning fokuslari orasidagi masofa katta va kichik o'qlarining uchlari orasidagi masofaga teng. Ellipsning ekssentrisitetini toping.

248. $9x^2+25y^2=225$ ellipsda shunday $M(x;y)$ nuqta topingki, undan o'ng fokusgacha bo'lgan masofa chap fokusgacha bo'lgan masofadan 4 marta katta bo'lsin.

249. $x^2+y^2=36$ aylanadagi barcha nuqtalarning ordinatalari 3 barobar qisqartirishdan hosil bo'lgan yangi egri chlziq tenglamasini yozing.

250. $M(x;y)$ nuqta $x=-4$ to'g'ri chiziqqa nisbatan $F(-1;0)$ nuqtaga ikki barobar yaqinroqda harakat qiladi. Uning trayektoriyasini aniqlang.

251. $x^2+4y^2=16$ ellipsni yasang, uning fokuslari va ekssentrisitetini toping.

252. Quyidagi berilgan parametrlarga nisbatan ellipsning eng sodda tenglamasini yozing:

1) yarim o'qlari 4 va 2 ga teng;
2) fokuslari orasidagi masofa 6 va katta yarim o'qi 5 ga teng;
3) katta yarim o'qi 10 va ekssentrisiteti $\varepsilon = 0,8$ ga teng;
4) kichik yarim o'qi 3 va ekssentrisiteti $\varepsilon = \frac{\sqrt{2}}{2}$ ga teng;
5) yarim o'qlari yig'indisi 8 va fokuslar orasidagi masofa 8 ga teng.

253. $25x^2+169y^2=4225$ ellipsining o'qlari uzunliklari, fokuslarining koordinatalari va ekssentrisitetini toping.

254. Ellips fokuslarining biridan katta o'qining uchlarigacha bo'lgan masofalar 7 va 1 ga teng. Uning eng sodda tenglamasini tuzing.

255. Ellipsdagi ikki nuqtaning koordinatalari (1;4) va (−6;1). Bu ellipsning tenglamasini tuzing.

256. Katta o'qi kichik o'qidan uch marta katta bo'lgan ellipsning ekssentrisitetini toping.

257. Ellipsning tenglamasi berilgan: $9x^2+25y^2=225$. Uning abssissasi 3 bo'lgan nuqtasining radius-vektorlarini aniqlang.

258. Ellipsning tenglamasi berilgan: $7x^2+18y^2=126$. Uning abssissasi 3 va ordinatasi musbat bo‘lgan nuqtasining radius-vektorlari orasidagi burchakni toping.

259. $5x^2+9y^2=180$ ellipsda shunday nuqtani topingki, undan o‘ng fokusgacha bo‘lgan masofa chap fokusgacha bo‘lgan masofadan ikki marta kichik bo‘lsin.

260. $16x^2+25y^2=400$ ellips va markazi ellips kichik o‘qining yuqori uchida bo‘lib, uning fokusidan o‘tuvchi aylana berilgan. Ellips va aylananing kesishish nuqtalarini toping.

261. $8x^2+10y^2 = 160$ ellipsga to‘g‘ri to‘rtburchak shunday ichki chizilganki, uning ikkita qarama-qarshi tomoni ellipsning fokuslaridan o‘tadi. Bu to‘g‘ri to‘rtburchak yuzini toping.

3- §. Giperbola

Giperbola *deb shunday nuqtalarning geometrik o‘rniga aytiladiki, ularning har biridan berilgan* F_1 *va* F_2 *nuqtalargacha (fokuslargacha) bo‘lgan masofalar ayirmasining absolut qiymati o‘zgarmas* $2a$ $(0<2a<F_1F_2)$ *miqdordan iborat.*

Giperbolaning kanonik (eng sodda) tenglamasi quyidagi ko‘rinishga ega:

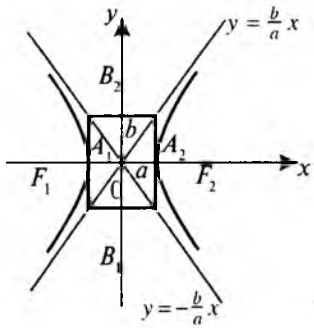
$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1. \tag{1}$$

(1) tenglama bilan berilgan giperbola koordinata o‘qlariga nisbatan simmetrikdir (3- chizma). Giperbola Ox o‘qini *uchlar* deb ataluvchi $A_1(a;0)$, $A_2(-a;0)$ nuqtalarda kesadi, Oy o‘q bilan esa kesishmaydi. a parametr *haqiqiy yarim o‘q*, b esa *mavhum yarim o‘q* deyiladi. $c=\sqrt{a^2+b^2}$ parametr markazdan fokusgacha bo‘lgan masofani bildiradi. $\frac{c}{a}=\varepsilon>1$ — giperbolaning *ekssentrisiteti* deyiladi. $y=\pm\frac{b}{a}x$ to‘g‘ri chiziqlar giperbolaning *asimptotalari* deyiladi. $M(x;y)$ nuqtadan fokuslargacha bo‘lgan masofalar (fokal radius- vektorlar)

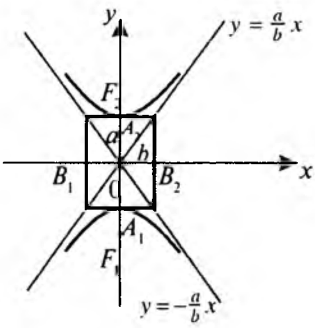
$$r=|\varepsilon x-a|,\quad r=|\varepsilon x+a| \tag{2}$$

formulalar bilan aniqlanadi.

$x=\pm\frac{a}{\varepsilon}$ chiziqlar giperbolaning *direktrisalari* deyiladi.



3-chizma.



4-chizma.

Agar $a=b$ bo'lsa, giperbola *teng tomonli* giperbola deb ataladi. Uning tenglamasi $x^2-y^2=a^2$, asimptotalarining tenglamalari esa $y = \pm x$ bo'ladi.

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ va $\frac{y^2}{b^2}-\frac{x^2}{a^2}=1$ giperbolalar *qo'shma* giperbolalar deyiladi. Bu holda o'qlarning roli almashadi (4- chizma).

262. $16x^2-9y^2-144=0$ giperbolaning o'qlari, uchlari, ekssentrisiteti va asimptotalarining tenglamalarini tuzing.

263. Quyidagi giperbolalarning uchlari koordinatalari, o'qlari, fokuslari va ekssentrisitetini toping:

1) $4x^2-5y^2-100=0$; 2) $9x^2-4y^2-144=0$;
3) $16x^2-9y^2+144=0$; 4) $9x^2-7y^2-252=0$.

264. Haqiqiy o'qi abssissalar o'qida yotadigan va $M_1(3;-2)$, $M_2(-6;\ 2\sqrt{10})$ nuqtalardan o'tadigan giperbolaning ekssentrisiteti va fokuslarining koordinatalarini toping.

265. Haqiqiy o'qi 6 ga, fokuslar orasidagi masofa 8 ga teng bo'lgan giperbolanmg eng sodda tenglamasini tuzing. Qo'shma giperbolaning tenglamasini tuzing.

266. Giperbolaning yarim o'qlari yig'indisi 17 ga, ekssentrisiteti $\varepsilon=\frac{13}{12}$ ga teng. Giperbolaning eng sodda tenglamasini tuzing va fokuslarining koordinatalarini toping.

267. $7x^2-5y^2=35$ giperbola fokuslaridan o'tuvchi va Ox o'q bilan 60^0 li burchak tashkil etuvchi to'g'ri chiziqlarning tenglamalarini tuzing.

268. $M(-5;2)$ nuqta orqali $9x^2-4y^2=36$ giperbola asimptotalariga parallel bo'lgan to'g'ri chiziqlar o'tkazing.

269. Giperbolaning ekssentrisiteti $\sqrt{3}$ ga teng, fokuslari (6;0) va (-6;0) nuqtalarda joylashgan. Giperbola va uning asimptotalari tenglamalarini yozing.

270. $x^2-3y^2=27$ giperbola asimptotalari orasidagi o'tkir burchak va ekssentrisitetini toping.

271. $M(6;\ \frac{3}{2}\sqrt{5})$ nuqtadan o'tuvchi, koordinata o'qlariga nisbatan simmetrik bo'lgan giperbolaning haqiqiy yarim o'qi $a=4$ ga teng. Giperbolaning chap fokusidan asimptotalariga tushirilgan perpendikularning tenglamalarini yozing.

272. M nuqta $9x^2-16y^2=144$ giperbolaning fokuslari orasidagi masofani $F_2M : MF_1=2 : 3$ nisbatda bo'ladi. F_2- giperbolaning chap fokusi va M nuqtadan Ox o'qi bilan 135° li burchak tashkil etuvchi to'g'ri chiziq o'tkazilgan. Shu to'g'ri chiziqning giperbola asimptotalari bilan kesishgan nuqtalarini toping.

273. $x=-2$ to'g'ri chiziqqa nisbatan $F(-8;0)$ nuqtadan ikki barobar uzoqlikda harakat qiluvchi M nuqtaning trayektoriyasini aniqlang.

274. Quyidagi berilgan parametrlarga ko'ra giperbolaning sodda tenglamasini yozing:

1) uchlarining orasidagi masofa 8 ga , fokuslar orasidagi masofa 10 ga teng;

2) haqiqiy yarim o'q 5 ga teng va uchlari markaz bilan fokus oralig'ini teng ikkiga bo'ladi;

3) haqiqiy o'q 6 ga teng va giperbola (9; –4) nuqtadan o'tadi;

4) giperbola $P(-5;2)$ va $Q(2\sqrt{5};\ \sqrt{2})$ nuqtalardan o'tadi.

275. Fokuslari $F_1(10;\ 0)$, $F_2(-10;0)$ bo'lgan va $M(12;3\sqrt{5})$ nuqtadan o'tuvchi giperbolaning tenglamasini tuzing.

276. Fokusi $\frac{x^2}{49}+\frac{y^2}{24}=1$ ellipsning fokusi bilan umumiy bo'lgan va ekssentrisiteti $\varepsilon=1,25$ ga teng giperbolaning tenglamasini tuzing.

277. Uchlari $\frac{x^2}{169}+\frac{y^2}{144}=1$ ellipsning fokuslarida, fokuslari esa uning uchlarida bo'lgan giperbolaning tenglamasini tuzing.

278. $\frac{x^2}{49}-\frac{y^2}{25}=1$ giperbolaning fokuslarini va asimptotalarini yasang.

279. $\frac{x^2}{9}-\frac{y^2}{16}=1$ giperbola berilgan. Talab qilinadi:

1) fokuslarining koordinatalarini hisoblash;

2) ekssentrisitetini hisoblash;

3) asimptotalari va direktrisalarining tenglamalarini yozish;

4) qo‘shma giperbola tenglamalarini yozish va uning ekssentrisitetini hisoblash.

280. Mavhum o‘qi $2\sqrt{2}$ ga teng bo‘lgan giperbolada direktrisalarining tenglamalari: $x\pm 2=0$. Giperbolaning tenglamasini tuzing.

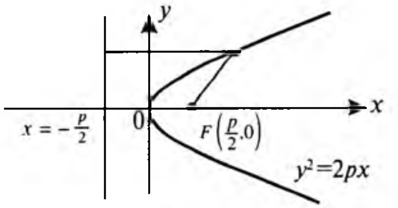
4- §. Parabola

Berilgan nuqta (fokus)dan va berilgan to‘g‘ri chiziq (direktrisa) dan bir xil uzoqlikda bo‘lgan nuqtalarning geometrik o‘rniga ***parabola*** *deyiladi.*

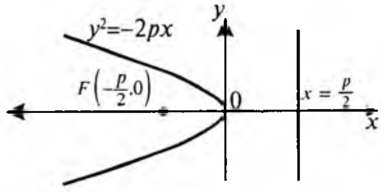
Parabolaning kanonik tenglamasi quyidagi ikki ko‘rinishga ega:

1) $y^2=2px$ – Ox o‘qqa nisbatan simmetrik parabola (5,6-chizmalar);

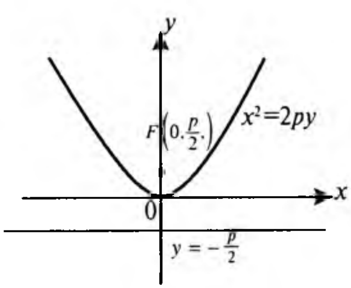
2) $x^2=2py$ – Oy o‘qqa nisbatan simmetrik parabola (7,8-chizmalar). Har ikki holda ham parabolaning uchi, ya’ni simmetriya o‘qidan o‘tuvchi nuqtasi, koordinatalar boshida bo‘ladi.



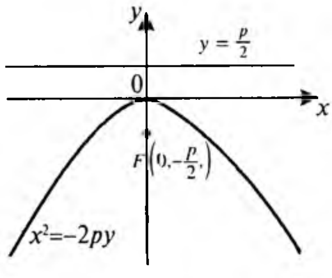
5-chizma.



6-chizma.



7-chizma.



8-chizma.

$x^2=2py$ parabola $F(0;\frac{p}{2})$ fokus va $y=-\frac{p}{2}$ direktrisaga ega; uning $M(x;y)$ nuqtasining fokal radius-vektori $r=y+\frac{p}{2}$.

$y^2=2px$ parabola $F(\frac{p}{2};0)$ fokus va $x=-\frac{p}{2}$ direktrisaga ega; uning $M(x;y)$ nuqtasining fokal radius-vektori: $r=x+\frac{p}{2}$.

281. Quyidagi parabolalarning fokusi koordinatalarini toping va direktrisasi tenglamasini yozing:

1) $y^2=8x$; 2) $y^2=-12x$; 3) $x^2=10y$; 4) $x^2=-16y$.

282. Quyidagilarga asoslanib, parabolaning tenglamasini tuzing:

1) uchidan fokusigacha bo‘lgan masofa 3 ga teng;

2) fokusining koordinatasi — (5;0), direktrisasi — ordinatalar o‘qi;

3) $M(1;-4)$ nuqtadan o‘tuvchi Ox o‘qiga simmetrik bo‘lgan parabola;

4) fokusi (0;2) da, Oy o‘qiga simmetrik va uchi koordinata boshida bo‘lgan parabola;

5) koordinatalar boshidan va $M(6;-2)$ nuqtadan otuvchi, Oy o‘qiga simmetrik bo‘lgan parabola.

283. $y^2=8x$ parabolada fokal radius-vektori 20 ga teng bo‘lgan nuqtani toping.

284. $y^2=4{,}5x$ parabolada direktrisasidan $d=9{,}125$ masofada bo‘lgan $M(x;y)$ nuqta berilgan. Shu nuqtadan parabola uchigacha bo‘lgan masofani toping.

285. $y^2=48x$ parabolaning fokusi orqali o‘tuvchi va $y=\sqrt{3}x+1$ to‘g‘ri chiziqqa parallel qilib to‘g‘ri chiziq o‘tkazilgan. Hosil bo‘lgan vatarning uzunligini toping.

286. $y^2=6x$ parabolaning: 1) $3x+y-6=0$; 2) $2x-y+5=0$; 3) $y-6=0$ to‘g‘ri chiziqlar bilan kesishish nuqtasini toping.

287. $y^2=6x$ parabolaning $\frac{x^2}{100}+\frac{y^2}{64}=1$ ellips bilan kesishish nuqtalarini toping.

288. $y^2=36x$ parabola hamda $(x+12)^2+y^2=400$ aylananing umumiy vatari tenglamasini tuzing va uzunligini aniqlang.

289. Uchlari koordinatalar boshida, fokuslari $F_1(-6;0)$ va $F_2(0;-6)$ nuqtalarda bo‘lgan ikkita parabolaning umumiy vatari uzunligini toping.

290. Uchi (7;2) nuqtada, fokusi (7;5) nuqtada bo‘lgan parabola tenglamasini tuzing.

291. Ox o‘qqa nisbatan simmetrik bo‘lgan parabolaning uchi (3;0) nuqtada. U ordinatalar o‘qidan uzunligi 24 ga teng bo‘lgan vatar ajratiladi. Parabola tenglamasini tuzing, uning fokusi va direktrisasini toping.

292. Fokusi $(\frac{9}{2};-1)$ nuqtada bo‘lib, direktrisasi $2x-3=0$ to‘g‘ri chiziqdan iborat bo‘lgan parabola tenglamasini tuzing.

293. Parabola uchi (–3;4) nuqtada, direktrisasi $2y-9=0$ to‘g‘ri chiziqdan iborat bo‘lgan parabola tenglamasini tuzing.

294. Ko‘prik arki tenglamasi $y^2=96x$ bo‘lgan parabola ko‘rinishga ega. Agar balandligi 6 m ga teng bo‘lsa, ark vatarining uzunligini toping.

295. Parabolik ko‘zguning diametri 125 sm ga, botiqligi 15 sm ga teng. Yorug‘lik manbayini parabola uchidan qanday masofada joylashtirganda qaytgan nur parabola o‘qiga parallel bo‘ladi?

296. Fontandan otilib chiqayotgan suv oqimining parametri $p=2$ ga teng bo‘lgan parabola shakliga ega. Agar suv ko‘pi bilan 4m ga ko‘tarila olishi ma’lum bo‘lsa, suv chiqayotgan joyidan qancha nariga tushishini aniqlang.

297. O‘tkir burchak ostida gorizontal otilgan jism parabola yoyi chizib, boshlang‘ich holatdan 32 m masofaga borib tushdi. Agar jism ko‘tarilgan eng yuqori balandlik 12 m bo‘lsa, parabolik trayektoriyaning parametrini aniqlang.

* * *

298. $F(0;2)$ nuqtadan va $y=4$ to‘g‘ri chiziqdan bir xil uzoqlashgan nuqtalar geometrik o‘rnining tenglamasini tuzing. Bu egri chiziqning koordinata o‘qlari bilan kesishgan nuqtalarini toping va uni yasang.

299. 1) $y^2=4x$; 2) $y^2=-4x$; 3) $x^2=4y$; 4) $x^2=-4y$ tenglamalar bilan berilgan parabolalar hamda ularning fokuslari, direktrisalarini yasang va direktrisalarning tenglamalarini tuzing.

300. 1) (0;0) va (1;–3) nuqtalardan o‘tuvchi Ox o‘qqa nisbatan simmetrik; 2) (0;0) va (2;–4) nuqtalardan o‘tuvchi Oy o‘qqa nisbatan simmetrik bo‘lgan parabolaning tenglamasini tuzing.

301. Uchi koordinatalar boshida va fokusining koordinatalari: 1) $F(0;4)$; 2) $F(0;-3)$; 3) $F(6;0)$; 4) $F(-2,5;0)$ bo‘lgan parabolaning tenglamasini tuzing.

302. Parabola Ox o‘qqa nisbatan simmetrik, uning uchi koordinatalar boshida, fokusidan uchigacha bo‘lgan masofa 12 ga teng. Parabolaning tenglamasini tuzing.

303. Parabolaning tenglamasi $y^2=24x$ va undagi nuqtaning radius-vektori 14 ga teng. Bu nuqtaning parabola boshidan uzoqligini toping.

304. Parabolaning tenglamasi $y^2=6x$. Uning shunday vatarini topingki, u (4;3) nuqtada teng ikkiga bo‘linsin.

305. $y=x$ to‘ g‘ri chiziq bilan $x^2+y^2+6x=0$ aylananing kesishgan nuqtalaridan o‘tuvchi va Ox o‘qqa nisbatan simmetrik bo‘lgan parabolaning va direktrisasining tenglamalarini yozing. To‘g‘ri chiziq, parabola, aylanani yasang.

5- §. Ikkinchi tartibli egri chiziqlarning direktrisalari, diametrlari va ularga o‘tkazilgan urinmalar

1°. Oy o‘qqa parallel va undan $\frac{a}{\varepsilon}$ masofada joylashgan to‘g‘ri chiziqlar $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $(a>b)$ ellipsning va $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ giperbolaning direktrisalari deyiladi, bunda ε— egri chiziqning ekssentrisiteti.

Direktrisalarning tenglamalari:

$$x=\pm\frac{a}{\varepsilon}. \tag{1}$$

Direktrisalarning xossasi: egri chiziq ixtiyoriy nuqtasining fokusgacha va mos direktrisagacha masofalarining nisbati ekssentrisitetga teng:

$$\frac{r}{d}=\varepsilon. \tag{2}$$

2°. Ikkinchi tartibli egri chiziqning *diametri* deb, parallel vatarlar o‘rtalarining geometrik o‘rniga aytiladi. Ellips bilan giperbolaning diametrlari ularning markazlaridan o‘tuvchi to‘g‘ri chiziqlar kesmalaridan va nurlaridan iborat bo‘lsa, parabolaning diametri esa uning o‘qiga parallel nurlardan iboratdir.

$\frac{x^2}{a^2}\pm\frac{y^2}{b^2}=1$ egri chiziqlar uchun og‘maliklari $k=\operatorname{tg}\alpha$ bo‘lgan vatarlarni teng bo‘luvchi diametrning tenglamasi

$$y=\pm\frac{b^2}{a^2k}x \tag{3}$$

bo‘lsa, $y^2=2px$ parabola uchun

$$y=\frac{p}{k} \tag{4}$$

bo‘ladi.

Ellips va giperbolaning bir diametri ikkinchi diametrga parallel bo‘lgan vatarlarni teng ikkiga bo‘lsa, bunday ikki diametr *o‘zaro qo‘shma* deyiladi. Qo‘shma diametrlarning k va k_1 burchak koeffitsiyentlari o‘zaro

$$kk_1 = -\frac{b^2}{a^2} \quad \text{(ellips uchun)},$$

$$kk_1 = \frac{b^2}{a^2} \quad \text{(giperbola uchun)}$$

tenglik bilan bog‘langandir.

3°. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ellipsga o‘tkazilgan urinmaning tenglamasi:

$$\frac{xx_0}{a^2}+\frac{yy_0}{b^2}=1;$$

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ giperbolaga o‘tkazilgan urinmaning tenglamasi:

$$\frac{xx_0}{a^2}-\frac{yy_0}{b^2}=1;$$

$y^2=2px$ parabolaga o‘tkazilgan urinmaning tenglamasi: $yy_0=p(x+x_0)$ dan iboratdir, bu yerda $(x_0;\ y_0)$ urinish nuqtasi.

306. $\frac{x^2}{25}+\frac{y^2}{9}=1$ ellips va uning direktrisalarini yasang. Ellipsning $x=-3$ abssissasidan o‘ng fokusgacha va o‘ng direktrisasigacha bo‘lgan masofalarni toping.

307. $\frac{x^2}{16}-\frac{y^2}{9}=1$ giperbola va uning direktrisalarini yasang, giperbolaning $x=5$ abssissasidan chap direktrisasigacha bo‘lgan masofalarni toping.

308. Katta yarim o‘qi 2 ga teng, direktrisalari esa $x=\pm\frac{4}{\sqrt{3}}$ to‘g‘ri chiziqlardan iborat ellipsning kanonik tenglamasini yozing.

309. Asimptotalari $y=\pm x$, direktrisalari esa $x=\pm\sqrt{6}$ bo‘lgan giperbolaning tenglamasini yozing.

310. $x^2+4y^2=16$ ellips, uning $y=\frac{x}{2}$ diametri va unga qo‘shma diametrini yasang. Yasalgan yarim diametrlarning a_1 va b_1 uzunliklarini toping.

311. Ellipsning tenglamasi $5x^2+9y^2=45$. Uning abssissasi 2 va ordinatasi musbat bo‘lgan nuqtasidan o‘tgan urinmaning tenglamasini tuzing.

312. Ellipsning tenglamasi $3x^2+4y^2=13$. Uning shunday nuqtasidan urinma o'tkazingki, u urinma $y=3x+4$ to'g'ri chiziqqa parallel bo'lsin.

313. Ellipsning tenglamasi $4x^2 + 7y^2 =56$. Uning shunday nuqtasidan urinma o'tkazingki, u $x-2y-5=0$ to'g'ri chiziqqa perpendikular bo'lsin.

314. Ellipsning tenglamasi $4x^2 + 25y^2 =100$. Uning $(4;\frac{6}{5})$ nuqtasida urinma va normal o'tkazilgan. Ularning tenglamasini tuzing.

315. Giperbolaning tenglamasi $x^2 -4y^2=1$. Unga qo'shma bo'lgan giperbolaning tenglamasini tuzing va uning ekssentrisitetini toping.

316. Giperbolaning tenglamasi $9x^2 -16y^2=144$. Uning abssissasi 8 bo'lgan nuqtasining radius-vektorlarini aniqlang.

317. Giperbolaning tenglamasi $16x^2 -25y^2 =400$. Uning asimptotalari va direktrisalarining tenglamalarini tuzing.

318. Giperbolaning tenglamasi $4x^2 -y^2 =15$. Uning shunday nuqtasidan urinma o'tkazingki, u $8x-y-3= 0$ to'g'ri chiziqqa parallel bo'lsin.

319. Giperbolaning tenglamasi $2x^2 -3y^2= 5$. Unga (1;3) nuqtadan o'tkazilgan urinmaning tenglamasini tuzing.

320. Giperbolaning tenglamasi $x^2 -y^2= 4$. Uning shunday nuqtasidan urinma o'tkazingki, u $2x+5y+1= 0$ to'g'ri chiziqqa perpendikular bo'lsin.

321. Giperbolaning asimptotalari $y = \pm\frac{2}{3}x$. Tenglamasi $x-4=0$ bo'lgan tog'ri chiziq unga urinma bo'lmoqda. Giperbolaning tenglamasini tuzing.

322. Giperbolaning tenglamasi $8x^2 -5y^2 - 40=0$, uning diametrlaridan birining tenglamasi $y=2x$. Bunga qo'shma bo'lgan diametrning tenglamasini tuzing.

323. Giperbolaning tenglamasi $4x^2 - 6y^2=24$. Uning ikki qo'shma diametrlari orasidagi burchak 45'. Ikkala diametrning tenglamasini tuzing.

324. $4x^2 - 9y^2=36$ giperbolaning $x+2y=0$ to'g'ri chiziqqa perpendikular bo'lgan urinmalarining tenglamalarini yozing.

325. Parabolaning tenglamasi $y^2=2x$. Uning (2; –2) nuqtasidan o'tkazilgan urinma va normalning tenglamalarini tuzing.

326. Parabolaning tenglamasi $y^2=4x$. Uning abssissasi 9 va ordinatasi musbat bo'lgan nuqtasidan urinma va normal o'tkazilgan. Ularning tenglamalarini tuzing.

327. Parabolaning tenglamasi $y^2 = 8x$. Uning shunday nuqtasidan urinma o‘tkazingki, u $2x+2y+5=0$ to‘g‘ri chiziqqa perpendikular bo‘lsin. Uning tenglamasini tuzing.

328. Parabolaning tenglamasi $y^2 = 6x$. Uning tekisligida (4;3) nuqta berilgan. Parabolaning bu nuqtadan o‘tgan diametrini toping.

329. Parabolaning tenglamasi $y^2=6x$. Uning shunday vatarini topingki, u (4;3) nuqtada teng ikkiga bo‘linsin.

* * *

330. $\frac{x^2}{16} - \frac{y^2}{9} = 1$ giperbola asimptotalarining direktrisalari bilan kesishgan nuqtalarini toping.

331. $x^2+4y^2=16$ ellips, uning $y=x$ diametri hamda unga qo‘shma diametrini yasang. Shu diametrlar orasidagi burchakni toping.

332. $\frac{x^2}{9} + \frac{y^2}{7} = 1$ ellipsning koordinata burchaklarining bissektrisalari bilan kesishish nuqtalarini toping.

333. $\frac{x^2}{16} + \frac{y^2}{4} = 1$ ellips va $x^2 + y^2-8y = 0$ aylananing umumiy vatari tenglamasini tuzing.

334. Ekssentrisiteti $\varepsilon =1{,}2$ ga teng degan shartda $\frac{x^2}{64} + \frac{y^2}{28} = 1$ ellips bilan umumiy fokuslarga ega bo‘lgan giperbolaning tenglamasini tuzing.

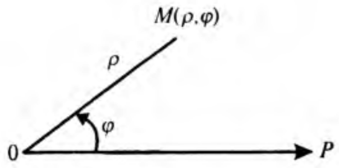
335. Ko‘prik arki tenglamasi $y^2 = 96x$ bo‘lgan parabola ko‘rinishiga ega. Agar ark balandligi 6 m ga teng bo‘lsa, ark vatarining uzunligini toping.

336. Parabolik ko‘zguning diametri 120 sm ga, botiqligi 15 sm ga teng. Yorug‘lik manbayini parabola uchidan qanday masofada joylashtirganda qaytgan nur parabola o‘qiga parallel bo‘ladi?

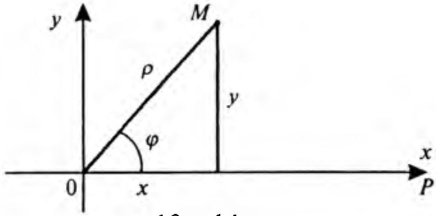
6- §. Qutb koordinatalari

Tekislikda O nuqta — *qutb* va OP nur — *qutb o‘qi* berilgan bo‘lsin (9- chizma). U holda M nuqtaning tekislikdagi o‘rni:

1) $\varphi=\angle MOP$ qutb burchak; 2) $r = OM$ radius-vektor bilan aniqlanadi, φ bilan r orasidagi bog‘lanishni ifodalovchi tenglamani o‘rganganda, qutb koordinatalar φ va r lar qanday musbat va manfiy qiymatlar qabul qiladi, deb qarash foydalidir. φ burchak soat strelkasi yurishi bo‘yicha hisoblansa, manfiy, r bo‘lsa, nurning o‘zi bo‘yicha emas, uning qutbning ikkinchi tomonidagi davomida joylashtiriladi.



9- chizma.



10- chizma.

Agar qutbni Dekart koordinatalari sistemasining boshi, *OP* qutb o‘qini esa *Ox* o‘q deb qabul etsak, ixtiyoriy *M* nuqtaning Dekart sistemasidagi $(x;y)$ koordinatalari bilan uning $(\varphi;\ r)$ qutb koordinatalari orasidagi bog‘lanish

$$x = r\cos\varphi,\ \ y = r\sin\varphi, \tag{1}$$

$$r = \sqrt{x^2 + y^2};\ \ \mathrm{tg}\varphi = \frac{y}{x} \tag{2}$$

tenglamalar bilan ifodalanadi (10- chizma).

Agar ellips, giperbola va parabola fokusini qutb deb olib, qutb o‘qi uchun esa qutbga eng yaqin uchiga qaratilgan yonalishga teskari yonaltirilgan fokal simmetriya o‘qini olsak, bu egri chiziqlarning qutb koordinatalaridagi tenglamalari bir xil

$$r = \frac{p}{1-\varepsilon\cos\varphi} \tag{3}$$

ko‘rinishda bo‘ladi, bunda ε— ekssentrisitet, p — parametr. Ellips va giperbola uchun: $p = \frac{b^2}{a}$.

337. Qutb koordinatalarida quyidagi nuqtalarni yasang:

$$A\left(3;\frac{\pi}{6}\right);\ B\left(1;\frac{5\pi}{3}\right);\ C\left(5;\frac{7\pi}{6}\right);\ D\left(0,5;\frac{\pi}{2}\right);\ E\left(2,5;\frac{2\pi}{3}\right);$$

$$F(6;\pi);\ M\left(3;\frac{\pi}{3}\right);\ N\left(\sqrt{3};-\frac{\pi}{6}\right);\ K\left(-2;\ \frac{\pi}{4}\right).$$

338. Dekart koordinatalar sistemasida $M_1(0;2)$, $M_2(-1;0)$, $M_3(-\sqrt{3};1)$, $M_4(-1;-1)$, $M_5(1;\sqrt{3})$ nuqtalar berilgan. Ularning qutb koordinatalarini toping.

339. Qutb koordinatalari ushbu: a) $p=1$; b) $p=5$; d) $p=a$; e) $\varphi=\frac{\pi}{6}$; f) $\varphi=\frac{\pi}{3}$; g) $\varphi=\frac{\pi}{2}$; h) $\varphi=\mathrm{const}$ tenglamalardan birini qanoatlantiradigan nuqtalar qanday joylashgan?

340. Ushbu nuqtalarga $\left(1;\frac{\pi}{4}\right)$; $\left(3;\frac{2\pi}{3}\right)$; $\left(\frac{2}{3};-\frac{\pi}{6}\right)$; $M(\rho;\varphi)$: a) qutbga nisbatan; b) qutb o'qiga nisbatan simmetrik bo'lgan nuqtalarning qutb koordinatalarini toping.

341. Qutb burchaklari 0°, 15°, 30°, 45°, 60°, 75°, 90° ga teng va ularga mos radius-vektorlari $\rho=a\sin2\varphi$ tenglama bilan hisoblanadigan nuqtalarni yasang. Olingan nuqtalarni silliq egri chiziq bilan birlashtiring.

342. Quyidagi chiziqlarni yasang:

1) $r=2+2\cos\varphi$; 2) $r=a\varphi$ (Arximed spirali);
3) $r=a(1-\cos\varphi)$ (kardioida); 4) $r^2=a^2\cos\varphi$ (lemniskata);
5) $r=\frac{a}{\varphi}$ (giperbolik spiral); 6) $r=a(1+2\cos\varphi)$ (Paskal chig'anog'i);

343. 1) $A\left(2;\frac{\pi}{26}\right)$ va $B\left(1;\frac{5\pi}{12}\right)$; 2) $C\left(4;\frac{\pi}{5}\right)$ va $F\left(4;\ \frac{\pi}{4}\right)$; 3) $D\left(6;\frac{6\pi}{5}\right)$ va $E\left(3;\frac{11\pi}{18}\right)$ nuqtalar orasidagi masofani toping.

344. Uchburchak uchlari berilgan: $A\left(5;\ \frac{\pi}{2}\right)$, $B\left(8;\ \frac{5\pi}{4}\right)$, $C\left(3;\ \frac{7\pi}{6}\right)$. Bu uchburchak teng tomonli ekanligini ko'rsating.

345. Qutb o'qida $A\left(4\sqrt{2};\ \frac{\pi}{4}\right)$ nuqtadan 5 birlik uzoqlikda yotgan nuqtani toping.

346. Bitta uchi qutbda, qolganlari $\left(4;\frac{\pi}{9}\right)$ va $\left(1;\frac{5\pi}{18}\right)$ nuqtalarda bo'lgan uchburchakning yuzini toping.

347. Uchlari $A\left(9;\frac{\pi}{10}\right)$, $B\left(12;\frac{4\pi}{15}\right)$, $C\left(10;\frac{3\pi}{5}\right)$ nuqtalarda bo'lgan uchburchakning yuzini toping.

348. 1) Qutb o'qiga perpendikular bo'lib, undan a kesma ajratuvchi to'g'ri chiziq; 2) $A(\alpha;a)$ nuqtadan o'tuvchi qutb o'qiga parallel bo'lgan to'g'ri chiziqning qutb koordinatalardagi tenglamalarini yozing.

349. Markazi $C(0;a)$ nuqtada va radiusi a ga teng aylananing qutb koordinatalardagi tenglamasini yozing.

350. Ushbu 1) $x^2-y^2=a$; 2) $x^2+y^2=a^2$; 3) $x\cos\varphi+y\sin\varphi-p=0$; 4) $y=x$; 5) $x^2+y^2=ax$; 6) $(x^2+y^2)^2=a^2(x^2-y^2)$

chiziqlarning tenglamalarini qutb koordinatalardagi tenglamalari bilan almashtiring.

K o ' r s a t m a . $x=p\cos\varphi$, $y=p\sin\varphi$ larni berilgan tenglamalarga qo'yib soddalashtiring.

III bob javoblari

221. 1) $(x-1)^2+(y-2)^2=9$; 2) $x^2+(y+3)^2-25=0$; 3) $x^2+y^2+8x-6y=0$. **222.** $x^2+y^2+4x-6y=0$. **223.** $(x+1)^2+(y+2)^2=34$. **224.** $(x+\frac{5}{2})^2+(y+6)^2=\frac{169}{4}$. **225.** a) $(-1;3)$, $R=\sqrt{5}$; b) $(-2;0)$, $R=3$; d) $(0;-2)$ $R=3$; e) $\left(\frac{5}{8};0\right)$, $R=\frac{1}{8}\sqrt{153}$. **226.** $(2;0)$, $(3;0)$, $(0;6)$, $(0;1)$. **227.** $x^2+(y+5)^2=25$. **228.** $(x-4^2+y^2=36$. **229.** $(x+9)^2+(y-4)^2=169$ va $(x-8)^2+(y+13)^2=169$. **230.** $x^2-y^2-6y-9=0$. **231.** 1) $(3;-2)$, $R=6$; 2) $\left(-\frac{5}{2};\frac{7}{2}\right)$, $R=4$; 3) $\left(0;-\frac{7}{2};\right)$, $R=\frac{7}{2}$. **232.** $(x+10)^2+(y+10)^2=100$ va $(x-8)^2+(y+13)^2=169$. **233.** $(x-\sqrt{5})^2+(y+\sqrt{5})^2=5$. **234.** $(x+1)^2+(y-1)^2=1$ va $(x+3)^2+(y-3)^2=9$. **235.** $(x-2)^2+(y+1)^2=25$. **236.** $(x-1)^2+(y-5)^2=25$. **237.** $2x-y\pm5=0$. **238.** $(x-\frac{7}{2})^2+(y-\frac{7}{2})^2=\frac{25}{2}$ va $(x-\frac{13}{18})^2+(y-\frac{13}{18})^2=\frac{25}{162}$. **239.** $y=0$, $15x+8y=0$. **240.** 90^0. **241.** 1) $2a=10$, $2b=8$; $A_1(-5;0)$, $A_2(5;0)$, $B_1(0;-4)$, $B_2(0;4)$, $F_1(-3;0)$, $F_2(3;0)$, $\varepsilon=\frac{3}{5}$. 2) $2a=6$, $2b=4$; $A_1(-3;0)$, $A_2(3;0)$, $B_1(0;-2)$, $B_2(0;2)$, $F_1(-\sqrt{5};0)$, $F_2(\sqrt{5};0)$, $\varepsilon=\frac{\sqrt{5}}{3}$. 3) $2a=6$, $2b=8$; $A_1(-3;0)$, $A_2(3;0)$, $B_1(0;-4)$, $B_2(0;4)$, $F_1(0;-\sqrt{7})$, $F_2(0;\sqrt{7})$, $\varepsilon=\frac{\sqrt{7}}{4}$. 4) $2a=12$, $2b=20$, $A_1(-6;0)$, $A_2(6;0)$, $B_1(0;10)$, $B_2(0;10)$, $F_1(0;-8)$, $F_2(0;8)$, $\varepsilon=\frac{4}{5}$. **242.** A va D nuqtalar ellipsda, B, C va F nuqtalar ellipsdan tashqarida, E ellips ichida. **243.** $\frac{x^2}{16}+\frac{y^2}{9}=1$; $4\pm\frac{\sqrt{7}}{2}$. **244.** $\frac{x^2}{169}+\frac{y^2}{144}=1$. **245.** $\frac{x^2}{100}+\frac{y^2}{676}=1$. **246.** $\frac{x^2}{81}+\frac{y^2}{45}=1$. **247.** $\varepsilon=\frac{\sqrt{10}}{4}$. **248.** $\left(-\frac{15}{4};\ \pm\frac{\sqrt{63}}{4}\right)$. **249.** $\frac{x^2}{36}+\frac{y^2}{4}=1$. **250.** $\frac{x^2}{4}+\frac{y^2}{3}=1$. **251.** $a=4$, $b=2$, $c=2\sqrt{3}$, $\varepsilon=\frac{\sqrt{3}}{2}$. **252.** 1) $\frac{x^2}{16}+\frac{y^2}{4}=1$; 2) $\frac{x^2}{25}+\frac{y^2}{16}=1$; 3) $\frac{x^2}{100}+\frac{y^2}{36}=1$; 4) $\frac{x^2}{18}+\frac{y^2}{9}=1$; 5) $\frac{x^2}{25}+\frac{y^2}{9}=1$. **253.** $2a=26$, $2b=10$; $\varepsilon=\frac{12}{13}$; $F_1(12;0)$, $F_2(-12;0)$.

254. $\frac{x^2}{16}+\frac{y^2}{7}=1$. **255.** $3x^2+7y^2=115$. **256.** $\frac{3\sqrt{2}}{4}$. **257.** $2\frac{3}{5};7\frac{2}{5}$. **258.** $\approx 83°$. **259.** $(3;\sqrt{5})$, $(3;-\sqrt{5})$. **260.** $\left(\frac{5\sqrt{65}}{9};\frac{16}{9}\right)$ va $\left(-\frac{5\sqrt{65}}{9};\frac{16}{9}\right)$. **261.** $\frac{64\sqrt{5}}{5}$ kv bir. **262.** $2a=6$, $2b=8$; $A_1(-3;0)$, $A_2(3;0)$, $F_1(-5;0)$, $F_2(5;0)$; $\varepsilon=\frac{5}{2}$; $y=\pm\frac{4}{3}x$. **263.** 1) $2a=10$, $2b=4\sqrt{5}$; $A_1(-5;0)$, $A_2(5;0)$, $F_1(-3\sqrt{5};0)$, $F_2(3\sqrt{5};0)$; $\varepsilon=\frac{3\sqrt{5}}{5}$; $y=\pm\frac{2\sqrt{5}}{5}x$; 2) $2a=8$, $2b=12$; $A_1(-4;0)$, $A_2(4;0)$, $F_1(-2\sqrt{13};0)$, $F_2(2\sqrt{13};0)$; $\varepsilon=\frac{\sqrt{13}}{2}$; $y=\pm\frac{3}{2}x$; 3) $2a=6$, $2b=8$; $A_1(0;-4)$, $A_2(0;4)$, $F_1(0;-5)$, $F_2(0;5)$; $\varepsilon=\frac{5}{4}$; $y=\pm\frac{4}{3}x$; 4) $2a=4\sqrt{7}$, $2b=12$, $A_1(0;-6)$; $A_2(0;6)$; $F_1(0;-8)$, $F_2(0;8)$; $\varepsilon=\frac{4}{3}$; $y=\pm\frac{3\sqrt{7}}{7}x$; **264.** $\frac{x^2}{6}-\frac{y^2}{8}=1$, $\varepsilon=\frac{5}{3}$, $F_1(-5;0)$, $F_2(5;0)$. **265.** $\frac{x^2}{9}-\frac{y^2}{7}=1$; $\frac{x^2}{9}+\frac{y^2}{7}=1$. **266.** $\frac{x^2}{144}-\frac{y^2}{25}=1$; $F_1(-13;0)$, $F_2(13;0)$ yoki $-\frac{x^2}{144}+\frac{y^2}{25}=1$; $F_1(0;-13)$, $F_2(0;13)$. **267.** $3x-2y+19=0$, $3x+2y+11=0$. **268.** $\frac{x^2}{64}-\frac{y^2}{36}=1$; $\varepsilon=\frac{5}{4}$. **269.** $\frac{x^2}{12}+\frac{y^2}{24}=1$; $y=\pm\sqrt{2}x$. **270.** $60°$; $\varepsilon=\frac{2\sqrt{3}}{3}$. **271.** $y=\pm\frac{4}{3}(x-5)$. **272.** $(-4;-3)$ va $\left(-\frac{4}{7};-\frac{3}{7}\right)$. **273.** $\frac{x^2}{16}+\frac{y^2}{48}=1$. **274.** 1) $\frac{x^2}{16}-\frac{y^2}{9}=1$, 2) $\frac{x^2}{25}-\frac{y^2}{75}=1$, 3) $\frac{x^2}{9}-\frac{y^2}{2}=1$, 4) $\frac{x^2}{15}-\frac{y^2}{6}=1$. **275.** $\frac{x^2}{64}-\frac{y^2}{36}=1$. **276.** $\frac{x^2}{16}-\frac{y^2}{9}=1$. **277.** $\frac{x^2}{25}-\frac{y^2}{144}=1$. **279.** 1) $F_1(5;0)$, $F_2(-5;0)$; 2) $\varepsilon=\frac{5}{3}$. 3) $y=\pm\frac{4}{3}x$, $x=\pm\frac{9}{5}$; 4) $\frac{x^2}{16}-\frac{y^2}{9}=1, \varepsilon=\frac{5}{4}$. **280.** $x^2-2y^2=6$. **281.** 1) $F(2;0)$, $x+2=0$; 2) $F(-3;0)$, $x-3=0$; 3) $F(0;\frac{5}{2})$, $2y+5=0$; 4) $F(0;-4)$, $y-4=0$; **282.** 1) $y^2=12x$; 2) $y^2=10x-5$; 3) $y^2=16x$; 4) $x^2=8y$; 5) $x^2=-18y$; **283.** $A(18;12)$; $B(18;12)$. **284.** $OM=10$. **285.** 64. **286.** 1) (1;3) va (4;−6). 2) Kesishish nuqtalari yo‘q. 3) (4;6). **287.** $\left(\frac{5}{2};2\sqrt{15}\right)$ va $\left(\frac{5}{2};-2\sqrt{15}\right)$. **288.**

$x-4=0$; 24; **289.** $24\sqrt{2}$. **290.** $(x-7)^2=12(y-2)$. **291.** $y^2=-48(x-3)$; $F(-9;0)$, $x-15=0$. **292.** $(y+1)^2=6(x-3)$. **293.** $(x+3)^2=-2(y-4)$. **294.** 48 m. **295.** 60 sm. **296.** 16 m. **297.** $P=10\frac{2}{3}$. **298.** $y=3-\frac{x^2}{4}$. **300.** 1) $y^2=9x$; 2) $y=-x^2$. **301.** 1) $x^2=16y$; 2) $x^2=-12y$; 3) $y^2=24x$; 4) $y^2=-10x$. **302.** $y^2=48x$ yoki $y^2=-48x$. **303.** 16. **304.** $x-y-1=0$. **305.** $y^2=-3x$. **306.** $R=7{,}4$; $d=9{,}25$. **307.** Direktrisasi $x=\pm 3{,}2$; $\varepsilon=1{,}25$; $r=10{,}25$; $d=8{,}2$. **308.** $\frac{x^2}{4}+y^2=1$. **309.** $x^2-y^2=12$. **310.** Qo'shma diametr $y=-\frac{x}{2}$; $a_1=b_1=\sqrt{10}$. **311.** $2x+3y-9=0$. **312.** $2x-6y=13$ va $6x-2y=13$. **313.** $2x-y\pm 8=0$. **314.** Urinma $8x+15y-50=0$; normal: $75x-40y-252=0$. **315.** $x^2-4y^2=-1$; $\varepsilon=\sqrt{5}$. **316.** 6 va 24. **317.** Asimptotalari: $y=\pm\frac{4}{5}x$, direktrisalari: $\sqrt{41}x\pm 25=0$. **318.** $y=8x\pm 15$. **319.** $3y-4x=15$ va $16x+3y=25$. **320.** $5x-2y=\pm 2\sqrt{21}$. **321.** $4x^2-9y^2=64$. **322.** $y=\frac{4}{5}x$. **323.** $x\pm 3y=0$, $2x\pm y=0$. **324.** $y=2x\pm 4\sqrt{2}$. **325.** Urinma: $x+2y+2=0$; normal: $2x-y-6=0$. **326.** Urinma: $x-3y+9=0$; normal: $3x+y-33=0$. **327.** $x-y+2=0$. **328.** $y=3$. **329.** $x-y-1=0$. **330.** $(\pm 3{,}2;\ \pm 2{,}4)$. **331.** Diametrlar: $y=x$, $y=-\frac{x}{4}$; burchak: $59°02'$. **332.** $\left(\pm\frac{3\sqrt{7}}{4};\pm\frac{3\sqrt{7}}{4}\right)$. **333.** $3y-4=0$. **334.** $\frac{x}{25}-\frac{y}{11}=1$. **335.** 45m. **336.** 60 sm. **338.** $M_1(2;\frac{\pi}{2})$, $M_2(1;\pi)$, $M_3(2;\frac{5\pi}{6})$, $M_4(\sqrt{2};\frac{5\pi}{4})$, $M_5(2;\frac{\pi}{3})$. **339.** *a*), *b*), *d*)— markazi qutbda va radiuslari 1,5 va a bo'lgan aylanada. *e*), *f*), *g*), *h*) — qutbdan chiqqan va qutb o'qi bilan 30°, 60°, 90°, φ burchak tashkil qilgan nuqtalarda. **340.** *a*) $(1;\frac{5\pi}{4})$, *b*) $(3;\frac{5\pi}{3})$, $(\frac{2}{3};\frac{5\pi}{6})$, $(\rho;\varphi;\pi)$; b) $(1;\frac{7\pi}{4})$, $(3;\frac{4\pi}{3})$, $(\frac{2}{3};\frac{\pi}{6})$, $(\rho;2\pi-\varphi)$. **343.** $AB=\sqrt{3}$, $CD=10$, $EF=5$. **344.** $AB=BC=CA=7$. **345.** $M_1(1;0)$ va $M_2(7;0)$. **347.** $S=6(5\sqrt{3}-3)$. Ko'rsatma. *OAB, OBC* va *OAC* qutb bilan hosil qilingan uchburchaklarning yuzlari yig'indisi orqali topiladi. **348.** 1) $r=\frac{a}{\cos\varphi}$; 2) $r=\frac{a\sin\alpha}{\sin\varphi}$. **349.** $r=2a\cos\varphi$. **350.** 1) $r^2=\frac{a^2}{\cos 2\varphi}$; 2) $r=a$; 3) $r=\frac{p}{\cos(\varphi-\pi)}$; 4) $\operatorname{tg}\varphi=1$; 5) $r=\alpha\cos\varphi$; 6) $r^2=a^2\cos^2\varphi$.

IV BOB | VEKTORLAR ALGEBRASI

1- §. Vektor tushunchasi. Vektor proyeksiyalari

1°. Ta'riflar. *Yonaltirilgan* $\overline{AB}$ *kesma vektor deyiladi* (11-chizma). Bunda A nuqta vektorning boshi, B nuqta esa uning oxiri deb qaraladi. Vektor *boshi* va *oxiri* ko'rsatilib, $\overline{AB}$ yoki $\vec{a}$ ko'rinishda belgilanadi.

Vektorlarning moduli (uzunligi) $|\overline{AB}|$ yoki $|\vec{a}|$, yoki AB bilan belgilanadi.

Bir to'g'ri chiziqqa parallel bo'lgan vektorlar *kollinear* vektorlar deyiladi. Bir tekislikka parallel bo'lgan vektorlar *komplanar* vektorlar deyiladi. Agar ikki $\vec{a}$ va $\vec{b}$ vektorlar: 1) teng modulga ega; 2) o'zaro kollinear; 3) bir tomonga yo'nalgan bo'lsa, ular o'zaro teng deyiladi.

2°. $\overline{AB}$ vektorning biror u o'qqa proyeksiyasi deb A_1B_1 kesmaning uzunligiga teng bo'lgan songa aytiladi. Bu yerda A_1 nuqta A ning, B_1 esa B nuqtaning u o'qdagi proyeksiyalari. $\overline{AB}$ vektorning u o'qdagi proyeksiyasi $PR_u\ \overline{AB}$ simvol bilan belgilanadi. Agar vektor $\vec{a}$ bilan belgilangan bo'lsa, proyeksiya $PR_u\ \vec{a}$ kabi yoziladi.

$\vec{a}$ vektorning u o'qdagi proyeksiyasini uning moduli va o'qqa og'ish burchagi φ orqali

$$PR_u\vec{a} = |\vec{a}|\cos\varphi \qquad (1)$$

formula bilan beriladi.

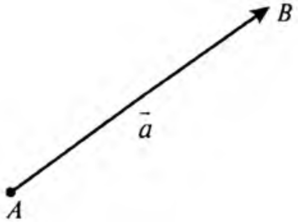
Ixtiyoriy $\vec{a}$ vektor koordinatalar sistemasida X, Y, Z harflar bilan belgilanadi. $A = \{X, Y, Z\}$ tenglikda X, Y, Z sonlar $\vec{a}$ vektorning koordinatalar o'qidagi proyeksiyalarini bildiradi.

Agar vektor $\vec{a} = \overline{M_1M_2}$, $M_1(x_1;y_1;z_1)$ va $M_2(x_2;\ y_2;\ z_2)$ nuqtalari bilan berilgan bo'lsa, $X=x_2-x_1$, $Y=y_2-y_1$, $Z=z_2-z_1$ formulalar bilan aniqlanadi.

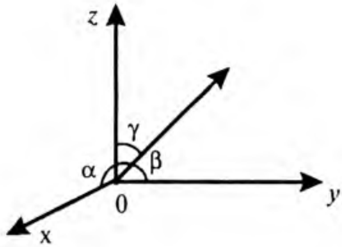
Ushbu

$$|\vec{a}| = \sqrt{X^2+Y^2+Z^2} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2} \qquad (2)$$

formula vektorning koordinatalarga nisbatan modulini aniqlaydi.



11- chizma.



12- chizma.

Agar α, β, γ lar $\vec{a}$ vektorning koordinata o'qlari bilan tashkil qilgan burchaklari bo'lsa, $\cos\alpha, \cos\beta, \cos\gamma$ lar vektorning *yo'naltiruvchi kosinuslari* deyiladi.

(1) formuladan $X = |\vec{a}|\cos\alpha$, $Y = |\vec{a}|\cos\beta$, $Z = |\vec{a}|\cos\gamma$ va (2) dan $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ tenglamalarga ega bo'lamiz.

351. $\vec{a} = \{6; 3; -2\}$ vektorning modulini hisoblang.

352. Vektorning X=4, Y=–12 koordinatalari berilgan. Agar $|\vec{a}| = 13$ bo'lsa, uning uchinchi Z koordinatasini toping.

353. Agar $\vec{a} = \{3; -1; 4\}$ vektorning boshlang'ich nuqtasi M(1;2;3) bo'lsa, uning oxirgi N nuqtasini toping.

354. Vektorning moduli $|\vec{a}| = 2$ va burchaklari $\alpha = 45^0$, $\beta = 60^0$, $\gamma = 12^\circ$. Uning koordinata o'qlaridagi proyeksiyalarini hisoblang.

355. $\vec{a} = \left\{\frac{3}{13}; \frac{4}{13}; \frac{12}{13}\right\}$ vektorning yo'naltiruvchi kosinuslarini hisoblang.

356. Vektor ikkita koordinata o'qlari bilan quyidagi burchaklarni tashkil qilishi mumkinmi: 1) $\alpha = 30^\circ$, $\beta = 45^\circ$; 2) $\beta = 60^\circ$, $\gamma = 60^\circ$; 3) $\alpha = 150^\circ$, $\gamma = 30^\circ$?

357. $\vec{a}$ vektor Ox , Oy o'qlar bilan $\alpha = 60^\circ$, $\beta = 120^\circ$ burchak tashkil qiladi. Agar $|\vec{a}| = 2$ bo'lsa, uning koordinatalarini hisoblang.

358. Koordinata o'qlari bilan bir xil burchaklar tashkil qiluvchi va moduli 3 bo'lgan radius-vektorning M nuqtasi koordinatalarini toping.

* * *

359. $A(3;-1;2)$ va $B(-1;2;1)$ nuqtalar berilgan. $\overline{AB}$ va $\overline{BA}$ vektorlarning koordinatalarini toping.

360. Oxiri $(1;-1;2)$ nuqtada bo‘lgan $\vec{a}=\{2;-3;-1\}$ vektorning boshlang‘ich nuqtasini toping.

361. $\vec{a}=\{12;-15;-16\}$ vektorning yo‘naltiruvchi kosinuslarini hisoblang.

362. Vektor koordinata o‘qlari bilan

1) $\alpha=45^\circ$, $\beta=60^\circ$, $\gamma=120^\circ$; 2) $\alpha=45^\circ$, $\beta=135^\circ$, $\gamma=60^\circ$;

3) $\alpha=90^\circ$, $\beta=150^\circ$, $\gamma=60^\circ$ burchaklarni tashkil qilishi mumkinmi?

363. Vektor Ox va Oz o‘qlari bilan $\alpha=120^\circ$, $\gamma=45^\circ$ burchak tashkil qilsa, u Oy o‘qi bilan qanday burchak tashkil qiladi?

2- §. Vektorlar ustida chiziqli amallar

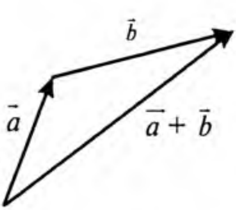
1°. Vektorlarni qo‘shish. $\vec{a}$ va $\vec{b}$ vektorlarning yig‘indisini *uchburchak usuli* bilan (13- chizma) va *parallelogramm usuli* bilan (14- chizma) topish mumkin.

2°. Bir necha vektorning *yig‘indisi* $\vec{a}+\vec{b}+\vec{c}+\vec{d}$ deb shu siniq chiziqning yopuvchisidan iborat vektorga aytiladi (15- chizma).

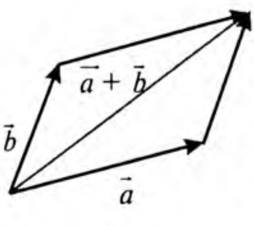
Ikki vektorning *ayirmasi* $\vec{a}-\vec{b}$ deb bu vektorlarda quril-gan parallelogrammning ikkinchi diagonaliga aytiladi, bu holda vektorning yo‘naluvchi „kamayuvchi“ vektorning oxiriga qo‘yiladi.

3°. Vektorni songa ko‘paytirish. $\vec{a}$ vektorning biror α songa *ko‘paytmasi* deb uzunligi $\alpha\,|\vec{a}|$ ga teng bo‘lgan, yo‘nalishi esa berilgan vektor yo‘nalishiday $(\alpha>0)$ yoki unga qarama-qarshi $(\alpha<0)$ bo‘lgan yangi vektorga aytiladi.

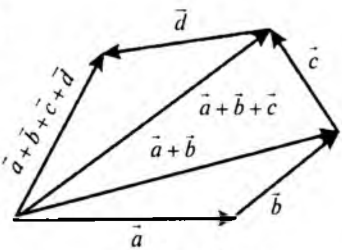
4°. Vektor proyeksiyalari haqida quyidagi ikki asosiy teoremalar o‘rinlidir:



13- chizma.



14- chizma.



15- chizma.

1. *Vektorlar yig'indisining biror o'qqa nisbatan proyeksiyasi, ularning shu o'qqa nisbatan proyeksiyalarining yig'indisidan iborat:*

$$PR_u(\vec{a}_1 + \vec{a}_2 + \ldots + \vec{a}_n) = PR_u\vec{a}_1 + PR_u\vec{a}_2 + \ldots + PR_u\vec{a}_n.$$

2. *Vektor biror songa ko'paysa, uning proyeksiyasi ham shu songa ko'payadi:*

$$PR_u(\alpha\vec{a}) = \alpha PR_u\vec{a}.$$

Agar $\vec{a} = \{x_1; y_1; z_1\}$, $\vec{b} = \{x_2; y_2; z_2\}$ bo'lsa, u holda $\vec{a} + \vec{b} = \{x_1 + x_2; y_1 + y_2; z_1 + z_2\}$ va $\vec{a} - \vec{b} = \{x_1 - x_2; y_1 - y_2; z_1 - z_2\}$. *Agar* $\vec{a} = \{x; y; z\}$ *bo'lsa, u holda ixtiyoriy* α *soni uchun* $\alpha\vec{a} = \{\alpha x; \alpha y; \alpha z\}$ *bo'ladi.*

5°. Uchta ***i, j, k*** vektorlar uchun quyidagi shartlar bajarilsa, ular koordinata *bazislari* deyiladi:

1) ***i*** vektor *Ox* o'qida, ***j*** vektor *Oy* o'qida, ***k*** vektor *Oz* o'qida yotsa;

2) har bir ***i, j, k*** vektorlar o'z o'qida musbat tomonga yo'nalgan bo'lsa;

3) $|\boldsymbol{i}| = 1$, $|\boldsymbol{j}| = 1$ va $|\boldsymbol{k}| = 1$, ya'ni ular birlik vektorlar bo'lsa.

Har bir $\vec{a} = \{x; y; z\}$ vektorni ***i, j, k*** bazis bo'yicha yoyish mumkin, ya'ni

$$\vec{a} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}.$$

364. Berilgan $\vec{a}$ va $\vec{b}$ vektorlarga ko'ra ushbu 1) $\vec{a} + \vec{b}$; 2) $\vec{a} - \vec{b}$; 3) $\vec{b} - \vec{a}$; 4) $-\vec{a} - \vec{b}$; vektorlarni yasang.

365. $|\vec{a}| = 13$, $|\vec{b}| = 19$ va $|\vec{a} + \vec{b}| = 24$ berilgan. $|\vec{a} - \vec{b}|$ ni hisoblang.

366. $|\vec{a}| = 5$, $|\vec{b}| = 8$ va ular o'zaro $\varphi = 60°$ li burchak ostida kesishadi. $|\vec{a} + \vec{b}|$ va $|\vec{a} - \vec{b}|$ larni aniqlang.

367. *ABCD A'B'C'D'* (16- chizma) parallelepipedda uning qirralari bilan ustma-ust tushuvchi

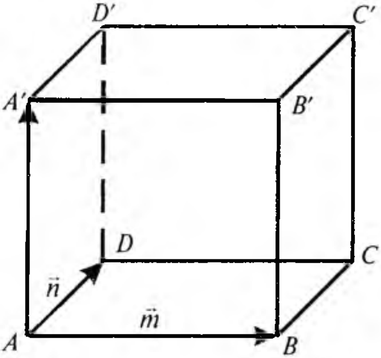
$$\overrightarrow{AB} = \vec{m},\ \overrightarrow{AD} = \vec{n},\ \overrightarrow{AA_1} = \vec{p}$$

vektorlar berilgan. Quyidagi har bir vektorlarni yasang:

1) $\vec{m}+\vec{n}+\vec{p}$; 2) $\vec{m}+\vec{n}+\frac{1}{2}\vec{p}$;

3) $\frac{1}{2}\vec{m}+\frac{1}{2}\vec{n}+\vec{p}$; 4) $\vec{m}+\vec{n}-\vec{p}$;

5) $-\vec{m}-\vec{n}+\frac{1}{2}\vec{p}$.



16- chizma.

368. O‘zaro perpendikular yonalishdagi uchta M,N va P kuchlar bir nuqtaga qo‘yilgan. Agar $|M|=2\text{kg}$, $|N|=10\text{kg}$ va $|P|=11\text{kg}$ bo‘lsa, ularning teng ta’sir etuvchisi R ning qiymatini toping.

369. Ikkita $\vec{a}=\{3;-2;6\}$va $\vec{b}=\{-2;1;0\}$vektorlar berilgan. Quyidagi 1) $\vec{a}+\vec{b}$; 2) $\vec{a}-\vec{b}$; 3) $2\vec{a}$; 4) $-\frac{1}{2}\vec{b}$; 5) $2\vec{a}+3\vec{b}$; 6) $\frac{1}{3}\vec{a}-\vec{b}$ vektorlarning har birini koordinata o‘qlaridagi proyeksiyasini toping.

370. $\vec{a}=\{2;\ -1;\ 3\}$va $\vec{b}=\{-6;\ 3;-9\}$vektorlarning kollinearligini tekshiring. Ularning uzunliklarini va uzunliklar orasidagi farqni hamda ularning yo‘nalishlarini aniqlang.

371. α va β ning qanday qiymatlarida $\vec{a}=-2\boldsymbol{i}+3\boldsymbol{j}+\beta\boldsymbol{k}$, $\vec{b}=\alpha\boldsymbol{i}-6\boldsymbol{j}+2\boldsymbol{k}$ vektorlar kollinear bo‘ladi?

* * *

372. $\vec{a}=11$, $|\vec{b}|=23$ berilgan. $|\vec{a}+\vec{b}|$ni hisoblang.

373. $|\vec{a}|=5$ va $|\vec{b}|=12$ berilgan. Agar $\vec{a},\vec{b}$ vektorlar o‘zaro perpendikular bo‘lsa, $|\vec{a}+\vec{b}|$ va $|\vec{a}-\vec{b}|$ larni aniqlang.

374. $\vec{a}$ va $\vec{b}$ vektorlar o‘zaro $120°$ burchak tashkil qiladi. $|\vec{a}|=3$ va $|\vec{b}|=5$ bo‘lsa, $|\vec{a}+\vec{b}|,|\vec{a}-\vec{b}|$ larni aniqlang.

375. ABC uchburchakda vektorlar $\overline{AB}=\vec{m}$, $\overline{AC}=\vec{n}$. Quyidagi vektorlarning har birini yasang:

1) $\frac{\vec{m}+\vec{n}}{2}$; 2) $\frac{\vec{m}-\vec{n}}{2}$; 3) $\frac{\vec{n}-\vec{m}}{2}$; 4) $-\frac{\vec{m}+\vec{n}}{2}$. Masshtab birligini $\frac{1}{2}|\vec{n}|$ deb olib, quyidagi vektorlarni yasang:

$$5)\ |\vec{n}|-\vec{m}+|\vec{m}|-\vec{n};\ \ 6)\ |\vec{n}|-\vec{m}-|\vec{m}|-\vec{n}.$$

376. ABC uchburchakning og'irlik markazi O nuqtada. $\overline{OA}+\overline{OB}+\overline{OC}=0$ ekanligini isbot qiling.

377. $A(-1;5;-10)$, $B(5;-7;8)$, $C(2;2;-7)$ va $D(5;-4;2)$ nuqtalar berilgan. $\overline{AB}$ va $\overline{CD}$ vektorlarning kollinearligini tekshiring, ularning uzunliklari orasidagi farq va yo'nalishlarini ko'rsating.

3- §. Vektorlarning skalyar ko'paytmasi

1°. Ta'rif. *Ikki vektorning skalyar ko'paytmasi deb shu vektorlar modullarining ular orasidagi burchak kosinusi bilan ko'paytmasiga aytiladi.*

$\vec{a}$ va $\vec{b}$ vektorning skalyar ko'paytmasi $\vec{a}\cdot\vec{b}$ ko'rinishda belgilanadi ($\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}$). Demak,

$$\vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{b}\cos\varphi. \quad (1)$$

Skalyar ko'paytmani quyidagi formulalar bilan ham ifodalash mumkin:

$$\vec{a}\cdot\vec{b}=|\vec{a}|\,PR_a\vec{b} \text{ yoki } \vec{a}\cdot\vec{b}=|\vec{b}|\,PR_b\vec{a}. \quad (2)$$

(1) formuladan ko'rinadiki, agar φ o'tkir burchak bo'lsa, $\vec{a}\cdot\vec{b}>0$, agar φ o'tmas burchak bo'lsa, $\vec{a}\cdot\vec{b}<0$; agar vektorlar perpendikular bo'lsa, $\vec{a}\cdot\vec{b}=0$. Xususiy holda, $\vec{a}^2=\vec{a}\cdot\vec{a}=|\vec{a}|\cdot|\vec{a}|\cos 0°=|\vec{a}|^2$, bunda $|\vec{a}|=\sqrt{\vec{a}^2}$.

2°. Agar $\bar{a}$ va $\bar{b}$ vektorlar $\bar{a}=\{a_x;a_y;a_z\}$, $\bar{b}=\{b_x;b_y;b_z\}$ koordinatalari bilan berilgan bo'lsa, ularning skalyar ko'paytmasi:

$$\vec{a}\cdot\vec{b}=a_xb_x+a_yb_y+a_zb_z. \quad (3)$$

3°. Ikki vektor orasidagi burchak:

$$\cos\varphi=\frac{\vec{a}\cdot\vec{b}}{ab}=\frac{a_xb_x+a_yb_y+a_zb_z}{\sqrt{a_x^2+a_y^2+a_z^2}\cdot\sqrt{b_x^2+b_y^2+b_z^2}}. \quad (4)$$

Parallellik sharti: $\vec{b} = m\vec{a}$ yoki $\frac{b_x}{a_x} = \frac{b_y}{a_y} = \frac{b_z}{a_z} = m.$

Perpendikularlik sharti: $\vec{a} \cdot \vec{b} = 0$ yoki $a_x b_x + a_y b_y + a_z b_z = 0.$

Ixtiyoriy $\vec{S}\{X, Y, Z\}$ vektorning biror u o‘qqa proyeksiyasi $PR_u \vec{S} = \vec{S}\vec{e}$ formula bilan hisoblanadi, bu yerda $\vec{e}$ o‘q u bo‘yicha yonalgan birlik vektor. Agar u o‘qning koordinata o‘qlari bilan hosil qilgan α, β, γ burchaklari berilgan bo‘lsa, u holda $\vec{e} = \{\cos\alpha,\ \cos\beta,\ \cos\gamma\}$ va $\vec{S}$ ning proyeksiyasini hisoblash uchun

$$PR_u\ \vec{S} = X\cos\alpha + Y\cos\beta + Z\cos\gamma$$

formula o‘rinli.

378. $\vec{a} = \{2; -4;\ 4\}$ va $\vec{b} = \{-3; 2;\ 6\}$ vektorlar orasidagi burchakni hisoblang.

379. Uchburchakning $A(3;\ 2;\ -3)$, $B(5;\ 1;\ -1)$ va $C(1;\ -2;\ 1)$ uchlari berilgan. Uning A uchidagi tashqi burchagini aniqlang.

380. $\vec{a} = \{4; -2;\ -4\}$, $\vec{b} = \{6; -3;\ 2\}$ vektorlar berilgan. Hisoblang:

1) $\vec{a} \cdot \vec{b}$; 2) $\sqrt{\vec{a}^2}$; 3) $\sqrt{\vec{b}^2}$; 4) $(2\vec{a} - 3\vec{b})(\vec{a} + 2\vec{b})$;

5) $(\vec{a} + \vec{b})^2$; 6) $(\vec{a} - b)^2$.

381. α ning qanday qiymatida $\vec{a} = \alpha\boldsymbol{i} - 3\boldsymbol{j} + 2\boldsymbol{k}$ va $\vec{b} = \boldsymbol{i} - 2\boldsymbol{j} + \alpha\boldsymbol{k}$ vektorlar o‘zaro perpendikular bo‘ladi?

382. $\vec{a} = \{6; -8;\ -7{,}5\}$ vektorga kollinear bo‘lgan $\vec{x}$ vektor Oz o‘qi bilan o‘tkir burchak tashkil etadi. Agar $|\vec{x}| = 50$ bo‘lsa, uning koordinatalarini toping.

383. $\vec{a} = 3\boldsymbol{i} + 2\boldsymbol{j} + 2\boldsymbol{k}$ va $\vec{b} = 18\boldsymbol{i} - 22\boldsymbol{j} - 5\boldsymbol{k}$ vektorlarga perpendikular bo‘lgan $\vec{x}$ vektor Oy o‘qi bilan o‘tmas burchak tashkil etadi. $|\vec{x}| = 14$ bo‘lsa, uning koordinatalarini toping.

384. Uchta $\vec{a} = 2\boldsymbol{i} + \boldsymbol{j} + 3\boldsymbol{k}$, $\vec{b} = \boldsymbol{i} - 3\boldsymbol{j} - 2\boldsymbol{k}$ va $\vec{c} = 3\boldsymbol{i} + 2\boldsymbol{j} - 4\boldsymbol{k}$ vektorlar berilgan. $\vec{x} \cdot \vec{a} = -5$, $\vec{x} \cdot \vec{b} = -11$, $\vec{x} \cdot \vec{c} = 20$ shartlarni qanoatlantiruvchi $\vec{x}$ vektorni toping.

385. $\vec{S} = \{4; -3; 2\}$ vektorning koordinata o‘qlari bilan bir xil o‘tkir burchak tashkil etuvchi o‘qdagi proyeksiyasini toping.

386. l o‘qi koordinata o‘qlari: Ox bilan $\alpha = 45°$, Oz bilan $\gamma = 60°$ va Oy bilan o‘tkir β burchak tashkil etadi. $\vec{S} = \{\sqrt{2}; -3; -5\}$ vektorning l o‘qdagi proyeksiyasini toping.

387. Agar $\vec{f} = \{3; -2; -5\}$ kuchning qo‘yilish nuqtasi to‘g‘ri chiziqli harakatda $A(2;-3;5)$ dan $B(3;-2;-1)$ holatga ko‘chsa, bu kuch qancha ish bajaradi?

* * *

388. Uchburchakni $A(-1;-2;4)$, $B(-4;-2;0)$ va $C(3;-2;1)$ uchlari berilgan. Uning B uchidagi ichki burchagini aniqlang.

389. Uchlari $A(1;2;1)$, $B(3;-1;7)$, $C(7;4;-2)$ bilan berilgan uchburchakning ichki burchaklarini hisoblab, uning teng tomonli ekanligini ko‘rsating.

390. $\vec{a} = \{2; 1; -1\}$ vektorga kollinear bo‘lgan va $\vec{x} \cdot \vec{a} = 3$ shartni bajaruvchi $\vec{x}$ vektorni toping.

391. Ikkita $A(3;-4;-2)$, $B(2;5;-2)$ nuqta berilgan. l o‘qi koordinata o‘qlari Ox, Oy bilan $\alpha = 60°$, $\beta = 120°$, Oz o‘qi bilan γ o‘tmas burchak tashkil etadi. $\overrightarrow{AB}$ ning l o‘qqa proyeksiyasini toping.

392. Bir nuqtaga uchta $\vec{M} = \{3; -4; -2\}$, $\vec{N} = \{2; 3; -5\}$, $\vec{P} = \{-3; -2; 4\}$ kuchlar qo‘yilgan. Ularning teng ta’sir etuvchi kuchining qo‘yilish nuqtasi to‘g‘ri chiziqli harakat qilib, $M_1(5;3;-7)$ dan $M_2(4;-1;-4)$ holatga o‘tganda, bu kuch qancha ish bajarishini hisoblang.

4- §. Ikki vektorning vektor ko‘paytmasi

1°. Ta’rif. *$\vec{a}$ va $\vec{b}$ vektorlarning vektor ko‘paytmasi deb shunday uchinchi vektorga aytiladiki:*

1) u son qiymati bo‘yicha berilgan $\vec{a}$ va $\vec{b}$ vektorlarda yasalgan parallellogramm yuziga teng modulga ega;

2) u parallelogramm tekisligiga perpendikular;

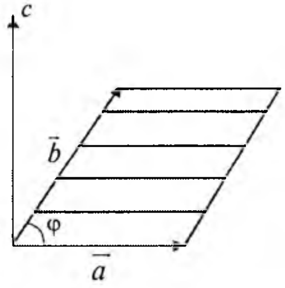
3) u shunday tomonga yonaltirilganki, uning uchidan qaraganda $\vec{a}$ vektordan $\vec{b}$ vektorga qarab eng kichik burilish soat strelkasi harakat yo‘nalishiga qarama-qarshi tomonga yo‘nalgan bo‘ladi. $\vec{a}$, $\vec{b}$ va $\vec{c}$ vektorlarning bu xildagi joylashishiga o‘ng bog‘lam deyiladi(17-chizma).

Ikki vektorning vektor ko'paytmasi $\vec{a}\times\vec{b}$ *ko'rinishda belgilanadi. Shunday qilib, agar*

1) $|\vec{a}\times\vec{b}| = ab\sin\varphi$,

2) $\vec{c}\perp\vec{a}$ va $\vec{c}\perp\vec{b}$,

3) $\vec{a},\vec{b},\vec{c}$ lar o'ng bog'lam hosil qilsa, $\vec{a}\times\vec{b}=\vec{c}$ bo'ladi.



17- chizma.

2°. Vektor kopaytmaning xossalari:

I. $\vec{a}\times\vec{b} = -\vec{b}\times\vec{a}$.

II. $\vec{a}\times(\vec{b}+\vec{c}) = \vec{a}\times\vec{b}+\vec{a}\times\vec{c}$ – taqsimot qonuni.

III. Agar $\vec{a}\,||\,\vec{b}$ bo'lsa, $\vec{a}\times\vec{b}=0$, xususiy holda $\vec{a}\times\vec{a}=0$.

3°. Ortlarnimg vektor ko'paytmalari

$$\boldsymbol{i}\times\boldsymbol{j}=\boldsymbol{k},\ \boldsymbol{j}\times\boldsymbol{k}=\boldsymbol{i},\ \boldsymbol{k}\times\boldsymbol{i}=\boldsymbol{j}. \tag{1}$$

Umuman, har ikki qo'shni vektorning quyidagi tartibdagi

$\xrightarrow{\qquad\qquad\quad +}$

$i \quad j \quad k \quad i \quad j$ ko'paytmasi (+) ishora bilan olingan

$\xleftarrow{- \qquad\qquad\quad}$

uchinchi vektorga, teskari tartibdagi ko'paytmasi esa (–) ishora bilan olingan uchinchi vektorga teng.

4°. Vektor ko'paytmani ko'paytuvchilar koordinatalari $\vec{a}\{a_x, a_y, a_z\}$ va $\vec{b}\{b_x, b_y, b_z\}$ orqali ifodalash:

$$\vec{a}\times\vec{b} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}. \tag{2}$$

5°. $\vec{a}$ va $\vec{b}$ vektorlarga yasalgan parallelogrammning yuzi:

$$S_{\text{par.}} = |\vec{a}\times\vec{b}|, \tag{3}$$

shu vektorlarda yasalgan uchburchakning yuzi:

$$S_{\Delta} = \frac{1}{2}|\vec{a}\times\vec{b}|.$$

393. Agar 1) $\vec{a}=\{2;3;0\}$, $\vec{b}=\{0;3;1\}$; 2) $\vec{a}=\{-2;3;0\}$, $\vec{b}=\{-2;\ 0;\ 4\}$ bo'lsa, $\vec{c}=\vec{a}\times\vec{b}$ vektorni aniqlang va yasang. Har bir hol uchun berilgan vektorlarda yasalgan parallelogramm yuzini hisoblang.

394. Uchlari 1) $A(7;3;\ 4)$, $B(1;\ 0;6)$ va $C(4;\ 5;\ -2)$; 2) $A(1;\ 1;\ 1)$, $B(2;\ 3;\ 4)$ va $C(4;\ 3;\ 2)$ nuqtalarda bo'lgan uchburchakning yuzini hisoblang.

395. $\vec{a} = 2\boldsymbol{j} + \boldsymbol{k}$ va $\vec{b} = \boldsymbol{i} + 2\boldsymbol{k}$ vektorlardan parallelogramm yasang hamda uning yuzi va balandligini aniqlang.

396. Uchburchakning $A(1;\ -1;\ 2)$, $B(5;\ -6;\ 2)$ va $C(1;\ 6;\ -1)$ uchlari berilgan. Uning B uchidan AC tomonga tushurilgan balandligini toping.

397. $\vec{a} = \{2;-2;1\}$ va $\vec{b} = \{2;3;6\}$ vektorlar tashkil qilgan burchak sinusini hisoblang.

398. $\vec{a} = \{4;-2;3\}$ va $\vec{b} = \{0;1;3\}$ vektorlarga perpendikular bo'lgan $\vec{x}$ vektor Oy o'qi bilan o'tmas burchak tashkil etadi. Agar $|\vec{m}| = 26$ bo'lsa, uning koordinatalarini toping.

399. Oz o'qiga va $\vec{a} = \{8;-15;3\}$ vektorga perpendikular bo'lgan $\vec{m}$ vektor Ox o'qi bilan o'tkir burchak tashkil etadi. Agar $|\vec{m}| = 51$ bo'lsa, uning koordinatalarini toping.

400. $\vec{a} = \{2;-3;1\}$ va $\vec{b} = \{1;-2;3\}$ vektorlarga perpendikular bo'lib, $\vec{x} \cdot (\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}) = 10$ shartni qanoatlantiruvchi $\vec{x}$ vektorni toping.

* * *

401. Agar 1) $\vec{a} = 3\boldsymbol{k}$, $\vec{b} = 2\boldsymbol{k}$; 2) $\vec{a} = \boldsymbol{i} + \boldsymbol{j}$; $b = \boldsymbol{i} - \boldsymbol{j}$ bo'lsa, $\vec{c} = \vec{a} \times \vec{b}$ vektorni aniqlang va yasang. Har bir hol uchun berilgan vektorlarda yasalgan parallelogramm yuzini hisoblang.

402. $\vec{a}$ va $\vec{b}$ vektorlar $\varphi = \frac{\pi}{6}$ burchak tashkil etadi. Agar $|\vec{a}| = 6$, $|\vec{b}| = 5$ bo'lsa, $|\vec{a} \times \vec{b}|$ ni hisoblang.

403. $\vec{a} = \{0;3;-2\}$, $\vec{b} = \{3;-2;0\}$ va $\vec{c} = \vec{a} \times \vec{b}$ vektorlarni yasang. $\vec{c}$ vektorning moduli hamda $\vec{a}$ va $\vec{b}$ vektorlarda yasalgan uchburchak yuzini hisoblang.

404. Uchlari $A(1;\ -1;\ 2)$, $B(5;\ -6;\ 2)$ va $C(1;\ 3;\ -1)$ bo'lgan uchburchakning $h = |\overline{BD}|$ balandligini toping.

405. $|\vec{a}| = |\vec{b}| = 5$, $(\vec{a}, \vec{b}) = \frac{\pi}{4} \cdot \vec{a} - 2\vec{b}$ va $3\vec{a} + 2\vec{b}$ vektorlarda yasalgan uchburchak yuzini hisoblang.

406. $\vec{m}$ va $\vec{n}$ o'zaro 30° burchak tashkil etuvchi birlik vektorlar bo'lsa, $\vec{a} = \vec{m} + 2\vec{n}$ va $\vec{b} = \vec{m} + \vec{n}$ vektorlarda yasalgan parallelogramm yuzini toping.

5- §. Uch vektorning aralash ko'paytmasi

1°. Ta'rif. $\vec{a}, \vec{b}$ va $\vec{c}$ *vektorlarning aralash ko'paytmasi deb* $(\vec{a} \times \vec{b}) \cdot \vec{c}$ *ko'rinishdagi ifodaga aytiladi.*

Agar $\vec{a}, \vec{b}$ va $\vec{c}$ vektorlar o'zlarining koordinatalari bilan berilsa, u holda

$$(\vec{a} \times \vec{b}) \cdot \vec{c} = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}. \qquad (1)$$

2°. Aralash ko'paytmaning xossalari

I. *Aralash ko'paytmaning istalgan ikkita ko'paytuvchisining o'rinlari o'zaro almashtirilsa, ko'paytmaning ishorasi o'zgaradi:*

$$(\vec{a} \times \vec{b}) \cdot \vec{c} = -(\vec{a} \times \vec{c}) \cdot \vec{b} = -(\vec{c} \times \vec{b}) \cdot \vec{a}. \qquad (2)$$

II. *Agar berilgan uchta vektordan ikkitasi o'zaro teng yoki parallel bo'lsa, aralash ko'paytma 0 ga teng bo'ladi.*

III. *„Nuqta" bilan ko'rsatilgan va „krest" (×) bilan ko'rsatilgan amallarning o'rinlarini almashtirish mumkin:*

$$(\vec{a} \times \vec{b}) \cdot \vec{c} = \vec{a} \cdot (\vec{b} \times \vec{c}).$$

Shuning uchun ham aralash ko'paytmani ***abc*** ko'rinishda, ya'ni qavslarni va amallarni ko'rsatmasdan yozish qabul qilingan.

3°. $\vec{a}, \vec{b}$ va $\vec{c}$ vektorlarda yasalgan parallelepipedning hajmi:

$$V = \pm \boldsymbol{abc} \begin{cases} +, & \text{vektorlar o'ng bog'lam tashkil etsa,} \\ -, & \text{vektorlar chap bog'lam tashkil etsa.} \end{cases} \qquad (3)$$

$\vec{a}, \vec{b}$ va $\vec{c}$ vektorlarda yasalgan piramidaning hajmi:

$$V_{\text{pir}} = \pm \frac{1}{6} \vec{a}\vec{b}\vec{c}.$$

4°. Komplanarlik sharti. Agar $\vec{a}, \vec{b}$ va $\vec{c}$ vektorlar o'zaro komplanar bo'lsa, $\vec{a}\vec{b}\vec{c} = 0$, va aksincha, so'nggi tenglik bajarilsa, berilgan uch vektor o'zaro komplanar bo'ladi. Shuning bilan birga

$\vec{a}, \vec{b}$ va $\vec{c}$ orasidagi $\vec{c} = m\vec{a} + n\vec{b}$ ko'rinishdagi chiziqli bog'lanish mavjud bo'ladi.

407. O'ng bog'lam tashkil etuvchi $\vec{a}, \vec{b}$ va $\vec{c}$ vektorlar o'zaro perpendikular. Agar $|\vec{a}| = 4$, $|\vec{b}| = 2$, $|\vec{c}| = 3$ bo'lsa, $\vec{a}\vec{b}\vec{c}$ ni hisoblang.

408. Uchta $\vec{a} = \{1;-1;3\}$, $\vec{b} = \{-2;2;1\}$ va $\vec{c} = \{3;-2;5\}$ vektorlar berilgan. $\vec{a}\vec{b}\vec{c}$ ni hisoblang.

409. Quyidagi vektorlarning komplanarligini ko'rsating:

1) $\vec{a} = \{2;3;-1\}$, $\vec{b} = \{1;-1;3\}$ va $\vec{c} = \{1;9;-1\}$;

2) $\vec{a} = \{3;-2;1\}$, $\vec{b} = \{2;1;2\}$ va $\vec{c} = \{3;-1;-2\}$;

3) $\vec{a} = \{2;-1;2\}$, $\vec{b} = \{1;2;-3\}$ va $\vec{c} = \{3;-4;7\}$.

410. To'rtta $A(1;\ 2;\ -1)$, $B(0;\ 1;\ 5)$, $C(-1;\ 2;\ 1)$ va $D(2;\ 1;\ 3)$ nuqtalarning bir tekislikda yotishini ko'rsating.

411. Uchlari $A(2;\ -1;\ 1)$, $B(5;\ 6;\ 4)$, $C(3;\ 2;\ -1)$ va $D(4;\ 1;\ 3)$ nuqtalarda bo'lgan tetraedrning hajmini hisoblang.

412. Tetraedrning $A(2;\ 3;\ 1)$, $B(4;\ 11;\ -2)$, $C(6;\ 3;\ 7)$ va $D(-5;\ -4;\ 8)$ uchlari berilgan. Uning D nuqtasidan tushirilgan balandligining uzunligini toping.

413. Tetraedrning hajmi $V=5$, uning uchta uchlari $A(2;\ 1;\ -1)$, $B(3;\ 0;\ 1)$, $C(2;\ -1;\ 3)$ nuqtalarda. Agar to'rtinchi D uchi Oy o'qida yotsa, uning koordinatalarini toping.

* * *

414. $\vec{c}$ vektor $\vec{a}$ va $\vec{b}$ vektorlarga perpendikular, $\vec{a}$ va $\vec{b}$ lar orasidagi burchak 30°. Agar $|\vec{a}| = 6$, $|\vec{b}| = 3$, $|\vec{c}| = 3$ bo'lsa, $\vec{a}\vec{b}\vec{c}$ ni hisoblang.

415. $\vec{a} = 3\boldsymbol{i} + 4\boldsymbol{j}$, $\vec{b} = -3\boldsymbol{j} + \boldsymbol{k}$, $\vec{c} = 2\boldsymbol{i} + 5\boldsymbol{k}$ vektorlardan parallelepiped yasang hamda uning hajmini hisoblang. Berilgan ($\vec{a}\vec{b}\vec{c}$) vektorlar qaysi bog'lamni tashkil etadi?

416. Uchlari $O(0;\ 0;\ 0)$, $A(5;\ 2;\ 0)$, $B(2;\ 5;\ 0)$ va $C(1;\ 2;\ 4)$ nuqtalarda bo'lgan piramida yasang hamda uning hajmini, ABC yoqning yuzini va shu yoqqa tushirilgan balandlikni hisoblang.

417. $A(2;\ -1;\ -2)$, $B(1;\ 2;\ 1)$, $C(2;\ 3;\ 0)$ va $D(5;\ 0;\ -6)$ nuqtalarning bir tekislikda yotishini ko'rsating.

418. $\vec{a} = \boldsymbol{i} + \boldsymbol{j} + 4\boldsymbol{k}$, $\vec{b} = \boldsymbol{i} - 2\boldsymbol{j}$ va $\vec{c} = 3\boldsymbol{i} - 3\boldsymbol{j} + 4\boldsymbol{k}$ vektorlarni yasang va ular o'zaro komplanar ekanligini ko'rsating. Bu vektorlar orasidagi chiziqli bog'lanishni toping.

IV bob javoblari

351. $|a| = 7$. **352.** $z = \pm 3$. **353.** $N(4;1;1)$. **354**. $x = \sqrt{2}$, $y=1$, $z=1$. **355**. $\cos\alpha = \frac{3}{13}$, $\cos\beta = \frac{4}{13}$, $\cos\gamma = \frac{12}{13}$. **356**. 1) mumkin emas; 2) mumkin; 3) mumkin emas. **357**. $A=\{1;-1;\sqrt{2}\}$ yoki $a=\{1;-1;-\sqrt{2}\}$. **358**. $M_1(\sqrt{3};\sqrt{3};\sqrt{3})$, $M_2(-\sqrt{3};-\sqrt{3};-\sqrt{3})$. **359.** $\overline{AB} = \{-4;3;-1\}$, $\overline{BA} = \{4;-3;1\}$. **360**. (–1;2;3). **361**. $\cos\alpha = \frac{12}{25}, \cos\beta = -\frac{3}{5}, \cos\gamma = -\frac{16}{25}$. **362.** 1) mumkin; 2) mumkin emas; 3) mumkin. **363.** 60° yoki 120°. **365.** $|\vec{a} - \vec{b}| = 22$. **366.** $|\vec{a} - \vec{b}| = 129$, $|\vec{a} - \vec{b}| = 7$. **368.** $|R| = 15$. **369**. 1) {1;–1;6}; 2) {5; –3; 6}; 3) {6; –4; 12}; 4) $\{1; -\frac{1}{2}; 0\}$; 5) {0; –2; 12}; 6) $\{3; -\frac{5}{3}; 2\}$. **370.** *b* vektor *a* vektordan 3 marta uzun, ular qarama-qarshi yo'nalgan. **371.** $\alpha = 4$; $\beta = -1$. **372.** $|\vec{a} + \vec{b}| = 20$. **373.** $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}| = 13$. **374.** $|\vec{a} + \vec{b}| = \sqrt{19}$; $|\vec{a} - \vec{b}| = 7$. **375.** $\overline{AB}$ vektorning uzunligi $\overline{CD}$ nikidan ikki barobar uzun; ular bir xil tomonga yo'nalgan. **378**. $\cos\varphi = \frac{5}{21}$. **379.** $\arccos\left(-\frac{4}{9}\right)$. **380.** 1) 22; 2) 6; 3) 7; 4) –200; 5) 41. **381.** $\alpha=-6$. **382**. $x=\{-24; 32; 30\}$. **383.** $x=-4\boldsymbol{i}-6\boldsymbol{j}+12\boldsymbol{k}$. **384**. $x=2\boldsymbol{i}+3\mathbf{j}-2\boldsymbol{k}$. **385.** $\sqrt{3}$. **386.** –3. **387**. 31. **388.** 45°. **390**. $\{1; \frac{1}{2}; -\frac{1}{2}\}$. **391**. –5. **392**. 13. **393**. 1) $6i-4j+6k$. $2\sqrt{22}$; 2) $12\boldsymbol{i}+8\boldsymbol{j}+6\boldsymbol{k}$, $2\sqrt{61}$. **394**. 1) 24,5; 2) $2\sqrt{6}$. **395**. $\sqrt{21}; h = \sqrt{4,2}$. **396**. 5. **397**. $\sin\varphi = \frac{5\sqrt{17}}{21}$. **398**. $x=\{-6; -24; 8\}$. **399**. $m=\{45; 24; 0\}$. **400**. $x=\{7; 5; 1\}$. **401**. 1) $-6j$; 6; 2) $-k$; 2. **402**. 15. **403**. $3\sqrt{17}$, $S_\Delta = \frac{1}{2}3\sqrt{17}$. **404**. 5. **405.** $50\sqrt{2}$. **406**. 1,5. **407**. $abc=24$. **408**. $abc=-7$. **409**. 1) komplanar; 2) komplanar emas; 3) komplanar. **411**. 3 kv bir. **412**. 11. **413.** $D_1(0; 8; 0)$, $D_2(0; -7; 0)$. **414**. $abc=\pm 27$. **415.** $V=51$, chap. **416**. $v=14$, $h=\frac{7}{3}\sqrt{3}$. **418**. $c=a+2b$.

V BOB FAZODAGI ANALITIK GEOMETRIYA

1- §. Tekislik tenglamasi

1°. $M_1(x_1;y_1;z_1)$ nuqtadan o'tuvchi va $\overline{N}\{A;B;C\}$ vektorga perpendikular tekislik tenglamasi.

$M(x;y;z)$ — tekislikning ixtiyoriy nuqtasi bo'lsin (18- chizma). U holda $\overline{M_1M}\perp\overline{N}$ va ikki vektorning perpendikularlik shartiga ko'ra:

$$A(x-x_1)+B(y-y_1)+C(z-z_1)=0. \qquad (1)$$

2°. Tekislikning umumiy tenglamasi quyidagicha yoziladi:

$$Ax+By+Cz+D=0. \qquad (2)$$

$\overline{N}(A;B;C)$ vektor (1) yoki (2) tekislikda *normal vektor* deyiladi.

3°. $Ax+By+Cz+D=0$ tenglamaning maxsus hollari:

1) $D=0$ bo'lganda, $Ax+By+Cz=0$ tekislik koordinatalar boshidan o'tadı,

2) $C=0$ bo'lganda, $Ax+By+D=0$ tekislik Oz o'qqa parallel;

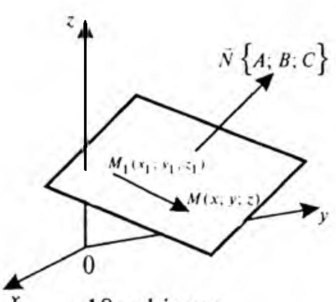
3) $C=D=0$ bo'lganda, $Ax+By=0$ tekslik Oz o'qdan o'tadi;

4) $B=C=0$ bo'lganda, $Ax+D=0$ tekislik Oyz tekislikka parallel;

5) koordinata tekisliklarining tenglamalari: $x=0$, $y=0$ va $z=0$.

4°. Tekislikning koordinata o'qlaridan ajratgan kesmalar bo'yicha tenglamasi:

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1. \qquad (3)$$



18- chizma.

419. Quyidagi tekisliklarni yasang: 1) $3x-2y+6z-12=0$; 2) $2x+3y-z=0$; 3) $3x+2z-6=0$; 4) $3z-8=0$.

420. a) $M_1(2;-3;3)$ nuqtadan o'tib va Oxy tekislikka parallel bo'lgan; b) $M_3(-5;2;-1)$ nuqtadan o'tib va Oyz tekislikka parallel bo'lgan tekislik tenglamasini yozing!

421. 1) Ox va $M_1(4; -1; 2)$ nuqtadan; 2) Oy va $M_2(1; 4; -3)$ nuqtadan; 3) Oz va $M_3(3; -4; 6)$ nuqtadan o‘tuvchi tekislik tenglamasini yozing.

422. 1) $M_1(7; 2; -3)$ va $M_2(5; 6; -4)$ nuqtalardan o‘tib, Ox o‘qiga parallel; 2) $P_1(2; -1; 1)$ va P_2 (3; 1; 2) nuqtalardan o‘tib, Oy o‘qiga parallel; 3) $Q_1(3; -2; 5)$ va $Q_2(2; 3; 1)$ nuqtalardan o‘tib, Oz o‘qiga parallel bo‘lgan tekislik tenglamasini yozing.

423. $2x-3y-4z-24=0$ tekislikning koordinata o‘qlari bilan kesishgan nuqtalarini toping.

424. $5x-6y+3z+120=0$ tekislikning Oxy koordinata burchagi bo‘yicha ajratgan ucliburchakning yuzini toping.

425. $2x-3y+6z-12=0$ tekislik va koordinita tekisliklari bilan chegaralangan piramidaning hajmini toping.

426. Tekislik $M(6; -10; 1)$ nuqtadan o‘tib, abssissalar o‘qidan $a=3$ va applikatalar o‘qidan $c=2$ kesmalar ajratadi. Bu tekislikning „kesmalardagi“ tenglamasini tuzing.

427. $M_1(4; 3; 2)$ nuqtadan o‘tib, koordinata o‘qlari bilan noldan farqli, bir xil uzunlikdagi kesmalarni ajratuvchi tekislik tenglamasini tuzing.

428. Oz o‘qidan $c=-5$ kesma ajratuvchi va $\bar{n}(-2; 1; 3)$ vektorga perpendikular bo‘lgan tekislik tenglamasini tuzing.

* * *

429. Oz o‘qi va $M(1; -2; 1)$ nuqtadan o‘tuvchi tekislik tenglamasini yozing.

430. $M(2; 3; -4)$ nuqtadan o‘tib, Oyz tekislikka parallel bo‘lgan tekislik tenglamasini tuzing.

431. Tekislik $P(3; 8; -4)$ nuqtadan o‘tib, abssissalar o‘qidan $a=-3$, applikatalar o‘qidan $c=2$ kesma ajratadi. Bu tekislik tenglamasini tuzing.

432. $x+2y-3z+2=0$ tekislik va koordinata tekisliklari bilan chegaralangan piramidaning hajmini topmg.

433. Koordinatalar boshidan tekislikka tushirilgan perpendikularning asosi $M(2; -1; 2)$ nuqtada. Bu tekislik tenglamasini toping.

434. Oz o‘qqa parallel liamda $M_1(2; 2; 0)$ va $M_2(4; 0; 0)$ nuqtalardan o‘tuvchi tekislik tenglamasini yozing.

435. $M(1; -3; 5)$ nuqtadan o‘tib, Oy va Oz o‘qlardan Ox o‘qdagiga ko‘ra ikki marta katta kesma ajratuvchi tekislik tenglamasini yozing.

2- §. Tekislikka doir asosiy masalalar

1°. Ikki tekislik orasidagi burchak

$$\cos\varphi = \pm\frac{AA_1+BB_1+CC_1}{NN_1} \qquad (1)$$

formuladan topiladi, bunda N va N_1, mos ravishda, $Ax+By+$ $+Cz+D=0$ va $A_1x+B_1y+C_1z+D_1=0$ tekisliklarga normal vektorlar.

Parallellik sharti: $\frac{A}{A_1}=\frac{B}{B_1}=\frac{C}{C_1}$. (2)

Perpendikulyarlik sharti: $AA_1+BB_1+CC_1=0$. (3)

2°. $M_0(x_0;\ y_0;z_0)$ nuqtadan $Ax+By+Cz+D=0$ tekislikgacha bo'lgan masofa:

$$d=\frac{|Ax_0+By_0+Cz_0+D|}{N}. \qquad (4)$$

3°. Berilgan ikki tekislikning kesishgan chizig'idan o'tuvchi barcha tekisliklar dastasining tenglamasi quyidagicha yoziladi:

$$\alpha(Ax+By+Cz+D)+\beta(A_1x+B_1y+C_1z+D_1)=0, \qquad (5)$$

bu yerda $\alpha=1$ deb olish mumkin, u holda (5) dastadan berilgan tekisliklardan ikkinchisini chiqarib tashlagan bo'lamiz.

436. a) $4x-5y+3z-1=0$ va $x-4y-z+9=0$;
b) $3x-y+2z+15=0$ va $5x+9y-3z-1=0$;
d) $6x+2y-4z+17=0$ va $9x+3y-6z-4=0$;
e) $x+y-1=0$ va $2x-y+\sqrt{3}z+1=0$
tekisliklar orasidagi burchakni toping.

437. Berilgan $M(x;y;z)$ nuqtadan $Ax+By+Cz+D=0$ tekislikgacha bo'lgan masofani toping: 1) $M_1(-2;-4;3)$, $2x-y+2z+3=0$; 2) $M_2(2;-1;-1)$, $16x-12y+15z-4=0$; 3) $M_3(1;2;-3)$, $5x-3y+z+4=0$; 4) $M_4(3;-6;7)$, $4x-3z-1=0$; 5) $M_5(9;2;-2)$, $12y-5z+5=0$.

438. $P(-1;1;-2)$ nuqtadan $M_1(1;-1;1)$, $M_2(-2;1;3)$ va $M_3(4;-5;-2)$ nuqtalardan o'tuvchi tekislikgacha bo'lgan masofani toping.

439. Ikkita $x+2y-2z+2=0$, $3x+6y-6z-4=0$ parallel tekisliklar orasidagi masofani toping.

440. $7x-6y+6z+7=0$ tekislikdan 2 birlik uzoqlikda bo'lgan parallel tekislik tenglamasini toping.

441. $M_0(2;-3;-7)$ nuqtadan o'tib, $2x-6y-3z+5=0$ tekislikka parallel bo'lgan tekislik tenglamasini toping.

442. $M_0(-2;7;3)$ nuqtadan o'tib, $x-4y+5z+1=0$ tekislikka parallel bo'lgan tekislik tenglamasini toping.

443. $M_0(2;-3;1)$ nuqtadan o'tib, $\bar{a}\{-3;2;-1\}$ va $\bar{b}\{1;2;3\}$ vektorlarga parallel bo'lgan tekishk tenglamasini toping.

444. $M_0(2;2;-2)$ nuqtadan o'tib, $3x-2y-z+1=0$ va $x-y-z=0$ tekisliklarning kesishish chizig'iga perpendikular bo'lgan tekislik tenglamasini toping.

445. $M_1(2;\ -1;3)$, $M_2(3;\ 1;\ 2)$ nuqtalardan o'tib, $3x-y-4z=0$ tekislikka perpendikular bo'lgan tekislik tenglamasini toping.

446. $M_1(3;\ -1;2)$, $M_2(4;\ -1;\ -1)$ va $M_3(2;\ 0;\ 2)$ nuqtalardan o'tuvchi tekislik tenglamasini toping.

447. $A(0;0;2)$, $B(3;\ 0;\ 5)$, $C(1;\ 1;\ 0)$ va $D(4;\ 1;\ 2)$ — tetraedr uchlarining koordinatalari. Uning yoqlarining tenglamalarini toping.

* * *

448. $x-y+\sqrt{2}\,z-5=0$ va (yz) tekisliklar orasidagi burchakni toping.

449. $P(3;-6;2)$ nuqta — koordinatalar boshidan tekislikka tushirilgan perpendikularning asosi. Shu tekislik tenglamasini toping.

450. a) $M_1(3;1;-1)$ nuqtadan $22x+4y-20z-45=0$ tekislikgacha;

b) $M_2(4;3;-2)$ nuqtadan $3x-y+5z+1=0$ tekislikgacha;

d) $M_3(2;0;-\frac{1}{2})$ nuqtadan $4x-4y+2z+17=0$ tekislikgacha bo'lgan masofani hisoblang.

451. Piramidaning $S(0;6;4)$, $A(3;5;3)$, $B(-2;11;-5)$, $C(1;-1;\ 4)$ uchlari berilgan. Uning (h_s) balandligini hisoblang.

452. $A(1;3;\ -2)$ va $B(7;-4;4)$ nuqtalar berilgan. AB kesmaga perpendikular bo'lgan va B nuqtadan o'tuvchi tekislik tenglamasini toping.

453. 1) $(2;-5;3)$ nuqtadan o'tib, (xz) tekisligiga parallel bo'lgan;

2) z o'qi va $(-3;1;-2)$ nuqtadan o'tuvchi;

3) $(4;0;-2)$ va $(5;1;7)$ nuqtalardan o'tib, Ox o'qiga parallel bo'lgan tekislik tenglamasini toping.

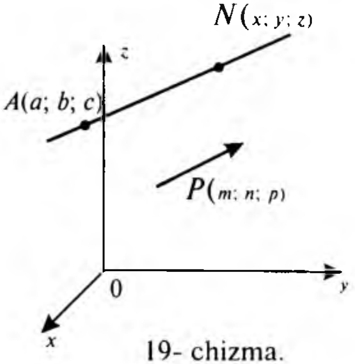
454. $4x-y+3z-6=0$ va $x+5y-z+10=0$ tekisliklarning kesishish chizig'idan o'tuvchi va $2x-y+5z-5=0$ tekislikka perpendikular bo'lgan tekislik tenglamasini yozing.

3- §. To‘g‘ri chiziq tenglamasi

1°. $A(a;b;c)$ nuqtadan o‘tuvchi va $\overrightarrow{P}\{m;n;p\}$ vektorga parallel bo‘lgan to‘g‘ri chiziq tenglamalari. $N(x;y;z)$ — to‘g‘ri chiziqning ixtiyoriy nuqtasi bo‘lsin (19- chizma). U holda $\overrightarrow{AN} \parallel \overrightarrow{P}$ va ikki vektorning parallellik shartiga ko‘ra:

$$\frac{x-a}{m}=\frac{y-b}{n}=\frac{z-c}{p}. \qquad (1)$$

(1) tenglamalar to‘g‘ri chiziqning *kanonik tenglamasi* deyiladi.



19- chizma.

2°. (1) tenglamadagi har bir nisbatni t parametrga tenglab, to‘g‘ri chiziqning

$$\begin{cases} x = mt + a, \\ y = nt + b, \\ z = pt + c \end{cases} \qquad (2)$$

ko‘rinishdagi *parametrik tenglamalariga* ega bo‘lamiz.

3°. Ikki nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasi:

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}. \qquad (3)$$

4°. To‘g‘ri chiziqning umumiy tenglamasi:

$$\begin{cases} Ax + By + Cz + D = 0, \\ A_1x + B_1y + C_1z + D_1 = 0. \end{cases} \qquad (4)$$

5°. (4) tenglamalardan bir marta y ni, ikkinchi marta x ni yo‘qotib, **to‘g‘ri chiziqning proyeksiyalari bo‘yicha yozilgan tenglamalariga** ega bo‘lamiz:

$$\begin{cases} x = mz + a, \\ y = nz + b. \end{cases} \qquad (5)$$

(5) tenglamalarni ushbu $\frac{x-a}{m}=\frac{y-b}{n}=\frac{z-0}{1}$ kanonik ko‘rinishda yozish mumkin.

455. $M(-1;1;-3)$ nuqtadan o‘tib, $\overline{S}\{1;-3;4\}$ vektorga parallel bo‘lgan to‘g‘ri chiziqning kanonik va parametrik tenglamalarini yozing.

456. $M_1(2;-1;-1)$ va $M_2(3;3;-1)$ nuqtalardan o‘tuvchi to‘g‘ri chiziqning kanonik va parametrik tenglamalarini yozing.

457. $M(-1;-2;2)$ nuqtadan o‘tib, Ox o‘qiga parallel bo‘lgan to‘g‘ri chiziqning kanonik va parametrik tenglamalarini yozing.

458. $M(1;-5;3)$ nuqtadan o‘tib, koordinata o‘qlari bilan $\alpha=\frac{\pi}{4}$, $\beta=\frac{\pi}{3}$, $\gamma=\frac{2\pi}{3}$ burchaklar tashkil qiluvchi to‘g‘ri chiziqning kanonik tenglamalarini yozing.

459. Uchburchakning $A(-5;7;1)$, $B(2;4;-1)$, $C(-1;3;5)$ uchlari berilgan. B uchidan AC tomonga tushirilgan medziananing kanonik tenglamasini toping.

460. Uchburchakning $A(1;-1;3)$, $B(3;-4;9)$, $C(-5;11;7)$ uchlari berilgan. A uchidan tushirilgan bissektrisasining kanonik tenglamasini toping.

461. 1) $\begin{cases} x-3y+2=0, \\ 2y-z+1=0. \end{cases}$ 2) $\begin{cases} 2x-3y-3z=0, \\ x-2y+z+3=0. \end{cases}$

3) $\begin{cases} x-2y+3z+1=0, \\ 2x+y-4z-8=0 \end{cases}$

to‘g‘ri chiziq tenglamalarini kanonik ko‘rinishga keltiring.

462. $M_0(1;-3;5)$ nuqtadan o‘tib, $\begin{cases} 3x-y+2z+7+0, \\ x+3y-2z+3=0 \end{cases}$ to‘g‘ri chiziqqa parallel to‘g‘ri chiziqning kanonik tenglamasini toping.

463. $M(2;1;-1)$ nuqtadan o‘tib, $x-y+z+1=0$ tekislikka perpendikular bo‘lgan to‘g‘ri chiziqning kanonik tenglamasini toping.

464. $\begin{cases} 3x-4y-2x=0, \\ 2x+y-2z+1=0 \end{cases}$ va $\begin{cases} 4x+y-6y-2=0, \\ y-3z-2=0 \end{cases}$ to‘g‘ri chiziqlar orasidagi burchakni toping.

465. 1) $\frac{x+3}{1}=\frac{y+2}{-1}=\frac{z}{\sqrt{2}}$ va $\frac{x+2}{1}=\frac{y-3}{1}=\frac{z-5}{\sqrt{5}}$;

2) $\begin{cases} x+3z-7=0, \\ y=0 \end{cases}$ va $\begin{cases} x-2z-5=0, \\ y=0 \end{cases}$ to‘g‘ri chiziqlar orasidagi burchakni toping.

466. Quyidagi to‘g‘ri chiziqlarning perpendikularligini ko‘rsating:

1) $\frac{x}{1}=\frac{y-1}{-2}=\frac{z}{3}$ va $\begin{cases} 3x+y-5z+1=0, \\ 2x+3y-8z+3=0; \end{cases}$

2) $\begin{cases} 2x + y - 4z + 2 = 0, \\ 4x - y - 5z + 4 = 0 \end{cases}$ va $\begin{cases} x = 1 + 2t, \\ y = -2 + 3t, \\ z = 1 - 6t. \end{cases}$

467. $\frac{x+2}{3} = \frac{y-1}{-2} = \frac{z}{1}$ va $\begin{cases} x + y + z = 0, \\ x - y - 5z - 8 = 0 \end{cases}$ to‘g‘ri chiziqlarning parallelligini ko‘rsating.

468. $\frac{x+1}{2} = \frac{y}{-1} = \frac{z-2}{3}$ to‘g‘ri chiziqqa $A(2;3;1)$ nuqtadan o‘tkazilgan perpendikularning tenglamasini yozing.

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469. $\begin{cases} x - 2y + 3z - 4 = 0, \\ 3x + 2y - 5z - 4 = 0 \end{cases}$ to‘g‘ri chiziq tenglamalarini: 1) proyeksiyalar bo‘yicha; 2) kanonik ko‘rinishda yozing.

470. Berilgan ikki nuqtadan o‘tuvchi to‘g‘ri chiziqning kanonik tenglamasini tuzing: 1) (1;–2;1) va (3;1;–1); 2)(3;–1;0) va (1;0;3); 3) (0;–2;3) va (3;–2;1); 4) (–1;2;–4) va (0;2;–4).

471. $M_1(-1;1;3)$ nuqtadan o‘tib, 1) $\vec{a}\{2;-2;4\}$ vektorga; 2) $\frac{x-1}{2} = \frac{y+2}{4} = \frac{z-1}{0}$ to‘g‘ri chiziqqa; 3) $x=3t-1$; $y=-2t+3$; $z = 5t + 2$ to‘g‘ri chiziqqa parallel bo‘lgan to‘g‘ri chiziqning parametrik tenglamasini tuzing.

472. $M_1(-6;6;-5)$ va $M_2(12;-6;1)$ nuqtalardan o‘tuvchi to‘g‘ri chiziqning koordinata o‘qlari bilan kesishgan nuqtalarini toping.

473. Quyidagi to‘g‘ri chiziqlarning parallelligini ko‘rsating:

1) $\frac{x+2}{3} = \frac{y-1}{-2} = \frac{z}{1}$ va $\begin{cases} x + y - z = 0, \\ x - y - 5z - 8 = 0; \end{cases}$

2) $\begin{cases} x = 2t + 5, \\ y = -t + 2, \\ z = t - 7 \end{cases}$ va $\begin{cases} x + 3y + z + 2 = 0, \\ x - y - 3z - 2 = 0; \end{cases}$

3) $\begin{cases} x + y - 3z + 1 = 0, \\ x - y + z + 3 = 0 \end{cases}$ va $\begin{cases} x + 2y - 5z - 1 = 0, \\ x - 2y + 3z - 9 = 0. \end{cases}$

474. Quyidagi to‘g‘ri chiziqlarning perpendikularligini ko‘rsating:

1) $\begin{cases} x + y - 3z - 1 = 0, \\ 2x - y - 9z - 2 = 0 \end{cases}$ va $\begin{cases} 3x + y - 5z + 1 = 0, \\ 2x + 3y - 8z + 3 = 0; \end{cases}$

2) $\begin{cases} x = 2t + 1, \\ y = 3t - 2, \\ z = -6t + 1 \end{cases}$ va $\begin{cases} 2x + y - 4z + 2 = 0, \\ 4x - y - 5z + 4 = 0; \end{cases}$

3) $\begin{cases} x + y - 3z - 1 = 0, \\ 2x - y - 9z - 2 = 0 \end{cases}$ va $\begin{cases} 2x + y + 2z + 5 = 0, \\ 2x - 2y - z + 2 = 0. \end{cases}$

4- §. To‘g‘ri chiziq va tekislik

1°. $\frac{x-a}{m} = \frac{y-b}{n} = \frac{z-c}{p}$ to‘g‘ri chiziq bilan $Ax+By+Cz+D=0$ tekislik orasidagi burchak:

$$\sin\theta = \frac{|\overline{NP}|}{NP} = \frac{|Am+Bn+Cp|}{NP}. \quad (1)$$

Ularning parallellik sharti: $(\vec{N} \parallel \vec{P}): Am+Bn+Cp=0.$ (2)

Ularning perpendikularlik sharti: $\left(\bar{N} \perp \vec{P}\right): \frac{A}{m} = \frac{B}{n} = \frac{C}{p}.$ (3)

2°. Tekislik hilan to‘g‘ri chiziqning kesishgan nuqtasi. To‘g‘ri chiziq tenglamalari: $x=mt+a$, $y=nt+b$, $z=pt+c$ ni parametrik ko‘rinishda yozib, tekislikning $Ax+By+Cz+D=0$ tenglamasidagi x, y, z larning o‘rniga ularning t ga nisbatan yozilgan qiymatlarini qo‘yamiz. Hosil bo‘lgan tenglamadan t_0 ni, so‘ngra kesishgan nuqta koordinatalari x_0, y_0, z_0 larni topamiz.

3°. Ikki to‘g‘ri chiziqning bir tekislikda yotish sharti:

$$\begin{vmatrix} a-a_1 & b-b_1 & c-c_1 \\ m & n & p \\ m_1 & n_1 & p_1 \end{vmatrix} = 0. \quad (4)$$

475. $\frac{x-1}{4} = \frac{y}{12} = \frac{z-1}{-3}$ to‘g‘ri chiziq bilan $6x-3y+2z=0$ tekislik orasidagi burchakni hisoblang.

476. $A(-1;0;-5)$ va $B(1;2;0)$ nuqtalardan o‘tuvchi to‘g‘ri chiziq bilan $x-3y+z+5=0$ tekislik orasidagi burchakni toping.

477. $M_0(3;-2;4)$ nuqtadan o‘tib, $5x+3y-7z+1=0$ tekislikka perpendikular bo‘lgan to‘g‘ri chiziq tenglamasini tuzing.

478. $x=3t-2$, $y=-4t+1$, $z=4t-5$ to‘g‘ri chiziqning $4x-3y-6z-5=0$ tekislikka parallelligini ko‘rsating.

479. $\begin{cases} 5x - 3y + 2z - 5 = 0, \\ 2x - y - z - 1 = 0 \end{cases}$ to‘g‘ri chiziqning $4x–3y+7z–7=0$ tekislikda yotishini isbot qiling.

480. To‘g‘ri chiziq bilan tekislikning kesishish nuqtasini toping:

1) $\frac{x-1}{1} = \frac{y+1}{-2} = \frac{z}{6}$, $2x + 3y + z - 1 = 0$;

2) $\frac{x+3}{3} = \frac{y-2}{-1} = \frac{z+1}{-5}$, $x - 2y + z - 15 = 0$;

3) $\frac{x+2}{-2} = \frac{y-1}{3} = \frac{z-3}{2}$, $x + 2y - 2z + 6 = 0$.

481. $M_0(2;–3;–5)$ nuqtadan o‘tib, $6x–3y–5z+2=0$ tekislikka perpendikular bo‘lgan to‘g‘ri chiziq tenglamasini yozing.

482. $M_0(1;–1;–1)$ nuqtadan o‘tib, $\frac{x+3}{2} = \frac{y-1}{-3} = \frac{z+2}{4}$ to‘g‘ri chiziqqa perpendikular bo‘lgan tekislik tenglamasini yozing.

483. $M_0(1;–2;1)$ nuqtadan o‘tib, $\begin{cases} x - 2y + z - 3 = 0, \\ x + y - z + 2 = 0 \end{cases}$ to‘g‘ri chiziqqa perpendikular bo‘lgan tekislik tenglamasini yozing.

484. n ning qanday qiymatida $\frac{x+1}{3} = \frac{y-2}{n} = \frac{z+3}{-2}$ to‘g‘ri chiziq $x–3y+6z+7=0$ tekislikka parallel bo‘ladi?

485. C ning qanday qiymatida $\begin{cases} 3x - 2y + z + 3 = 0, \\ 4x - 3y + 4z + 1 = 0 \end{cases}$ to‘g‘ri chiziq $2x–y+Cz–2=0$ tekislikka parallel bo‘ladi?

486. A va D ning qanday qiymatida $x = 3 + 4t$, $y = 1–4t$, $z = –3+7t$ to‘g‘ri chiziq $Ax+2y–4z+D=0$ tekislikda yotadi?

487. A va B ning qanday qiymatida $Ax+By+3z–5=0$ tekislik $x = 3+2t$, $y = 5–3t$, $z = –2–2t$ to‘g‘ri chiziqqa perpendikular bo‘ladi?

488. m va C ning qanday qiymatida $\frac{x-2}{m} = \frac{y+1}{4} = \frac{z-5}{-3}$ to‘g‘ri chiziq $3x–2y+Cz+1=0$ tekislikka perpendikular bo‘ladi?

489. $A(4;–3;1)$ nuqtaning $x+2y–z–3=0$ tekislikdagi proyeksiyasini toping.

490. $A(1;2;1)$ nuqtaning $\frac{x+2}{3} = \frac{y}{-1} = \frac{z-1}{2}$ to‘g‘ri chiziqdagi proyeksiyasini toping.

491. $P(2;–1;3)$ nuqtaning $x = 3t$, $y = 5t–7$, $z = 2t+2$ to‘g‘ri chiziqdagi proyeksiyasini toping.

492. $P(4;1;6)$ nuqtaga $\begin{cases} x - y - 4z + 12 = 0, \\ 2x + y - 2z + 3 = 0 \end{cases}$ to'g'ri chiziqqa nisbatan simmetrik nuqtani toping.

493. $P(2;-5;7)$ nuqtaga $M_1(5;4;6)$ va $M_2(-2;-17;\ -8)$ nuqtalardan o'tuvchi to'g'ri chiziqqa nisbatan simmetrik nuqtani toping.

494. $P(5;2;-1)$ nuqtaning $2x-y+3z+23=0$ tekislikdagi proyeksiyasini toping.

495. $M(3;1;-2)$ nuqtadan va $\frac{x-4}{5} = \frac{y+3}{2} = \frac{z}{1}$ to'g'ri chiziqdan o'tuvchi tekislik tenglamasini yozing.

496. $\frac{x-2}{5} = \frac{y-3}{1} = \frac{z+1}{2}$ to'g'ri chiziqdan $x+4y-3z+7=0$ tekislikka perpendikular tekislik o'tkazing.

497. $\frac{x}{4} = \frac{y-4}{3} = \frac{z+1}{-2}$ to'g'ri chiziqning $x-y+3z+8=0$ tekislikdagi proyeksiyasini toping.

498. $\frac{x-3}{5} = \frac{y+1}{2} = \frac{z-2}{4}$ va $\frac{x-8}{3} = \frac{y-1}{1} = \frac{z-6}{-2}$ to'g'ri chiziqlarning kesishuvchi ekanligini ko'rsating, ular orqali o'tgan tekislik tenglamasini yozing.

499. $M(-3;2;5)$ nuqtadan $4x+y-3z+13=0$ va $x-2y+z-11=0$ tekisliklarga tushirilgan perpendikular orqali tekislik o'tkazing.

500. $\frac{x}{7} = \frac{y+2}{3} = \frac{z-1}{2}$ va $\frac{x-1}{7} = \frac{y-3}{3} = \frac{z+2}{5}$ parallel to'g'ri chiziqlardan o'tuvchi tekislik tenglamasini yozing.

501. $P(7;9;7)$ nuqtadan $\frac{x-2}{4} = \frac{y-1}{3} = \frac{z}{2}$ to'g'ri chiziqqacha bo'lgan masofani toping.

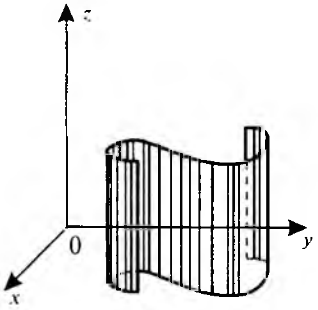
502. $\frac{x-3}{3} = \frac{y+1}{4} = \frac{z}{2}$ va $\frac{x-7}{3} = \frac{y-1}{4} = \frac{z-3}{2}$ parallel to'g'ri chiziqlar orasidagi masofani toping.

503. $\frac{x+3}{4} = \frac{y-6}{-3} = \frac{z-3}{2}$ va $\frac{x-4}{8} = \frac{y+1}{-3} = \frac{z+4}{3}$ to'g'ri chiziqlar orasidagi masofani toping.

5- §. Sirtlar

1. Silindrik sirtlar

Berilgan L chiziqni kesib o'tib, berilgan l to'g'ri chiziqqa parallel bo'lgan to'g'ri chiziqlar hosil qilgan sirtga *silindrik sirt* deyiladi. L chiziq uning *yonaltiruvchisi*, harakatlanayotgan to'g'ri chiziqning



20- chizma.

har bir holati — *tashkil etuvchi* deyiladi. Har doim l to'g'ri chiziqni koordinatalar sistemasining biror o'qi bilan ustma-ust tushadigan qilib tanlash mumkin. Tashkil etuvchilari Oz o'qiga parallel bo'lgan silindrik sirtning (20-chizma) tenglamasida z qatnashmaydi, ya'ni

$$F(x,y)=0. \qquad (1)$$

Tashkil etuvchilari Ox va Oy o'qlariga parallel bo'lgan silindrik sirtlarning tenglamalari mos ravishda

$$F(y,z)=0 \qquad (2)$$

va

$$F(x,z)=0 \qquad (3)$$

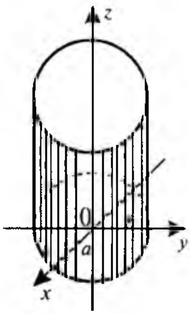
ko'rinishda bo'ladi:

(1) sirtning xOy tekkislik bilan kesishgan chiziqni uning L yo'naltiruvchisi sifatida olish mumkin. U holda tashkil etuvchining tenglamasi:

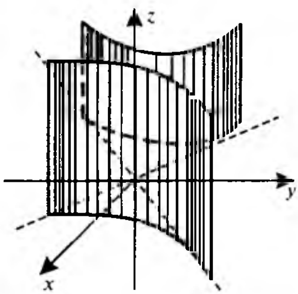
$$L\begin{cases} F(x,y)=0, \\ z=0 \end{cases} \text{va} \quad L\begin{cases} F(x,z)=0, \\ y=0. \end{cases}$$

Agar (1) tenglama ikkinchi darajali algebraik tenglama bo'lsa, unga mos keladigan silindrik sirt *ikkinchi tartibli silindr* deyiladi. 21, 22, 23- chizmalarda *elliptik, giperbolik, parabolik silindrlar* tasvirlangan, ularning tenglamalari:

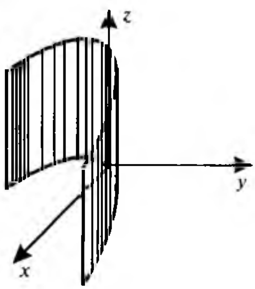
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, \quad \frac{x^2}{a^2}-\frac{y^2}{b^2}=1, \quad y^2=2px.$$



21- chizma.



22- chizma.



23- chizma.

Ba'zi hollarda (1) tenglama ikkita chiziqli ko'paytuvchiga ajraladi. Bu holda silindr Oz o'qqa parallel bo'lgan ikkita tekislikdan iborat bo'ladi.

Fazoda ikkita sirtning kesishuvidan hosil bo'lgan L chiziq berilgan bo'lsin:

$$L \begin{cases} F_1(x, y, z) = 0, \\ F_2(x, y, z) = 0. \end{cases} \qquad (4)$$

Bu tenglamalardan z ni yo'qotsak, $F(x,y)=0$ tenglama hosil bo'ladi. Bu tenglama L chiziqni xOy tekislikka proyeksiyalaydigan sirtning tenglamasi. L chiziqning xOz (yoki yOz) tekisliklarga proyeksiyalaydigan sirtning tenglamalarini topish uchun (4) tenglamadan y yoki x ni yo'qotiladi.

Quyidagi tenglamalar qanday sirtlarni aniqlaydi (chizmasini chizing):

504. $x^2+z^2=9$.

505. $16y^2-25z^2=400$.

506. $y^2=-6z$.

507. $x^2=z^2$.

508. $xz=2$.

509. $x^2+y^2=2ax$.

510. $z^2+4z-2x+6=0$.

511. $4x^2+9y^2-8x+36y+4=0$.

512. $x^2+y^2=4y$.

513. $z^2+2z-4x+1=0$ tenglama qanday sirtni ifodalaydi?

514. $9y^2-16z^2+64z-18y-199=0$ tenglama qanday sirtni ifodalaydi?

515. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$ tenglama qanday sirtni ifodalaydi?

516. L chiziq $x^2+z^2=R^2$, $y^2+z^2=r^2$ $(R>r)$ tenglamalar bilan berilgan. Bu chiziqning xOy tekislikdagi proyeksiyasini toping.

517. $L \begin{cases} 9y^2 - 6xy - 2xz + 24x - 9y - 3z - 63 = 0, \\ 2x - 3y + z - 9 = 0 \end{cases}$

chiziqning xOy tekislikdagi proyeksiyasini toping.

518. O'qi Oy o'qqa parallel va $P(1;2;-1)$ nuqtadan o'tuvchi, radiusi 3 ga teng bo'lgan doiraviy silindrning tenglamasini tuzing.

519. $x^2+y^2=9$, $x-z=0$ tenglamalar sistemasi fazoda qanday chiziqni ifodalaydi?

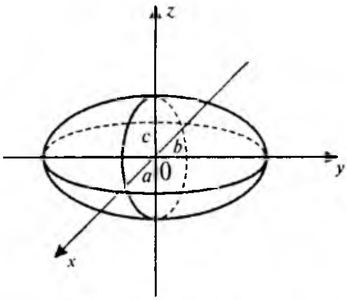
2. Ikkinchi tartibli sirtlarning kanonik tenglamalari

Ikkinchi tartibli sirtlar dekart koordinata sistemasida ikkinchi darajali tenglama bilan yoziladi:

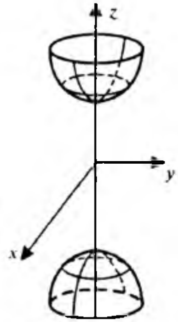
$$Ax^2+By^2+Cz^2+2Dxy+2Exz+2Fyz+2Gx+2Hy+2Kz+L=0. \qquad (5)$$

Maxsus koordinatalar sistemasini tanlash bilan bu tenglama sodda kanonik ko'rinishga keltiriladi.

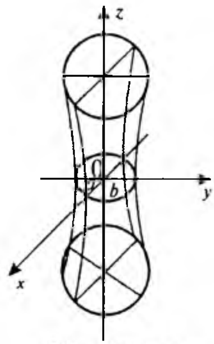
1- bandda ko'rilgan ikkinchi tartibli silindrik sirtdan tashqari quyidagi sirtlar ham ikkinchi tartibli sirtlarga kiradi:



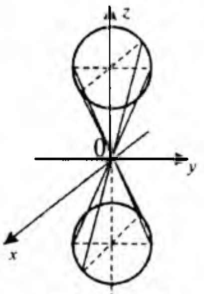
24- chizma.



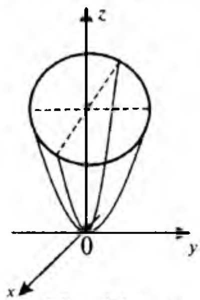
25- chizma.



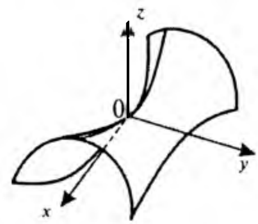
26- chizma.



27- chizma.



28- chizma.



29- chizma.

$x^2+y^2+z^2=R^2$ — sfera;

$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ — ellipsoid (24- chizma);

$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$ — ikki kovakli giperboloid (25- chizma);

$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ — bir kovakli giperboloid (26- chizma);

$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$ — konus (27- chizma);

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=2z$ — elliptik paraboloid (28- chizma);

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=2z$ —giperbolik paraboloid (29- chizma).

Agar (5) tenglamaning o‘ng tarafi ikkita chiziqli ko‘paytuvchilarga ajralsa, juft tekisliklar bo‘ladi.

3. Sfera. Sferaning umumiy tenglamasi (5) tenglamada $A=B=C\neq 0$, $D=E=F=0$ qiymatlarda hosil qilinadi: $Ax^2+Ay^2+Az^2+2Cx+2Hy+$ $+2Kz+L=0$, bu tenglamani A koeffitsiyentga bo‘lib, x, y, z larga nisbatan to‘la kvadratlar ajratiladi: $(x-a)^2+(y-b)^2+(z-c)^2=R^2$, bu yerda R — sfera radiusi, $(a;\ b;\ c)$ — markazining koordinatalari. Xususiy holda, markazi koordinatalar boshida bo'lsa, $x^2+y^2+z^2=R^2$— sferaning sodda tenglamasini olamiz.

520. $M(-2;3;6)$ nuqtadan o'tuvchi, markazi koordinatalar boshida bo'lgan sfera tenglamasini tuzing.

521. $M(4;2;2)$ nuqtalardan o'tuvchi, markazi $C(1;-1;-1)$ nuqtada bo'lgan sferaning tenglamasini tuzing.

522. Markazi $C(1;-1;4)$ nuqtada bo'lgan sfera $2x+y-3z-3=0$ tekislikka urinadi. Sferaning tenglamasini tuzing.

523. Markazi $C(0;4;0)$ nuqtada bo'lgan sfera $2x+6y-3z-3=0$ tekislikka urinadi. Sferaning tenglamasini tuzing.

524. Quyidagi sferalarning markazlari va radiusini toping:

a) $x^2+y^2+z^2-6x+8y+2z+10=0$;

b) $x^2+y^2+z^2+2x-4y-4=0$;

d) $x^2+y^2+z^2-6x+10=0$;

e) $x^2+y^2+z^2-4x+12y-2z+41=0$;

f) $36x^2+36y^2+36z^2-36x+24y-72z-95=0$;

g) $x^2+y^2+z^2-2x+4y-4z-7=0$.

525. Biror diametrining uchlari $A(2;5;-7)$ va $B(6;-1;3)$ nuqtalarda bo'lgan sfera tenglamasini tuzing.

526. $M_1(1;-2;-1)$, $M_2(-5;10;-1)$, $M_3(-8;-2;2)$ nuqtalardan o'tuvchi, radiusi $R=9$ bo'lgan sfera tenglamasini tuzing.

4. Parallel kesimlar usuli. Biror sirtning tenglamasi berilgan bo'lsa, bu sirtni koordinata o'qlariga nisbatan joylashtirish va ko'rinishlarini tekshirish masalalari kelib chiqadi. Bu masalalarni hal etish uchun parallel kesimlar usuli qo'llaniladi, ya'ni sirt koordinata tekisliklariga parallel bo'lgan bir nechta tekisliklar bilan kesiladi. Hosil bo'lgan kesimlarning ko'rinishi va o'lchamlariga nisbatan sirtning ko'rinishi kelib chiqadi.

527. Koordinatalar sistemasiga nisbatan $4-z=x^2+y^2$ ning joylashishi va ko'rinishini tekshiring.

Quyidagi sirtlarni kesimlar usuli bilan koordinatalar sistemasiga nisbatan joylashishi va ko'rinishini aniqlang (chizmasini chizing).

528. $\frac{x^2}{4}+\frac{y^2}{9}-\frac{z^2}{4}=1$.

529. $\frac{x^2}{1}+\frac{y^2}{2}-\frac{z^2}{4}=-1$.

530. $\frac{x^2}{4}+\frac{y^2}{1}-\frac{z^2}{2}=0$.

532. $x^2+y^2=2(z-1)^2$.

533. $2y^2+z^2=1-x$.

534. $3x^2+y^2-z^2=3$.

531. $\frac{x^2}{3}+\frac{y^2}{1}=2z.$ **535**. $x^2-2y^2+z^2=1.$

536. $x^2-y^2=2z.$

537. Ushbu:

1) $x^2+y^2+z^2=2az;$ 2) $x^2+y^2=2az;$

3) $x^2+y^2=2az;$ 4) $x^2-y^2=2az;$

5) $x^2-y^2=z^2;$ 6) $x^2=2az;$

7) $x^2=2yz;$ 8) $z=2+x^2+y^2;$

9) $(z-a)^2=xy;$ 10) $(z-2x)^2+4(z-2x)=y^2$

sirtlardan har birining nomini aniqlang va ularni yasang.

5. Boshqa masalalar. Agar (5) tenglamada koordinatalarga ko‘paytirilgan hadlar ($D=E=F=0$) qatnashmasa, u holda (5) ni x,y,z larga nisbatan to‘liq kvadrat ajratish va parallel ko‘chirish yordamida kanonik ko‘rinishga keltiriladi.

Ikkinchi tartibli sirtlarning to‘g‘ri chiziq bilan kesishish nuqtasi quyidagicha topiladi: to‘g‘ri chiziq tenglamasi $x=a+mt,\ y=b+nt,\ z=c+pt$ parametrik ko‘rinishga keltiriladi va x, y, z larning qiymatlari sirt tenglamasiga qo‘yiladi. Hosil bo‘lgan kvadrat tenglamadan t ni topamiz. Agar bu tenglamaning ildizi musbat bo‘lsa, to‘g‘ri chiziq sirtga urinma bo‘ladi, agar ildiz manfiy bo‘lsa, kesishish nuqtasi yo‘q.

538. $2x^2-y^2+2z^2+4x+2y+8z+1=0$ tenglama qanday sirtni aniqlaydi?

539. $x^2+y^2+4z^2=2$ ellipsoid bilan $\frac{x-1}{1}=\frac{y-1}{1}=\frac{z}{\frac{\sqrt{2}}{2}}$ to‘g‘ri chiziqning kesishish nuqtasini toping.

540. Ushbu tenglamalar qanday sirtni aniqlaydi?

1) $2x^2+y^2+z^2-4x+4y+4z+7=0;$

2) $x^2-6y^2+3z^2+8x+12y+1=0;$

3) $x^2+y^2+2x-2y-2z-2=0.$

541. $\frac{x-4}{2}=\frac{y+6}{-3}=\frac{z+2}{-2}$ to‘g‘ri chiziq va $\frac{x^2}{16}+\frac{y^2}{12}+\frac{z^2}{4}=1$ ellipsoid qaysi nuqtalarda kesishishadi?

542. $\frac{x-4}{4}=\frac{y+3}{3}=\frac{z-2}{2}$ to‘g‘ri chiziq va $\frac{x^2}{16}+\frac{y^2}{9}-\frac{z^2}{4}=1$ bir kovakli giperboloidning kesishish nuqtalarini toping.

V bob javoblari

420. a) $z-3=0$; b) $x+5=0$. **421.** 1) $2y+z=0$; 2) $3x+z=0$; 3) $4x+3y=0$. **422.** 1) $y+4z+10=0$; 2) $x-y-1=0$; 3) $5x+y-13=0$. **423.** (12; 0; 0), (0; −8; 0), (0; 0; 6). **424.** 240 kv bir. **425.** 8 kv bir. **426.** $\frac{x}{-3}=\frac{y}{-4}=\frac{z}{2}$. **427.** $x+y+z-9=0$, $x-y-z+1=0$, $x-y+z-3=0$, $x+y-z-5=0$. **428.** $2x-y-3z-15=0$. **429.** $2x+y=0$. **430.** $x-2=0$. **431.** $2x-3y-3z+6=0$. **432.** $V=\frac{2}{9}$. **433.** $2x-y+2z-9=0$. **434.** $x+y=4$. **435.** $\frac{x}{2}+\frac{y}{4}+\frac{z}{4}=1$. **436.** a) $\varphi=\arccos 0{,}7$; b) $\varphi=\frac{\pi}{2}$; d) $\varphi=0$; e) $\varphi=\arccos\frac{1}{4}$. **437.** 1) $d=3$; 2) $d=1$; 3) $d=0$; M_3 nuqta tekislikda yotadi; 4) $d=2$; 5) $d=3$. **438.** $d=4$. **439.** $d=\frac{10}{3}$. **440.** $7x-6y+6z-15=0$ yoki $7x-6y+6z+29=0$. **441.** $2x-6y-3z-43=0$. **442.** $x-4y+5z+15=0$. **443.** $x+y-z+2=0$. **444.** $x+2y-z-8=0$. **445.** $9x-y+7z-40=0$. **446.** $3x+3y+z-8=0$. **447.** $x-3y-z+2=0$ (ABC); $x-4y-z+2=0$ (ABD); $2x-8y-3z+6=0$ (ACD); $2x-11y-3z+9=0$ (BCD). **448.** $\alpha=\frac{\pi}{3}$. **449.** $3x-6y+2z-49=0$. **450.** a) $d=\frac{3}{4}$; b) d=0, nuqta tekislikda yotadi; d) $d=4$. **451.** $h_s=3$. **452.** $6x-7y+6z-94=0$. **453.** a) $y+5=0$; b) $x+3y=0$; d) $9y-z-2=0$. **454.** $7x+14y+24=0$. **455.** $\frac{x+1}{1}=\frac{y-1}{-3}=\frac{z+3}{4}$; $x=-1+t$, $y=1-3t$, $z=-3+4t$. **456.** $\frac{x-2}{1}=\frac{y+1}{4}=\frac{z+1}{0}$; $x=2+t$, $y=-1+4i$, $z=-1$. **457.** $\frac{x+1}{1}=\frac{y+2}{0}=\frac{z-2}{0}$; $x=-1+t$, $y=-2$, $z=2$. **458.** $\frac{x-1}{\sqrt{2}}=\frac{y+5}{1}=\frac{z-3}{1}$. **459.** $\frac{x-2}{-5}=\frac{y-4}{1}=\frac{z+1}{4}$. **460.** $\frac{x-1}{-1}=\frac{y+1}{3}=\frac{z-3}{8}$.

461. 1) $\frac{x+2}{3}=\frac{y}{1}=\frac{z-1}{2}$; 2) $\frac{x}{9}=\frac{y}{5}=\frac{z+3}{1}$; 3) $\frac{x-2}{1}=\frac{y}{2}=\frac{z+1}{1}$. **462.** $\frac{x-1}{-2}=\frac{y+3}{4}=\frac{z-5}{5}$. **463.** $\frac{x-2}{1}=\frac{y-1}{-1}=\frac{z+1}{1}$. **464.** 1) $\varphi=\frac{\pi}{3}$; 2) $\varphi=\frac{3\pi}{4}$.

465. $\varphi \approx 60^0$. **469**. $x=2+2t,\ y=-1+7t,\ z=4t;\ \frac{x-2}{2}=\frac{y+1}{7}=\frac{z}{4}$. **470**. 1) $\frac{x-1}{2}=\frac{y+2}{3}=\frac{z-1}{-2}$; 2) $\frac{x-3}{2}=\frac{y+1}{-1}=\frac{z}{3}$; 3) $\frac{x}{3}=\frac{y+2}{0}=\frac{z-3}{-2}$; 4) $\frac{x+1}{1}=\frac{y-2}{0}=\frac{z+4}{0}$. **471**. 1) $x=2t+1,\ y=-2t-1,\ z=4t-3$; 2) $x=2t+1,\ y=4t-1,\ z=-3$; 3) $x=3t+1,\ y=-2t-1,\ z=5t-3$. **472**. (9; –4; 0), (3; 0; –2), (0; 2; –3). **475**. $Q=\arcsin\frac{18}{91}$. **476**. $Q=\arcsin\frac{1}{11\sqrt{13}}$. **477**. 4) $\frac{x-3}{5}=\frac{y+2}{3}=\frac{z-4}{-7}$. **480**. 1) (2; –3; 6); 2) to‘g‘ri chiziq tekislikka parallel; 3) to‘g‘ri chiziq tekislikda yotadi. **481**. $\frac{x-2}{6}=\frac{y+3}{-3}=\frac{z+5}{-5}$. **482**. $2x-3y+4z-1=0$. **483**. $x+2y+3z=0$. **484**. $m=-3$. **485**. $C=-2$. **486**. $A=3$, $D=-23$. **487**. $A=-3$ $B=4\frac{1}{2}$. **488**. $m=-6$. $C=\frac{3}{2}$. **489**. (5; –1; 0). **490**. $\left(-\frac{5}{13};-\frac{7}{13};-\frac{27}{13}\right)$. **491**. (3; –2; 4). **492**. (2; –3; 2). **493**. (1; 4; –7). **494**. (–5; 1; 0). **495**. $8x-9y-22z-59=0$. **496**. $11x-17y-19z+10=0$. **497**. $\frac{x+9}{7}=\frac{y+1}{4}=\frac{z}{-1}$. **498**. $8x-22y+z-48=0$. **499**. $5x+7y+9z-44=0$. **500**. $17x-13y-16z-10=0$. **501**. $d=\sqrt{22}$. **502**. $d=3$. **503**. $d=13$. **504**. *Oy* o‘qli $R=3$ bo‘lgan doiraviy silindr. **505**. *Ox* o‘qqa parallel bo‘lgan giperbolik silindr. **506**. *Oz* ga qarama-qarshi yo‘nalgan parabolik silindr. **507**. *Oy* o‘qini kesuvchi $x=\pm z$ tekisliklar. **508**. *Oy* o‘qqa parallel bo‘lgan giperbolik silindr. **509**. Markazi $M(a;\ 0;\ 0)$ da, $R=|a|$ bo‘lgan aylana. **510**. $(z+2)^2=2(x-1)$ parabolik silindr. **511**. $\frac{(x-1)^2}{9}+\frac{(y+2)^2}{4}=1$ elliptik silindr. **512**. O‘qlari $x=0$, $y=2$ to‘g‘ri chiziq bilan ustma-ust tushadigan doiraviy silindr. **513**. $\begin{cases}(z+1)^2=4x,\\ y=0\end{cases}$ parabolik silindr. **514**. $\begin{cases}\frac{(y-1)^2}{16}-\frac{(z-1)^2}{9}=1,\\ x=0\end{cases}$ giperbolik silindr.

515. $\frac{x}{a}-\frac{y}{b}=0$; $\frac{x}{a}+\frac{y}{t}=0$ tekisliklar. **516**. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, $z=0$ teng tomonli giperbola. **517**. $\frac{x^2}{9}-\frac{y^2}{4}=1, z=0$ giperbola. **518**. $(x-1)^2+$ $+(z+1)^2=9$. **519**. Ellips. **520.** $x^2+y^2+z^2=49$. **521**. $(x-1)^2+(y+1)^2+(z+1)^2=27$. **522**. $(x-1)^2+(y+1)^2+(z-4)^2=14$. **523**. $x^2+(y-4)^2+z^2=9$. **525**. $(x-4)^2+$ $+(y-2)^2+(z+2)^2=38$. **526**. $(x+2)^2+(y-4)^2+(z-5)^2=81$. **527**. Elliptik paraboloid. **528.** $x=0$ va $y=0$ da giperbola, $z=h$ — eilips. **529**. $x=0$ va $y=0$ da — haqiqiy o‘qi Oy bo‘lgan giperbola, $z=h(|h\geq 2)|)$ da — ellips. **530**. $x=0$ va $y=0$ koordinata boshidan o‘tuvchi ikkita to‘g‘ri chiziq. $z=0$— nuqta, $z=h\neq 0$ ellips. **531**. $x=0$ va $y=0$ — Oz o‘qining musbat yo‘nalishi bo‘yicha yo‘nalgan parabola, $z=0$ da — nuqta, $z=h>0$ ellipslar. **532**. $y=0$ —Oz o‘qining musbat tomoniga yo‘nalgan parabola, $x=0$—Oz o‘qining manfiy tomoniga yo‘nalgan parabola, $z=0$ — ikkita to‘g‘ri chiziq, $z=h\neq 0$—$h>0$ bo‘lganda haqiqiy o‘qi Ox o‘qiga, $h<0$ da o‘qi Oy o‘qiga parallel bo‘lgan giperbolalar. **533**. Uchi (0; 0;1) da bo‘lgan va Oz o‘qi atrofida aylanma konus. **534**. Uchi (1; 0;0)da bo‘lgan va Ox o‘qining manfiy tomonida elliptik paraboloid. **535**. Ox o‘qi atrofida aylanma ikki kovakli giperboloid. **536**. Oy o‘qi atrofida aylanma bir kovakli giperboloid. **537**. 1) markazi (0; 0; a) da va radiusi $R=a$ bo‘lgan sfera; 2) Oz o‘qi atrofida aylanma paraboloid; 3) silindr; 4) giperbolik paraboloid; 5) konus; 6) parabolik silindr; 7) konus; 8) aylanma paraboloid; 9) konus; 10) silindr. **538.** Markazi (−1; 1;−2) da bo‘lgan $\frac{(x+1)^2}{4}-\frac{(y-1)^2}{4}+\frac{(z+2)^2}{4}=1$ paraboloid. **539.** M_1 $(0;0;-\frac{\sqrt{2}}{2})$, $M_2\left(\frac{4}{5};\frac{4}{5};-\frac{\sqrt{2}}{10}\right)$. **540.** 1)Markazi (−2;1;0) bo‘lgan ellipsoid; 3) uchi (−1;1;−2) bo‘lgan aylanma paraboloid. **541.** M_1 (2;−3;0), M_2(0;0;2) **542.** M_1 (4;−3;2), M_2(12;3;6).

VI BOB | ANALIZGA KIRISH

1- §. O'zgaruvchi miqdorlar va funksiyalar

1°. Intervallar. $a<x<b$ tengsizliklarni qanoatlantiruvchi x sonlar to'plami *oraliq* deyiladi va $(a;b)$ bilan belgilanadi. $a \le x \le b$ tengsizliklarni qanoatlantiruvchi x sonlar to'plami *segment* deyiladi va $[a;b]$ bilan belgilanadi.

O'zaro ekvivalent $x^2<a^2$ yoki $|x| < a$, yoki $-a<x<a$ tengsizliklar ($a>0$ bo'lganda) nolga nisbatan simmetrik oraliqlarni bildiradi.

2°. O'zgaruvchi miqdorlar va funksiyalar. Son qiymati o'zgarib turadigan miqdorga *o'zgaruvchan miqdor* deyiladi. O'zgaruvchi x ning qabul qiladigan qiymatlari to'plami X, o'zgaruvchi y ning qabul qiladigan qiymatlari to'plami Y haqiqiy sonlar to'plamidan iborat bo'lsin.

Ta'rif. *Agar X to'plamdan olingan har bir x songa biror f qoidaga yoki qonunga ko'ra Y to'plamning bitta y soni mos qo'yilgan bo'lsa, u holda X to'plamda funksiya aniqlangan (berilgan) deyiladi.*

x ning y funksiyasi simvolik $y=f(x)$, $y=F(x)$ yoki $y=\varphi(x)$ va shunga o'xshash ko'rinishda yoziladi. $f(x)$ yoki $F(x)$ va shunga o'xshash simvol x va y o'zgaruvchilarning moslik qonunini belgilaydi, xususiy holda x ning qiymatiga mos keladigan y ning qiymatini topish uchun x ustida bajarish kerak bo'lgan amallar yoki operatsiyalar to'plamini bildirishi mumkin.

543. 1) $|x|<4$; 2) $x^2 \le 9$; 3) $|x-4|<1$;
4) $-1<x-3\le 2$; 5) $x^2> 9$; 6) $(x-2)^2\le 4$

tengsizliklarni qanoatlantiruvchi x ning o'zgarish intervallarini yasang.

544. O'zgaruvchilarning $[-1;3]$; $(0;4)$; $[-2;1]$ o'zgarish intervallarini tengsizliklar orqali yozing va yasang.

545. $x=1-\frac{1}{t}$ o'zgaruvchining o'zgarish intervalni aniqlang, bunda t birdan kichik bo'lmagan har qanday qiymatni qabul qiladi $(t \ge 1)$.

546—548- masalalarda $|x| \le 3$ segmentda berilgan funksiyalarning grafiklarini nuqtalar bo'yicha yasang:

546. 1) $y=2x$; 2) $y=2x+2$; 3) $y=2x-2$.

547. 1) $y=x^2$; 2) $y=x^2+1$; 3) $y=x^2-1$.

548. 1) $y=\frac{x^3}{3}$; 2) $y=\frac{x^3}{3}+1$; 3) $y=\frac{x^3}{3}-1$.

549. 1) $y=\frac{6}{x}$; 2) $y=2^x$; 3) $y=\log_2 x$ funksiyalarning grafiklarini yasang. Bu egri chiziqlarning koordinata o‘qlariga nisbatan vaziyatlarida qanday xususiyatlarni ko‘rish mumkin?

550. 1) $y=\sin x$; 2) $y=\cos x$ funksiyalarning grafiklari y ning eng katta, eng kichik va nolga teng qiymatlar qabul etuvchi nuqtalari bo‘yicha yasang. Bu egri chiziqlar ordinatalarini qo‘shib, o‘sha chizmaning o‘zida $y=\sin x+\cos x$ funksiya grafigini yasang.

Quyidagi funksiyalarning grafiklarini yasang:

551. 1) $y=x^2+2x-1$; 2) $y=\sin 2x$; 3) $y=\cos 2x$;

4) $y=\sin\frac{x}{2}$; 5) $y=\cos\frac{x}{2}$; 6) $y=3^x$;

7) $y=2^{-x^2}$; 8) $y=\frac{1}{x-1}$; 9) $y=\frac{1}{1-x^2}$;

10) $y=\log_2(1/x)$.

552. $y=4x-x^2$ funksiyaning ildizlari x_1 va x_2 ni toping hamda uning $[x_1-1;\ x_2+1]$ segmentdagi grafigini yasang.

553. 1) $y=|x|$; 2) $y=-|x-2|$; 3) $y=|x|-x$ funksiyalarning grafiklarini yasang.

554. 1) $y=|x-1|$; $y=|\sin 2x|$; $y=|3^x|$ funksiyalarning grafiklarini yasang.

555—556- misollardagi funksiyalarning aniqlanish sohalarini toping va ularning grafiklarini yasang:

555. 1) $y=\sqrt{x+2}$; 2) $y=\sqrt{9-x^2}$; 3) $y=\sqrt{4x-x^2}$.

556. 1) $y=\sqrt{-x}+\sqrt{x+4}$; 2) $y=\arcsin\frac{x-1}{2}$.

557. 1) $f(x)=x^2-x+1$ bo‘lsa, $f(0)$, $f(1)$, $f(-1)$, $f(2)$, $f(a+1)$ larni hisoblang; 2) $\varphi(x)=\frac{2x-3}{x^2+1}$ bo‘lsa, $\varphi(0)$, $\varphi(-1)$, $\varphi(3/2)$, $\varphi(1/x)$, $1/\varphi(x)$ larni hisoblang.

558. $\varphi(x)=\sqrt{4+x^2}$ bo'lsa, $\varphi(2x)$ va $\varphi(0)$ ni hisoblang.

559. Agar $f(\varphi)=\operatorname{tg}\varphi$ bo'lsa, $f(2\varphi)=\frac{2f(\varphi)}{1-[f(\varphi)]^2}$ tenglikni tekshirib ko'ring.

560. $f(x)=\lg x$; $\varphi(x)=x^3$. Quyidagi ifodalarni yozing:
a) $f[\varphi(2)]$; *b*) $f[\varphi(a)]$; *d*) $\varphi[f(a)]$.

561. $F(x)=x^2$ bo'lsa, 1) $\frac{F(b)-F(a)}{b-a}$; 2) $F\left[\frac{a+h}{2}\right]-F\left[\frac{a-h}{2}\right]$ ni hisoblang.

562. $f(x)=x^2$; $\varphi(x)=x^3$ bo'lsa, $\frac{f(b)-f(a)}{\varphi(b)-\varphi(a)}$ ni hisoblang.

563. $F(x,y)=x^3-3xy-x^2$ bo'lsa, $f(4,3)$ va $f(3,4)$ ni hisoblang.

564. Agar $f(-x)=f(x)$ bo'lsa, $f(x)$ funksiya juft, agar $f(-x)=-f(x)$ bo'lsa, toq funksiya deyiladi. Ushbu

1) $f(x)=\frac{\sin x}{x}$; 2) $\varphi(x)=\frac{a^x-1}{a^x+1}$; 3) $F(x)=a^x+\frac{1}{a^x}$;

4) $\varphi(x)=a^x-\frac{1}{a^x}$; 5) $\varphi(x)=x\sin^2 x-x^3$; 6) $f_1(x)=x+x^2$

funksiyalardan qaysilari juft, qaysilari toq ekanligini ko'rsating.

565. Elementar funksiyalardan qaysisi $f(1)=0$, $f(a)=1$, $f(xy)=f(x)+f(y)$ xossalarga ega?

566. Elementar funksiyalardan qaysisi $f(1)=0$, $f(1)=a$, $f(x+y)=f(x)f(y)$ xossalarga ega?

* * *

567. 1) $|x|<3$; 2) $x^2\leq 4$; 3) $|x-2|<2$; 4) $(x-1)^2\leq 4$ tengsizliklarni qanoatlantiruvchi x ning o'zgarisli intervallarini yasang.

568. $x=2+\frac{1}{t}$ o'zgaruvchining ixtiyoriy $t\geq 1$ qiymatlar qabul qilgandagi o'zgarish intervalini aniqlang.

569. Quyidagi funksiyalarning grafiklarini yasang:

1) $y=4-\frac{x^3}{2}$ ning $|x|\leq 2$ segmentda;

2) $y=3,5+3x-\frac{x^2}{2}$ ning abssissalar o'qi bilan kesishgan nuqtalari orasida.

570. Quyidagi funksiyalarning grafiklarini yasang:

1) $y=x-4+|x-2|$ ning $[-2;5]$ segmentda;

2) $y=1-\cos x$ ning $|x|\le 2\pi$ segmentda.

571. 1) $y=-\frac{4}{x}$; 2) $y=2^{-x}$ funksiyalarning grafiklarini yasang.

572. 1) $y=\sqrt{4-x^2}$; 2) $y=\sqrt{x+1}-\sqrt{3-x}$; 3) $y=1-\sqrt{2\cos 2x}$;

4) $y=\frac{4}{1+\sqrt{x^2-4}}$ funksiyalar haqiqiy qiymatlarining aniqlanish sohalarini toping.

573. 1) Agar $f(x)=\frac{2x+1}{x^2+1}$ bo‘lsa, $f(0)$; $f(-2)$; $f(-1/2)$; $f(x-1)$; $f(1/2)$ larni hisoblang;

2) agar $\varphi(x)=x^3$ bo‘lsa, $\frac{\varphi(x+h)-\varphi(x-h)}{h}$ ni hisoblang;

3) agar $f(x)=4x-x^2$ bo‘lsa, $f(a+1)-f(a-1)$ ni hisoblang.

574. Agar $\varphi(x)=\frac{x-1}{3x+5}$ bo‘lsa, $\varphi\left(\frac{1}{x}\right)$ va $1/\varphi(x)$ ifodalarni yozing.

2- §. Sonlar ketma-ketligi. Cheksiz kichik va cheksiz katta miqdorlar. Funksiya limiti

1°. Sonlar ketma-ketligi. Aytaylik, N to‘plamda biror $f(n)$ funksiya berilgan bo‘lsin. Bu funksiya qiymatlarini x_n bilan belgilaymiz:

$$f(n)=x_n.$$

Qaralayotgan funksiya qiymatlaridan tashkil topgan ushbu

$$x_1,\ x_2,\ x_3,\ldots,\ x_n,\ldots \qquad (1)$$

to‘plam *sonlar ketma-ketligi* deyiladi. (1) ketma-ketlikning tuzilish qonuni n- had formulasi bilan beriladi.

Masalan: $x_n=\frac{1}{3^n}$ bo‘lsa, $n=1, 2, 3, \ldots$ deb olsak,

$$\frac{1}{3},\frac{1}{9},\frac{1}{27},\frac{1}{81},\ldots$$

ketma-ketlik hosil bo‘ladi.

2°. Funksiyaning limiti. Biror haqiqiy sonlar to‘plami X berilgan bo‘lsin.

1- ta’rif. Agar $a \in R$ nuqtaning ixtiyoriy ε atrofida $(\varepsilon > 0)$ X to‘plamning cheksiz ko‘p elementlari yotsa, a nuqta X to‘plamning *limit nuqtasi* deyiladi.

2- ta’rif. Agar X to‘plamning nuqtalaridan tuzilgan, a ga yaqinlashuvchi har qanday $\{x_n\}$ ketma-ketlik olinganda ham, funksiya qiymatlaridan iborat $\{f(x_n)\}$ ketma-ketlik yagona (chekli yoki cheksiz) b limitga intilsa, shu b ga $f(x)$ funksiyaning *a nuqtadagi limiti* deyiladi va

$$\lim_{x \to a} f(x) = b$$

kabi belgilanadi.

Funksiya limitiga berilgan bu ta’rif *Geyne ta’rifi* deyiladi.

Funksiya limitiga quyidagicha ham ta’rif berish mumkin.

3- ta’rif. Agar $\forall \varepsilon > 0$ son uchun shunday $\delta(\varepsilon) > 0$ son topilsaki, argument x ning $0 < |x - a| < \delta$ tengsizlikni qanoatlantiruvchi barcha qiymatlarida $|f(x) - b| < \varepsilon$ tengsizlik bajarilsa, b son $f(x)$ funksiyaning *a nuqtadagi* ($x \to a$ dagi) *limiti* deyiladi va

$$\lim_{x \to a} f(x) = b$$

kabi belgilanadi.

Funksiya limitiga berilgan bu ta’rif *Koshi ta’rifi* deyiladi.

4- ta’rif. Agar $\forall \varepsilon > 0$ son uchun shunday $\delta(\varepsilon) > 0$ son topilsaki, argument x ning $0 < x - a < \delta(\varepsilon)$ $(-\delta(\varepsilon) < x - a < 0)$ tengsizlikni qanoatlantiruvchi barcha qiymatlarida $|f(x) - b| < \varepsilon$ tengsizlik bajarilsa, b son $f(x)$ funksiyaning *a nuqtadagi o‘ng (chap) limiti* deyiladi va

$$\lim_{x \to a+0} f(x) = b \quad \left(\lim_{x \to a-0} f(x) = b\right)$$

kabi belgilanadi.

5- ta’rif. Agar $\forall \varepsilon > 0$ son uchun shunday $\Delta > 0$ son topilsaki, argument x ning $|x| > \Delta$ $(x > \Delta,\ -x > \Delta)$ tengsizlikni qanoatlantiruvchi barcha qiymatlarida $|f(x) - b| < \varepsilon$ tengsizlik bajarilsa, b son $f(x)$ funksiyaning $x \to \infty$ $(x \to +\infty, x \to -\infty)$ dagi *limiti* deyiladi va

$$\lim_{x \to \infty} f(x) = b \quad \left(\lim_{x \to +\infty} f(x) = b;\ \lim_{x \to -\infty} f(x) = b\right)$$

kabi belgilanadi.

3°. X to‘plamda $\alpha(x)$ funksiya berilgan bo‘lib, a nuqta X ning limit nuqtasi bo‘lsin.

Ta’rif. Agar $x \to a$ da $\alpha(x)$ funksiyaning limiti nolga teng, ya’ni

$$\lim_{x \to a} \alpha(x) = 0$$

bo‘lsa, $\alpha(x)$ funksiya a nuqtada (yoki $x \to a$ da) *cheksiz kichik* deyiladi.

4°. X to‘plamda $\beta(x)$ funksiya berilgan bo‘lib, a nuqta X ning limit nuqtasi bo‘lsin.

Ta’rif. Agar $x \to a$ da $\beta(x)$ funksiyaning limiti cheksiz, ya’ni

$$\lim_{x \to a} \beta(x) = \infty$$

bo‘lsa, $\beta(x)$ funksiya a nuqtada (yoki $x \to a$ da) *cheksiz katta* deyiladi.

575. $n = 0,\ 1, 2, \ldots$ deb

$$\alpha = \frac{1}{2^n},\quad \alpha = -\frac{1}{2^n},\ \alpha = \left(-\frac{1}{2}\right)^n$$

o‘zgaruvchilar qiymatlarining ketma-ketligini yozing va ularning o‘zgarishini grafik usulda tasvirlang. n ning qanday qiymatlaridan boshlab o‘zgaruvchilardan har qaysisi 0,001 dan, berilgan musbat ε dan kichik bo‘ladi va shunday bo‘lib qola beradi?

576. $x = 1 + \frac{(-1)^n}{2n+1}$ o‘zgaruvchi qiymatlarining ketma-ketligini yozing va uning o‘zgarishini grafik usulda tasvirlang. n ning qaysi qiymatidan boshlab $x - 1$ ning moduli 0,01 dan, berilgan musbat ε dan kichik bo‘ladi va shunday bo‘lib qola beradi?

577. 3 ga oldin 1 ni, so‘ngra 0,1 ni, undan so‘ng 0,01 ni va hokazo qo‘shib (yoki ayirib) o‘zgaruvchi x ning $x \to 3+0$ yoki $x \to 3-0$ limitlarga yaqinlashishlarini „o‘nli“ ketma-ketliklari bilan yozing.

Ketma-ketliklarning umumiy hadi formulasini yozing:

578. $-\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, \ldots$ **579.** $0,\ 2,\ 0,\ 2,\ \ldots$

580. $2, \frac{4}{3}, \frac{6}{5}, \frac{8}{7}, \ldots$ **581.** $1,\ 0,\ -3,\ 0,\ 5,\ 0,\ -7,\ 0,\ \ldots$

582. $-3, \frac{5}{3}, -\frac{7}{5}, \frac{9}{7}, -\frac{11}{9}, \ldots$ **583.** $\frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \ldots$

584. $-\frac{1}{2}, \frac{1}{5}, -\frac{1}{10}, -\frac{1}{17}, \ldots$

585—590- misollarda yuqoridan (quyidan) chegaralangan $(x_n)_{n\in N}$ ketma-ketlikning ehg katta (kichik) hadini toping.

585. $x_n = 6n - n^2 - 5$. **586**. $x_n = e^{10n-n^2-24}$.

587. $x_n = 3n^2 - 10n - 14$. **588**. $x_n = 2n + \frac{512}{n^2}$.

589. $x_n = -\frac{n^2}{2^n}$. **590**. $x_n = -\frac{\sqrt{n}}{9+n}$.

591. $\lim\limits_{x\to\infty} \frac{3x+4}{x} = 3$ ekanini isbot qiling. $x=1, 10, 100, 1000, \ldots$ bo'lganda x va $\frac{3x+4}{x}$ larning qiymatlarini jadvallar bilan tushuntiring.

592. $\lim\limits_{x\to 2+0} \frac{3}{x-2}$ va $\lim\limits_{x\to 2-0} \frac{3}{x-2}$ larni toping va jadvallar bilan tushuntiring.

593. $\lim\limits_{x\to 0+0} 2^{\frac{1}{x}}$ va $\lim\limits_{x\to 0-0} 2^{\frac{1}{x}}$ ni toping va jadvallar bilan tushuntiring.

594. Ushbu 1) $\frac{2}{\infty} = 0$; 2) $-\frac{2}{0} = \pm\infty$; 3) $3^{\infty} = \infty$; 4) $3^{-\infty} = 0$; 5) $\lg 0 = -\infty$ „shartli" yozishlarning aniq ma'nolarini tushuntiring.

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Ketma -ketllklarning birinchi beshta hadini yozing:

595. 1) $x_n = 1 + (-1)^n \frac{1}{n}$; 2) $x_n = n(1-(-1)^n)$.

596. $x_n = \frac{3n+5}{2n-3}$. **597**. $x_n = (-1)^n \arcsin\frac{\sqrt{3}}{2} + \pi n$.

Ketma-ketliklarning umumiy hadi formulasini yozing:

598. $-\frac{1}{2}, \frac{1}{4}, -\frac{1}{6}, \frac{1}{8}, \ldots$ **599**. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$

600. $-2, \frac{3}{2}, -\frac{4}{3}, \frac{5}{4}, -\frac{6}{5}, \ldots$ **601**. $\frac{1}{e}, e^2, \frac{1}{e^3}, e^4, \frac{1}{e^4}, \ldots$

602. O'zgaruvchilarning limitlariga yaqinlashishlarining „o'nli" ketma-ketliklarini yozing: $x \to 4+0$; $x \to 4-0$; $x \to -1{,}5+0$; $x \to -1{,}5-0$.

603. $\lim_{x\to\infty}\frac{5x+2}{2x}=2,5$ ekanini isbot qiling.

$x=1,\ 10,\ 100,\ 1000,\ ...$

bo‘lganda x va $\frac{5x+2}{2x}$ larning qiymatlarini jadvallar bilan tushuntiring.

604. O‘zgaruvchilar qiymatlari ketma-ketligini yozing:

1) $x_n=1+\left[-\frac{1}{2}\right]^n$; 2) $x_n=(-1)^n+\frac{1}{2^n}$;

3) $x_n=(-1)^n(2n+1)$; 4) $x_n=\frac{2n\sin\frac{nx}{2}}{n+1}$.

$n\to\infty$ da bu o‘zgaruvchilarning qaysi biri limitga ega?

605. 1) $\lim_{x\to 1-0}\frac{1}{2^{x-1}}$; 2) $\lim_{x\to 1+0}\frac{1}{2^{x-1}}$; 3) $\lim_{x\to\frac{\pi}{4}-0}3\mathrm{tg}2x$;

4) $\lim_{x\to\frac{\pi}{2}+0}3^{\mathrm{tg}2x}$; 5) $\lim_{x\to\frac{\pi}{2}+0}\frac{2}{1+2^{\mathrm{tg}x}}$; 6) $\lim_{x\to\frac{\pi}{2}-0}\frac{2}{1+2^{\mathrm{tg}x}}$;

7) $\lim_{x\to+\infty}\frac{a}{1+a^x}$ limitlarni toping.

3- §. Limitlarning xossalari. $\frac{0}{0}$ va $\frac{\infty}{\infty}$ ko‘rinishdagi aniqmasliklarni ochish

1°. O‘zgarmas miqdorning limiti o‘ziga teng.

2°. Agar $\lim_{x\to a}u(x)$ va $\lim_{x\to a}v(x)$ mavjud bo‘lsa:

$$\lim_{x\to a}[u(x)+v(x)]=\lim_{x\to a}u(x)+\lim_{x\to a}v(x).$$

3°. $\lim_{x\to a}[u(x)\times v(x)]=\lim_{x\to a}u(x)\times\lim_{x\to a}v(x).$

4°. Agar $\lim_{x\to a}u(x)$ va $\lim_{x\to a}v(x)$ mavjud bo‘lib, $\lim_{x\to a}v(x)\neq 0$

bo‘lsa, $\lim_{x\to a}\frac{u(x)}{v(x)}=\frac{\lim_{x\to a}u(x)}{\lim_{x\to a}v(x)}$.

5⁰. Agar a nuqtaning qandaydir bir atrofidagi x ning, balki faqat $x=a$ dan boshqa, barcha qiymatlarida $f(x)$ va $\varphi(x)$ funksiyalar bir-biriga teng bo‘lsa va ulardan biri $x\to a$ da limitga ega bo‘lsa, ikkinchisi ham o‘sha limitga ega bo‘ladi. Bu xossa $\frac{0}{0}$ va $\frac{\infty}{\infty}$

ko‘rinishdagi aniqmasliklarni ochishiga tatbiq etiladi. Masalan, x ning 3 dan boshqa barcha qiymatlari uchun: $\frac{x^2-5x+6}{x-3} = x-2.$

5^0 xossaga ko‘ra:

$$\lim_{x\to3}\frac{x^2-5x+6}{x-3} = \lim_{x\to3}\frac{(x-3)(x-2)}{x-3} = \lim_{x\to3}(x-2) = 1.$$

Quyidagi limitlarni toping:

606. 1) $\lim_{x\to2}\frac{x^2-4x+1}{2x+1}$; 2) $\lim_{x\to\pi/4}\frac{1+\sin 2x}{1-\cos 4x}$.

607. $\lim_{x\to2}\frac{x^2-4}{x-2}$ (jadval bilan tushuntiring.)

608. $\lim_{x\to2}\frac{x-2}{x^2-3x+2}$.

609. $\lim_{x\to3}\frac{x^2-9}{x^2-2x-3}$.

K o ‘ r s a t m a . 608- misolni ikki usul: 1) $x=2+\alpha$ deb olib;

2) maxrajini ko‘paytuvchilarga ajratish bilan yeching.

610. $\lim_{x\to1}\frac{x^3-1}{x-1}$.

611. $\lim_{x\to2}\frac{x^2-5x+6}{x^2-12x+20}$.

612. $\lim_{x\to2}\frac{x^2+3x-10}{3x^2-5x-2}$.

613. $\lim_{x\to-2}\frac{x^3+3x^2+x}{x^2-x-6}$.

614. $\lim_{x\to-2}\frac{x^3+4x^2+4x}{(x+2)(x-3)}$.

615. $\lim_{h\to0}\frac{(x+h)^3-x^3}{h}$.

616. $\lim_{x\to\pi}\frac{\operatorname{tg}x}{\sin 2x}$.

617. $\lim_{x\to\pi/4}\frac{\sin x-\cos x}{\cos 2x}$.

618. $\lim_{x\to0}\frac{x}{\sqrt{1+3x}-1}$.

619. $\lim_{x\to a}\frac{\sqrt{ax}-x}{x-a}$.

620. $\lim_{x\to1}\frac{\sqrt[3]{x}-1}{\sqrt{x}-1}$.

621. $\lim_{x\to0}\frac{\sqrt[3]{1+mx}-1}{x}$.

K o ‘ r s a t m a . 620- misolda $x=t^6$, 621-da $1+mx=t^3$ deb oling.

622. $\lim_{x\to0}\frac{\sqrt{1+x}-1}{x}$.

623. $\lim_{x\to4}\frac{\sqrt{2x+1}-3}{\sqrt{x-2}-\sqrt{2}}$.

624. $\lim_{x\to0}\frac{\sin x}{\operatorname{tg} x}$.

625. $\lim_{x\to64}\frac{\sqrt{x}-8}{4-\sqrt[3]{x}}$.

626. $\lim\limits_{\varphi\to 0}\frac{16\varphi-1}{1-4\varphi}$. **627.** $\lim\limits_{x\to 0}\frac{\sqrt{1x+1}-\sqrt{1-x}}{x}$.

628. $\lim\limits_{x\to\pi}\frac{\sqrt{1-\operatorname{tg}x}-\sqrt{1+\operatorname{tg}x}}{\sin 2x}$. **629.** 1) $\lim\limits_{x\to\infty}\frac{2x^2-1}{3x^2-4x}$; 2) $\lim\limits_{x\to\infty}\frac{5x^3-7x}{1-2x^3}$.

K o ' r s a t m a. 629- misolni ikki usul: 1) surat va maxrajini x ning eng katta darajasiga bo'lish; 2) $x=\frac{1}{\alpha}$ deb olish bilan yechish mumkin.

630. $\lim\limits_{x\to\infty}\frac{2x^3+1}{3x^4+2x^2-1}$. **631.** $\lim\limits_{x\to+\infty}\frac{\sqrt{x^2-8}}{\sqrt[3]{x^3+1}}$. **632.** $\lim\limits_{x\to\infty}\frac{x^4-5x}{x^2-3x+1}$.

633. $\lim\limits_{x\to\infty}\frac{3x-1}{x^2+1}$. **634.** $\lim\limits_{x\to\infty}\frac{x^3-1}{x^2+1}$. **635.** $\lim\limits_{x\to\infty}\frac{\sqrt{x}-6x}{3x+1}$.

636. $\lim\limits_{n\to\infty}\frac{3n}{1-2n}$. **637.** $\lim\limits_{n\to\infty}\frac{\sqrt{2n^2+1}}{2n-1}$. **638.** $\lim\limits_{x\to\infty}\frac{1+2+3+...+n}{\sqrt{9n^4+1}}$.

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639. $\lim\limits_{x\to -2}\frac{3x+6}{x^3+8}$. **640.** $\lim\limits_{x\to 3}\frac{9-x^2}{\sqrt{3x}-3}$. **641.** $\lim\limits_{x\to -1}\frac{x^2-x-2}{x^3+1}$.

642. $\lim\limits_{x\to\pi+0}\frac{\sqrt{1+\cos x}}{\sin x}$. **643.** $\lim\limits_{x\to 2}\frac{x^2-5x+6}{x^2-3x+2}$. **644.** $\lim\limits_{x\to 0}\frac{3-\sqrt{x+9}}{x}$.

645. $\lim\limits_{x\to 81}\frac{3-\sqrt[4]{x}}{9-\sqrt{x}}$. **646.** $\lim\limits_{x\to 1}\frac{\sqrt{x}-\sqrt{2-x}}{x-1}$. **647.** $\lim\limits_{x\to 2}\frac{\sqrt{4x+1}-3}{\sqrt{x+2}-2}$.

648. $\lim\limits_{x\to\infty}\frac{5x^2-3x+2}{2x^2+4x+1}$. **649.** $\lim\limits_{n\to\infty}\frac{3n+1}{\sqrt{3n^2+1}}$. **650.** $\lim\limits_{x\to\infty}\frac{2x^3+1}{3x^4+2x^2-1}$.

651. $\lim\limits_{x\to\infty}\left(\frac{5-x^2}{1-x^2}+2^{\frac{1}{x}}\right)$. **652.** $\lim\limits_{n\to\infty}\frac{1+3+5+...+(2n-1)}{1+2+3+...+n}$.

653. $\lim\limits_{x\to 7}\frac{2-\sqrt{x-3}}{x^2-49}$. **654.** $\lim\limits_{x\to\pi/4}\frac{\sin 2x-\cos 2x-1}{\cos x-\sin x}$.

4- §. $\frac{\sin\alpha}{\alpha}$ nisbatning $\alpha\to 0$ dagi limiti

Agar α burchak radian o'lchovi bilan berilgan bo'lsa, u holda quyidagi tengliklar o'rinlidir:

$$\lim_{\alpha\to 0}\frac{\sin\alpha}{\alpha}=1;\quad \lim_{\alpha\to 0}\frac{\alpha}{\sin\alpha}=1.$$

Limitlarni toping:

655. 1) $\lim\limits_{x\to 0}\frac{\sin 4x}{x}$; 2) $\lim\limits_{x\to 0}\frac{\sin\frac{x}{3}}{x}$.

Ko'rsatma. 655- misolda kasrning surati va maxrajini 4 ga ko'paytiring (yoki $4x=a$ deb oling).

656. $\lim\limits_{x\to 0}\frac{\mathrm{tg}x}{x}$.

657. $\lim\limits_{x\to 0}\frac{\sin^2\frac{x}{2}}{x^2}$.

658. $\lim\limits_{x\to 0}\frac{1-\cos 2x}{x\sin x}$.

659. $\lim\limits_{x\to 0}\frac{\sin 3x}{\sqrt{x+2}-\sqrt{2}}$.

660. $\lim\limits_{h\to 0}\frac{\sin(x+h)-\sin(x-h)}{h}$.

661. $\lim\limits_{x\to 0}\frac{\sin^2 x}{x}$.

662. $\lim\limits_{x\to 1/3}\frac{4\sin(3x-1)}{3x-1}$.

663. $\lim\limits_{x\to 0}\frac{\sin(\sin x)}{x}$.

664. $\lim\limits_{x\to 0}\frac{\mathrm{tg}2x}{\mathrm{tg}3x}$.

665. $\lim\limits_{x\to 0}\frac{\sin 10\pi x}{\mathrm{tg}5x}$.

666. $\lim\limits_{x\to 0}\frac{1-\cos 3x}{\mathrm{tg}^2 6x}$.

667. $\lim\limits_{x\to 1}\frac{\sin(1-x)}{x^2-1}$.

668. $\lim\limits_{x\to 0}\mathrm{ctg}^2\frac{x}{8}\,\mathrm{tg}^2\frac{5x}{4}$.

669. 1) $\lim\limits_{x\to 0}\frac{\mathrm{arctg}x}{x}$; 2) $\lim\limits_{x\to 1/2}\frac{\arcsin(1-2x)}{4x^2-1}$.

Ko'rsatma. 669- misol: 1) misolda $\mathrm{arctg}x=\alpha$, 2) misolda esa $\arcsin(1-2x)=\alpha$ deb olish kerak.

670. 1) $\lim\limits_{x\to 0}\frac{1-\cos x}{x^2}$.

671. 1) $\lim\limits_{x\to 0}\frac{\mathrm{tg}x-\sin x}{x^3}$.

672. $\lim\limits_{x\to -2}\frac{\mathrm{arctg}x(x+2)}{4-x^2}$.

673. $\lim\limits_{x\to 0}\frac{x}{\sin 3x}$.

674. $\lim\limits_{x\to \pi}\frac{\sin 7x}{\sin 3x}$.

675. $\lim\limits_{x\to 0}\frac{\sin 5x}{7x}$.

676. $\lim\limits_{x\to 0}\frac{\arcsin x}{x}$.

677. $\lim\limits_{x\to 0}\frac{\mathrm{tg}10x}{\sin^2 2x}$.

678. $\lim\limits_{x\to 0}\frac{\sin 4x}{\sqrt{x+1}-1}$.

679. $\lim\limits_{x\to 0}\frac{\sqrt{1-\cos 2x}}{x}$.

680. $\lim\limits_{x\to 0}\frac{2x\sin x}{\sec x-1}$.

681. $\lim\limits_{x\to 0}\frac{1-\cos mx}{x^2}$.

682. $\lim\limits_{x\to 0}\frac{1-\cos 2x+\mathrm{tg}^2 x}{x\sin x}$.

683. $\lim\limits_{x\to 2}\left[\frac{\sin(x-2)}{x^2-2}+2^{-\frac{1}{(x-2)^2}}\right]$

($x=2+\alpha$ deb oling).

684. 1) $\lim\limits_{h\to 0}\frac{\cos(x+h)-\cos(x-h)}{h}$; 2) $\lim\limits_{x\to -2}\frac{\arcsin(x+2)}{x^2+2x}$.

685. $\lim\limits_{x\to 0}\frac{x^2-1}{\arcsin(x-1)}$.

686. $\lim\limits_{x\to 0}\frac{\sin^2 x}{\sqrt{1+x\sin x}-\cos x}$.

5- §. $\infty-\infty$ va $0\cdot\infty$ ko'rinishdagi aniqmasliklar

Limitlarni toping:

687. $\lim\limits_{x\to+\infty}\left(\sqrt{x^2+3x}-x\right)$.

688. $\lim\limits_{x\to 0}\left(\sqrt{x+1}-\sqrt{x-1}\right)$.

689. $\lim\limits_{x\to\infty}\left(\sqrt{x^2+1}-\sqrt{x^2-1}\right)$.

690. $\lim\limits_{x\to 1}\left(\frac{1}{x-1}-\frac{2}{x^2-1}\right)$.

691. $\lim\limits_{x\to 2}\left(\frac{1}{2-x}-\frac{3}{8-x^3}\right)$.

692. $\lim\limits_{x\to+\infty}\left(\sqrt{x^2+x+1}-\sqrt{x^2-x}\right)$.

693. $\lim\limits_{x\to 0}\left(\sqrt{4x^2-7x+4}-2x\right)$.

694. $\lim\limits_{x\to 2}\left(\frac{1}{\sin^2 x}-\frac{1}{4\sin^2\frac{x}{2}}\right)$.

695. $\lim\limits_{x\to\infty}\left(\frac{1+3+\ldots+(2n-1)}{n+3}-n\right)$.

696. $\lim\limits_{x\to 1}(1-x)\,\mathrm{tg}\frac{\pi}{2}x$ ($x=1-\alpha$ deb oling).

697. $\lim\limits_{x\to 0}x\,\mathrm{ctg}\,ax$.

698. $\lim\limits_{x\to\alpha}\mathrm{tg}\frac{\pi x}{2\alpha}\sin\frac{x-\alpha}{2}$.

699. $\lim\limits_{x\to-\infty}\left(\sqrt{x^2+1}-\sqrt{x^2-4x}\right)$.

700. $\lim\limits_{x\to-2}\left(\frac{1}{x+2}+\frac{4}{x^2-4}\right)$.

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Limitlarni toping:

701. $\lim\limits_{x\to+\infty}\left(x-\sqrt{x^2-x+1}\right)$.

702. $\lim\limits_{x\to+\infty}\left(x-\sqrt{x^2-a}\right)$.

703. $\lim\limits_{x\to\pi/2}\left(\frac{\sin x}{\cos^2 x}-\mathrm{tg}^2 x\right)$. **704.** $\lim\limits_{n\to\infty}\left(\frac{1+2+3+...+n}{n+2}-\frac{n}{2}\right)$.

705. $\lim\limits_{x\to\pi/2}\left(x-\frac{\pi}{2}\right)\mathrm{tg}x$ $\left(\begin{matrix} x=\pi/2+\alpha \text{ deb} \\ \text{oling}\end{matrix}\right)$.

706. 1) $\lim\limits_{x\to0} x\mathrm{ctg}\pi x$; 2) $\lim\limits_{x\to0}\left(\frac{1}{\sin x}-\mathrm{ctg}x\right)$;

3) $\lim\limits_{x\to1}\left(\frac{1}{1-x}-\frac{3}{1-x^3}\right)$.

6- §. Limitlarni hisoblashga doir aralash misollar

Limitlarni toping:

707. 1) $\lim\limits_{x\to0}\frac{\sqrt{x+4}-2}{\sin 5x}$; 2) $\lim\limits_{x\to0}\frac{1+x\sin x-\cos 2x}{\sin^2 x}$.

708. 1) $\lim\limits_{x\to1}\frac{\sqrt[4]{x}-1}{\sqrt[3]{x}-1}$ 2) $\lim\limits_{x\to0}\frac{x}{\sqrt[4]{1+2x}-1}$.

709. $\lim\limits_{x\to-\infty}\left(\sqrt{x^2+ax}-\sqrt{x^2-ax}\right)$.

710. 1) $\lim\limits_{x\to\infty}\left(\frac{1+2x}{\sqrt[3]{1+8x^3}}+2^{-x^2}\right)$; 2) $\lim\limits_{x\to\infty}\frac{x-\sin x}{1-5x}$.

711. 1) $\lim\limits_{x\to-1}\frac{x^3+1}{\sin(x+1)}$; 2) $\lim\limits_{x\to-2}\frac{x^2+x-2}{x^2+2x}$.

712. 1) $\lim\limits_{x\to0}\frac{1-\cos x}{x(\sqrt{x+1}-1)}$, 2) $\lim\limits_{x\to\pi}\frac{\cos\frac{x}{2}}{x-\pi}$.

713. 1) $\lim\limits_{x\to1}\frac{\sin(1-x)}{\sqrt{x}-1}$; 2) $\lim\limits_{n\to+\infty}\frac{1-10^n}{1+10^{n+1}}$.

714. 1) $\lim\limits_{x\to\infty}\left(\frac{3x^4}{1-2x^4}-2^{\frac{1}{x}}\right)$; 2) $\lim\limits_{n\to-\infty}\frac{3-10^n}{2+10^{n+1}}$.

715. 1) $\lim\limits_{x\to\pi/2+0}\frac{\sqrt{1+\cos 2x}}{\sqrt{\pi}-\sqrt{2x}}$; 2) $\lim\limits_{x\to-1}\frac{\pi(x+1)}{\sqrt[3]{x}+1}$.

716. $\lim\limits_{x\to 0}\frac{\operatorname{tg}x-\sin x}{x^3}$.

717. $\lim\limits_{x\to 0}\frac{\sin x}{\sqrt{x+1}-1}$.

718. $\lim\limits_{z\to -1}(1-z)\operatorname{tg}\frac{\pi z}{2}$.

719. $\lim\limits_{x\to 10}\frac{\sqrt{x-1}-3}{x-10}$.

7- §. Cheksiz kichiklarni taqqoslash

1°. Ta'riflar. $x\to a$ da $\alpha(x)$ va $\beta(x)$ funksiyalar cheksiz kichik bo'lsin. U vaqtda:

I. Agar $\lim\limits_{x\to a}\frac{\beta}{\alpha}=0$ bo'lsa, β α ga nisbatan *yuqori tartibli cheksiz kichik* deyiladi.

II. Agar $\lim\limits_{x\to a}\frac{\beta}{\alpha^n}=A$ (chekli va 0 dan farqli) bo'lsa, β α *ga nisbatan n-tartibli cheksiz kichik* deyiladi.

III. Agar $\lim\limits_{x\to a}\frac{\beta}{\alpha}=1$ bo'lsa, β va α *ekvivalent cheksiz kichiklar* deyiladi. Ekvivalentlik $\beta\approx\alpha$ ko'rinishida yoziladi.

2°. Ekvivalent cheksiz kichiklarning xossalari:

a) ekvivalent cheksiz kichiklarning ayirmasi ularning har biriga nisbatan ham yuqori tartibli cheksiz kichik bo'ladi;

b) agar bir nechta har xil tartibli cheksiz kichiklar yig'indisidan yuqori tartiblilari chiqarib tashlansa, u holda qolgan qismi *bosh qism* deyiladi va u umumiy yig'indiga ekvivalent bo'ladi.

720. Cheksiz kichik x ga nisbatan : 1) $1-\cos x$; 2) $\operatorname{tg}x-\sin x$ cheksiz kichiklarning tartiblarini aniqlang.

721. Cheksiz kichik x ga nisbatan:

1) $2\sin^4x-x^5$; 2) $\sqrt{\sin^2 x+x^4}$; 3) $\sqrt{1+x^3}-1$ cheksiz kichiklarining tartiblarini aniqlang.

$x\to 0$ da $\beta(x)=x$ ga nisbatan cheksiz kichik $\alpha(x)$ ning tartibini aniqlang:

722. $\alpha(x)=\frac{3\sqrt{x^3}}{1-x}$.

723. $\alpha(x)=\frac{1-\cos x}{x}$.

724. $\alpha(x)=\sin(\sqrt{x+2}-\sqrt{2})$.

725. $\alpha(x)=\sqrt{1+2x}-1-\sqrt{x}$.

726. x nolga intilganda ($x\to 0$):

1) $\sin mx \approx mx$, 2) $\operatorname{tg} mx \approx mx$; 3) $\sqrt[3]{1+x}-1 \approx \frac{1}{3}x$ ekanligini isbot qiling.

727. Agar $\alpha \approx \alpha_1$, $\beta \approx \beta_1$ va $\lim \frac{\beta}{\alpha}$ yoki $\lim \frac{\beta_1}{\alpha_1}$ limitlardan biri mavjud bo'lsa, $\lim \frac{\beta_1}{\alpha_1} = \lim \frac{\beta}{\alpha}$ bo'ladi, degan teoremaga asoslanib,

1) $\lim\limits_{x \to 0} \frac{\sin 5x}{\sin 2x}$; 2) $\lim\limits_{x \to 0} \frac{\sin ax + x^2}{\operatorname{tg} bx}$; 3) $\lim\limits_{x \to 0} \frac{3x + \sin^2 x}{\sin 2x - x^3}$ limitlarni toping.

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728. Cheksiz kichik x ga nisbatan:

1) $\sqrt{1+x^2}-1$; 2) $\sin 2x - 2\sin x$; 3) $1-2\cos(x+\frac{\pi}{3})$ cheksiz kichiklarning tartiblarini aniqlang.

$x \to 0$ da $\beta(x)=x$ ga nisbatan cheksiz kichik $\alpha(x)$ ning tartibini aniqlang:

729. $\alpha(x) = \sqrt[3]{x^2} - \sqrt{x^3}$. **730.** $\alpha(x) = \operatorname{tg} x - \sin x$.

731. $\alpha(x) = 3\sin^3 x - x^4$. **732.** $\alpha(x) = 2^x - \cos x$.

733. x nolga intilganda ($x \to 0$): 1) $\operatorname{arctg} mx \approx mx$;

2) $\sqrt{1+x}-1 \approx \frac{1}{2}x$; 3) $1-\cos^3 x \approx 1{,}5\sin^2 x$ ekanini isbot qiling.

8- §. Funksiyaning uzluksizligi

1°. Ta'rif. Agar *f(x)* funksiya *a* ning biror atrofida aniqlangan va

$$\lim_{x \to a} f(x) = f(a)$$

bo'lsa, u *x=a nuqtada uzluksiz* deyiladi. Bu ta'rif quyidagi to'rtta uzluksizlik shartini o'z ichiga oladi:

1) *f(x)* funksiya *a* ning qandaydir atrofida aniqlangan bo'lishi kerak;

2) chekli $\lim\limits_{x \to a-0} f(x)$ va $\lim\limits_{x \to a+0} f(x)$ limitlar mavjud bo'lishi kerak;

3) bu (chap va o'ng) limitlar bir xil bo'lishi kerak;

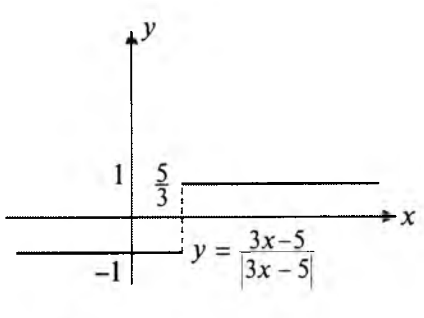
4) bu limitlar *f(a)* ga teng bo'lishi kerak.

Agar funksiya $[x_1; x_2]$ segmentning har bir ichki nuqtasida uzluksiz bo'lsa va uning chegaralarida esa $\lim\limits_{x \to x_1+0} f(x) = f(x_1)$ va $\lim\limits_{x \to x_2-0} f(x) = f(x_2)$ bo'lsa, u shu *segmentda uzluksiz* deyiladi.

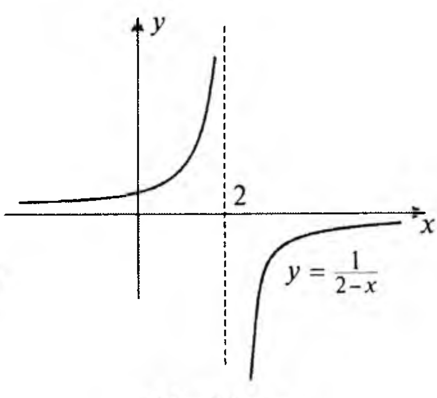
2^0. Funksiyalarning uzilishlari. Agar funksiya a dan o'ngda va chapda aniqlangan bo'lsa, ammo a nuqtada uzluksizlikning to'rtta shartidan aqalli bittasi bajarilmasa, $f(x)$ funksiya $x=a$ bo'lganda *uzilishga* ega bo'ladi. Uzilishlar ikki asosiy turga ajratiladi.

Birinchi tur uzilish. Agar chekli $\lim\limits_{x \to a-0} f(x)$ va $\lim\limits_{x \to a+0} f(x)$ limitlar mavjud bo'lib, ya'ni uzluksizlik shartlaridan ikkinchisi bajarilib, qolganlari (yoki ulardan aqalli bittasi) bajarilmasa, funksiya $x=a$ nuqtada 1- *tur uzilishga* ega bo'ladi. Masalan, $x<\frac{5}{3}$ bo'lganda -1 ga, $x>\frac{5}{3}$ bo'lganda 1 ga teng bo'lgan $y=\frac{3x-5}{|3x-5|}$ funksiya $x=\frac{5}{3}$ da 1- tur uzilishiga ega (30- chizma), chunki $\lim\limits_{x \to \frac{5}{3}-0} y = -1$ va $\lim\limits_{x \to \frac{5}{3}+0} y = 1$ limitlar mavjud, ammo bu limitlar o'zaro teng emas.

Ikkinchi tur uzilish. Agar ushbu $\lim\limits_{x \to a} f(x)$ limit (o'ngdan yoki chapdan) $\pm\infty$ ga teng bo'lsa, funksiya $x=a$ nuqtada 2- *tur*



30- chizma.



31- chizma.

uzilishga ega bo'ladi. Masalan, $y=f(x)=\frac{1}{2-x}$ funksiya (31- chizma) $x=2$ nuqtada ikkinchi tur uzilishiga ega.

734. $y=\frac{4}{x-2}$ funksiyaning uzilish nuqtasini ko'rsating, $\lim\limits_{x\to 2-0} y$; $\lim\limits_{x\to 2+0} y$; $\lim\limits_{x\to\pm\infty} y$ larni toping va $x=-2,\ 0,\ 1,\ 3,\ 4$ va 6 nuqtalar bo'yicha egri chiziq yasang.

735. 1) $y=-\frac{6}{x}$; 2) $y=\operatorname{tg}x$; 3) $y=\frac{4}{4-x^2}$ funksiyalarning uzilish nuqtalarini toping va grafiklarini yasang.

736. 1) $y=\frac{1}{x+3}$ va 2) $y=\frac{1}{1+2^{1/x}}$ funksiyalar berilgan. Uzilish nuqtalarini toping va ularning xarakterini tekshiring. Uzilish nuqtalari atrofida funksiyalarning sxematik grafiklarini yasang.

737. $y=\begin{cases} x/2, & x=2 \text{ bo'lganda}, \\ 0, & x\neq 2 \text{ bo'lganda} \end{cases}$

funksiyaning grafigini yasang va uning uzilish nuqtasini ko'rsating. Nuqtada uzluksizlikning to'rtta shartidan qaysilari bajariladi va qaysilari bajarilmaydi?

738. 1) $y=\frac{x+1}{|x+1|}$ va 2) $y=x+\frac{x+1}{|x+1|}$ funksiyalarning grafiklarini yasang. Bu funksiyalarning uzilish nuqtalarida uzluksizlik shartlaridan qaysilari bajariladi va qaysilari bajarilmaydi?

739—744- misollarda har qaysi funksiya uchun uzilish nuqtalarini toping va ularning xarakterini tekshiring.

739. $y=-\frac{1}{2-x}$.

740. $y=\frac{1}{(x+5)^2}$.

741. $y=\frac{2}{x^2-1}$.

742. $y=\frac{1}{x^2-4x+3}$.

743. $y=\frac{3}{2^x-1}$.

744. $y=5^{1/x}+2$.

745. $y=2^{1/x}$ funksiyaning uzilish nuqtasini ko'rsating, $\lim\limits_{x\to 2-0} y$; $\lim\limits_{x\to 2+0} y$; $\lim\limits_{x\to\pm\infty} y$ larni toping va funksiyaning grafigini yasang.

Uzilish nuqtasida uzluksizlik shartlarining qaysilari bajariladi, qaysilari bajarilmaydi?

746. $y=f(x)=\begin{cases} 0,5x^2, |x|<2 & \text{bo'lganda,} \\ 2,5, \ |x|=2 & \text{bo'lganda,} \\ 3, \ |x|>2 & \text{bo'lganda} \end{cases}$

funksiyaning grafigini yasang va uning uzilish nuqtalarini ko'rsating.

747. 1) $y=\dfrac{1}{1+2^{1/x}}$; 2) $y=\dfrac{x^3-x^2}{2|x-1|}$

funksiyalarning uzilish nuqtalarini toping va grafiklarini yasang.

748 va **749**- misollarda funksiyaning grafigini yasang va uning uzilish nuqtalarini ko'rsating.

748. $y=\begin{cases} x<-1 \ \text{da} \ -x-1, \\ -1\le x<0 \ \text{da} \ 0, \\ x\ge 0 \ \text{da} \ \sqrt{x}. \end{cases}$ **749.** $y=\begin{cases} x<0 \ \text{da} \ 1/x, \\ 0\le x<\pi/2 \ \text{da} \ \sin x, \\ x\ge \pi/2 \ \text{da} \ 0. \end{cases}$

750. $y=\dfrac{x}{x+2}$ funksiyaning uzilish nuqtasini ko'rsating, $\lim\limits_{x\to -2-0} y$, $\lim\limits_{x\to -2+0} y$, $\lim\limits_{x\to \pm\infty} y$ larni toping va $y=-6, -4, -3, -1, 0, 2$ nuqtalar bo'yicha grafigini yasang.

751. $y=f(x)=\begin{cases} 2, \ x=0 \ \text{va} \ x=\pm 2 & \text{bo'lsa,} \\ 4-x^2, 0<|x|<2 & \text{bo'lsa,} \\ 4, \ |x|>2 & \text{bo'lsa,} \end{cases}$

funksiyaning grafigini yasang va uzilish nuqtalarini ko'rsating.

752. 1) $y=2-\dfrac{|x|}{x}$; 2) $y=2^{\frac{1}{x-2}}$; 3) $y=1-2^{\frac{1}{x}}$; 4) $y=\dfrac{x^3+x}{2|x|}$;

5) $y=\dfrac{4-x^2}{|4x-x^3|}$ funksiyalarning uzilish nuqtalarini toping va grafiklarini yasang.

753 va **754**- misollarda funksiyaning grafigini yasang va uning uzilish nuqtalarini ko'rsating.

753. $y=\begin{cases} x\le 0 \ \text{da} \ 2-x, \\ 0<x<\pi/2 \ \text{da} \ \cos x, \\ x\ge \pi/2 \ \text{da} \ 0. \end{cases}$ **754.** $y=\begin{cases} x\le 0 \ \text{da} \ x, \\ x>0 \ \text{da} \ 1/x. \end{cases}$

9- §. Asimptotalar

Egri chiziqning *asimptotasi* deb shunday to‘g‘ri chiziqqa aytiladiki, egri chiziqning nuqtasi egri chiziq bo‘yicha cheksiz uzoqlashganda, u to‘g‘ri chiziqqa cheksiz yaqinlashib boradi.

1°. Agar $\lim\limits_{x\to a} f(x)=\pm\infty$ bo‘lsa, u holda $x=a$ to‘g‘ri chiziq $y=f(x)$ egri chiziqning *vertikal asimptotasi* bo‘ladi.

Masalan, $y=\dfrac{1}{2-x}$ egri chiziq $x=2$ vertikal asimptotaga ega (31- chizma).

2°. Agar $y=f(x)$ egri chiziq tenglamasining o‘ng tomonidan chiziqli qismini shunday ajratish mumkin bo‘lsaki, ya’ni $y=f(x)=kx+b+\alpha(x)$, $x\to\pm\infty$ da qolgan qismi $\alpha(x)\to 0$ bo‘lsa, u holda $y=kx+b$ to‘g‘ri chiziq egri chiziqning *og‘ma yoki gorizontal* asimptotasi bo‘ladi.

M i s o l l a r. 1) $y=\dfrac{2x^4-x^3+5}{x^3}=2x-1+\dfrac{5}{x^3}$ egri chiziq $y=2x-1$ og‘ma asimptotaga ega ($x=0$ vertikal asimptotasi bo‘ladi).

2) $y=\dfrac{1}{2-x}=0+\dfrac{1}{2-x}$ egri chiziq $y=0$ (31- chizma) gorizontal asimptotaga ega.

3°. Agar chekli $\lim\limits_{x\to\pm\infty}\dfrac{f(x)}{x}=k$ va $\lim\limits_{x\to\pm\infty}[f(x)-kx]=b$ limitlar mavjud bo‘lsa, u holda $y=kx+b$ to‘g‘ri chiziq $f(x)$ funksiya grafigining (og‘ma yoki gorizontal) asimptotasi bo‘ladi.

755. $y=1-\dfrac{4}{x^2}$ egri chiziqnimg asimptotalarini aniqlang va $x=\pm1, \pm2, \pm4$ nuqtalar bo‘yicba egri chiziqni yasang.

756—758- misollarda, kasrning chiziqli butun qismini ajratib, egri chiziqning asimptotalarini toping; asimptotalar 'va egri chiziqlarni yasang.

756. 1) $y=\dfrac{x^2+1}{x}$; 2) $y=\dfrac{x^2}{x+1}$; 3) $y=\dfrac{x^2}{x^2+1}$.

757. 1) $y=\dfrac{2}{|x|}-1$; 2) $y=\dfrac{x^2-x-1}{x}$; 3) $y=\dfrac{ax+b}{mx+n}$.

758. 1) $y=\dfrac{1-4x}{1+2x}$; 2) $y=\dfrac{x^3}{x^2+1}$; 3) $y=\dfrac{4x-x^3}{x^2+4}$.

Egri chiziqlarning asimptotalarini toping va egri chiziqlarni yasang:

759. $x^2 - y^2 = a^2$.

760. 1) $y = \sqrt{x^2+1} - \sqrt{x^2-1}$;

2) $y = \sqrt{x^2+1} + \sqrt{x^2-1}$; 3) $y = x - \frac{1}{\sqrt{x}}$.

761. 1) $y = x^2 + \frac{3}{x}$; 2) $y = \frac{x^2}{2(x-2)^2}$;

3) $y = \frac{x}{x^2-9}$; 4) $y = \frac{x^3}{x^2-1}$.

762. 1) $y = \frac{3x^5+1}{x^4}$; 2) $y = x^3 + \frac{3}{x}$.

763. 1) $y = e^{2x-x^2}$; 2) $y = x + \ln(x^2-4)$;

3) $y = 2xe^{2/x}$; 4) $y = (4e^{x^2}+3)/e^{x^2}$.

* * *

764. 1) $y = \left(1 - \frac{2}{x}\right)^2$; 2) $y = -x + \frac{1}{x^2}$ egri chiziqlarning asimptotalarini toping va $x = \pm\frac{1}{2}$, ± 1, ± 2 nuqtalar bo‘yicha egri chiziqlarni yasang.

765. 1) $y = \frac{x-4}{2x+4}$; 2) $y = \frac{x^2}{2-2x}$; 3) $y = \frac{x^2}{x^2-4}$;

4) $y = \frac{x^3}{1-x^2}$ egri chiziqlarning asimptotalarini toping va egri chiziqlarni yasang.

766—768- misollarda egri chiziqlarning asimptotalarini toping va egri chiziqlarni yasang.

766. $y = \frac{2x^2+1}{x}$.

767. $y = \frac{x^2}{x-1}$.

768. $y = \frac{x^3}{1+x^3}$.

10- §. *e* soni

$\lim\limits_{n\to\infty}(1+\frac{1}{n})^n = \lim\limits_{n\to-\infty}(1+\frac{1}{n})^n = \lim\limits_{\alpha\to 0}(1+\alpha)^{\frac{1}{\alpha}} = e$ limit *e* soni deyiladi.

Bu son irratsional bo'lib, $e\approx 2{,}71828...$. Asosi *e* ga teng logarifmlar *natural logarifmlar* deyiladi va $\log_e x = \ln x$ ko'rinishda belgilanadi.

O'nli logarifm: $\lg x = M\ln x$, bunda $M=0{,}43429...$.

Limitlarni toping:

769. $\lim\limits_{n\to\infty}\left(1-\frac{5}{n}\right)^n$, $(-\frac{5}{n}=\alpha$ belgilash kiritng).

770. 1) $\lim\limits_{n\to\infty}\left(1-\frac{1}{3n}\right)^n$; 2) $\lim\limits_{n\to-\infty}\left(1+\frac{4}{n}\right)^{n+3}$.

771. 1) $\lim\limits_{x\to\infty}(1+2x)^{1/x}$; 2) $\lim\limits_{x\to 0}(1-4x)^{\frac{1-x}{x}}$.

772. 1) $\lim\limits_{n\to\infty}\left(\frac{n}{n+1}\right)^n$; 2) $\lim\limits_{x\to\infty}\left(\frac{2x-1}{2x+1}\right)^{2x}$.

773. 1) $\lim\limits_{x\to\infty}\left(1+\frac{1}{x-1}\right)^x$; 2) $\lim\limits_{x\to\infty}\left(1-\frac{1}{x+1}\right)^x$; 3) $\lim\limits_{x\to\infty}\left(1+\frac{1}{x+1}\right)^{2x}$.

774. 1) $\lim\limits_{x\to\infty}\left(\frac{x+5}{x+2}\right)^{x-1}$; 2) $\lim\limits_{x\to\infty}\left(\frac{x-5}{x-2}\right)^x$; 3) $\lim\limits_{x\to\infty}\left(\frac{2x-1}{2x+1}\right)^{x+1}$.

775. $\lim\limits_{x\to\infty}\left(\frac{ax+b}{ax}\right)^x$. **776.** $\lim\limits_{x\to\infty}\left(1+\frac{m}{x}\right)^{x/n}$.

777. $\lim\limits_{n\to\infty}\left(\frac{n+5}{n}\right)^{1/n}$. **778.** $\lim\limits_{x\to 0}(1+\cos x)^{a\sec x}$.

779. $\lim\limits_{x\to\infty}\left(1+\frac{3}{x}\right)^{5x}$. **780.** $\lim\limits_{x\to 1} x^{\frac{1}{x-1}}$.

781. $\lim\limits_{x\to\infty}(x-1)^{\frac{1}{x-2}}$. **782.** 1) $\lim\limits_{n\to\infty} n[\ln(n+3)-\ln n]$;

2) $\lim\limits_{x\to 0}(1+3\text{tg}^2 x)^{\text{ctg}^2 x}$. **783.** $\lim\limits_{x\to 0}(\cos x)^{\text{ctg}^2 x}$, $(\sin^2 x=\alpha$ deb oling).

784. 1) $\lim\limits_{x\to 0}\frac{\ln(1+x)}{x}$; 2) $\lim\limits_{x\to 0}\frac{e^{-x}-1}{x}$; 3) $\lim\limits_{x\to 0}\frac{a^{2x}-1}{x}$.

Ko'rsatma. 784-misol: 2) misolda $e^{-x}-1=\alpha$ deb oling.

785. 1) $\lim\limits_{n\to\infty}\left(1+\frac{2}{n}\right)^{3n}$; 2) $\lim\limits_{n\to\infty}\left(\frac{n-3}{n}\right)^{n/2}$.

786. 1) $\lim\limits_{x\to\infty}\left(\frac{3x-2}{3x+1}\right)^{2x}$; 2) $\lim\limits_{x\to\infty}\frac{e^{-3x}-1}{x}$.

787. $\lim\limits_{n\to\infty}\left(1+\frac{a}{n}\right)^{n}$. **788.** $\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^{x+3}$.

789. $\lim\limits_{x\to\infty}\left(\frac{x}{x+1}\right)^{x}$. **790.** $\lim\limits_{x\to\infty}\left(\frac{x+2}{x+1}\right)^{x}$.

791. $\lim\limits_{x\to\infty}\left(\frac{2x+3}{2x+1}\right)^{x}$. **792.** $\lim\limits_{x\to\infty}\left(\frac{x}{x-1}\right)^{2x-3}$.

793. $\lim\limits_{x\to\pi/4}(\sin 2x)^{\operatorname{tg}^2 2x}$, ($\cos^2 2x=\alpha$ deb oling).

794. 1) $\lim\limits_{t\to 0}\frac{t}{\ln(1+xt)}$; 2) $\lim\limits_{n\to\infty} n[\ln n-\ln(n+2)]$.

VI bob javoblari

545. $0\le x<1$. **552.** $x_1=0$, $x_2=4$. **555.** 1) $x\le -2$; 2) $-3\le x\le 3$; 3) $0x\le 4$. **556.** 1) $-4\le x\le 0$; 2) $1\le x\le 3$. **557.** 1) $f(0)=1$, $f(1)=1$, $f(-1)=3$, $f(2)=3$, $f(a+1)=a^2+a+1$. **558.** $\varphi(2x)=2\sqrt{x^2+1}$, $\varphi(0)=2$. **560.** *a*) $3\lg 2$; *b*) $3\lg a$; *d*) $(\lg a)^3$. **561.** 1) $b+a$; 2) $2ah$. **562.** $\frac{b+a}{b^2+ab+a^2}$. **563.** $F(4,3)=19$, $F(3,4)=-25$. **564.** 1) juft; 2) toq; 3) juft; 4) toq; 5) toq; 6) juft ham emas, toq ham emas. **565.** $\log_a x$. **566.** a^x **568.** $2<x\le 3$. **572.** 1) $|x|\le 2$; 2) $-1\le x\le 3$; 3) $-\frac{\pi}{4}+k\pi\le x\le\frac{\pi}{4}+k\pi$; 4) $|x|\ge 2$. **573.** 2) $6x^2+2h^2$; 3) $4(2-a)$. **574.** $\varphi(1/x)=\frac{1-x}{3+5x}$, $\frac{1}{\varphi(x)}=\frac{3x+5}{x-1}$. **577.** $x=4$; 3, 1; 3, 01; …→3+0, $x=2$;2,9;2,99…→3−0.

592.

x	3;	2,1;	2,01; ⋯→	2+0
$\frac{3}{x-2}$	3;	30;	300; ⋯→	$+\infty$

$\lim\limits_{x\to 2+0}\frac{3}{x-2}=\infty$.

x	1;	1,9;	1,09; ⋯→	2−0
$\frac{3}{x-2}$	−3;	−30;	−300; ⋯→	$-\infty$

$\lim\limits_{x\to 2-0}\frac{3}{x-2}=-\infty$.

594. 1) $\lim\limits_{x\to\infty}\frac{2}{x}=0$; 2) $\lim\limits_{x\to+0}\frac{2}{x}=+\infty$; $\lim\limits_{x\to-0}\frac{2}{x}=-\infty$; 3) $\lim\limits_{x\to+\infty}3^x=\infty$;
4) $\lim\limits_{x\to\infty}3^x=0$; 5) $\lim\limits_{x\to+0}\lg x=-\infty$.

602. x=5; 4, 1; 4, 01; 4, 001;...→4+0;

x=3; 3, 9; 3, 99; 3, 999; ...→4−0;

x=−0,5; −1, 4; −1, 49; −1, 499; ...→−0,5+0;

x=−2,5; −1,6;−1,51;−1,501; ...→−1,5−0. **604.** Faqat birinchi o‘zgaruvchigina $\lim\limits_{x\to\infty}x_n=1$ limitga ega. Qolgan misollarda $\lim\limits_{x\to\infty}x_n$ mavjud emas. **605.** 1) 0; 2) ∞; 3) ∞; 4) 0; 5) 2; 6) 0; 7) a>1 bo‘lganda 0, a=1 bo‘lganda $\frac{1}{2}$, 0<a<1 bo‘lganda a. **606.** 1) –0,6; 2) 1. **607.** 4. **608.** 1. **609.** $\frac{3}{2}$. **610.** 3. **611.** $\frac{1}{8}$. **612.** 1. **613.** $-\frac{2}{3}$. **614.** 0. **615.** $3x^2$. **616.** $\frac{1}{2}$. **617.** $-\frac{1}{\sqrt{2}}$. **618.** $\frac{2}{3}$. **619.** a>0 bo‘lganda $-\frac{1}{2}$ va a<0 bo‘lganda ∞. **620.** $\frac{2}{3}$. **621.** $\frac{m}{3}$. **622.** $\frac{1}{2}$. **623.** $\frac{2\sqrt{2}}{3}$. **624.** 1. **625.** –3. **626.** –2. **627.** 1. **628.** $-\frac{1}{2}$. **629.** 1) $\frac{2}{3}$; 2) −2,5. **630.** 0. **631.** 1. **632.** ∞. **633.** 0. **634.** ∞. **635.** –2. **636.** $\frac{3}{2}$. **637.** $-\frac{1}{\sqrt{2}}$ **638.** $\frac{1}{6}$. **639.** $\frac{1}{4}$ **640.** 12. **641.** –1. **642.** $\lim\frac{|\sin x|}{\sin x\sqrt{1-\cos x}}=-\frac{1}{\sqrt{2}}$. **643.** –1. **644.** $-\frac{1}{6}$. **645.** $\frac{1}{6}$. **646.** 1. **647.** $\frac{8}{3}$. **648.** 2,5.x→π+0. **649.** $\sqrt{3}$. **650.** 0. **651.** −4. **652.** 2. **653.** $-\frac{1}{56}$.

654. $-\sqrt{2}$. **655.** 1) 4; 2) $\frac{1}{3}$. **656.** 1. **657.** $\frac{1}{4}$. **658.** 2. **659.** $6\sqrt{2}$. **660.** 2cos$x$. **661.** 0. **662.** 4. **663.** 1. **664.** $\frac{2}{3}$. **665.** 2π. **666.** $\frac{1}{8}$. **667.** $-\frac{1}{2}$. **668.** 100. **669.** 1) 1; 2) $-\frac{1}{2}$. **670.** $\frac{1}{2}$. **671.** $\frac{1}{8}$. **672.** $\frac{1}{4}$. **673.** $\frac{1}{3}$. **674.** $\frac{7}{3}$. **675.** $\frac{5}{7}$. **676.** 1. **677.** ∞. **678.** 8. **679.** $\lim\frac{\sqrt{2}|\sin x|}{x}=-\sqrt{2}$. **680.** 4. **681.** $m^2/2$. **682.** 3. **683.** $\frac{1}{4}$. **684.** 1) −2sin$x$; 2) $-\frac{1}{2}$. **685.** 2, x→–0. **686.** 1. **687.** 1,5. **688.** 0. **689.** 0. **690.** $\frac{1}{2}$. **691.** ∞. **692.** 1. **693.** $-\frac{7}{4}$.

694. $\frac{1}{4}$. **695.** –3. **696.** $\frac{2}{\pi}$. **697.** $\frac{1}{4}$. **698.** $-\frac{\alpha}{\pi}$. **699.** –2. **700.** $-\frac{1}{4}$.

701. $\frac{1}{2}$. **702.** 0. **703.** $\frac{1}{2}$. **704.** $-\frac{1}{2}$. **705.** –1. **706.** 1) $\frac{1}{\pi}$; 2) 0; 3) –1. **707.** 1) $\frac{1}{20}$; 2) 3. **708.** 1) $\frac{3}{4}$; 2) 2. **709.** $-a$. **710.** 1) –1; 2) –0,2. **711.** 1) 3; 2) $\frac{3}{2}$. **712.** 1) 1; 2) $-\frac{1}{2}$. **713.** 1) –2; 2) –0,1. **714.** 1) –2,5; 2) 1,5. **715.** 1) $-\sqrt{2\pi}$; 2) –1. **716.** $\frac{1}{2}$. **717.** 2. **718.** $\frac{2}{\pi}$.

719. $\frac{1}{6}$. **720.** 1) 2; 2) 3. **721.** 1) 4; 2) 1; 3) 3. **722.** $\frac{3}{2}$. **723.** 1. **724.** 1.

725. $\frac{1}{2}$. **727.** 1) 2,5; 2) $\frac{a}{b}$; 3) 1,5. **728** 1) 2; 2) 3; 3) 1. **729.** $\frac{2}{3}$.

730. 3. **731.** 3. **732.** $\frac{1}{2}$. **735.** 1) $x=0$; 2) $x=\frac{2n-1}{2}\pi$; 3) $x=\pm 2$. **736.** 1) $x=-3$ nuqtada II tur uzilishga ega; 2) $x=0$ nuqtada I tur uzilishga ega. **737.** $x=0$ da uzluksizlikning 4- sharti bajarilmaydi.

738. 1) $y=\begin{cases}-1, x<-1 & \text{bo‘lsa,}\\ 1, x>-1 & \text{bo‘lsa,}\end{cases}$ 2) $y=\begin{cases}x-1, x<-1 & \text{bo‘lsa,}\\ x+1, x>-1 & \text{bo‘lsa.}\end{cases}$

739. $x=2$, II tur uzilish nuqtasi. **740.** $x=-5$, II tur uzilish nuqtasi. **741.** $x=-1$ va $x=1$ I tur uzilish nuqtasi. **742.** $x=1$ va $x=3$ II tur uzilish nuqtasi. **743.** $x=0$ II tur uzilish nuqtasi. **744.** $x=0$ II tur uzilish nuqtasi. **745.** $x=0$ da uzilishga ega, $\lim y=\infty$, $\lim y=0$, $\lim y=1$. **746.** $x=\pm 2$ da uzilishlarga ega. **747.** 1) $x=0$ da I tur $x\to+0$, $x\to-0$, $x\to\infty$ uzilishga ega; 2) $x=1$ da I tur uzilishga ega. **748.** Funksiya uzluksiz. **749.** $x=a$ II tur uzilish nuqtasi; $x=\frac{\pi}{2}$ I tur uzilish nuqtasi. **750.** $x=-2$ da II tur uzilishga ega; $\lim\limits_{x\to-2-0} y=+\infty$, $\lim\limits_{x\to-2+0} y=-\infty$, $\lim\limits_{x\to\pm\infty}=1$. **751.** $x=0$ da uzluksizlikning 4- sharti bajarilmaydi, $x=\pm 2$ da ham uzilishga ega, uzluksizlikning 3- sharti bajarilmaydi. **752.** 1) $x=0$; 2) $x=2$; 3) $x=0$; 4) $x=0$; 5) $x=\pm 2$ va $x=0$ uzilish nuqtasi. **753.** $x=0$ da I tur uzilishda ega. **755.** $x=0$ va $y=1$. **756.** 1) $x=0$ va $y=x$; 2) $x=-1$ va $y=x-1$; 3) $y=1$. **757.** 1) $x=0$, $y=-1$; 2) $x=0$ va $y=x-1$; 3) $x=n/m$, $y=a/m$. **758.** 1) $x=-\frac{1}{2}$ va

$y=-2$; 2) $y=x$; 3) $y=-x$. **759.** $y=\pm x$. **760.** 1) $y=0$; 2) $y=\pm 2x$; 3) $x=0$ va $y=x$. **764.** 1) $x=0$ va $y=1$; 2) $x=0$ va $y=x$. **765.** 1) $x=-2$; $y=1/2$; 2) $x=1$; $y=-\frac{x+1}{2}$; 3) $x=2$, $x=-2$, $y=1$; 4) $x=1$, $x=-1$, $y=-x$. **766.** $x=0$, $y=2x$. **767.** $x=1$, $y=x+1$. **768.** $x=-1$, $y=1$. **769.** $1/e^5$. **770.** 1) $e^{-1/3}$; 2) e^4. **771.** 1) e^2; 2) e^4. **772.** 1) e^{-1}; 2) e^{-2}. **773.** 1) e; 2) $1/e$; 3) e^2. **774.** 1) e^3; 2) $1/e^3$; 3) $1/e$; **775.** $e^{b/a}$. **776.** $e^{m/n}$. **777.** $\sqrt[3]{e}$. **778.** e^a. **779.** e^{15}. **780.** e. **781.** e. **782.** 1) 3; 2) e^3. **783.** $1/\sqrt{e}$. **784.** 1) 1; 2) -1; 3) $2\ln a$. **785.** 1) e^b; 2) $\frac{1}{e\sqrt{e}}$. **786.** 1) $1/e^2$; 2) -3. **787.** e^a. **788.** e. **789.** $1/e$. **790.** e. **791.** e. **792.** e^2. **793.** $1/\sqrt{e}$. **794.** 1) $1/x$; 2) -2.

VII BOB HOSILA VA DIFFERENSIAL

1- §. Algebraik va trigonometrik funksiyalar hosilasi

1°. Ta'rif. $y=f(x)$ funksiya $(a;b)$ intervalda berilgan bo'lsin. x_0 shu intervalning biror nuqtasi bo'lsin. Bu x_0 nuqtaga biror Δx ($\Delta x\neq 0$, $x_0+\Delta x\in(a;\ b)$) orttirma berib, berilgan funksiyaning Δy orttirmasini topamiz:

$$\Delta y=f(x_0+\Delta x)-f(x_0).$$

Funksiya orttirmasining argument orttirmasiga nisbatini tuzamiz:

$$\frac{\Delta y}{\Delta x}=\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}.$$

Bu nisbatning $\Delta x\to 0$ dagi limitini topamiz. Agar

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}$$

limit mavjud va chekli bo'lsa, bu limit $f(x)$ funksiyaning x_0 nuqtadagi *hosilasi* deyiladi va

$$f'(x_0)\quad \text{yoki}\quad \frac{df(x_0)}{dx}$$

kabi belgilanadi.

Hosila uchun $f'(x)$ belgi qatorida boshqacha belgilar ham ishlatiladi, masalan,

$$y',\ \frac{dy}{dx}.$$

Berilgan $f(x)$ funksiyadan hosila topish amali shu *funksiyani differensiallash* deyiladi.

2°. Differensiallashning asosiy formulalari

1) $(C)'=0$, $C=\text{const}$; 2) $(x^n)'=nx^{n-1}$;

3) $(Cu)'=Cu'$; 4) $(u+v)'=u'+v'$;

5) $(u\cdot v)'=u'v+uv'$; 6) $\left(\frac{u}{v}\right)'=\frac{u'v-uv'}{v^2}$;

7) $\left(\sqrt{x}\right)'=\frac{1}{2\sqrt{x}}$; 8) $(\sin x)'=\cos x$;

9) $(\cos x)' = -\sin x$; 10) $(\mathrm{tg}x)' = \frac{1}{\cos^2 x}$; 11) $(\mathrm{ctg}x)' = -\frac{1}{\sin^2 x}$.

795. Quyidagi funksiyalarning hosilalarini $\lim\limits_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$ yordamida toping:

a) $y = x^3$; *b*) $y = \sqrt{x}$; d) $y = x^4$; e) $y = \cos x$;

f) $y = \frac{1}{x}$; g) $y = \frac{1}{\sqrt{x}}$; h) $y = \frac{1}{x^2}$; i) $y = \mathrm{tg}x$;

j) $y = \frac{1}{x^3}$; k) $y = \sqrt{3x+2}$; l) $y = \frac{1}{5x-1}$; m) $y = \sqrt{1+x^3}$.

Quyidagi funksiyalardan hosila oling:

796. $y = \frac{x^4}{5} + 3x^3 - x^2 + 2x - 4$. **797.** $y = \frac{x^6}{6} - \frac{2x^4}{4} + x$.

798. $y = 3x + 4\sqrt{x}$. **799.** $y = \frac{5}{x^4}$.

800. $y = \frac{1}{2x} + \frac{1}{x^2} + \frac{1}{x^3}$. **801.** $y = 3x + \frac{1}{x^2} - \frac{1}{5x^5}$.

802. $y = 3x - 6\sqrt{x}$. **803.** $y = 6\sqrt[3]{x} - 4\sqrt[4]{x}$.

804. $y = \left(1 - \frac{1}{\sqrt[3]{x}}\right)^2$. **805.** $y = \frac{1}{2x^2} - \frac{1}{3x^3}$.

806. $y = \frac{8}{\sqrt[4]{x}} - \frac{6}{\sqrt[3]{x}}$. **807.** $y = 2x + \cos x$.

808. $y = x^3 \sin x$. **809.** $y = x^2 \mathrm{ctg}x$.

810. $y = \frac{\cos x}{x^2}$. **811.** $y = \frac{x^2}{x^2+1}$.

812. $y = \frac{x}{1-4x}$. **813.** $y = \frac{\mathrm{tg}x}{\sqrt{x}}$.

814. $y = \frac{\cos x}{1-\sin x}$. **815.** $y = \frac{\sqrt{x}}{\sqrt{x}+1}$.

816. $y = \frac{2x^4}{b^2-x^2}$. **817.** $y = (2x-1)(x^2-6x+3)$.

818. $y = e^x(1-x^2)$. **819.** $y = \frac{e^x-1}{e^x+1}$.

820. $y=\frac{x^3}{3}-x^2+x$ berilgan, $y'(0)$, $y'(1)$, $y'(-1)$ ni hisoblang.

821. $f(x)=x^2-\frac{1}{2x^2}$ berilgan, $f'(2)-f'(-2)$ ni hisoblang.

822. $f(x)=\frac{(\sqrt{x}-1)^2}{x}$ berilgan, $0,01\cdot f'(0,01)$ ni hisoblang.

* * *

823. $y=\left(2x^2-3\right)^2$.

824. $y=\left(1+\sqrt[3]{x}\right)^2$.

825. $y=\frac{x^5}{a+b}-\frac{x^2}{a-b}-x.$

826. $y=6x^{7/2}+4x^{5/2}+2x.$

827. $y=\sqrt{x}+\sqrt[3]{x}+\frac{1}{x}.$

828. $y=\frac{1}{10x^5}-\frac{1}{4x^4}.$

829. $y=\frac{3}{\sqrt[3]{x}}-\frac{2}{\sqrt{x}}.$

830. $y=x^2\cdot\sin x.$

831. $y=x^2\cdot \mathrm{tg}x.$

832. $y=\sqrt{x}\cdot\cos x.$

833. $S=\frac{t^2}{4}-\frac{2}{t^3}.$

834. $y=x^3-\frac{2}{x^2}-\frac{1}{6x^3}.$

835. $y=\frac{x^2-1}{x^2+1}.$

836. $y=\left(1+\sqrt[3]{x}\right)^3$.

837. $y=\frac{2\sin x}{3+4\cos x}.$

838. $f(x)=\sqrt[3]{x^2}$ berilgan, $f'(-8)$ ni hisoblang.

839. $f(x)=\frac{3x+1}{x}$ berilgan, $f'(1)$, $f'(-1)$, $f'(0,1)$ larni hisoblang.

2- §. Murakkab funksiyaning hosilasi

Agar $y=f(u)$, $u=\varphi(x)$ bo'lsa, u holda *y=f(u) murakkab funksiya* deyiladi. Unda *y=f(u)* funksiya hosilasi $\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$ yoki $y'=f'(u)\cdot u'$ formula bo'yicha topiladi.

Oldingi paragrafdagi formulalar quyidagi ko'rinishda bo'ladi:

1) $(u^n)' = nu^{n-1} \cdot u'$; 2) $(\sin u)' = \cos u \cdot u'$;

3) $(\cos u)' = -\sin u \cdot u'$; 4) $(\sqrt{u})' = \frac{u'}{2\sqrt{u}}$;

5) $(\text{tg} u)' = \frac{u'}{\cos^2 u}$; 6) $(\text{ctg} u)' = -\frac{u'}{\sin^2 u}$.

Quyldagi funksiyalarning hosilasini toping.

840. 1) $y = \sin 7x$; 2) $y = e^{a+bx}$.

841. 1) $y = \sin\frac{x}{3} + 2\cos\frac{x}{2}$; 2) $y = \frac{1}{4}\cos 4x$.

842. $y = (x^2 + 5x + 7)^8$. **843.** $y = \sqrt[3]{(4+3x)^2}$.

844. 1) $y = \frac{2}{(x^2 - x + 1)^2}$; 2) $y = \sqrt{1 - x^2}$; 3) $y = \sqrt{\sin 2x}$.

845. $y = \sqrt{3x + \cos 3x}$. **846.** 1) $y = \cos^2 x$; 2) $y = \frac{1}{4}\text{tg}^4 x$.

847. $y = \cos x - \frac{1}{3}\cos^3 x$. **848.** $y = 3\sin^2 x - \sin^3 x$.

849. $y = \frac{1}{3}\text{tg}^3 x - \text{tg}x + x$. **850.** $y = x \cdot \sec^2 x - \text{tg}x$.

851. $y = \text{tg}\frac{x+1}{2}$. **852.** $y = \sqrt{1 + 2\text{tg}x}$.

853. $y = \sqrt[4]{1 + \cos^2 x}$. **854.** $y = \sin\sqrt{x}$.

855. $y = \frac{1}{(1 + \cos 4x)^5}$. **856.** $y = \frac{\sin^2 x}{\cos x}$.

857. $y = \text{ctg}\sqrt[3]{1 + x^2}$. **858.** $y = \sin(\sin x)$.

859. $y = \cos^2\frac{1-\sqrt{x}}{1+\sqrt{x}}$. **860.** $y = \sin^2(\cos 3x)$.

861. $r = a\sqrt{\cos 2\varphi}$. **862.** $f(t) = \sqrt{a^2 + b^2 - 2ab\cos t}$ berilgan, $f'\left(\frac{\pi}{2}\right)$ ni hisoblang.

863. $f(x)=\sqrt{x+2\sqrt{x}}$ berilgan, $f'(1)$ ni hisoblang.

864. $y(x)=\sqrt{\frac{x+1}{x-1}}$ berilgan, $y'(2)$ ni hisoblang.

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865. $y=\sqrt{1+\sin 2x}$.

866. $y=x^2\sqrt{1-x^2}$.

867. $y=\cos 4x$.

868. $y=\sqrt{x}\cdot\cos^2 x$.

869. $y=\operatorname{tg}\left(x^2+1\right)$.

870. $y=\frac{1}{4}\operatorname{tg}^4 x-\frac{1}{2}\operatorname{tg}^2 x-\ln\cos x$.

871. $y=\sin^2 x^3$.

872. $y=\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}-x}$.

873. $y=\sqrt{\frac{x}{2}-\sin\frac{x}{2}}$.

874. $y=\operatorname{tg}^4\left(x^2+1\right)$.

875. $y=\frac{\sin^3 x}{1+2^{x^2}}$.

876. $y=\frac{\sqrt{4x+1}}{x^2}$.

877. $u(x)=\left(x^2+x+2\right)^{\frac{3}{2}}$ berilgan, $u(1)$ ni hisoblang.

878. $z(t)=\left(\sqrt{t^3}+1\right)t$ berilgan, $z(0)$ ni hisoblang.

3- §. Logarifmik va ko'rsatkichli funksiyalarning hosilalari

Asosiy formulalar:

$(\ln u)'=\frac{u'}{u}$; $(e^u)'=e^u\cdot u'$; $(a^u)'=a^u\cdot\ln a\cdot u'$.

Quyidagi funksiyalarning hosilasini toping:

879. 1) $y=x\ln x$; 2) $y=\ln^2 x$; 3) $y=x^2\cdot\log_3 x$.

880. $y=\sqrt{\ln x}$.

881. $y=\frac{x-1}{\log_2 x}$.

882. $y=x\cdot\sin x\cdot\ln x$.

883. $y=\frac{1-\ln x}{1+\ln x}$.

884. $y=\sqrt{1+\ln^2 x}$.

885. $y=\ln^4\sin x$.

886. $y=\sqrt[3]{\ln\sin\frac{x+3}{4}}$.

887. $y=\ln\left(\sqrt{x}+\sqrt{x+1}\right)$.

888. $y=\ln\operatorname{tg}\left(\frac{\pi}{2}+\frac{x}{2}\right)$.

889. $y=\log_3\left(x^2-1\right)$.

890. $y=\frac{\cos x}{\sin^2 x}+\ln\operatorname{tg}\frac{x}{2}$.

891. $y=\ln\frac{x^2}{\sqrt{1-ax^4}}$.

892. $y=\ln\frac{\sqrt{5}+\operatorname{tg}\frac{x}{2}}{\sqrt{5}-\operatorname{tg}\frac{x}{2}}$.

893. $y=\ln x$ egri chiziqning Ox o‘qi bilan kesishish nuqtasidan o‘tkazilgan urinma tenglamasini yozing. Egri chiziq va urinma grafigini chizing.

Quyidagi funksiyalarning hosilasini toping:

894. 1) $y=x^3+2^x$; 2) $y=x^4\cdot 5^x$; 3) $y=x^3\cdot e^x$.

895. 1) $y=a^{\sin x}$; 2) $y=e^{-x^3}$; 3) $y=x^3\cdot e^{4x}$.

896. $y=2^{\frac{x}{\ln x}}$.

897. $y=\frac{1+e^x}{1-e^x}$.

898. $y=\frac{1-10^x}{1+10^x}$.

899. $y=10^{2x-3}$.

900. $y=\sin(2^x)$.

901. $y=a^{\sin^3 x}$.

902. $y=e^{\sqrt{\ln\left(ax^2+bx+c\right)}}$.

903. $y=ae^{-b^2x^2}$.

904. $y=Ae^{-k^2x}\cdot\sin(\omega x+\alpha)$.

905. 1) $y=x^x$; 2) $y=x^{\sin x}$.

906. 1) $y=x^{x^2}$; 2) $y=2x^{\sqrt{x}}$; 3) $y=\left(\frac{x}{1+x}\right)^x$.

* * *

907. $y=\ln\cos x-\frac{1}{2}\cos^2 x$.

908. $y=\ln\operatorname{tg}x$.

909. $y=\ln(x^2+3x+4)$.

910. $y=\ln(\sqrt{x}-\sqrt{x-1})$.

911. $y = \ln(\sin x + \sqrt{1 + \sin^2 x})$. **912.** $y = \ln \frac{1+\sqrt{x^2+1}}{x}$.

913. $y = \ln \frac{x}{\sqrt{1-x^2}}$. **914.** $y = \ln(e^x \cos x + e^{-x} \sin x)$.

915. $y = \frac{\ln \sin x}{\ln \cos x}$. **916.** $y = x \cdot 10^{\sqrt{x}}$.

917. $y = \frac{a}{2}(e^{\frac{x}{a}} + e^{-\frac{x}{a}})$. **918.** $y = \frac{e^x + e^{-x}}{e^x - e^{-x}}$.

919. $y = \ln(e^{2x} + \sqrt{e^{4x} + 1})$. **920.** $y = \ln(x \sin x \sqrt{1 - x^2})$.

921. $y = x^{\frac{1}{x}}$. **922.** $y = 2^{\frac{x}{\ln x}}$.

923. $y = (3 + x^2)^{\sqrt{x}}$. **924.** $y = 10^{x \operatorname{tg} x}$.

4- §. Teskari trigonometrik funksiyalarning hosilalari

Misollarni yechishda quyidagi formulalardan foydalaniladi:

$(\arcsin u)' = \frac{u'}{\sqrt{1-u^2}}$; $(\arccos u)' = -\frac{u'}{\sqrt{1-u^2}}$;

$(\operatorname{arctg} u)' = \frac{u'}{1+u^2}$; $(\operatorname{arcctg} u)' = -\frac{u'}{1-u^2}$.

925. $y = \arcsin \frac{1}{x}$. **926.** $y = \operatorname{arcctg}\sqrt{x}$.

927. $y = (\arcsin x)^2$. **928.** $y = \arcsin \sqrt{1 - 4x}$.

929. $y = \operatorname{arctg} \frac{x}{a}$. **930.** $y = \arccos(1 - 2x)$.

931. $y = \operatorname{arctg} \frac{x+1}{x-1}$. **932.** $y = \arcsin(n \sin x)$.

933. $y = \operatorname{arctg}\sqrt{\frac{1-x}{1+x}}$. **934.** $y = \ln(\operatorname{arctg} \frac{1}{1+x})$.

935. $y = \arccos \frac{1}{\sqrt{x}}$. **936.** $y = x \operatorname{arc\,tg} \frac{x}{a} - \frac{a}{2} \ln(x^2 + a^2)$.

* * *

Quyidagi funksiyalarning hosilasini toping:

937. $y = \arcsin \sqrt{x}$. **938.** $y = \arcsin(e^{3x})$.

939. $y = \arccos(1 - x^2)$. **940**. $y = e^x\sqrt{1 - e^{2x}} + \arcsin e^x$.

941. $y = 4^{\mathrm{arctg}\sqrt{x^2-1}}$. **942.** $y = \mathrm{arctg}e^{2x} + \ln\sqrt{\dfrac{e^{2x}+1}{e^{2x}-1}}$.

943. $s = \sqrt{4t - t^2} + 4\arcsin\dfrac{\sqrt{t}}{2}$.

944. $y = \dfrac{1}{m\sqrt{ab}}\mathrm{arctg}\left(e^{mx}\sqrt{\dfrac{a}{b}}\right)$.

945. $y = \arccos\sqrt{1 - 2x} + \sqrt{2x - 4x^2}$.

946. $f(z) = (z + 1)\mathrm{arctg}e^{-2z}$ berilgan, $f'(0)$ ni hisoblang.

947. $y = \sqrt[x]{\mathrm{arcctg}x}$. **948.** $y = (\cos x)^{\mathrm{arctg}x}$.

5- §.Giperbolik funksiyalarning hosilalari

1°.Ta'rif. $\dfrac{e^x - e^{-x}}{2}$ va $\dfrac{e^x + e^{-x}}{2}$ ifodalar va ularning nisbati, mos ravishda, *giperbolik sinus, giperbolik kosinus, giperbolik tangens* va *giperbolik kotangens* deyiladi.

2°.Giperbolik funksiyalarning xossalari:

1) $\mathrm{ch}^2x - \mathrm{sh}^2x = 1$; 4) $\mathrm{sh}0 = 0$, $\mathrm{ch}0 = 1$;

2) $\mathrm{ch}^2x + \mathrm{sh}^2x = \mathrm{ch}2x$; 5) $(\mathrm{sh}x)' = \mathrm{ch}x$, $(\mathrm{ch}x)' = \mathrm{sh}x$;

3) $\mathrm{sh}2x = 2\mathrm{sh}x \cdot \mathrm{ch}2x$;

6) $(\mathrm{th}x)' = \dfrac{1}{\mathrm{ch}^2x}$; $(\mathrm{cth}x)' = -\dfrac{1}{\mathrm{sh}^2x}$.

Quyidagi funksiyalarning hosilasini toping:

949. 1) $y = \dfrac{x}{\mathrm{sh}x}$; 2) $y = \mathrm{ch}^2x + \mathrm{sh}^2x$;

3) $y = x^2\mathrm{ch}x$; 4) $y = \ln\mathrm{th}x$.

950. $y = 2^{\mathrm{sh}x}$. **951.** $y = \mathrm{sh}^4(x^2 + 2x + 1)$.

952. $y = 4^{\mathrm{ch}^3x}$. **953.** $y = (\mathrm{ch}x)^{e^x}$.

954. $y = (\mathrm{ch}x)^{\mathrm{sh}x}$ funksiya hosilasining $x=0$ nuqtadagi qiymatini toping.

955. $f(x) = \mathrm{sh}\dfrac{x}{2} + \mathrm{ch}\dfrac{x}{2}$ berilgan, $f(0) + f'(0)$ ni toping.

956. 1) $y = \arcsin[\mathrm{th}x]$; 2) $y = \sqrt{1 + \mathrm{sh}^2 4x}$.

6- §. Differensiallashga oid aralash misollar

Funksiyalarning hosilasini toping:

957. $y = (1+\sqrt[3]{x})^3$.

958. $y = \operatorname{arctg}(\frac{x}{k} + b)$.

959. $y = \sqrt{1+\sqrt{2px}}$.

960. $y = \operatorname{arctg}(x^2 - 3x + 2)$.

961. $y = \lg(x - \cos x)$.

962. $y = 3\cos^2 x - \cos^3 x$.

963. $y = 5\operatorname{tg}\frac{x}{5} + \operatorname{tg}\frac{\pi}{8}$.

964. $y = \frac{1}{\sqrt[3]{x+\sqrt{x}}}$.

965. $y = \sin\frac{x}{2}\sin 2x$.

966. $y = \sin x \cdot e^{\cos x}$.

967. $y = x^5\sqrt[3]{x^6 - 8}$.

968. $y = e^{-x^2} \cdot \ln x$.

969. $y = (\sqrt{x} + \frac{1}{\sqrt{x}})^{10}$.

970. $y = \operatorname{arctg}\frac{x+1}{x-1}$.

971. $y = \frac{1}{99}(1-x)^{-99} - \frac{1}{49}(1-x)^{-98} + \frac{1}{97}(1-x)^{-97}$.

972. $y = (0,4\cos(8x+9) - 0,6\sin 0,8x)^2$.

973. $y = \sqrt{2x^2 + \sqrt{x^2+1}}$.

974. $y = \sqrt[13]{9 + 7\sqrt[5]{2x}}$.

975. $y = \operatorname{ctg}x^2 - \frac{1}{3}\operatorname{tg}^3 2x$.

976. $y = \frac{1-\cos(8x-3\pi)}{\operatorname{tg}2x - \operatorname{ctg}2x}$.

977. $y = \frac{1}{2}\operatorname{arctg}\frac{x}{2} - \frac{1}{3}\operatorname{arctg}\frac{x}{3}$.

978. $y = \ln\ln(x/2)$.

979. $y = \log_2^3(2x+3)^2$.

980. $y = 3^{\operatorname{arctg}(2x+\pi)}$.

981. $y = 10^{x/\log_3 x}$.

982. $y = \sin\cos^2 x \cdot \cos\sin^2 x$.

983. $y = \frac{\operatorname{ch}x^2}{\operatorname{sh}^2 x^2} - \ln\operatorname{cth}\frac{x^2}{2}$.

984. $y = \arccos(\sin x^4 - \cos x^4)$.

985. $f(u) = e^{-\frac{u}{a}} \cdot \cos\frac{u}{a}$; $f(0) + af'(0) = 0$ ekanligini isbotlang.

986. $s = \frac{1}{t\ln ct}$ funksiya $t\frac{ds}{dt} + s = -ts^2$ differensial tenglamaning yechimi ekanligini ko‘rsating.

7- §. Yuqori tartibli hosilalar

987. Funksiyalarning 2-tartibli hosilasini toping:

1) $y=\sin^2 x$; 2) $y=\mathrm{tg}x$; 3) $y=\sqrt{1+x^2}$.

988. $y=1-x^2-x^4$; $y'''=?$ **989.** $f(x)=(x+10)^6$; $f'''(2)=?$.

990. Funksiyalarning 3-tartibli hosilasini toping:

1) $y=x\ln x$; 2) $s=te^{-t}$; 3) $y=\mathrm{arctg}\,\frac{x}{a}$.

991. $s=\frac{t}{2}\sqrt{2-t^2}+\arcsin\frac{t}{\sqrt{2}}$; $\frac{d^3s}{dt^3}$ ni toping.

992. $y=x^3\ln x$; $y^{IV}=?$ **993.** $\rho=a\sin 2\varphi$; $\frac{d^4\rho}{d\varphi^4}=?$

Quyidagi funksiyalarning n- tartibli hosilasini toping:

994. 1) $e^{-\frac{x}{a}}$; 2) $\ln x$; 3) $\sqrt{x}$.

995. 1) x^n; 2) $\sin x$; 3) $\cos^2 x$.

996. 1) $\sin ax+\cos bx$; 2) xe^x; 3) $x\ln x$; 4) $\frac{x}{x^2-1}$.

997. Ketma-ket differensiallab Leybnis formulasini chiqaring:

$(uv)''=u''v+2u'v'+uv''$;

$(uv)'''=u'''v+3u''v'+3u'v''+uv'''$;

$(uv)^{IV}=u^{IV}v+4u'''v'+6u''v''+4u'v'''+uv^{IV}$.

998. Leybnis formulasi yordamida quyidagi funksiyalarning 2-tartibli hosilasini toping:

1) $y=e^x\cos x$; 2) $y=a^x\cdot x^3$; 3) $y=x^2\sin x$.

999. Leybnis formulasi yordamida quyidagi funksiyalarning 3-tartibli hosilasini toping:

1) $y=e^{-x}\sin x$; 2) $y=x^2\ln x$; 3) $y=x\cos x$.

1000. $f(x)=xe^{\frac{x}{a}}$; $f'''(x), f^{(n)}(x), f^{(n)}(0)$ ni toping.

1001. $f(x)=(1+x)^m$; $f(0), f'(0), f''(0), f'''(0), \dots, f^{(n)}(0)$ ni toping.

1002. $f(x)=\frac{x}{\sqrt{1+x}}$ berilgan, $n\geq 2$ bo‘lganda,

$f^{(n)}(0)=(-1)^{n-1}\frac{1\cdot 3\cdot 5\cdot\ldots\cdot(2n-3)}{2^{n-1}}\cdot n$ ekanligini ko‘rsating.

1003. $f(x)=\frac{1}{1-x^2}$; $f^{(n)}(0)=\begin{cases} n!, & n=2m \text{ bo‘lganda}, \\ 0, & n=2m-1 \text{ bo‘lganda}. \end{cases}$

Ko‘rsatma. Ushbu ayniyatni qo‘llang:

$$\frac{1}{1-x^2}=\frac{1}{2}\left(\frac{1}{1+x}+\frac{1}{1-x}\right).$$

1004. $(x-1)(x^2+x^3+...+x^n)=x^{n+1}-x^2$ ayniyatni x bo‘yicha 3 marta differensiallab, keyin $x=1$ qiymatni qo‘yib,

$$\sum_{k=1}^{n} k(k-1)=\frac{(n+1)n(n-1)}{3} \text{ va}$$

$$\sum_{k=1}^{n} k^2=1^2+2^2+...+n^2=\frac{n(n+1)(2n+1)}{6}$$ yig‘indini toping.

1005. $S=\frac{y'''}{y'}-\frac{3}{2}\left(\frac{y''}{y'}\right)^2$ ifodada y ni $\frac{1}{y}$ bilan almashtirilganda ifoda o‘zgarmasligini, ya’ni $y=\frac{1}{y_1}$ bo‘lsa, u holda

$$\frac{y_1'''}{y_1'}-\frac{3}{2}\left(\frac{y_1''}{y_1'}\right)^2=S$$ bo‘lishini isbotlang.

1006. $F(x)=f(x)\cdot\varphi(x)$ funksiya berilgan. Agar $f'(x)\cdot\varphi'(x)=C$ shart o‘rinli bo‘lsa, $\frac{F''}{F}=\frac{f''}{f}+\frac{\varphi''}{\varphi}+\frac{2C}{f\cdot\varphi}$ va $\frac{F'''}{F}=\frac{f'''}{f}+\frac{\varphi'''}{\varphi}$ tenglikni isbotlang.

* * *

1007. Quyidagi funksiyalarning 2- tartibli hosilasini toping:

1) $y=e^{-x^2}$; 2) $y=\operatorname{ctg}x$.

1008. $f(x)=\frac{1}{1-x}$; $f^{V}(x)=?$

1009. $y=x^2\cdot\sin 2x$, $y'''=?$

1010. $y=x^4\ln x$, $y^{IV}=?$

1011. Quyidagi funksiyalarning n-tartibli hosilasini toping:

1) $y=a^x$; 2) $y=\frac{1}{1+2x}$; 3) $y=\sin^2 x$.

1012. $f(x)=\arcsin\frac{1}{x}$ bo‘lsa, $f(2)$, $f'(2)$ va $f''(2)$ ni toping.

1013. Leybnis formulasi bo'yicha quyidagi funksiyalarning 3-tartibli hosilasini toping:

1) $y = x^3e^x$; 2) $y = x^2 \sin\frac{x}{a}$; 3) $y = xf'(a-x)+3f(a-x)$.

1014. $y = e^x \cos x$ funksiya $y^{IV} + 4y = 0$ differensial tenglamani qanoatlantirishini ko'rsating.

1015. $y = xe^{-\frac{1}{x}}$ funksiya $x^3y'' - xy' + y = 0$ tenglamani qanoatlantirishini ko'rsating.

1016. $f(x) = x^2e^{-\frac{x}{a}}$ berilgan. $f^{(n)}(0) = \frac{n(n-1)(-1)^n}{a^{n-2}}$ ekanligini ko'rsating.

1017. $f(x) = e^{-x^2}$ berilgan. $f^{(n)}(0) = -2(n-1)f^{(n-2)}(0)$,

$f^{(2m-1)}(0) = 0$, $f^{(2m)}(0) = (-2)^m \cdot (2m-1)(2m-3)...5\cdot 3\cdot 1$ ekanligini ko'rsating.

1018. $f(x) = x^n$ berilgan. $f(1)+\frac{f'(1)}{1!}+\frac{f''(1)}{2!}+...+\frac{f^{(n)}(1)}{n!} = 2^n$ ekanligini ko'rsating.

8- §. Oshkormas funksiyaning hosilasi

Agar $F(x,y)=0$ tenglamadagi y o'zgaruvchi x ning bir qiymatli funksiyasi bo'lsa, u holda y x ning *oshkormas funksiyasi* deylladi. Oshkormas funksiyaning y' hosilasini topish uchun $F(x,y)=0$ tenglamada y ni x ning funksiyasi deb qarab, uning ikki tomonidan x bo'yicha hosila olinadi. Olingan tenglamadan y' hosila topiladi. y'' ni topish uchun $F(x,y)=0$ tenglamani x bo'yicha ikki marta differensiallanadi va h. k.

Oshkormas funksiyalarning birinchi tartibli hosilasini toping:

1019. 1) $x^2+y^2=a^2$; 2) $x^2 = 2px$; 3) $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

1020. 1) $x^2+xy+x^2=6$; 2) $x^2+y^2-xy=0$.

1021. 1) $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$; 2) $e^y - e^{-x}+xy=0$.

1022. $e^x \cdot \sin y - e^{-y} \cdot \cos x = 0$.

1023. $x = y + \text{arcctg}y$.

1024. $x^3y - 3x^2y^2 + 5y^3 - 3x + 4 = 0.$

1025. $x^2y + \operatorname{arctg}\frac{y}{x} = 0.$

1026. $e^{xy} - x^2 + x^3 = 0$ bo‘lsa, $\frac{dy}{dx}$ ning $x = 0$ dagi qiymatini toping.

1027. Quyidagi tenglamalardan y'' ni toping:

1) $x^2 + y^2 = a^2$; 2) $ax + by - xy = c$; 3) $x^m y^n = 1$;

4) $xy^3 + 2y - 1 = 0.$

1028. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ berilgan, y'' ning $(0;b)$ nuqtadagi qiymatini toping.

1029. $x^2 + y^2 + 4x - 2y - 3 = 0$ egri chiziqning Oy o‘qi bilan kesishish nuqtasidagi urinma tenglamasini yozing.

1030. Quyidagi egri chiziqlarning $(x_0;y_0)$ nuqtasidagi urinma tenglamasini yozing:

1) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$; 2) $y^2 = 2px.$

1031. $x^{2/3} + x^{2/3} = a^{2/3}$ astroidaning $y=x$ to‘g‘ri chiziq bilan kesishish nuqtasidan o‘tuvchi urinma tenglamasini yozing.

1032. $x^2 + y^2 = 5$ va $y^2 = 4x$ egri chiziqlar qanday burchak ostida kesishadi?

* * *

1033. Quyidagi tenglamalardan y' ni toping:

1) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$; 2) $x^3 + y^3 - 3axy = 0$; 3) $x^3y^2 + 5xy + 4 = 0.$

1034. Quyidagi tenglamalardan y'' ni toping:

1) $x^2 - y^2 = a^2$; 3) $\operatorname{arctg} y = x + y$;

2) $(x-\alpha)^2 + (y-\beta)^2 = R^2$; 4) $x^2 + xy + y^2 = a^2.$

1035. $x^2 + y^2 + 4x - 4y + 3 = 0$ aylananing Ox o‘qi bilan kesishish nuqtasidan o‘tuvchi urinma tenglamasini yozing. Aylana va urinmani yasang.

1036. $t \cdot e^{-s/2} + se^{-t/2} = 2$ berilgan. $\frac{ds}{dt}$ ning $t = 0$ dagi qiymatini toping.

1037. $t\ln x - x\ln t = 1$ berilgan. $\frac{dx}{dt}$ ning $t = 1$ dagi qiymatini toping.

1038. $x^2 \sin y - \cos y + \cos 2y = 0$ berilgan. y' ning $y = \frac{\pi}{2}$ dagi qiymatini toping.

1039. $x^3 - 2x^2y^2 + 5x + y - 5 = 0$ egri chiziqning $(1;1)$ nuqtasidan o'tuvchi urinma tenglamasini yozing.

9- §. Funksiyaning differensiali

Faraz qilaylik, $y=f(x)$ funksiya $[a;b]$ kesmada differensiallanuvchi bo'lsin. Bu funksiyaning $[a;b]$ kesmaga tegishli biror x nuqtadagi hosilasi

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = f'(x) \tag{1}$$

tenglik bilan aniqlanadi. $\Delta x \to 0$ da $\frac{\Delta y}{\Delta x}$ nisbat ma'lum songa intiladi va demak, $f'(x)$ hosiladan cheksiz kichik miqdorga farq qiladi:

$$\frac{\Delta y}{\Delta x} = f'(x) + \alpha, \tag{2}$$

bu yerda $\Delta x \to 0$ da $\alpha \to 0$.

Oxirgi tenglikning barcha hadlarini Δx ga ko'paytirib, ushbu tenglikni hosil qilamiz:

$$\Delta y = f'(x)\Delta x + \alpha\Delta x. \tag{3}$$

Shunday qilib, funksiyaning Δy orttirmasi ikkita qo'shiluvchidan iborat bo'lib, bulardan orttirmaning bosh qismi deb ataladigan birinchisi ($f'(x) \neq 0$ da)Δx orttirmaga nisbatan chiziqlidir. $f'(x)\Delta x$ ko'paytma *funksiyaning differensiali* deb ataladi va dy yoki $df(x)$ bilan belgilanadi:

$$dy = f'(x)\Delta x. \tag{4}$$

$y = x$ funksiyaning differensialini topamiz, bu holda

$$y' = (x)' = 1$$

va,demak, $dy = dx = \Delta x$. Shunday qilib, erkli o'zgaruvchi x ning differensiali dx uning orttirmasi bilan bir xil bo'lar ekan.

(4) formulani quyidagi holda yoza olamiz:

$$dy = f'(x)dx.$$

(3) ifodani (4) ni hisobga olgan holda quyidagicha yozamiz:

$$\Delta y = dy + \alpha \cdot \Delta x \, .$$

Agar $f'(x) \neq 0$ bo'lsa, u holda $\alpha \cdot \Delta x$ ko'paytma dy ga nisbatan yuqori tartibli cheksiz kichik miqdordir:

$$\lim_{\Delta x \to 0} \frac{\Delta y}{dy} = 1 + \lim_{\Delta x \to 0} \frac{\alpha \cdot \Delta x}{f'(x)\Delta x} = 1 + \lim_{\Delta x \to 0} \frac{\alpha}{f'(x)} = 1.$$

Shuning uchun taqribiy hisoblashlarda ushbu

$$\Delta y \approx dy$$

taqribiy tenglikdan foydalaniladi.

Quyidagi funksiyalarning differensialini toping:

1040. 1) $y = x^n$; 2) $y = x^3 - 3x^2 + 3x$.

1041. 1) $y = \sqrt{1 + x^2}$; 2) $S = \frac{gt^2}{2}$.

1042. 1) $y = \operatorname{arctg} \frac{1}{x}$; 2) $y = 2^{\operatorname{tg}^2 x}$.

1043. 1) $y = \operatorname{ctg}^5 \left(x^3 + x^2\right)$; 2) $y = \sqrt[3]{(2 + \cos x)^2}$.

1044. 1) $y = \cos^2 \sqrt{x}$; 2) $y = \arccos(2^x)$.

1045. 1) $y = \frac{\operatorname{arctg} x}{\sqrt{1+x^2}}$; 2) $y = \left(1 + x^2\right)\operatorname{arcctg} x$.

1046. 1) $d\left(\frac{a}{x} + \operatorname{arctg} \frac{x}{a}\right)$; 2) $d(\alpha + \ln \alpha)$;

3) $d\left(\cos \frac{\varphi}{2}\right)$; 4) $d\left(\arcsin \frac{1}{x}\right)$.

1047. 1) $y = x^2$ berilgan. Agar x 2 dan 2,01 gacha o'zgarsa, y $(\Delta y \approx dy)$ ning taqribiy o'zgarishini toping;

2) $y = \sqrt{x}$ berilgan. Agar x 100 dan101 gacha o'zgarsa, y ning taqribiy o'zgarishini toping.

1048. Kubning tomoni $x = 5\text{m} \pm 0,01\text{m}$ ga teng. Kub hajmini hisoblashdagi absolut va nisbiy xatolikni toping.

1049. $\sqrt[3]{26,19}$ ning taqribiy qiymatini toping.

1050. $\sqrt[4]{16,64}$ ning taqribiy qiymatini toping.

1051. $\ln 0,9$ ning taqribiy qiymatini toping.

1052. $\sin 29^\circ$ ning taqribiy qiymatini toping.

* * *

Funksiya differensialini toping:

1053. 1) $y = \frac{1}{x} - \frac{1}{x^2}$; 2) $r = \cos(a - b\varphi)$;

3) $S = \sqrt{1 - t^2}$.

1054. 1) $y = \ln\cos x$; 2) $z = \operatorname{arctg}\sqrt{4u - 1}$;

3) $S = e^{-2t}$.

1055. 1) $d\left(\sqrt{x} + 1\right)$; 2) $d(\operatorname{tg}\alpha - \alpha)$;

3) $d\left(bt - e^{-bt}\right)$.

1056. $y = x^3$ bo'lsa, x 2 dan 1,98 gacha o'zgarganda, Δy va dy ni aniqlang.

1057. $\sin 46^\circ$ ning taqribiy qiymatini toping.

1058. 1) arctg 1,02; 2) arctg 0,97; 3) arcsin 0,4983 larning taqribiy qiymatini toping.

10- §. Egri chiziqning parametrik tenglamasi

Egri chiziq $x = f(t)$ va $y = \varphi(t)$ parametrik tenglamalar bilan berilgan bo'lsin. Parametr bo'yicha hosilani nuqta orqali belgilab, quyidagini topamiz:

$$\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}}; \qquad \frac{d^2y}{dx^2} = \frac{d\left(\frac{\dot{y}}{\dot{x}}\right)}{dx} = \frac{\ddot{y}\dot{x} - \ddot{x}\dot{y}}{\dot{x}^3}.$$

Egri chiziq parametrik tenglamalarini $F(x, y) = 0$ (yoki $y = f(x)$) ko'rinishga keltiring:

1059. 1) $\begin{cases} x = a\cos t, \\ y = b\sin t. \end{cases}$ 2) $\begin{cases} x = a\cos^3 t, \\ y = a\sin^3 t. \end{cases}$

1060. 1) $\begin{cases} x = \dfrac{e^t + e^{-t}}{2}, \\ y = \dfrac{e^t - e^{-t}}{2}. \end{cases}$ 2) $\begin{cases} x = \operatorname{tg} t, \\ y = \cos^2 t. \end{cases}$

1061. $\left.\begin{aligned} x &= \frac{1}{t+1} \\ y &= \frac{t}{t+1} \end{aligned}\right\}$ bo'lsa, $\frac{dy}{dx}$ ni toping.

1062. $\left.\begin{aligned} x &= \frac{3at}{1+t^3} \\ y &= \frac{3at^2}{1+t^3} \end{aligned}\right\}$ bo'lsa, $\frac{dy}{dx}$ ni toping.

1063. $\left.\begin{aligned} x &= e^t \cos t \\ y &= e^t \sin t \end{aligned}\right\}$ bo'lsa, $\frac{d^2y}{dx^2}$ ni toping.

1064. $\left.\begin{aligned} x &= a(\sin t - t\cos t) \\ y &= a(\cos t + t\sin t) \end{aligned}\right\}$ bo'lsa, $\frac{d^2y}{dx^2}$ ni toping.

1065. $\left.\begin{aligned} x &= e^t \\ y &= \arcsin t \end{aligned}\right\}$ bo'lsa, $\frac{d^2y}{dx^2}$ ni toping.

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1066. $\left.\begin{aligned} x &= a\cos\varphi \\ y &= b\sin\varphi \end{aligned}\right\}$ bo'lsa, $\frac{dy}{dx}$ ni toping.

1067. $\left.\begin{aligned} x &= 1 - t^2 \\ y &= t - t^3 \end{aligned}\right\}$ bo'lsa, $\frac{dy}{dx}$ ni toping.

1068. $x = \ln(1+t^2),\ y = t - \mathrm{arctg}t$ bo'lsa, $\frac{dy}{dx}$ ni toping.

1069. $\left.\begin{aligned} x &= \varphi(1 - \sin\varphi) \\ y &= \varphi\cos\varphi \end{aligned}\right\}$ bo'lsa, $\frac{dy}{dx}$ ni toping.

1070. $\frac{d^2y}{dx^2}$ ni toping:

1) $\begin{cases} x = 2\cos t, \\ y = \sin t. \end{cases}$ 2) $\begin{cases} x = t^2, \\ y = t + t^3. \end{cases}$ 3) $\begin{cases} x = e^{2t}, \\ y = e^{3t}. \end{cases}$

1071. $\frac{d^2y}{dx^2}$ ni toping:

1) $\begin{cases} x = a\cos^3 t, \\ y = a\sin^3 t. \end{cases}$ 2) $\begin{cases} x = \ln t, \\ y = \sin 2t. \end{cases}$

VIII BOB | HOSILA TATBIQLARI

1- §. Tezlik va tezlanish

1072. Moddiy nuqta $s=\frac{4}{3}t^3-t+5$ qonun bo'yicha harakatlanmoqda. $t=2$ sekunddagi a tezlanishni toping.

1073. Snaryad V m/s boshlang'ich tezlik bilan vertikal yuqoriga otildi. Necha t sekunddan keyin snaryad x balandlikda bo'ladi? Snaryadning tezligi va tezlanishini aniqlang.

1074. Moddiy nuqta $x=a\cos\omega t$ qonun bo'yicha tebranma harakatlanmoqda. $x=0$ va $x=\pm a$ nuqtada harakat tezligi va tezlanishini aniqlang.

1075. a radiusli g'ildirak to'g'ri chiziq bo'ylab harakatlanmoqda. G'ildirakning t sekunddan keyin φ burilish burchagi $\varphi=t+\frac{t^2}{2}$ tenglama bilan aniqlanadi. G'ildirak markazining harakat tezligi va tezlanishini aniqlang.

1076. Ox o'qi bo'ylab harakatlanayotgan nuqtaning tezligi v, tezlanishi w bo'lsin. $wdx=vdv$ tenglik o'rinliligini ko'rsating.

1077. Nuqta $v^2=2ax$ qonun bo'yicha to'g'ri chiziqli harakatlanmoqda, bu yerda v — tezlik, x — bosib o'tilgan yo'l va a — o'zgarmas. Harakat tezlanishini aniqlang.

* * *

1078. Jism 10 m balandlikdan 20 m/s boshlang'ich tezlik bilan vertikal tashlab yuborildi. Necha t sekunddan keyin jism x balandlikda bo'ladi? Harakat tezligi va tezlanishini aniqlang.

1079. Nuqtaning to'g'ri chiziqli harakati $s=t^2-4t+1$ qonunga bo'ysunadi. Nuqtaning harakati tezligi va tezlanishmi toping.

1080. Burchak tezlik $\frac{d\varphi}{dt}=\omega$, burchak tezlanish $\frac{d\omega}{dt}=\varepsilon$ ga teng bo'lsin. $\frac{d(\omega^2)}{d\varphi}=2\varepsilon$ tenglik o'rinli ekanligini ko'rsating.

2-§. O'rta qiymat haqida teoremalar

1°. Roll teoremasi. *Agar f(x) funksiya [a;b] kesmada uzluksiz, bu kesmaning barcha ichki nuqtalarida differensiallanuvchi bo'lib,* $x=a$ va $x=b$ *uchlarida nolga aylansa* ($f(a)=f(b)=0$ *bo'lsa*), *u holda* $[a;b]$ *kesma ichida hech bo'lmaganda shunday bir* $x=c$, $a<c<b$ *nuqta mavjudki, bu nuqtada* $f'(c)$ *hosila nolga aylanadi, ya'ni* $f'(c)=0$ bo'ladi.

2°. Lagranj teoremasi. *Agar f(x) funksiya [a;b] kesmada uzluksiz va bu kesmaning barcha ichki nuqtalarida differensiallanuvchi bo'lsa, u holda [a;b] kesma ichida hech bo'lmaganda shunday bir c,* $a<c<b$ *nuqta topiladiki, unda*

$$f(b)-f(a)=f'(c)(b-a)$$

bo'ladi.

3°. Koshi teoremasi. *Agar f(x) va φ(x) funksiyalar [a;b] kesmada uzluksiz va uning ichida differensiallanuvchi bo'lsa, shu bilan birga,* $\varphi'(x)$ *shu kesmada nolga aylanmasa, u holda [a;b] kesma ichida shunday* $x=c$, $a<c<b$ *nuqta topiladiki, unda*

$$\frac{f(b)-f(a)}{\varphi(b)-\varphi(a)}=\frac{f'(c)}{\varphi'(c)}$$

bo'ladi.

1081. $y=x^3+4x^2-7x-10$ funksiya uchun $[-1;2]$ intervalda Roll teoremasi shartlari o'rinli ekanligini tekshiring.

1082. $y=\ln\sin x$ funksiya uchun $\left[\frac{\pi}{6};\frac{5\pi}{6}\right]$ intervalda Roll teoremasi shartlari o'rinli ekanligini tekshiring.

1083. $y=4^{\sin x}$ funksiya uchun $[0;\pi]$ intervalda Roll teoremasi shartlari o'rinli ekanligini tekshiring.

1084. $y=\sqrt[3]{x^2-3x+2}$ funksiya uchun $[1;2]$ intervalda Roll teoremasi shartlari o'rinli ekanligini tekshiring.

1085. $y=\sin 3x$ funksiya uchun $[x_1;x_2]$ intervalda Lagranj formulasini yozing.

1086. $y=x(1-\ln x)$ funksiya uchun $[a;b]$ intervalda Lagranj formulasini yozing.

1087. $[0;a]$ intervalda $y=x^n$ funksiya uchun Lagranj teoremasi o'rinli ekanligini tekshiring *($a>0$, $n>0$)*.

1088. $[1;e]$ intervalda $y = \ln x$ funksiya uchun Lagranj teoremasi o'rinli bo'lishini tekshiring.

1089. $[1;\ 4]$ kesmada $f(x)=\sqrt{x}$ funksiya uchun Lagranj formulasini yozing va c ning qiymatini toping.

1090. $\frac{f(b)-f(a)}{\varphi(b)-\varphi(a)}=\frac{f'(c)}{\varphi'(c)}$ Koshi formulasini $f(x)=x^3$ va $\varphi(x)=x^2$ funksiyalar uchun yozing va c ning qiymatini toping.

1091. $f(x)=\sin x$ va $\varphi(x)=\cos x$ funksiyalar uchun $[0;\frac{\pi}{2}]$ intervalda Koshi formulasini yozing va c ning qiymatini toping.

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1092. $f(x)=x^3$ funksiya uchun $f(b)-f(a)=(b-a)f'(c)$ Lagranj formulasini yozing va c ning qiymatini toping.

1093. Quyidagi funksiyalar uchun Lagranj formulasini yozing va c ning qiymatini toping:

1) $f(x)=\text{arctg}x$, $[0;1]$ kesmada;

2) $f(x)=\arcsin x$, $[0;1]$ kesmada;

3) $f(x)=\ln x$, $[1;2]$ kesmada.

1094. $f(x)=x^2$ va $\varphi(x)=\sqrt{x}$ funksiyalar uchun $[1;4]$ intervalda Koshi formulasini yozing va c ning qiymatini toping.

3- §. Aniqmasliklarni ochish. Lopital qoidasi

1°. $\frac{0}{0}$ ko'rinishdagi aniqmaslik. Lopitalning birinchi qoidasi

Agar $\lim\limits_{x\to a} f(x)=\lim\limits_{x\to a}\varphi(x)=0$ va $\lim\limits_{x\to a}\frac{f'(x)}{\varphi'(x)}$ mavjud bo'lsa, u holda $\lim\limits_{x\to a}\frac{f(x)}{\varphi(x)}=\lim\limits_{x\to a}\frac{f'(x)}{\varphi'(x)}$ bo'ladi.

2°. $\frac{\infty}{\infty}$ ko'rinishdagi aniqmaslik. Lopitalning ikkinchi qoidasi

Agar $\lim\limits_{x\to a} f(x)=\lim\limits_{x\to a}\varphi(x)=\infty$ va $\lim\limits_{x\to a}\frac{f'(x)}{\varphi'(x)}$ mavjud bo'lsa, u holda $\lim\limits_{x\to a}\frac{f(x)}{\varphi(x)}=\lim\limits_{x\to a}\frac{f'(x)}{\varphi'(x)}$ bo'ladi.

3°. $0\cdot\infty$, $\infty-\infty$, 1^{∞} va 0^{0} ko'rinishdagi aniqmasliklar algebraik almashtirishlar yordamida $\frac{0}{0}$ va $\frac{\infty}{\infty}$ ko'rinishdagi aniqmasliklarga keltiriladi.

Quyidagi limitlarni toping:

1095. $\lim\limits_{x\to 0}\frac{\sin 2x}{3x}$.

1096. $\lim\limits_{x\to 0}\frac{e^{x}-1}{\sin 2x}$.

1097. $\lim\limits_{x\to a}\frac{x-a}{x^{n}-a^{n}}$.

1098. $\lim\limits_{x\to 0}\frac{1-\cos x}{2x^{2}}$.

1099. $\lim\limits_{x\to 1}\frac{\sin 3\pi x}{\sin 2\pi x}$.

1100. $\lim\limits_{x\to 0}\frac{\operatorname{tg}x-\sin x}{x-\sin x}$.

1101. $\lim\limits_{x\to 0}\frac{x-\sin x}{x^{3}}$.

1102. 1) $\lim\limits_{x\to +\infty}\frac{e^{x}}{x^{3}}$; 2) $\lim\limits_{x\to -\infty}\frac{e^{x}}{x^{3}}$.

1103. $\lim\limits_{x\to \infty}\frac{\ln x}{x}$.

1104. $\lim\limits_{x\to 0}\frac{\ln x}{\operatorname{ctg}x}$.

1105. $\lim\limits_{x\to \frac{\pi}{2}}\frac{\operatorname{tg}x}{\operatorname{tg}3x}$.

1106. $\lim\limits_{x\to \pi}(\pi-x)\cdot\operatorname{tg}\frac{x}{2}$.

1107. $\lim\limits_{x\to 0}x\ln x$.

1108. $\lim\limits_{x\to 1}\left(\frac{1}{\ln x}-\frac{1}{x-1}\right)$.

1109. $\lim\limits_{x\to \infty}\left(e^{x}-x^{2}\right)$.

1110. $\lim\limits_{x\to +\infty}x^{2}\cdot e^{-2x}$.

1111. $\lim\limits_{x\to 0}x^{x}$.

1112. $\lim\limits_{x\to 0}(\sin x)^{\operatorname{tg}x}$.

1113. $\lim\limits_{x\to +\infty}(\ln 2x)^{\frac{1}{\ln x}}$.

1114. $\lim\limits_{x\to 0}(\cos x)^{\operatorname{ctg}^{2}x}$.

1115. $\lim\limits_{x\to \infty}\left(1+\frac{3}{x}\right)^{x}$.

1116. $\lim\limits_{x\to \infty}\frac{x+\cos x}{x+\sin x}$.

* * *

1117. $\lim\limits_{x\to 0}\frac{e^{ax}-e^{bx}}{\sin x}$.

1118. $\lim\limits_{x\to 0}\frac{x-\operatorname{arctg}x}{x^{3}}$.

1119. $\lim\limits_{x\to \frac{\pi}{2a}}\frac{1-\sin ax}{(2ax-\pi)^{2}}$.

1120. $\lim\limits_{x\to 0}\frac{a^{x}-e^{x}}{\operatorname{tg}x}$.

1121. $\lim\limits_{x\to\frac{\pi}{6}}\frac{1-2\sin x}{\cos 3x}$.

1122. $\lim\limits_{x\to\frac{\pi}{4}}\frac{1-\mathrm{tg}x}{\cos 2x}$.

1123. $\lim\limits_{x\to 0}\frac{e^{2x}-1}{\ln(1+2x)}$.

1124. $\lim\limits_{x\to 0}\frac{\ln x}{1-x^3}$.

1125. $\lim\limits_{x\to 0}\left(1-e^{2x}\right)\cdot\mathrm{ctg}x$.

1126. $\lim\limits_{x\to 1}x^{\frac{1}{1-x}}$.

1127. $\lim\limits_{x\to 0}\left(\frac{1}{x\cdot\sin x}-\frac{1}{x^2}\right)$.

1128. $\lim\limits_{x\to 0}(e^{2x}+x)^{\frac{1}{x}}$.

4- §. Funksiyaning o‘sishi va kamayishi. Maksimum va minimum

1°. Ta’rif. I. Agar x_0 nuqtaning ε atrofida istalgan musbat $h<\varepsilon$ uchun $f(x_0-h)<f(x_0)<f(x_0+h)$ bo‘lsa, $f(x)$ funksiya x_0 nuqtada *o‘suvchi* deyiladi.

II. Agar $[a;b]$ segmentdagi istalgan $x_1$va x_2 uchun $x_1<x_2$ bo‘lganda $f(x_1)<f(x_2)$ bo‘lsa, $f(x)$ funksiya shu segmentda *o‘suvchi* deyiladi.

Funksiyaning nuqtada va segmentda kamayuvchi bo‘lishi ham shuning singari ta’riflanadi.

2°. $y=f(x)$ funksiyaning (nuqtada va kesmada) o‘suvchi va kamayuvchi bo‘lishining yetarli alomatlari:

agar $y''>0$ bo‘lsa, funksiya *o‘suvchi* bo‘ladi;

agar $y''<0$ bo‘lsa, funksiya *kamayuvchi* bo‘ladi.

3°. Ekstremumning zaruriy sharti. Agar differensiallanuvchi $y=f(x)$ funksiya $x=x_0$ nuqtada maksimumga yoki minimumga ega bo‘lsa, uning hosilasi shu nuqtada nolga aylanadi, ya’ni $f'(x_0)=0$. Bunday nuqtalar *kritik nuqtalar* deyiladi. Bu nuqtalarda urinma yoki gorizontal ($y'=0$), yoki vertikal (qaytish nuqtasida), yoki aniq urinma mavjud emas (masalan, burchak nuqtada). Oxirgi ikki holatda y' mavjud bo‘lmaydi.

4°. Ekstremumning yetarli sharti. $f(x)$ funksiya kritik nuqta x_0 ni o‘z ichiga olgan bironta intervalda uzluksiz va shu intervalning hamma (balki x_0 nuqtaning o‘zidan boshqa) nuqtalarida differensiallanuvchi bo‘lsin. Agar x_0 nuqtadan o‘tishda:

y' ishorasini „+“ dan„—” ga o‘zgartirsa, u holda $f(x_0)=y_{max}$,

y' ishorasini „—” dan „+” ga o‘zgartirsa, u holda $f(x_0)=y_{min}$,

y' ishorasini o'zgartirmasa, u holda ekstremum mavjud bo'lmaydi.

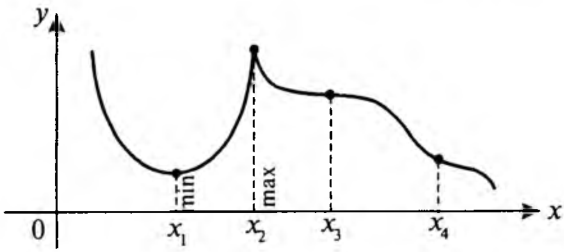
Shunday qilib, funksiya ekstremumini topish uchun:

1) y' va $y'=0$ yoki hosila mavjud bo'lmaydigan kritik nuqtalar topiladi;

2) y'ning ishorasi har bir kritik nuqtadan chap va o'ng tomonlarda aniqlanadi hamda quyidagi ko'rinishdagi jadval tuziladi:

x		x_1		x_2		x_3		x_4	
y'	$-$	0	+	Mavjud emas	$-$	0	$-$	$-\infty$	$-$
y	kamayadi	min	o'sadi	max	kamayadi	egilish	kamayadi	egilish	kamayadi

Keyin y_{max} va y_{min} topiladi va egri chiziq yasaladi. Yuqorlda keltirilgan jadval asosida 32- chizmada egri chiziq yasalgan.



32- chizma.

5°. Ekstremumning yetarli sharti (ikkinchi usul)

Agar biror $x=x_0$ nuqtada:

1) $y'=0$, $y''<0$ bo'lsa, u holda $f(x_0)=y_{max}$;

2) $y'=0$, $y''>0$ bo'lsa, u holda $f(x_0)=y_{min}$;

3) $y'=0$ va $y''=0$ bo'lsa, ekstremum mavjud bo'lishi noaniq bo'lib, tekshirishni birinchi usul bilan olib borish kerak.

Quyidagi funksiyalarning o'sish va kamayish oraliqlarini aniqlang:

1129. 1) $y = x^2$; 2) $y = x^3$; 3) $y = \frac{1}{x}$; 4) $y = \ln x$.

1130. 1) $y = \mathrm{tg}\, x$; 2) $y = e^x$; 3) $y = 4x - x^2$.

1131. $y = (x-2)^5 (2x+1)^4$.

1132. $y = \sqrt[3]{(2x-a)(a-x)^2} \quad (a > 0)$.

1133. $y = \frac{1-x+x^2}{1+x+x^2}$.

1134. $y = \frac{10}{4x^3-9x^2+6x}$.

1135. $y = x - e^x$.

1136. $y = x^2 e^{-x}$.

1137. $y = \frac{x}{\ln x}$.

1138. $y = 2x^2 - \ln x$.

Quyidagi funksiyalarning ekstremumini toping va grafigini chizing:

1139. $y = x^2 + 4x + 5$.

1140. $y = 4x - \frac{x^3}{3}$.

1141. $y = \frac{x^3}{3} - x^2 - 3x$.

1142. $y = 1 + 2x^2 - \frac{x^4}{4}$.

1143. $y = \frac{x^4}{4} - x^3$.

1144. $y = \frac{x}{2} + \frac{2}{x}$.

1145. $y = \sqrt[3]{x^2} - 1$.

1146. $y = \frac{1}{1+x^2}$.

1147. $y = \frac{x^2-6x+13}{x-3}$.

1148. $y = x^2(1-x)$.

1149. $y = 1 - \sqrt[3]{(x-4)^2}$.

1150. $y = e^{-x^2}$.

1151. $y = x + \cos 2x$, $(0;\pi)$ intervalda.

1152. $y = 4x - \mathrm{tg} x$, $\left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$ intervalda.

1153. $y = \frac{1+\ln x}{x}$.

1154. $y = x - \mathrm{arctg} 2x$.

1155. 1) $y = x \cdot e^{-x^2}$; 2) $y = x \ln x$.

1156. 1) $y = \sqrt{\sin x^2}$. 2) $y = \sqrt{e^{x^2} - 1}$.

1157. $y = \sin^4 x + \cos^4 x$.

1158. $y = x\sqrt{1-x}$.

1159. $y = \frac{4\sqrt{x}}{x+2}$.

1160. $y = \frac{x}{(x-1)(x-4)}$.

1161. $y = \frac{x^2}{2} + \frac{1}{x}$.

1162. $y = x^{\frac{2}{3}} + (x-2)^{\frac{2}{3}}$.

1163. $y = \frac{x^5}{5} - x^4 + x^3$.

1164. $y = x^3 (x+2)^2$.

1165. $y = 2\left(\frac{1}{x} - \frac{1}{x^2}\right)$.

1166. $y = \frac{x^3}{x^2-3}$.

1167. $y = 2\text{tg}x - \text{tg}^2 x$.

1168. $y = x + \ln(\cos x)$.

1169. $y = \ln\sqrt{1+x^2} - \text{arctg}\, x$.

1170. $y = x^2 e^{-x}$.

1171. $y = 3\sqrt[3]{(x+1)^2} - 2x$.

* * *

Quyidagi funksiyalarning ekstremumini toping va grafigini chizing:

1172. $y = 4x - x^2$.

1173. $y = x^2 + 2x - 3$.

1174. $y = \frac{x^3}{3} + x^2$.

1175. $y = x^3 + 6x^2 + 9x$.

1176. $y = \frac{x^2}{x-2}$.

1177. $y = x^3 + \frac{x^4}{4}$.

1178. $y = \frac{x^4}{4} - 2x^2$.

1179. $y = 2x - 3\sqrt[3]{x^2}$.

1180. $y = \frac{(x-1)^2}{x^2+1}$.

1181. $y = xe^{-x^2/2}$.

1182. $y = x - 2\ln x$.

1183. $y = x^{2/3} \cdot (x-5)$.

1184. $y = \sin 2x - x$, $\left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$ intervalda.

1185. $y = 2x + \text{ctg}x$, $(0;\pi)$ intervalda.

1186. $y = x + \text{arcctg}2x$.

1187. $y = 1 + \sqrt[3]{(x-1)^2}$.

1188. $y = 2\sin x + \cos 2x$, $(0;\pi)$ intervalda.

1189. $y = 3x^4 - 8x^3 + 6x$.

1190. $y = \frac{\ln x}{x}$.

1191. $y = \frac{3-x^2}{x+2}$.

1192. $y = x + \frac{1}{x}$.

1193. $y = 3x^5 - 5x^3$.

1194. $y = \frac{(4-x)^3}{9(2-x)}$.

5- §. Miqdorlarning eng katta va eng kichik qiymatlariga doir masalalar

1195. Uzunligi 120 metrlik panjara bilan bir tomondan uy bilan chegaralangan eng katta yuzga ega to‘g‘ri to‘rtburchak shaklidagi maydon o‘rab olinishi kerak. To‘g‘ri to‘rtburchakli maydon o‘lchamlarini aniqlang.

1196. 10 sonini shunday ikkita qo‘shiluvchiga ajratingki, ularning ko‘paytmasi eng katta bo‘lsin.

1197. 8 sonini shunday ikkita qo‘shiluvchiga ajratingki, ularning kublari yig‘indisi eng kichik bo‘lsin.

1198. Asosi a va balandligi h bo‘lgan uchburchakka eng katta yuzli to‘g‘ri to‘rtburchak ichki chizilgan. To‘g‘ri to‘rtburchak yuzini aniqlang.

1199. Tomoni a bo‘lgan kvadrat shaklidagi karton qog‘ozning to‘rtta uchidan kattaligi bir xil kvadratlar kesib olinib, qolgan qismidan to‘g‘ri burchakli quti yasalgan. Qutining hajmi eng katta bo‘lishi uchun kesib olingan kvadratning tomoni qanday bo‘lishi kerak?

1200. Tagi kvadrat shaklida, hajmi 32 m^3 ga teng ochiq hovuzning o‘lchamlarini shunday belgilash kerakki, uning devorlari bilan tagini qoplash uchun mumkin qadar oz material sarf etilsin.

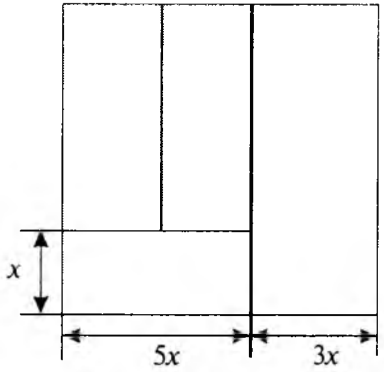
1201. Trapetsiyaning kichik asosi va yon tomonlarining har biri 10 smga teng. Uning katta asosini shunday aniqlansinki, trapetsiya yuzi eng katta bo‘lsin.

1202. Tunnelning kesimi bir tomoni yarim doiradan iborat to‘g‘ri to‘rtburchak shakliga ega. Kesim perimetri 18 m. Yarim doira radiusi qanday bo‘lsa, kesim yuzi eng katta bo‘ladi?

1203. Eni 2,4 va 1,6 m bo‘lgan ikki dahliz to‘g‘ri burchak ostida kesishadi. Bir dahlizdan ikkinchi dahlizga (gorizontal holatda) ko‘chirish mumkin bo‘lgan narvonning eng katta uzunligini aniqlang.

* * *

1204. Asosining radiusi 4 dm, balandligi 6 dm bo‘lgan konusga hajmi eng katta silindr ichki chizilgan. O‘sha silindrning hajmini toping.



33-chizma.

1205. Radiusi R bo'lgan yarim doiraga yuzi eng katta to'g'ri to'rtburchak ichki chizilgan. To'g'ri to'rtburchak o'lchamlarini aniqlang.

1206. Rejada ko'rsatilgan uy devorlarining umumiy uzunligi 90 m ga teng. Dahlizning eni x qanday bo'lsa, qolgan uch xonaning yuzi eng katta bo'ladi (33- chizma).

1207. R radiusli sharni ichki chizish mumkin bo'lgan eng katta hajmga ega konus balandligini toping.

6- §. Egri chiziqning qavariqlik va botiqlik oraliqlari. Funksiyani to'la tekshirish

1208. Egri chiziqning qavariqlik va botiqlik oraliqlarini toping va grafigini chizing:

1) $y = x^2$; 2) $y = x^3$; 3) $y = e^x$;

4) $y = \ln x$; 5) $y = x^{5/3}$.

1209. Egri chiziqnmg ekstremum hamda bukilish nuqtalarini aniqlang va grafigini yasang:

1) $y = \frac{x^3}{6} - x^2$; 2) $y = e^{-x^2}$;

3) $y = \frac{2x}{1+x^2}$; 4) $y = 2^{\frac{1}{x}}$.

1210. Egri chiziqlarning ekstremum va bukilish nuqtalarini aniqlang:

1) $y = -x^2 - 4x$; 2) $y = \ln(x+2)$; 3) $y = e^{-x}$.

1211—1234- misollarda berilgan funksiyalarni to'la tekshiring.

1211. $y = \frac{x}{1+x^2}$. **1212.** $y = \frac{1}{1-x^2}$.

1213. $y=\frac{x}{x^2-1}$.

1214. $y=\frac{1}{(x-1)(x-2)(x-3)}$.

1215. $y=\frac{x^2}{x^2-1}$.

1216. $y=\left(x^2-1\right)^3$.

1217. $y=32x^2\left(x^2-1\right)^3$.

1218. $y=\frac{1}{x}+4x^2$.

1219. $y=x^2+\frac{1}{x^2}$.

1220. $y=\frac{2x-1}{(x-1)^2}$.

1221. $y=\frac{x^2}{3-x^2}$.

1222. $y=\frac{x^3}{2(x+1)^2}$.

1223. $y(x-1)=x^3$.

1224. $y\left(x^3-1\right)=x^4$.

1225. $y=\frac{(x-1)^2}{(x+1)^3}$.

1226. $y=\frac{x^3+2x^2+7x-3}{2x^2}$.

1227. $y=\frac{x}{e^x}$.

1228. $y=x^2\cdot e^{-x}$.

1229. $y=\frac{e^x}{x}$.

1230. $y=x-\ln(x+1)$.

1231. $y=\ln\left(x^2+1\right)$.

1232. $y=x+\sin x$.

1233. $y=x\sin x$.

1234. $y=\ln\cos x$.

VII – VIII bob javoblari

795. a) $3x^2$; b) $\frac{1}{2\sqrt{x}}$; d) $4x^3$; e) $-\sin x$; f) $-\frac{1}{x^2}$; h) $-\frac{2}{x^3}$; i) $\frac{1}{\cos^2 x}$; j) $-\frac{3}{x^4}$; g) $-\frac{1}{2x\sqrt{x}}$; k) $\frac{3}{2\sqrt{3x+2}}$; l) $-\frac{5}{(5x-1)^2}$; m) $\frac{3x^2}{2\sqrt{1+x^3}}$. **796.** $\frac{4x^3}{5}+9x^2-2x+2$. **797.** x^5-2x^3+1. **798.** $3+\frac{2}{\sqrt{x}}$. **799.** $-\frac{20}{x^5}$. **800.** $-\frac{1}{2x^2}-\frac{2}{x^3}-\frac{3}{x^4}$. **801.** $3-\frac{2}{x^3}+\frac{1}{x^6}$. **802.** $3(1-\frac{1}{\sqrt{x}})$. **803.** $\frac{2}{\sqrt[3]{x^2}}-\frac{1}{\sqrt[4]{x^3}}$. **804.** $\frac{2}{3x}\left(\frac{1}{\sqrt[3]{x}}-\frac{1}{\sqrt[3]{x^2}}\right)$. **805.** $\frac{1-x}{x^4}$. **806.** $\frac{2}{x}\left(\frac{1}{\sqrt[3]{x}}-\frac{1}{\sqrt[4]{x}}\right)$. **807.** $2-\sin x$. **808.** $x^2(3\sin x+x\cos x)$. **809.** $\frac{x(\sin 2x-x)}{\sin^2 x}$. **810.** $-\frac{x\sin x+2\cos x}{x^3}$. **811.** $\frac{2x}{\left(x^2+1\right)^2}$. **812.** $\frac{1}{(1-4x)^2}$.

813. $\frac{4x-\sin 2x}{4x\sqrt{x}\cos^2 x}$. **814.** $\frac{1}{1-\sin x}$. **815.** $\frac{1}{2\sqrt{x}\left(\sqrt{x}+1\right)^2}$. **816.** $\frac{4x^3\left(2b^2-x^2\right)}{\left(b^2-x^2\right)^2}$. **817.** $6x^2-26x+12$. **818.** $e^x\left(1-2x-x^2\right)$. **819.** $\frac{2e^x}{\left(e^x+1\right)^2}$. **820.** 1;0;4. **821.** 8,25. **822.** −90. **823.** $8x(2x^2-3)$. **824.** $\frac{2}{3\sqrt[3]{x}}\left(\frac{1}{\sqrt[3]{x}}+1\right)$. **825.** $\frac{5x^4}{a+b}-\frac{2x}{a-b}-1$. **826.** $21x^{5/2}+10x^{3/2}+2$. **827.** $\frac{1}{2\sqrt{x}}+\frac{1}{3\sqrt[3]{x^2}}-\frac{1}{x^2}$. **828.** $\frac{2x-1}{2x^6}$. **829.** $\frac{1}{x}\left(\frac{1}{\sqrt{x}}-\frac{1}{\sqrt[3]{x}}\right)$. **830.** $x\left(2\sin x+x\cos x\right)$.

831. $\frac{x(\sin 2x+x)}{\cos^2 x}$. **832.** $\frac{\cos x-2x\sin x}{2\sqrt{x}}$. **833.** $\frac{ds}{dt}=\frac{t}{2}-\frac{6}{t^4}$. **834.** $3x^2+\frac{4}{x^3}+\frac{1}{2x^4}$.

835. $\frac{4x}{\left(x^2+1\right)^2}$. **836.** $\left(1+\frac{1}{\sqrt[3]{x}}\right)^2$. **837.** $\frac{2(4+3\cos x)}{(3+4\cos x)^2}$. **838.** $-\frac{1}{3}$. **839.** −1;−1;−100. **840.** 1) $7\cos 7x$; 2) be^{a+bx}. **841.** 1) $\frac{1}{3}\cos\frac{x}{3}-\sin\frac{x}{2}$; 2) $-\sin 4x$. **842.** $8(x^2+5x+7)^7(2x+5)$. **843.** $\frac{2}{\sqrt[3]{4+3x}}$. **844.** 1) $-\frac{4(2x-1)}{(x^2-x+1)^2}$; 2) $-\frac{x}{\sqrt{1-x^2}}$; 3) $\frac{\cos 2x}{\sqrt{\sin 2x}}$. **845.** $\frac{3(1-\sin 3x)}{2\sqrt{3x+\cos 3x}}$. **846.** 1) $-\sin 2x$; 2) $\mathrm{tg}^3 x\cdot\sec^2 x$. **847.** $-\sin^3 x$. **848.** $\frac{3}{2}\sin 2x(2-\sin x)$. **849.** $\mathrm{tg}^4 x$. **850.** $2x\frac{\sin x}{\cos^3 x}$. **851.** $\frac{1}{2\cos^2\frac{x+1}{2}}$. **852.** $\frac{1}{\sqrt{1+2\mathrm{tg}x}\cdot\cos^2 x}$. **853.** $-\frac{\sin 2x}{4\sqrt[4]{(1+\cos^2 x)^3}}$.

854. $\frac{\cos\sqrt{x}}{2\sqrt{x}}$. **855.** $\frac{20\sin 4x}{(1+\cos 4x)^6}$. **856.** $\sin x(1+\sec^2 x)$. **857.** $-\frac{2x}{3\sin^2\sqrt[3]{1+x^2}\cdot\sqrt[3]{(1+x^2)^2}}$. **858.** $\cos(\sin x)\cdot\cos x$. **859.** $\frac{\sin\left(2\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)}{\sqrt{x}(1+\sqrt{x})^2}$.

860. $-3\sin 3x\sin(2\cos 3x)$. **861.** $\frac{dr}{d\varphi}=-\frac{a\sin 2\varphi}{\sqrt{\cos 2\varphi}}$. **862.** $f'(\frac{\pi}{2})=\frac{ab}{\sqrt{a^2+b^2}}$.

863. $\frac{1}{\sqrt{3}}$. **864.** $y'(2)=-\frac{1}{\sqrt{3}}$. **865.** $\frac{\cos 2x}{\sqrt{1+\sin 2x}}$. **866.** $\frac{x(2-3x^2)}{\sqrt{1-x^2}}$. **867.** $-4\cos^3 x\cdot\sin x$. **868.** $\frac{\cos^2 x}{2\sqrt{x}}-2\sqrt{x}\cos x\cdot\sin x$. **869.** $\frac{2x}{\cos^2(x^2+1)}$. **870.**

$\mathrm{tg}^5 x$. **871.** $3x^2\cdot\sin 2x^3$. **872.** $\frac{2}{\sqrt{x^2+1}(\sqrt{x^2+1}-x^2)^2}$. **873.** $\frac{\sin^2\frac{x}{4}}{2\sqrt{\frac{x}{2}-\sin\frac{x}{2}}}$.

874. $\frac{8x\mathrm{tg}^3(x^2+1)}{\cos^2(x^2+1)}$. **875.** $\frac{\sin^2 x[3(1+2^{x^2})\cos x-2x\cdot 2^{x^2}\cdot\sin x\ln 2]}{(1+2^{x^2})^2}$. **876.** $-\frac{2(3x+1)}{x^3\sqrt{4x+1}}$. **877.** 9. **878.** 1. **879.** 1) $1+\ln x$; 2) $\frac{2}{x}\ln x$; 3) $2x\log_3 x+\frac{x}{\ln 3}$.

880. $\frac{1}{2x\sqrt{\ln x}}$. **881.** $\frac{x\ln x-x+1}{x\ln^2 x}$. **882.** $\sin x\cdot\ln x+x\cdot\cos x\cdot\ln x+\sin x$.

883. $-\frac{2}{x(1+\ln x)^2}$. **884.** $\frac{\ln x}{x\sqrt{1+\ln x^2}}$. **885.** $4\ln^3\sin x\cdot\mathrm{ctg}x$. **886.** $\frac{\mathrm{ctg}\frac{x+3}{4}}{12\sqrt[3]{\ln^2\sin\frac{x+3}{4}}}$. **887.** $\frac{1}{2\sqrt{x^2+x}}$. **888.** $\frac{1}{\cos x}$. **889.** $\frac{2x}{(x^2-1)\ln 3}$. **890.** $-\frac{2\mathrm{ctg}^2 x}{\sin x}$.

891. $\frac{2}{x-ax^5}$. **892.** $\frac{\sqrt{5}}{2+3\cos x}$. **893.** $y=x-1$. **894.** 1) $3x^2+2^x\ln 2$; 2) $(4+x\cdot\ln 5)\cdot 5^x\cdot x^3$; 3) $x^2\cdot e^x(3+x)$. **895.** 1) $\cos x\cdot a^{\sin x}\cdot\ln a$; 2) $-3x^2\cdot e^{-x^3}$; 3) $x^2\cdot e^{4x}(3+4x)$. **896.** $\frac{(\ln x-1)\ln 2}{\ln^2 x}\cdot 2^{\frac{x}{\ln x}}$. **897.** $\frac{2e^x}{(1-e^x)^2}$. **898.** $-\frac{2\cdot 10^x\ln 10}{(1+10^x)^2}$. **899.** $2\cdot 10^{2x-3}\cdot\ln 10$. **900.** $2^x\ln 2\cdot\cos(2^x)$.

901. $3\sin^2 x\cdot\cos x\cdot a^{\sin^3 x}\cdot\ln a$. **902.** $\frac{(2ax+b)e^{\sqrt{\ln(ax^2+bx+c)}}}{2(ax^2+bx+c)\sqrt{\ln(ax^2+bx+c)}}$. **903.** $-2ab^2xe^{-b^2x^2}$. **904.** $Ae^{-k^2x}[\omega\cos(\omega x+\alpha)-k^2\sin(\omega x+\alpha)]$. **905.** 1) $x^x(\ln x+1)$; 2) $x^{\sin x}[\cos x\ln x+\frac{\sin x}{x}]$. **906.** 1) $x^{x^2+1}(2\ln x+1)$; 2) $x^{\sqrt{x}-\frac{1}{2}}(2+\ln x)$; 3) $\left(\frac{x}{x+1}\right)^x\left(\frac{1}{x+1}+\ln\frac{x}{x+1}\right)$. **907.** $-\mathrm{tg}x\cdot\sin^2 x$. **908.** $\frac{1}{\sin x\cdot\cos x}$. **909.** $\frac{2x+3}{x^2+3x+4}$. **910.** $-\frac{1}{2\sqrt{x^2-x}}$. **911.** $\frac{\cos x}{\sqrt{1+\sin^2 x}}$. **912.** $-\frac{1}{x\sqrt{1+x^2}}$.

913. $\frac{1}{x(1-x^2)}$. **914.** $\frac{(\cos x-\sin x)(e^x+e^{-x})}{e^x\cos x+e^{-x}\sin x}$. **915.** $\frac{\mathrm{ctg}x\ln\cos x+\mathrm{tg}x\ln\sin x}{\ln^2\cos x}$. **916.** $10^{\sqrt{x}}(1+\frac{\sqrt{x}}{2}\ln 10)$. **917.** $\frac{1}{2}\left(e^{\frac{x}{a}}-e^{-\frac{x}{a}}\right)$. **918.** $-\frac{4}{(e^x-e^{-x})^2}$. **919.** $\frac{2e^{2x}}{\sqrt{e^{4x}+1}}$.

920. $\frac{1}{x}-\frac{x}{1-x^2}+\text{ctg}x$. **921.** $x^{\frac{1}{x}}\cdot\frac{1-\ln x}{x^2}$. **922.** $2^{\frac{x}{\ln x}}\cdot\frac{\ln x-1}{\ln^2 x}\cdot\ln 2$.

923. $\left(3+x^2\right)^{\sqrt{x}}\left[\frac{2x\sqrt{x}}{3+x^2}+\frac{\ln(3+x^2)}{2\sqrt{x}}\right]$. **924.** $10^{x\text{tg}x}\ln 10\left(\text{tg}x+\frac{x}{\cos^2 x}\right)$. **925.** $-\frac{1}{x\sqrt{x^2-1}}$. **926.** $-\frac{1}{2(1+x)\sqrt{x}}$. **927.** $\frac{2\arcsin x}{\sqrt{1-x^2}}$. **928.** $-\frac{1}{\sqrt{x-4x^2}}$. **929.** $\frac{a}{a^2+x^2}$. **930.** $\frac{1}{\sqrt{x-x^2}}$. **931.** $-\frac{1}{1+x^2}$. **932.** $\frac{n\cos x}{\sqrt{1-n^2\sin^2 x}}$. **933.** $-\frac{1}{2\sqrt{1-x^2}}$.

934. $-\frac{1}{(x^2+2x+2)\text{arctg}\frac{1}{1+x}}$. **935.** $\frac{1}{2x\sqrt{x-1}}$. **936.** $\text{arctg}\frac{x}{a}$. **937.** $\frac{1}{2\sqrt{x-x^2}}$.

938. $\frac{3e^{3x}}{\sqrt{1-e^{6x}}}$. **939.** $\frac{2x}{|x|\cdot\sqrt{2-x^2}}$. **940.** $2e^x\sqrt{1-e^{2x}}$. **941.** $4^{\text{arctg}\sqrt{x^2-1}}\ln 4\frac{1}{x\sqrt{x^2-1}}$. **942.** $\frac{4e^{2x}}{1-e^{8x}}$. **943.** $\sqrt{\frac{4}{t}-1}$. **944.** $\frac{1}{ae^{mx}+be^{-mx}}$.

945. $\sqrt{\frac{2}{x}-4}$. **946.** $\frac{\pi}{4}-1$. **947.** $-\sqrt[x]{\text{arcctg}x}\left[\frac{1}{x(1+x^2)\text{arcctg}x}+\frac{\ln \text{arcctg}x}{x^2}\right]$.

948. $(\cos x)^{\text{arctg}x}\left(\frac{\ln\cos x}{1+x^2}-\text{tg}x\cdot\text{arctg}x\right)$. **949.** 1) $\frac{\text{sh}x-x\text{ch}x}{\text{sh}^2 x}$; 2) $4\text{sh}x\text{ch}x$; 3) $x^2\text{sh}x+2x\text{sh}x$; 4) $\frac{1}{\text{sh}x\text{ch}x}$. **950.** $2^{\text{sh}x}\text{ch}x\cdot\ln 2$. **951.** $4\text{sh}^3 x(x^2+2x+1)\times$ $\times\text{ch}(x^2+2x+1)(2x+2)$. **952.** $4^{\text{ch}^3 x}\ln 4\cdot 3\text{ch}^2 x\cdot\text{sh}x$. **953.** $(\text{ch}x)^{e^x}\times$ $\times e^x\cdot(\ln\text{ch}x+\text{th}x)$. **954.** 0. **955.** 1,5. **956.** 1) $\frac{1}{\text{ch}x}$; 2) $4\text{sh}4x$. **957.** $\frac{(1+\sqrt[3]{x})^2}{\sqrt[3]{x^2}}$.

958. $\frac{a}{k\cos^2(\frac{x}{k}+b)}$. **959.** $\frac{p}{2\sqrt{1+\sqrt{2px}}\sqrt{2px}}$. **960.** $\frac{2x-3}{1+(x^2-3x+2)^2}$. **961.** $\frac{1+\sin x}{(x-\cos x)\ln 10}$.

962. $\frac{3}{2}\sin 2x(\cos x-2)$. **963.** $\sec^2\frac{x}{5}$. **964.** $-\frac{1+2\sqrt{x}}{6\sqrt{x}\sqrt[3]{(x+\sqrt{x})^4}}$. **965.** $2\sin\frac{x}{2}\cos 2x+\frac{1}{2}\cos\frac{x}{2}\sin 2x$. **966.** $e^{\cos x}(\cos x-\sin^2 x)$.

967. $\frac{x^4(7x^6-40)}{\sqrt[3]{(x^6-8)^2}}$. **968.** $e^{-x^2}(\frac{1}{x}-2x\ln x)$. **969.** $\frac{5(x-1)}{x\sqrt{x}}(\sqrt{x}+\frac{1}{\sqrt{x}})^9$.

970. $-\frac{1}{1+x^2}$. **971.** $\frac{x^2}{(1-x)^{100}}$. **972.** $-0,64(2\cos(8x+5)-3\sin 0,8x)\times$

$\times(2\sin(8x+5)+0{,}3\cos 0{,}8x)$. **973.** $\frac{1+4\sqrt{x^2+1}}{2\sqrt{2x^2+\sqrt{x^2+1}}}\cdot\frac{x}{\sqrt{x^2+1}}$. **974.** $\frac{14}{65\sqrt[5]{(2x)^4}\cdot\sqrt[13]{(9+7\sqrt[5]{2x})^{12}}}$. **975.** $-2(\frac{x}{\sin^2 x}+\frac{\operatorname{tg}^2 2x}{\cos^2 2x})$. **976.** $-4\cos 8x,\ x\neq\frac{\pi}{8}+\frac{\pi}{4}k,$ $x\neq\frac{\pi}{4}+\frac{\pi}{2}k,\quad x\neq\frac{\pi}{2}k,\quad k\in Z$ **977.** $\frac{5}{x^4+13x^2+36}$. **978.** $\frac{1}{x\ln\frac{x}{2}},\quad x>2.$

979. $\frac{12}{\ln 2}\cdot\frac{\log_2^2(2x+3)^2}{2x+3}$. **980.** $\frac{2\ln 3}{1+(2x+\pi)^2}\cdot 3^{\operatorname{arctg}(2x+3)}$. **981.** $\frac{(\ln x-1)\ln 10}{\ln x\cdot\log_3 x}\cdot 10^{\frac{x}{\log_3 x}}$.

982. $-\sin 2x\cdot\cos(\cos 2x)$. **983.** $-\frac{4x}{\operatorname{sh}^3 x^2}$. **984.** $-\frac{4x^3(\cos x^4+\sin x^4)}{\sqrt{\sin 2x^4}}$.

987. 1) $2\cos 2x$; 2) $2\operatorname{tg}x\cdot\sec^2 x$; 3) $\frac{1}{(1+x^2)^{\frac{3}{2}}}$. **988.** $-24x$. **989.** 207360.

990. 1) $-\frac{1}{x^2}$; 2) $e^{-t}(3-t)$; 3) $\frac{2a(3x^2-a^2)}{(x^2+a^2)^3}$. **991.** $-\frac{2}{(2-t)^{\frac{3}{2}}}$. **992.** $\frac{6}{x}$.

993. $16a\sin 2\varphi$. **994.** 1) $\left(-\frac{1}{a}\right)^n\cdot e^{-\frac{x}{a}}$; 2) $\frac{(-1)^n(n-1)!}{x^n}$; 3) $\frac{(-1)^n\cdot 1\cdot 3\cdot 5\cdot\ldots\cdot(2n-3)}{2^n\sqrt{x^{2n-1}}}$.

995. 1) $n!$; 2) $\sin(x+n\frac{\pi}{2})$; $\quad 2^{n-1}\cdot\cos(2x+n\frac{\pi}{2})$. **996.** 1) $a^n\sin(ax+n\frac{\pi}{2})+$ $+b^n\cos(bx+n\frac{\pi}{2})$; 2) $e^x(x+n)$; 3) $(-1)^n\frac{(n-2)!}{x^{n-1}}$ $(n\geq 2)$; 4) $(-1)^n\frac{n!}{2}$ $\left[\frac{1}{(x+1)^{n+1}}+\frac{1}{(x-1)^{n+1}}\right]$. **998.** 1) $-2e^x\sin x$; 2) $xa^x(x^2\ln^2 a+6x\ln a+6)$; 3) $2\sin x+4x\cos x-x^2\sin x$. **999.** 1) $2e^{-x}(\sin x+\cos x)$; 2) $\frac{2}{x}$; 3) $x\sin x-3\cos x$. **1000.** $f'''(x)=\frac{x+3a}{a^3}e^{\frac{x}{a}}$; $f^{(n)}(x)=\frac{x+na}{a^n}e^{\frac{x}{a}}$; $f^{(n)}(0)=\frac{n}{a^{n-1}}$.

1001. 1, m, $m(m-1)$, $m(m-1)(m-2)$, ..., $m(m-1)\ldots(m-n+1)$. **1007.** 1) $2e^{-x^2}(2x^2-1)$; 2) $\frac{2\operatorname{ctg}x}{\sin^2 x}$. **1008.** $\frac{5!}{(1-x)^6}$. **1009.** $y'''=-4(2x^2\cos 2x+$ $+6x\sin 2x-3\cos 2x)$. **1010.** $y^{IV}=24\ln x+50$. **1011.** 1) $a^x(\ln a)^n$; 2) $(-1)^n\frac{2^n\cdot n!}{(1+2x)^{n+1}}$; 3) $-2^{n-1}\cdot\cos(2x+n\cdot\frac{\pi}{2})$. **1012.** $\frac{\pi}{6}$; $-\frac{\sqrt{3}}{6}$; $\frac{7\sqrt{3}}{36}$. **1013.** 1) $e^x(x^3+9x^2+18x+6)$; 2) $\frac{1}{a^3}(6a^2\cos\frac{x}{a}-6ax\sin\frac{x}{a}-x^2\cos\frac{x}{a})$; 3)

$-x\cdot f^{IV}(a-x)$. **1019.** 1) $-\frac{x}{y}$; 2) $\frac{p}{y}$; 3) $\frac{b^2x}{a^2y}$. **1020.** 1) $-\frac{2x+y}{x-2y}$; 2) $\frac{2x-y}{x-2y}$. **1021.** 1) $-\sqrt[3]{\frac{y}{x}}$; 2) $\frac{e^{-x}+y}{e^y+x}$. **1022.** $-\frac{e^x\sin y+e^{-y}\sin x}{e^x\cos y+e^{-y}\cos x}$. **1023.** $\frac{1}{y^2}+1$. **1024.** $\frac{-3x^2y+6xy^2+3}{x^3-6x^2y+15y^2}$. **1025.** $\frac{-2x^3y-2xy^3+y}{x^4+x^2y^2+x}$. **1026.** $\frac{1}{3}$. **1027.** 1) $-\frac{a^2}{y^3}$; 2) $\frac{2(y-a)}{(x-b)^2}$; 3) $\frac{m(m+n)y}{n^2x^2}$; 4) $\frac{12y^5(xy^2+1)}{(3xy^2+2)^3}$. **1028.** $-\frac{b}{a^2}$. **1029.** $y=3-x$ va $y=x-1$. **1030.** 1) $\frac{xx_0}{a^2}+\frac{yy_0}{b^2}=1$; 2) $yy_0=p(x+x_0)$. **1031.** $x+y=\pm\frac{a}{\sqrt{2}}$. **1032.** arctg3. **1033.** 1) $-\frac{b^2x}{a^2y}$; 2) $\frac{x^2-ay}{ax-y^2}$; 3) $-\frac{3x^2y^2+5y}{2x^3y+5x}$. **1034.** 1) $-\frac{a^2}{y^3}$; 2) $-\frac{R^2}{(y-\beta)^3}$; 3) $-\frac{2(1+y^2)}{y^5}$; 4) $-\frac{6a^2}{(x+2y)^3}$. **1035.** $2y=-x-3$ va $2y=x+1$. **1036.** $1-\frac{1}{e}$. **1037.** $e(e-1)$. **1038.** ± 2. **1039.** $4x-3y-1=0$. **1040.** 1) $dy=nx^{n-1}dx$; 2) $dy=3(x-1)^2dx$. **1041.** 1) $dy=\frac{xdx}{\sqrt{1+x^2}}$; 2) $ds=gtdt$. **1042.** 1) $dy=-\frac{dx}{1+x^2}$; 2) $dy=2^{\mathrm{tg}^2x}\cdot\ln 2\cdot 2\mathrm{tg}x\cdot\frac{dx}{\cos^2 x}$. **1043.** 1) $dy=-5\mathrm{ctg}^4(x^3+x^2)\frac{1}{\sin^2(x^3+x^2)}\times$ $\times(3x^2+2x)dx$; 2) $dy=-\frac{2\sin xdx}{3\sqrt[3]{2+\cos x}}$. **1044.** 1) $dy=-2\cos\sqrt{x}\sin\sqrt{x}\,\frac{dx}{2\sqrt{x}}$; 2) $dy=-\frac{2^x\ln 2dx}{\sqrt{1-2^{2x}}}$. **1045.** 1) $dy=\frac{1-x\mathrm{arctg}x}{(1+x^2)^{\frac{3}{2}}}dx$; 2) $dy=(2x\ \mathrm{arcctg}x-1)dx$.

1046. 1) $-\frac{a^3dx}{x^2(a^2+x^2)}$; 2) $\frac{(\alpha+1)d\alpha}{\alpha}$; 3) $-\frac{1}{2}\sin\frac{\varphi}{2}\,d\varphi$; 4) $-\frac{dx}{x\sqrt{x^2-1}}$.

1047. 1) 0,04; 2) 0,05. **1048.** 1) $dV=3x^2dx=0{,}75$; $\frac{dV}{x^3}=0{,}006$ yoki 0,6%. **1049.** 2,97. **1050.** 2,02. **1051.** −0,1. **1052.** 0,485. **1053.** 1) $\frac{(2-x)dx}{x^3}$; 2) $b\sin(a-b\varphi)d\varphi$; 3) $-\frac{tdt}{\sqrt{1-t^2}}$. **1054.** 1) $-\mathrm{tg}x dx$; 2) $\frac{du}{2u\sqrt{4u-1}}$; 3) $-2e^{-2t}dt$. **1055.** 1) $\frac{dx}{2\sqrt{x}}$; 2) $\mathrm{tg}^2\alpha d\alpha$; 3) $b(1+e^{-bt})dt$. **1056.** $\Delta y=3x^2\Delta x+3x\Delta x^2+\Delta x^3=-0{,}2376$; $dy=3x^2dx=-0{,}24$. **1057.** 0,7194.

1058. 1) $\mathrm{arctg}1,02 \approx 0,795$; 2) $\mathrm{arctg}0,97 \approx 0,770$; 3) $\arcsin 0,4983 \approx$ $\approx 0,52164$. **1059.** 1) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$; 2) $x^{2/3}+y^{2/3}=a^{2/3}$. **1060.** 1) $x^2-y^2=1$; 2) $y=\frac{1}{1+x^2}$. **1061.** -1. **1062.** $\frac{t(2-t^3)}{1-2t^3}$. **1063.** $\frac{2}{e^t(\cos t-\sin t)^3}$. **1064.** $-\frac{1}{at\sin^3 t}$.

1065. $\frac{t^2+t-1}{e^{2t}(1-t^2)\sqrt{1-t^2}}$. **1066.** $-\frac{b}{a}\mathrm{ctg}\varphi$. **1067.** $\frac{3t^2-1}{2t}$. **1068.** $\frac{t}{4}$. **1069.** $\frac{\cos\varphi-\varphi\sin\varphi}{1-\sin\varphi-\varphi\cos\varphi}$. **1070.** 1) $-\frac{1}{4\sin^3 t}$; 2) $\frac{3t^2-1}{4t^3}$; 3) $\frac{3}{4e^t}$. **1071.** 1) $\frac{1}{3a\sin t\cdot\cos^4 t}$; 2) $2t(\cos 2t-2t\sin 2t)$. **1072.** 16 m/s^2. **1073.** $x=at-\frac{gt^2}{2}$; $\frac{dx}{dt}=a-gt$; $\frac{d^2x}{dt^2}=-g$. **1076.** $v=\frac{dx}{dt}$; $\frac{dv}{dt}=w$; hadma-had ko'paytiriladi. **1077.** $2v\frac{dv}{dt}=2a\frac{dx}{dt}=2av$; bundan $w=\frac{dv}{dt}=a$. **1078.** $x=10+20t-\frac{gt^2}{2}$; $\frac{dx}{dt}=20-gt$; $\frac{d^2x}{dt^2}=-g$. **1079.** $v=2t-4$; $a=2$. **1080.** $d(\omega^2)=2\omega d\omega$; $\frac{d(\omega^2)}{d\varphi}=2\omega\frac{d\omega}{d\varphi}=2\omega\frac{d\omega}{dt}\frac{dt}{d\varphi}=2\omega\varepsilon\frac{1}{\omega}=2\varepsilon$.

1089. $\frac{9}{4}$. **1090.** $\frac{b^3-a^3}{b^2-a^2}=\frac{3c^2}{2c}$; bundan $c=\frac{2(a^2+ab+b^2)}{3(a+b)}$. **1091.** $\frac{\pi}{4}$. **1092.** $c=\sqrt{\frac{a^2+ab+b^2}{3}}$. **1093.** 1) $\sqrt{\frac{4}{\pi}-1}$; 2) $\sqrt{1-\frac{4}{\pi^2}}$; 3) $\frac{1}{\ln 2}$. **1094.** $\sqrt[3]{\left(\frac{15}{4}\right)^2}\approx 2,4$. **1095.** $\frac{2}{3}$. **1096.** $\frac{1}{2}$. **1097.** $\frac{1}{na^{n-1}}$. **1098.** $\frac{1}{4}$. **1099.** $-\frac{3}{2}$. **1100.** 3. **1101.** $\frac{1}{6}$. **1102.** 1) ∞; 2) 0. **1103.** 0. **1104.** 0. **1105.** 1. **1106.** 2. **1107.** 0. **1108.** $\frac{1}{2}$. **1109.** $+\infty$. **1110.** 0. **1111.** 1. **1112.** 1. **1113.** 1. **1114.** $e^{-1/2}$. **1115.** e^3. **1116.** 1. **1117.** $a—b$. **1118.** $\frac{1}{3}$. **1119.** $\frac{1}{8}$. **1120.** $\ln\frac{a}{b}$. **1121.** $\frac{1}{\sqrt{3}}$. **1122.** 1. **1123.** 1. **1124.** $-\frac{1}{3}$. **1125.** -2. **1126.** $\frac{1}{e}$. **1127.** $\frac{1}{6}$. **1128.** e^3. **1131.** $\left(-\infty;\ -\frac{1}{2}\right)$ da o'sadi, $\left(-\frac{1}{2};\ \frac{11}{18}\right)$ da kamayadi, $\left(\frac{11}{18};\ \infty\right)$ da o'sadi. **1132.** $\left(-\infty;\ \frac{a}{2}\right)$ da o'sadi, $\left(\frac{a}{2};\ \frac{2}{3}a\right)$ da o'sadi, $\left(\frac{2}{3}a;\ a\right)$ da kamayadi, $(a;\ \infty)$ da o'sadi. **1133.** $(-\infty;\ -1)$ da o'sadi, $(-1;\ 1)$ da kamayadi, $(1;\ \infty)$ da o'sadi. **1134.** $(-\infty;\ 0)$ da kamayadi, $\left(0;\ \frac{1}{2}\right)$ da kamayadi, $\left(\frac{1}{2};\ 1\right)$ da o'sadi, $(1;\ \infty)$ da kamayadi. **1135.** $(-\infty;\ 0)$ da o'sadi, $(0;\ \infty)$ da kamayadi. **1136.** $(-\infty;\ 0)$

da kamayadi, (0; 2) da o'sadi, $(2;\infty)$ da kamayadi. **1137.** (0; 1) da kamayadi, (1; e) da kamayadi, $(e;\infty)$ da o'sadi. **1138.** $\left(0;\ \frac{1}{2}\right)$ da kamayadi, $\left(\frac{1}{2};\ \infty\right)$ da o'sadi. **1139.** $x=-2$ da $y_{min}=1$. **1140.** $y_{min}(-2)=-\frac{16}{3}$; $y_{max}(2)=\frac{16}{3}$; Ox o'qi bilan kesishish nuqtalari: $x_1=0$; $x_{2,3}=\pm 2\sqrt{3}\approx\pm 3,4$.

1141. $y_{max}(-1)=1\frac{2}{3}$; $y_{min}(3)=-9$; Ox o'qi bilan kesishish nuqtalari: $x_1=0$; $x_{2,3}\approx 1,5\pm 3,3$. **1142.** $y_{max}(\pm 2)=5$, $y_{min}(0)=1$; $x\approx\pm 2,9$ da $y=0$. **1143.** $x=3$ da $y_{min}=-6\frac{3}{4}$; $x=0$, $y=0$ – egilish nuqtasi. **1144.** $y_{max}(-2)=-2$; $y_{min}(2)=2$; $x=0$, $y=\frac{x}{2}$ — asimptotalari. **1145.** $y_{min}(0)=-1$ (qaytish nuqtasi); Ox o'qi bilan kesishish nuqtalari: $x=\pm 1$. **1146.** $y_{max}(0)=1$; $x\to\infty$ da $y\to 0$, $y=0$ – asimptota; egri chiziq Oy o'qiga nisbatan simmetrik. **1147.** $y_{max}(1)=-4$; $y_{min}(5)=4$; $x=3$ va $y=x-3$ — asimptotalari. **1148.** $y_{min}(0)=0$; $y_{max}\left(\frac{2}{3}\right)=\frac{4}{27}$.

1149. $y_{max}(4)=1$; $x=3$ yoki $x=5$ da $y=0$; $x=-4$ yoki 12 da $y=-3$. **1150.** $y_{max}(0)=1$; $y=0$ — asimptota; Oy o'qiga nisbatan simmetrik. **1151.** $y_{max}\left(\frac{\pi}{12}\right)=\frac{\pi}{12}+\frac{\sqrt{3}}{2}\approx 1,1$; $y_{min}\left(\frac{5\pi}{12}\right)\approx 0,4$. **1152.** $y_{max}\left(\frac{\pi}{3}\right)=\frac{4\pi}{12}-\sqrt{3}\approx 2,45$; $y_{min}\left(-\frac{\pi}{3}\right)=\sqrt{3}-\frac{4\pi}{3}\approx -2,45$; $x=\pm\frac{\pi}{2}$ asimptotalari. **1153.** $y_{max}(1)=1$; $x\to 0$, $y\to-\infty$; $x\to\infty$, $y\to 0$; $x=0$ va $y=0$ asimptotalari. Ox o'qi bilan kesishish nuqtasi: $1+\ln x=0$, $\ln x=-1$, $x=e^{-1}\approx 0,4$. **1154.** $y_{min}\left(\frac{1}{2}\right)=\frac{1}{2}-\frac{\pi}{4}\approx -0,28$; $y_{max}\left(-\frac{1}{2}\right)\approx 0,28$. Asimptotalari: $y=x\pm\frac{\pi}{2}$. **1155.** 1) $y_{max}(2)=\frac{2}{e}$. $y=0$ asimptota; 2) $y_{min}\left(\frac{1}{e}\right)=-\frac{1}{e}$; $\lim\limits_{x\to+0}y=0$; $x=1$ da $y=0$. **1156.** 1) $y_{min}(0)=0$ (burchak nuqta); $x=\pm\sqrt{\frac{4n+1}{2}\pi}$ da $y_{max}=1$; 2) $y_{min}(0)=0$ (burchak nuqta). **1157.** $x=\frac{\pi}{4}$; $\frac{3\pi}{4}$; $\frac{5\pi}{4}$;… nuqtalarda $y_{min}=\frac{1}{2}$. $x=0$; $\frac{\pi}{2}$; π; $\frac{3\pi}{2}$;… nuqtalarda $y_{max}=1$. **1158.** $y_{max}\left(\frac{1}{2}\right)=\frac{1}{2\sqrt{2}}$; $x_1=0$ va $x_2=1$ da $y=0$. **1159.** $y_{max}(2)=\sqrt{2}$. **1160.** $y_{min}(-2)=-\frac{1}{9}$; $y_{max}(2)=-1$; $x=1$ va $x=4$— asimptotalari. **1161.** $y_{min}(1)=1,5$. **1162.** $x=0$ va $x=2$ da $y_{min}=\sqrt[3]{4}\approx 1,6$; $y_{max}(1)=2$. **1163.** $y_{max}(1)=0,2$; $y_{min}(3)=-5,4$; $x=0$ da $y=0$. **1164.** $y_{max}(-2)=0$; $y_{min}(-1,2)\approx-1,1$; $x=0$ da $y=0$. **1165.** $y_{max}(2)=\frac{1}{2}$.

1166. $y_{max}(-3)=-4,5$; $y_{min}(3)=4,5$; $x=\pm\sqrt{3}$ va $y=x$ asimptotalari. **1167.** $x=\frac{\pi}{4}+k\pi$ da $y_{max}=1$; $x=\frac{\pi}{2}+k\pi$ da uzilishga ega. **1168.** $y_{max}\left(\frac{\pi}{4}+2k\pi\right)=\frac{\pi}{4}+2k\pi-\frac{1}{2}\ln 2$. **1169.** $y_{min}(1)=\frac{1}{2}\ln 2-\frac{\pi}{4}$. **1170.** $y_{min}(0)=0$; $y_{max}(2)=\frac{4}{e^2}\approx\frac{1}{2}$. $y=0$—asimptota. **1171.** $x=-1$ qaytish nuqtasida $y_{min}=2$; $y_{max}(0)=3$; $x\approx 4$, $y=0$. **1172.** $y_{max}(2)=4$; $x_1=0$ va $x_2=4$ da $y=0$. **1173.** $y_{min}(-1)=-4$; $x_1=1$ va $x_2=-3$ da $y=0$. **1174.** $y_{min}(0)=0$; $y_{max}(-2)=4/3$; $x_1=0$ va $x_2=-3$ da $y=0$. **1176.** $y_{max}(0)=0$; $x=2$ da $y=\pm\infty$; $x=4$ da $y_{min}=8$. $x=2$ va $y=x+2$— asimptotalari. **1177.** $y_{min}(-3)=-6,75$; $x=0$, $y=0$ — egilish nuqtasi; $x_1=0$ va $x_2=-4$ da $y=0$. **1178.** $y_{min}(\pm 2)=-4$; $y_{max}(0)=0$; $y=0$ da $x_1=0$, $x_{2,3}=\pm\sqrt{8}\approx\pm 2,8$. **1179.** $x=0$ (qaytish nuqtasi) da $y_{max}=0$; $y_{min}(1)=-1$; $y=0$ da $x_1=0$, $x_2=3\frac{3}{8}$. **1180.** $y_{max}(-1)=2$, $y_{min}(1)=0$; $x=0$ da $y=1$; $y=1$ asimptota. **1181.** $y_{min}(-1)=-\frac{1}{\sqrt{e}}\approx-0,6$; $y_{max}(1)\approx 0,6$; Ox o'qi asimptota. **1182.** $y_{min}(2)=2(1-\ln 2)\approx 0,6$; Oy o'qi asimptota; $x=1$ da $y=1$; $x=e^2\approx 7,4$ da $y\approx 3,4$. **1183.** $x=0$ (qaytish nuqtasi) da $y_{max}=0$; $y_{min}(2)=-3\sqrt[3]{4}\approx-4,8$; $x=5$ da $y=0$. **1184.** $y_{max}\left(\pi/6\right)=\frac{\sqrt{3}}{2}-\frac{\pi}{6}\approx 0,34$; $y_{min}\left(\pi/6\right)\approx-0,34$; $x=\pm\frac{\pi}{2}$ da $y=\pm\frac{\pi}{2}=\pm 1,57$. **1185.** $y_{min}\left(\frac{\pi}{4}\right)=\frac{\pi}{2}+1\approx 2,57$; $y_{max}\left(\frac{3\pi}{4}\right)=3,71$; $x=0$ va $x=\pi$— asimptotalari. **1186.** $y_{max}\left(-\frac{1}{2}\right)=-\frac{1}{2}+\frac{3\pi}{4}\approx 1,85$; $y_{min}\left(\frac{1}{2}\right)\approx 1,28$; $x=0$ da $y=\frac{\pi}{2}$. $y=x$ — asimptota. **1187.** $x=1$ qaytish nuqtasida $y_{min}=1$; $x=0$ da $y=2$; $x=2$ da $y=2$. **1188.** $x=\frac{\pi}{6}$ va $x=\frac{5\pi}{6}$ da $y_{max}=1,5$; $y_{min}\left(\frac{\pi}{2}\right)=1$. **1189.** $y_{min}(0)=0$; $x=1$, $y=1$ — egilish nuqtasi. **1190.** $y_{max}(e)=\frac{1}{e}\approx 0,4$; $y=0$ da $x=1$; $x=0$ va $y=0$ — asimptotalari. **1191.** $y_{min}(-3)=6$; $x=-2$ da $y=\infty$; $y_{max}(-1)=2$. **1192.** $y_{min}(1)=2$; $y_{max}(-1)=-2$. **1193.** $y_{max}(-1)=2$; $y_{min}(1)=-2$. **1194.** $y_{min}(1)=3$. **1195.** 30 m x 60 m. **1196.** 5 va 5. **1197.** 4 va 4. **1198.** $ah/4$. **1199.** $a/6$. **1200.** 4m x 4m x 2m. **1201.** 20 sm. **1202.** $\frac{18}{\pi+4}\approx 2,5$. **1203.** $l\approx 5,6$ m,

$l=\frac{2,4}{\sin\alpha}+\frac{1,6}{\cos\alpha}$. **1204.** x=2 dm balandlikda $\vartheta_{max}=\frac{128\pi}{9}\ dm^3$. **1205.** $x=\frac{R}{\sqrt{2}}$ balandlikda $S_{max}=R^2$. **1206.** x=2 m. **1207.** $\frac{4}{3}R$. **1208.** 1) $y=x^2$, $y''=2>0$, egri chiziq grafigi botiq; 2) $y=x^3$, $y''=6x$, $x>0$ da egri chiziq grafigi botiq, $x<0$ da qavariq, $x=0$ — bukilish nuqtasi; 3) $y=e^x$, $y''=e^x>0$, egri chiziq grafigi botiq, (0; 1)—Oy o'qi bilan kesishish nuqtasi; 4) $y=\ln x\ (x>0)$, $y''=-\frac{1}{x^2}<0$, egri chiziq graflgi qavariq, (1; 0) — Ox o'qi bilan kesishish nuqtasi; 5) (0; 0) — bukilish nuqtasi. **1209.** Egri chiziqning bukilish nuqtasi: 1) (2; —8/3); 2) $\left(\pm\frac{1}{\sqrt{2}};\ e^{-\frac{1}{2}}\right)$; 3) $\left(\pm\sqrt{3};\ \pm\frac{\sqrt{3}}{2}\right)$ va (0; 0); 4) $x=-\frac{\ln 2}{2}\approx-0,35$ da. **1211.** Aniqlanish sohasi: $(-\infty;\ \infty)$; $y_{max}(1)=\frac{1}{2}$; $y_{min}(-1)=-\frac{1}{2}$; grafik egilish nuqtalari: $\left(-\sqrt{3},\ -\frac{\sqrt{3}}{4}\right)$; $(0,0)$ va $\left(\sqrt{3},\frac{\sqrt{3}}{4}\right)$. $y=0$—asimptota. **1212.** $x=\pm1$ da aniqlanmagan. $y_{min}(0)=1$. Egilish nuqtasi yo'q. $x=\pm1$, $y=0$ — asimptotalari. **1213.** $x=\pm1$ da aniqlanmagan. Ekstremum nuqtasi yo'q. (0; 0) — egilish nuqtasi. $x=-1$, $x=1$, $y=0$—asimptotalari. **1214.** $x=1$, $x=2$ va $x=3$ da aniqlanmagan. $x\approx2,58$ da $y_{min}\approx-2,6$; $x\approx1,42$ da $y_{min}\approx2,6$. Egilish nuqtasi yo'q. Asimptotalari: $x=1$, $x=2$, $x=3$, $y=0$. **1215.** $x=\pm1$ da aniqlanmagan. $y_{max}(0)=0$; $x<-1$ da o'sadi, $x>1$ da kamayadi. Egilish nuqtasi yo'q. $x=\pm1$, $y=1$—asimptotalari. **1216.** Aniqlanish sohasi: $(-\infty;\ \infty)$; $y_{min}(0)=-1$; (1; 0) va (−1; 0) — egilish nuqtasi; $\left(\pm\frac{\sqrt{5}}{5};\ -\frac{64}{125}\right)$ — egilish nuqtasi. Asimptotasi yo'q. **1217.** Aniqlanish sohasi: $(-\infty;\ \infty)$; $y_{max}(0)=0$; $y\left(\pm\frac{1}{2}\right)=-\frac{27}{8}$; $(\pm1,\ 0)$ — grafikning gorizontal urinma bilan egilish nuqtasi. $x\approx\pm0,7$ va $x\approx\pm0,26$—grafik egilish nuqtalari. **1218.** $x=0$ da aniqlanmagan. $y_{min}\left(\frac{1}{2}\right)=3$; $\left(-\frac{\sqrt[3]{2}}{2};\ 0\right)$ — egilish nuqtasi. $x=0$—asimptota. **1219.** $x=0$ da aniqlanmagan. $y_{min}(\pm1)=2$. $x=0$—asimptota. **1220.** $x=1$ da aniqlanmagan. $y_{min}(0)=-1$; $\left(-\frac{1}{2},\ -\frac{8}{9}\right)$— egilish nuqtasi. $x=1$ va $y=0$ — asimptotalari. **1221.** $x\pm\sqrt{3}$ da aniqlanmagan. $y_{max}(3)=-4,5$; $y_{min}(-3)=4,5$;

(0; 0)— egilish nuqtasi. **1222.** $x=-1$ da aniqlanmagan. $y_{max}(-3)=3\frac{3}{8}$; (0; 0) — egilish nuqtasi; $x=-1$ va $y=\frac{1}{2}x-1$ — asimptotalari. **1223.** $x=1$ da aniqlanmagan. $y_{min}\left(\frac{3}{2}\right)=\frac{27}{4}$; (0; 0) — egilish nuqtasi. $x=1$ — asimptota.

1224. $x=1$ da aniqlanmagan. $y_{max}(0)=0$, $y_{min}=\left(\sqrt[3]{4}\right)=\frac{4}{3}\sqrt[3]{4}$. $\left(-\sqrt[3]{2};\ -\frac{2}{3}\sqrt[3]{2}\right)$ — egilish nuqtasi. $x=1$ va $y=x$ — asimptotalari. **1125.** $x=-1$ da aniqlanmagan. $y_{max}(5)=\frac{2}{27}$, $y_{min}(1)=0$; $x=5\pm2\sqrt{3}$ — egilish nuqtasi. $x=-1$ va $y=0$ — asimptotalari. **1226.** $x=0$ da aniqlanmagan. $y_{max}(1)=\frac{7}{2}$, $y_{max}(-3)=-\frac{11}{6}$, $y_{min}(2)=\frac{27}{8}$; $x=\frac{9}{7}$ — egilish nuqtasi. $x=0$ va $y=\frac{1}{2}x+1$ — asimptotalari.

1230. $x>-1$ da aniqlangan. $y_{min}(0)=0$. Egilish nuqtalari yo'q. $x=-1$ — asimptotasi. **1232.** $(-\infty;\ \infty)$ oraliqda aniqlangan. Ekstremumi mavjud emas. $x=\pm k\pi$ $(k=1, 3, 5, \ldots)$ — statsionar nuqtalar. Asimptotalari yo'q. $(k\pi;\ k\pi)$ $(k=0, \pm1, \pm2, \ldots)$ — egilish nuqtalari. **1233.** Aniqlanish sohasi:$(-\infty;\ \infty)$; Ekstremum nuqtalari $\mathrm{tg}x=-x$ tenglamani qanoatlantiradi. Egilish nuqtalarining abssissasi $x\mathrm{tg}x=2$ tenglamani qanoatlantiradi. Asimptotasi yo'q. **1234.** $\left(-\frac{\pi}{2}+2k\pi;\ \frac{\pi}{2}+2k\pi\right)$ intervalda aniqlangan, bu yerda $k=0$, $\pm1, \pm2, \ldots$ Davri 2π. $y_{max}(2k\pi)=0$. Grafikning egilish nuqtalari mavjud emas. $x=\frac{\pi}{2}+k\pi$ — asimptotalari.

IX BOB | ANIQMAS INTEGRAL

1°. 1- ta'rif. Hosilasi $f(x)$ ga teng bo'lgan $F(x)$ funksiya $f(x)$ funksiyaning *boshlang'ichi* (bo'shlang'ich funksiyasi) deyiladi.

2- ta'rif. Agar $F(x)$ funksiya $f(x)$ funksiyaning boshlang'ichi bo'lsa, u holda $F(x)+C$ ifoda $f(x)$ funksiyaning *aniqmas integrali* deyiladi va quyidagicha belgilanadi:

$$\int f(x)dx.$$

Shunday qilib, $\int f(x)dx = F(x)+ C$.

2°. Aniqmas integrallarning hossalari:

Agar $F'(x) = f(x)$ bo'lsa u holda:

1. $\left(\int f(x)dx\right)' = f(x)$.
2. $d\left(\int f(x)dx\right) = f(x)dx$.
3. $\int dF(x) = F(x)+C$.
4. $\int af(x)dx = a\int f(x)dx,\quad a = \text{const}$.
5. $\int\left[f_1(x)\pm f_2(x)\right]dx = \int f_1(x)dx \pm \int f_2(x)dx$.

3°. Asosiy integrallar jadvali

1. $\int x^m dx = \frac{x^{m+1}}{m+1} + C. \qquad (m \neq -1)$.

2. $\int \frac{dx}{x} = \ln|x| + C$.

3. $\int \sin dx = -\cos x + C$.

4. $\int \cos dx = \sin x + C$.

5. $\int a^x dx = \frac{a^x}{\ln a} + C$.

6. $\int e^x dx = e^x + C$.

7. $\int \frac{dx}{\cos^2 x} = \mathrm{tg}x + C$.

8. $\int \frac{dx}{\sin^2 x} = -\mathrm{ctg}x + C$.

9. $\int \frac{dx}{1+x^2} = \mathrm{arctg}x + C$.

10. $\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + C$.

11. $\int \mathrm{tg}x dx = -\ln|\cos x| + C$.

12. $\int \mathrm{ctg}x dx = \ln|\sin x| + C$.

13. $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\mathrm{arctg}\frac{x}{a} + C$.

14. $\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C.$

15. $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C.$

16. $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + C.$

1- §.Yoyib integrallash usuli

Integral ostidagi ifodani sodda qo'shiluvchilar yig'indisi ko'rinishiga keltirib integrallashga *yoyib integrallash* usuli deyiladi.

Integrallarni toping:

1235. $\int\left(x^2+2x+\frac{1}{x}\right)dx.$

1236. $\int \frac{10x^8+3}{x^4}dx.$

1237. $\int \frac{x-2}{x^3}dx.$

1238. $\int \frac{(x^2+1)^2}{x^3}dx.$

1239. $\int\left(\sqrt{x}+\sqrt[3]{x}\right)dx.$

1240. $\int\left(\frac{1}{\sqrt{x}}-\frac{1}{\sqrt[4]{x^3}}\right)dx.$

1241. $\int \frac{(\sqrt{x}-1)^3}{x}dx.$

1242. $\int \frac{x-1}{\sqrt[3]{x^2}}dx.$

1243. $\int e^x\left(1-\frac{e^{-x}}{x^2}\right)dx.$

1244. $\int a^x\left(1+\frac{a^{-x}}{\sqrt{x^3}}\right)dx.$

1245. $\int \frac{\cos 2x}{\cos^2 x\cdot\sin^2 x}dx.$

1246. $\int \mathrm{ctg}^2 x dx.$

1247. $\int \frac{dx}{\sin^2 x\cdot\cos^2 x}.$

1248. $\int \frac{3-2\mathrm{ctg}^2 x}{\cos^2 x}dx.$

1249. $\int \sin^2\frac{x}{2}dx.$

1250. $\int \cos^2\frac{x}{2}dx.$

1251. $\int\left(\frac{2}{1+x^2}-\frac{3}{\sqrt{1-x^2}}\right)dx.$

1252. $\int \frac{x^4}{1+x^2}dx.$

* * *

1253. $\int \frac{(x^2-1)^2}{x^3}dx.$

1254. $\int\left(\frac{1}{\sqrt[3]{x^2}}-\frac{1}{x\sqrt{x}}\right)dx.$

1255. $\int \frac{x-2}{\sqrt{x^3}}dx.$

1256. $\int \frac{(2\sqrt{x}+1)^2}{x^2}dx.$

1257. $\int \left(\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3}\right)dx.$

1258. $\int \left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)^2 dx.$

1259. $\int e^x\left(1+\frac{e^{-x}}{\cos^2 x}\right)dx.$

1260. $\int a^x\left(1+\frac{a^{-x}}{x^5}\right)dx.$

1261. $\int \frac{1-\sin^3 x}{\sin^2 x}dx.$

1262. $\int \mathrm{tg}^2 x dx.$

2- §. Bevosita integrallash va o‘rniga qo‘yish usullari

1- § da keltirilgan 1—5- xossalardan foydalanib, 1—16- formulalar (jadval iintegrallari) asosida integrallash *bevosita integrallash* deyiladi.

$\int f(x)dx$ integralda x o‘zgaruvchini

$$x = \varphi(t),\quad dx = \varphi'(t)dt$$

formula bo‘yicha t o‘zgaruvchi bilan almashtirib, berilgan integralni

$$\int f(x)dx = \int f\,[\varphi(t)]\,\varphi'(t)\,dt$$

ko‘rinishga keltirib integrallash *o‘rniga qo‘yish usuli* deyiladi.

Ba‘zi bir sodda hollarda

$$dx = \frac{1}{a}d(ax+b);\quad 2xdx = d(x^2);$$

$$\cos xdx = d(\sin x);\quad \frac{dx}{x} = d(\ln x)$$

va boshqa ayoniy tengliklardan foydalanib, qavs ichidagi ifodani dilda t bilan belgilab integrallashga ham bevosita integrallash deyiladi.

Integrallarni toping:

1263. $\int \cos 3xdx.$

1264. $\int \frac{dx}{\cos^2 5x}.$

1265. $\int e^{-3x}dx.$

1266. $\int \sin\frac{x}{2}dx.$

1267. $\int (e^{\frac{x}{2}}+e^{-\frac{x}{2}})dx.$

1268. $\int \sqrt{4x-1}dx.$

1269. $\int (3-2x)^4 dx.$

1270. $\int \sqrt[3]{5-6x}dx.$

1271. $\int \frac{dx}{\sqrt{3-2x}}.$

1272. $\int \sin(a-bx)dx.$

1273. $\int \frac{2x-5}{x^2-5x+7} dx .$

1274. $\int \frac{xdx}{x^2+1}.$

1275. $\int \frac{dx}{1-10x} .$

1276. $\int \frac{e^{2x}dx}{1-3e^{2x}}.$

1277. $\int \mathrm{ctg}x dx .$

1278. $\int \mathrm{tg}x dx.$

1279. $\int \frac{\cos 2x}{\sin x \cos x} dx.$

1280. $\int \frac{\sin xdx}{1+3\cos x} .$

1281. $\int \frac{\cos x}{1+2\sin x} dx.$

1282. $\int \frac{dx}{x(1+\ln x)} .$

1283. $\int \sin^2 x \cos xdx .$

1284. $\int \cos^3 x \sin xdx.$

1285. $\int \frac{\cos xdx}{\sin^4 x}.$

1286. $\int \frac{\sin xdx}{\cos^3 x}.$

1287. $\int \frac{1-2\cos x}{\sin^2 x} dx$

1288. $\int \sin x \cos xdx$.

1289. $\int e^{\cos x} \sin xdx .$

1290. $\int e^{x^3} x^2dx .$

1291. $\int e^{-x^2} xdx .$

1292. $\int \frac{e^{\sqrt{x}}dx}{\sqrt{x}} .$

1293. $\int \sqrt{x^2+1}\ xdx.$

1294. $\int \sqrt[3]{x^3-8} \cdot x^2dx .$

1295. $\int \frac{x^2dx}{\sqrt[3]{1+x^3}} .$

1296. $\int \frac{xdx}{\sqrt{1-x^2}} .$

1297. $\int \frac{\sin xdx}{\sqrt{1+2\cos x}} .$

1298. $\int \frac{\sqrt{1+\ln x}dx}{x} .$

1299. $\int \sqrt{1+4\sin x} \cdot \cos xdx$

1300. $\int \sqrt[3]{1-6x^5}x^4dx .$

1301. $\int \frac{\cos\sqrt{x}}{\sqrt{x}}dx \cdot$

1302. $\int \frac{\sqrt{1+\ln x}dx}{x \cdot \ln x} .$

1303. $\int \frac{\sqrt{x}dx}{\sqrt{x}+1} .$

1304. $\int \frac{dx}{\sqrt{e^x+1}} .$

1305. $\int(e^x+e^{-x})^2dx$.

1306. $\int\sin^3 x\cos xdx$.

1307. $\int\frac{dx}{\sqrt{1-4x}}$.

1308. $\int\cos(a-bx)dx$.

1309. $\int\sqrt[3]{1+3x}dx$.

1310. $\int\sqrt[6]{1-2x^3}\,x^2dx$.

1311. $\int\frac{xdx}{\sqrt{1+x^2}}$.

1312. $\int\frac{1-2\sin x}{\cos^2 x}dx$.

1313. $\int\frac{1+\sin 2x}{\sin^2 x}dx$.

1314. $\int e^{\sin x}\cos xdx$.

1315. $\int\frac{x^2}{1-x^3}dx$.

1316. $\int\frac{dx}{(a-bx)^3}$.

1317. $\int\frac{3^{\frac{1}{x}}dx}{x^2}$.

1318. $\int\frac{(\mathrm{arctg}x)^{100}dx}{1+x^2}$.

1319. $\int\frac{e^x dx}{\sqrt{4-e^{2x}}}$.

1320. $\int\frac{dx}{(\arccos x)^5\sqrt{1-x^2}}$.

3- §. $\int\frac{dx}{x^2\pm a^2}$, $\int\frac{dx}{\sqrt{a^2-x^2}}$, $\int\frac{dx}{\sqrt{x^2+k}}$ ko'rinishdagi va ularga keltiriladigan integrallar

1321. $\int\frac{dx}{a^2+x^2}$, ($x=a\mathrm{tg}t$).

1322. $\int\frac{dx}{\sqrt{a^2-x^2}}$, ($x=a\sin t$).

1323. $\int\frac{dx}{\sqrt{x^2-a^2}}$, $\left(\frac{1}{x^2-a^2}=\frac{1}{2a}\frac{a+x+a-x}{x^2-a^2}=\frac{1}{2a}\left(\frac{1}{x-a}-\frac{1}{x+a}\right)\right)$.

1324. $\int\frac{dx}{\sqrt{x^2+k}}$, $\left(\sqrt{x^2+k}=t-x\right)$.

1325. $\int\frac{dx}{x^2-25}$.

1326. $\int\frac{dx}{x^2+9}$.

1327. $\int\frac{dx}{\sqrt{4-x^2}}$.

1328. $\int\frac{dx}{\sqrt{x^2+5}}$.

1329. $\int\frac{dx}{\sqrt{x^2-4}}$.

1330. $\int\frac{dx}{x^2+3}$.

1331. $\int\frac{dx}{\sqrt{5-x^2}}$.

1332. $\int\frac{x^2dx}{4+x^6}$.

1333. $\int \frac{xdx}{\sqrt{3-x^4}}$. **1334.** $\int \frac{dx}{b^2x^2-a^2}$.

1335. $\int \frac{dx}{\sqrt{3-4x^2}}$. **1336.** $\int \frac{x^3dx}{\sqrt{x^8-1}}$.

1337. $\int \frac{5x-2}{x^2+4}dx$. **1338.** $\int \frac{3x-4}{x^2-4}dx$.

1339. $\int \frac{x+1}{\sqrt{x^2+1}}dx$. **1340.** $\int \frac{x+1}{\sqrt{1-x^2}}dx$.

1341. $\int \frac{x^2}{x^2+1}dx$. **1342.** $\int \frac{x^4}{x^2-3}dx$.

1343. $\int \frac{dx}{x^2+4x+5}$. **1344.** $\int \frac{dx}{x^2+6x+13}$.

1345. $\int \frac{dx}{\sqrt{x^2+2x+3}}$. **1346.** $\int \frac{dx}{\sqrt{1-2x-x^2}}$.

1347. $\int \frac{dx}{\sqrt{4x-x^2}}$. **1348.** $\int \frac{dx}{x^2+3x+3}$.

1349. $\int \frac{dx}{\sqrt{2+3x-2x^2}}$. **1350.** $\int \frac{dx}{\sqrt{3x^2-2x-1}}$.

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1351. $\int \left(\frac{3}{x^2+3}+\frac{6}{x^2-3}\right)dx$. **1352.** $\int \left(\frac{1}{\sqrt{2-x^2}}+\frac{1}{\sqrt{2+x^2}}\right)dx$.

1353. $\int \frac{4x-5}{x^2+5}dx$. **1354.** $\int \frac{x^2dx}{x^2-2}$.

1355. $\int \frac{x^4dx}{x^2+2}$. **1356.** $\int \frac{e^xdx}{\sqrt{1-e^{2x}}}$.

1357. $\int \frac{xdx}{x^4+0{,}25}$. **1358.** $\int \frac{dx}{x^2+4x+29}$.

1359. $\int \frac{dx}{x^2-2x+5}$. **1360.** $\int \frac{dx}{\sqrt{5-4x-x^2}}$.

1361. $\int \frac{xdx}{x^2+x+1}$. **1362.** $\int \frac{dx}{\sqrt{4x^2+4x+3}}$.

4- §. Bo‘laklab integrallash usuli

Agar $u(x)$ va $v(x)$ lar x ning differensiallanuvchi funksiyalari bo‘lsa, u holda

$$d(uv) = du \cdot v + dv \cdot u,$$

$$udv = d(uv) - vdu,$$
$$\int udv = uv - \int vdu. \qquad (*)$$

(*) tenglik *bo'laklab integrallash* formulasi deyiladi. Integral ostida algebraik va transsendent funksiyalar ko'paytmasi kelsa, u holda ko'pincha bo'laklab integrallash formulasidan foydalaniladi.

Integrallarni toping:

1363. $\int \ln x dx$.

1364. $\int x\ln(x-1)dx$.

1365. $\int xe^{2x}dx$.

1366. $\int x\mathrm{arctg}x$.

1367. $\int x^2 \cos x dx$.

1368. $\int e^x \sin x dx$.

1369. $\int \sqrt{x^2+k}\ dx = \frac{1}{2}\left[x\sqrt{x^2+k} + k\ln(x+\sqrt{x^2+k})\right] + C$

ekanligini ko'rsating.

1370. $\int (\ln x)^2 dx$.

1371. $\int \frac{xdx}{\sin^2 x}$.

1372. $\int \frac{\ln xdx}{x^2}$.

1373. $\int \frac{\arcsin x}{\sqrt{1+x}} dx$.

1374. $\int \arcsin x dx$.

1375. $\int x^3 e^{-x} dx$.

1376. $\int \ln(x^2+1)dx$.

1377. $\int \cos(\ln x)dx$.

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1378. $\int \sqrt{x} \cdot \ln x dx$.

1379. $\int x^2 e^{-\frac{x}{2}} dx$.

1380. $\int \mathrm{arctg}\, x dx$.

1381. $\int \frac{xdx}{\cos^2 x}$.

1382. $\int e^x \cos x dx$.

1383. $\int \frac{\arcsin \frac{x}{2}}{\sqrt{2-x}} dx$.

1384. $\int \frac{x \cos x dx}{\sin^3 x}$.

1385. $\int \mathrm{arctg}\sqrt{2x-1}\ dx$.

1386. $\int \frac{\arcsin x}{x^2} dx$.

1387. $\int \ln(x+\sqrt{1+x^2})dx$.

1388. $\int x\ \mathrm{arctg}\sqrt{x^2-1}\ dx$.

1389. $\int \frac{x\ \mathrm{arctg}x}{(x^2+1)^2} dx$.

5- §. Trigonometrik funksiyalarni integrallash

1°. Trigonometrik ifodalar qatnashgan integrallarni topishda quyidagi formulalardan foydalaniladi:

$$\sin^2 x = \frac{1-\cos 2x}{2}; \quad \cos^2 x = \frac{1+\cos 2x}{2}; \quad \sin x \cdot \cos x = \frac{\sin 2x}{2}.$$

2°. $\int \cos^m x \sin^n x dx$ integralda m va n lar juft bo'lsa, yuqoridagi formulalardan, m va n lar toq bo'lsa, u holda toq darajadan bittasini ajratib, yangi o'zgaruvchi kiritish yo'li bilan integrallanadi.

Integrallarni toping:

1390. $\int \sin^2 3x dx.$

1391. $\int (1+2\cos x)^2 dx.$

1392. $\int (1-\sin 2x)^2 dx.$

1393. $\int \cos^4 x dx.$

1394. $\int \sin^2 x \cos^2 x dx.$

1395. $\int \sin^4 x \cos^4 x dx.$

1396. $\int \sin^2 x \cos^4 x dx.$

1397. $\int \sin^5 x dx.$

1398. $\int \sin^2 x \cos^3 x dx.$

1399. $\int \sin^3 x \cos^3 x dx.$

1400. $\int \cos^7 x dx.$

1401. $\int (1+2\cos x)^3 dx.$

1402. $\int \frac{\cos^3 x dx}{\sin^2 x}.$

1403. $\int \frac{\sin^3 x dx}{\cos^2 x}.$

1404. $\int \frac{dx}{\sin 2x} = \int \frac{\sin^2 x + \cos^2 x}{2\sin x \cos x} dx.$

1405. $\int \frac{dx}{\sin x}.$

1406. $\int \frac{dx}{\cos x}.$

1407. $\int \frac{\cos x + \sin x}{\sin 2x} dx.$

1408. $\int \frac{dx}{\sin x - \cos x}.$

1409. $\int \mathrm{tg}^3 x dx.$

K o ' r s a t m a . 1409- misolda $\mathrm{tg}x = t$, $x = a$rctgt deb belgllab oling.

1410. $\int \mathrm{ctg}^3 x dx.$

1411. $\int \sin 3x \cos x dx.$

1412. $\int \cos mx \cdot \cos nx dx.$

1413. $\int \sin 3x \cdot \sin 5x dx.$

1414. $\int \sin mx \cdot \sin nx dx$.

1415. $\int \sin(5x - \frac{\pi}{4}) \cos(x + \frac{\pi}{4}) dx.$

K o ‘ r s a t m a . 1411—1415- misollarda

$$\sin\alpha\cdot\cos\beta=\frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)],$$

$$\cos\alpha\cdot\cos\beta=\frac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha-\beta)],$$

$$\sin\alpha\cdot\sin\beta=\frac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)]$$

formulalardan foydalaning.

1416. $\int\sin^n xdx=-\frac{1}{n}\cos x\cdot\sin^{n-1}x+\frac{n-1}{n}\int\sin^{n-2}xdx,$

$$\int\cos^n xdx=\frac{1}{n}\sin x\cdot\cos^{n-1}x+\frac{n-1}{n}\int\cos^{n-2}xdx$$

formulalarni isbotlang.

1417. $\int\sin^6 xdx.$

1418. $\int\cos^6 xdx.$

1419. $\int\frac{dx}{\sin^3 x}.$

1420. $\int\frac{dx}{\cos^3 x}.$

K o ‘ r s a t m a . $\int\frac{dx}{\sin x}$ va $\int\frac{dx}{\cos x}$ integrallarda 1416- misoldagi formulalardan foydalaning.

1421. $\int(1+3\cos 2x)^2dx.$

1422. $\int\sin^4 dx.$

1423. $\int\sin^4 x\cos^2 xdx.$

1424. $\int\cos^5 xdx.$

1425. $\int\sin^3 x\cdot\cos^2 xdx.$

1426. $\int(1+2\sin x)^3dx.$

1427. $\int\frac{(\sin x-\cos x)^2dx}{\sin 2x}.$

1428. $\int\sin 3x\cdot\sin xdx.$

1429. $\int\frac{\sin^3 x+1}{\cos^2 x}dx.$

1430. $\int\sin\left(x+\frac{\pi}{6}\right)\cos xdx.$

6- §. Ratsional algebraik funksiyalarni integrallash

1°. Integral ostida noto‘g‘ri kasr bo‘lsa, u holda uning butun qismini ajratish kerak.

2°. To‘g‘ri kasrning maxraji $(x-a)^\alpha$ va $(x^2+px+q)^\beta$ ko‘rinishdagi ko‘paytuvchilarga tarqatiladi, to‘g‘ri kasr esa elementar kasrlar yig‘indisi sifatida quyidagicha ifodalanadi:

$$\frac{P(x)}{(x-a)^{\alpha}(x^2+px+q)^{\beta}}=\frac{A_1}{x-a}+\frac{A_2}{(x-a)^2}+\ldots+\frac{A_{\alpha}}{(x-a)^{\alpha}}+$$

$$+\frac{M_1x+N_1}{x^2+px+q}+\frac{M_2x+N_2}{(x^2+px+q)^2}+\ldots+\frac{M_{\beta}x+N_{\beta}}{(x^2+px+q)^{\beta}},$$

bunda $P(x)$ — darajasi maxrajning darajasidan past ko‘phad.

Integrallarini toping:

1431. $\int\frac{x^3}{x-2}dx.$

1432. $\int\frac{x^4}{x^2+a^2}dx.$

1433. $\int\frac{x^5}{x^3-a^3}dx.$

1434. $\int\frac{x-4}{(x-2)(x-3)}dx.$

1435. $\int\frac{2x+7}{x^2+x-2}dx.$

1436. $\int\frac{3x^2+2x-3}{x^3-x}dx.$

1437. $\int\frac{(x+1)^3dx}{x^2-x}.$

1438. $\int\frac{x+2}{x^3-2x^2}dx.$

1439. $\int\frac{3x-2a}{x^4-ax^3}dx.$

1440. $\int\frac{2x^2-5x+1}{x^3-2x^2+x}dx.$

1441. $\int\frac{5x-1}{x^3-3x-2}dx.$

1442. $\int\frac{5x+2}{x^2+2x+10}dx.$

1443. $\int\frac{4x-2,4}{x^2-0,2x+0,17}dx.$

K o ‘ r s a t m a . 1442- misolda maxrajidan to‘la kvadratni ajratib, $x+1=t$ belgilash kiritish kerak.

1444. $\int\frac{2x^2+x+4}{x^3+x^2+4x+4}dx.$

1445. $\int\frac{7x-15}{x^3-2x^2+5x}dx.$

1446. $\int\frac{dx}{x^3+8}.$

1447. $\int\frac{3x^2+2x+1}{(x+1)^2(x^2+1)}dx.$

1448. 1) $\int\frac{dx}{(x^2+b^2)^2}$; 2) $\int\frac{dx}{(x^2+b^2)^3}.$

K o ‘ r s a t m a . $x=b\operatorname{tg}t$ belgilash kiritmg.

1449. $\int\frac{(2x+1)dx}{(x^2+2x+5)^2}dx.$

1450. $\int\frac{dx}{(x^2-6x+10)^3}dx.$

1451. $\int\frac{4x}{(1+x)(1+x^2)^2}dx.$

1452. $\int\frac{x+1}{x^4+4x^2+4}dx.$

Aniqmas koeffitsiyentlar usulidan foydalanmasdan integrallang:

1453. $\int \frac{dx}{x(x+a)}$.

1454. $\int \frac{dx}{(x+a)(x+b)}$.

1455. $\int \frac{dx}{x^2-2x}$.

1456. $\int \frac{dx}{(x^2-3)(x^2+2)}$.

1457. $\int \frac{dx}{x^4-x^2}$.

1458. $\int \frac{dx}{x^3+4x}$.

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Integrallarni toping:

1459. $\int \frac{2x-1}{(x-1)(x-2)}dx$.

1460. $\int \frac{3x+2}{2x^2+x-3}dx$.

1461. $\int \frac{5x-14}{x^3-x^2-4x+4}dx$.

1462. $\int \frac{11x+16}{(x-1)(x+2)^2}dx$.

1463. $\int \frac{5x-8}{x^3-4x^2+4x}dx$.

1464. $\int \frac{x+2}{x^3-2x^2+2x}dx$.

1465. $\int \frac{x-a}{x^3+a^2x}dx$.

1466. $\int \frac{dx}{x^3+x^2+2x+2}$.

1467. $\int \frac{dx}{x^3-8}$.

1468. $\int \frac{xdx}{(x^2+2x+2)^2}$.

1469—1472- misollarni aniqmas koeffitsiyentlar usulidan foydalanmasdan integrallang.

1469. $\int \frac{dx}{x^2+5x}$.

1470. $\int \frac{dx}{x^4+3x^2}$.

1471. $\int \frac{dx}{x^4-1}$.

1472. $\int \frac{dx}{x^4-x^2-2}$.

7- §. Irratsional algebraik funksiyalarni integrallash

1°. $\int R(x, \sqrt[n]{ax+b})dx$ integralni $ax+b=t^n$ almashtirish, $\int R(x^m, \sqrt[n]{ax^m+b})x^{m-1}dx$ ni esa $ax^m+b=t^n$ almashtirish bilan topiladi, bunda $R(x,y)$ — ratsional funksiya.

2°. $\int \frac{Mx+N}{(x-a)\sqrt{ax^2+bx+c}}dx$ integral $x-a=\frac{1}{t}$ almashtirish bilan topiladi.

3°. Trigonometrik almashtirishlar

a) $\int R(x,\sqrt{a^2-x^2})dx$ integral ostidagi ifoda $x = a\sin t$;

b) $\int R(x,\sqrt{a^2+x^2})dx$ integral ostidagi ifoda $x = a\mathrm{tg}t$ almashtirish yordamida ratsional trigonometrik ko‘rinishga keltiriladi.

4°. $\int \frac{a_0x^m + a_1x^{m-1} + ... + a_m}{\sqrt{ax^2+bx+c}}dx$ integralni

$$\int \frac{a_0x^m + a_1x^{m-1} + ... + a_m}{W}dx = (A_0x^{m-1} + ... + A_{m-1})W + A_m\int\frac{dx}{W}$$

ko‘rinishda yozish mumkin, bunda $W = \sqrt{ax^2+bx+c}$. A_0, ..., A_m koeffitsiyentlarni topish uchun oxirgi tenglik differensiallanib, maxrajdan qutqaziladi va x ning bir xil darajalari oldidagi koeffitsiyentlar tenglashtiriladi.

1° dan foydalanib integrallarni toping:

1473. $\int \frac{x+1}{\sqrt[3]{3x+1}}dx.$ **1474.** $\int \frac{x}{\sqrt{2x+1}+1}dx.$

1475. $\int \frac{dx}{\sqrt[3]{x}+\sqrt{x}}.$ **1476.** $\int x\sqrt{a-x}dx.$

1477. $\int \frac{x^3}{1+\sqrt[3]{x^4+1}}dx.$ **1478.** $\int \frac{x^3}{\sqrt{x^2+2}}dx.$

2° dan foydalanib integrallarni toping:

1479. $\int \frac{dx}{x\sqrt{x^2-1}}.$ **1480.** $\int \frac{dx}{x\sqrt{2x^2+2x+1}}.$

1481. $\int \frac{dx}{x\sqrt{2ax-x^2}}.$ **1482.** $\int \frac{dx}{(x+1)\sqrt{x^2+2x+2}}.$

3° dan foydalanib integrallarni toping:

1483. $\int \sqrt{a^2-x^2}dx.$ **1484.** $\int \frac{dx}{\sqrt{(4+x^2)^3}}.$

1485. $\int x^2\sqrt{4-x^2}\,dx.$ **1486.** $\int \frac{x^2dx}{\sqrt{(a^2+x^2)^5}}.$

1487. $\int \sqrt{3+2x-x^2}\,dx.$ **1488.** $\int \frac{x^2dx}{\sqrt{(2-x^2)^3}}.$

4° dan foydalanib integrallarni toping:

1489. $\int \frac{x^2+4x}{\sqrt{x^2+2x+2}}dx.$ **1490.** $\int \frac{xdx}{\sqrt{3-2x-x^2}}.$

1491. $\int \sqrt{x^2+k}\ dx.$ **1492.** $\int \sqrt{2ax-x^2}\ dx.$

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Integrallarni toping:

1493. $\int \frac{x-1}{\sqrt{2x-1}}dx.$ **1494.** $\int \frac{dx}{\sqrt[3]{3x+1}-1}.$

1495. $\int \frac{\sqrt{x}}{\sqrt{x}+1}dx.$ **1496.** $\int \frac{x}{\sqrt[3]{a-x}}dx.$

1497. $\int \frac{x+1}{x\sqrt{x-2}}dx.$ **1498.** $\int \frac{x^3dx}{\sqrt[3]{x^2+1}-1}.$

1499. $\int \frac{xdx}{x^2+2+2\sqrt{1+x^2}}.$ **1500.** $\int \frac{x^3dx}{2+\sqrt{4-x^2}}.$

1501. $\int \frac{dx}{x\sqrt{x^2+2x}}.$ **1502.** $\int \frac{dx}{(x-1)\sqrt{x^2-2x}}.$

1503. $\int \frac{x^2dx}{\sqrt{4-x^2}}.$ **1504.** $(x=2\sin^2 t).$

1505. $\int \sqrt{4x+x^2}\ dx.$ **1506.** $\int \frac{x^2dx}{\sqrt{5+4x-x^2}}.$

1507. 1) $\int \frac{dx}{x^3\sqrt{1+x^2}};$ 2) $\int \frac{dx}{x^2\sqrt{1+x^2}}.$

8- §. Ba'zi bir transsendent funksiyalarni integrallash

a) $\int R(e^x)dx$ ko'rinishdagi integrallarni $e^x=t, x=\ln t,$

$dx=\frac{dt}{t};$

b) $\int R(\mathrm{tg}x)dx$ ko'rinishdagi integrallarni $\mathrm{tg}x=t,\quad x=\mathrm{arctg}t,$

$dx=\frac{dt}{1+t^2};$

d) $\int R(\sin x, \cos x)dx$ ko'rinishdagi integrallarni $\operatorname{tg}\frac{x}{2} = t$,

$\sin x = \frac{2t}{1+t^2}$, $\cos x = \frac{1-t^2}{1+t^2}$, $dx = \frac{2dt}{1+t^2}$ almashtirishlar yordamida ratsional funksiyalardan olingan integrallarga keltiriladi.

Integrallarni toping:

1508. $\int \frac{e^{2x}-2e^x}{e^{2x}+1}dx.$

1509. $\int \operatorname{tg}^4 x dx.$

1510. $\int \frac{e^{3x}dx}{e^x+2}.$

1511. $\int \frac{dx}{\sin x}.$

1512. $\int \frac{dx}{5+3\cos x}.$

1513. $\int \frac{dx}{3\sin x+4\cos x}.$

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1514. $\int \frac{dx}{\sin^4 x}.$

1515. $\int \frac{dx}{1+3\cos^2 x}.$

1516. $\int \frac{e^{2x}dx}{e^x-1}.$

1517. $\int \operatorname{tg}^5 x dx.$

1518. $\int \frac{e^{3x}dx}{e^{2x}-1}.$

1519. $\int \frac{dx}{3+\cos x}.$

1520. $\int \frac{dx}{\cos^4 x}.$

1521. $\int \frac{dx}{1+3\sin^2 x}.$

1522. $\int \frac{dx}{2\sin x+\sin 2x}.$

1523. $\int \frac{1+\cos x}{\sin^3 x}dx.$

1524. $\int \frac{e^x+1}{e^x-1}dx.$

1525. $\int \frac{1+\operatorname{tg}x}{\sin 2x}dx.$

9- §. Integrallashga doir aralash misollar

Integrallarni toping:

1526. $\int \frac{\sqrt{1+x}}{x}dx.$

1527. $\int \frac{\operatorname{arctg}x dx}{1+x^2}.$

1528. $\int \frac{dx}{x^3+ax^2}.$

1529. $\int \frac{dx}{1+\sin x}.$

1530. $\int \frac{dx}{\sqrt{x(1-x)}}$.

1531. $\int \frac{dx}{\frac{\sin^2 x}{a^2}+\frac{\cos^2 x}{b^2}}$.

1532. $\int x\cos^2 xdx$.

1533. $\int \frac{dx}{e^{2x}+e^x}$.

1534. $\int \sqrt{\frac{1-x}{1+x}}dx$.

1535. $\int \frac{\cos^2 xdx}{\sin^4 x}$.

1536. $\int x\text{tg}^2 xdx$.

1537. $\int \frac{\cos^2 x}{\sin x}dx$.

1538. $\int \frac{\sin x}{b^2+\cos^2 x}dx$.

1539. $\int \frac{dx}{\sqrt[3]{x^2}+2\sqrt{x}}$.

1540. $\int \frac{ax-b}{(ax+b)^4}dx$.

1541. $\int \frac{dx}{x^4+x^2}$.

1542. $\int \frac{dx}{(\sin x+\cos x)^2}$.

1543. $\int \frac{dx}{x\sqrt{a+b\ln x}}$.

1544. $\int \frac{x^2dx}{(a-bx^3)^n}$.

1545. $\int \sqrt{1-2x-x^2}dx$.

1546. $\int \frac{dx}{(1+\sqrt{x})^3}$.

1547. $\int \frac{\text{arctg}xdx}{x^2}$.

1548. $\int \frac{e^x-2}{e^{2x}+4}dx$.

1549. $\int \frac{dx}{(2x+1)(1+\sqrt{2x+1})}$.

1550. $\int \text{ctg}^4 xdx$.

1551. $\int \frac{\sqrt{4-x^2}}{x^2}dx$.

1552. 1) $\int \frac{\cos x}{\cos 3x}dx$; 2) $\int \frac{\sin x}{\sin 3x}dx$.

1553. 1) $\int \frac{dx}{\sqrt{x+a}+\sqrt{x}}$. 2) $\int \frac{dx}{\sqrt{x^2+1}-x}$.

1554. $\int \frac{x^4+1}{x^3-x^2}dx$.

1555. $\int \frac{\sqrt{x^2+2x}}{x^3}dx$.

1556. $\int \frac{dx}{x\sqrt{x^3-1}}$.

1557. $\int \frac{dx}{1+\text{tg}x}$.

1558. $\int \frac{\arcsin\sqrt{x}}{\sqrt{x}}dx.$

1559. $\int \frac{\sin 2x}{\cos^4 x}dx.$

1560. $\int \frac{\cos 2x}{\sin^4 x}dx.$

1561. $\int \frac{\ln(\cos x)}{\sin^2 x}dx.$

1562. $\int \frac{dx}{e^{3x}-e^{x}}.$

1563. $\int \frac{\sin^3 xdx}{\cos^5 x}.$

1564. $\int \frac{\ln(x+1)}{x^2}dx.$

1565. $\int \sqrt{1-\sin x}\ dx.$

1566. $\int \frac{dx}{1+\sin^2 x}.$

1567. $\int \frac{xdx}{x^4-x^2-2}.$

1568. $\int e^{-\sqrt{x}}dx.$

1569. $\int \frac{\operatorname{arctg}\sqrt{x}}{\sqrt{x}}dx.$

1570. $\int \frac{\sqrt{\operatorname{tg}x}}{\sin 2x}dx.$

1571. $\int \frac{\ln(x^2+1)}{x^3}dx.$

1572. $\int \frac{a^x}{a^{2x}+1}dx.$

1573. $\int \frac{1-\sin\sqrt{x}}{\sqrt{x}}dx.$

1574. $\int \sqrt{\frac{(x+1)^3}{(x-1)^2}}dx.$

1575. $\int \frac{x\arcsin x}{\sqrt{1-x^2}}dx.$

1576. $\int \frac{dx}{x^2\sqrt{x^2-1}}.$

1577. $\int \frac{x^2dx}{(x+1)^4}.$

1578. $\int \frac{x-a}{\sqrt{2ax+x^2}}dx.$

1579. $\int \frac{4x+1}{2x^3+x^2-x}dx.$

1580. $\int \frac{\cos^3 x+1}{\sin^2 x}dx.$

1581. $\int \frac{dx}{x^2+4}.$

IX bob javoblari

1235. $\frac{x^3}{3}+x^2+\ln|x|+C.$ **1236.** $2x^5-\frac{1}{x^3}+C.$ **1237.** $\frac{1-x}{x^2}+C$

1238. $\frac{x^2}{2}+2\ln|x|-\frac{1}{2x^2}+C.$ **1239.** $x\left(\frac{2}{3}\sqrt{x}+\frac{3}{4}\sqrt[3]{x}\right)+C.$

1240. $2\sqrt{x}-4\sqrt[4]{x}+C.$ **1241.** $\frac{2x\sqrt{x}}{3}-3x+6\sqrt{x}-\ln|x|+C.$

1242. $\frac{3}{4}(x-y)\sqrt[3]{x}+C.$ **1243.** $e^x+\frac{1}{x}+C.$ **1244.** $\frac{e^x}{\ln a}-\frac{2}{\sqrt{x}}+C.$

1245. $-\mathrm{ctg}x-\mathrm{tg}x+C$. **1246.** $-\mathrm{ctg}x-x+C$. **1247.** $\mathrm{tg}x-\mathrm{ctg}x+C$.

1248. $3\mathrm{tg}x+2\mathrm{ctg}x+C$. **1249.** $\frac{x}{2}-\frac{\sin x}{2}+C$. **1250.** $\frac{x}{2}+\frac{\sin x}{2}+C$.

1251. $2\mathrm{arctg}x-3\arcsin x+C$. **1252.** $\frac{x^3}{3}-x+\mathrm{arctg}x+C$. **1253.** $\frac{x^4-1}{2x^2}-2\ln|x|+C$. **1254.** $3\sqrt[3]{x}+\frac{2}{\sqrt{x}}+C$. **1255.** $\frac{2(x+2)}{\sqrt{x}}+C$. **1256.** $4\ln|x|-\frac{8}{\sqrt{x}}-\frac{1}{x}+C$. **1257.** $\ln|x|-\frac{1}{x}-\frac{1}{2x^2}+C$. **1258.** $x+\cos x+C$.

1259. $e^x+\mathrm{tg}x+C$. **1260.** $\frac{1}{\ln a}-\frac{1}{4x^4}+C$. **1261.** $\cos x-\mathrm{ctg}x+C$.

1262. $\mathrm{tg}x-x+C$. **1263.** $\frac{1}{3}\sin 3x+C$. **1264.** $\frac{1}{5}\mathrm{tg}5x+C$. **1265.** $-\frac{1}{3}e^{-3x}+C$. **1266.** $-2\cos\frac{x}{2}+C$. **1267.** $2\left(e^{x/2}-e^{-x/2}\right)+C$. **1268.** $\frac{1}{6}(4x-1)^{\frac{3}{2}}+C$. **1269.** $-\frac{(3-2x)^5}{10}+C$. **1270.** $-\frac{1}{8}(5-6x)^{\frac{4}{3}}+C$. **1271.** $-\sqrt{3-2x}+C$. **1272.** $\frac{1}{b}\cos(a-bx)+C$. **1273.** $\ln(x^2-5x+7)+C$.

1274. $\frac{1}{2}\ln(x^2+1)+C$. **1275.** $-0{,}1\ln|1-10x|+C$. **1276.** $-\frac{1}{6}\ln\left|1-3e^{2x}\right|+C$.

1277. $\ln|\sin x|+C$. **1278.** $-\ln|\cos x|+C$. **1279.** $\ln|\sin 2x|+C$.

1280. $-\frac{1}{3}\ln|1+3\cos x|+C$. **1281.** $\frac{1}{2}\ln|1+2\sin x|+C$. **1282.** $\ln|1+\ln x|+C$.

1283. $\frac{\sin^3 x}{3}+C$. **1284.** $-\frac{\cos^4 x}{4}+C$. **1285.** $-\frac{1}{3\sin^3 x}+C$. **1286.** $\frac{1}{2\cos^2 x}+C$.

1287. $\frac{2-\cos x}{\sin x}+C$. **1288.** $\frac{\sin^2 x}{2}+C$. **1289.** $-e^{\cos x}+C$. **1290.** $\frac{1}{3}e^{x^3}+C$.

1291. $-\frac{1}{2}e^{-x^2}+C$. **1292.** $2e^{\sqrt{x}}+C$. **1293.** $\frac{1}{3}\sqrt{\left(x^2+1\right)^3}+C$. **1294.** $\frac{1}{4}\sqrt[3]{\left(x^3-8\right)^4}+C$. **1295.** $0{,}5\sqrt[3]{\left(1+x^3\right)^2}+C$. **1296.** $-\sqrt{1-x^2}+C$.

1297. $-\sqrt{1+2\cos x}+C$. **1298.** $\frac{2}{3}\sqrt{(1+\ln x)^3}+C$. **1299.** $\frac{1}{6}(1+4\sin x)^{\frac{3}{2}}+C$.

1300. $-\frac{1}{40}\left(1-6x^5\right)^{\frac{4}{3}}+C$. **1301.** $2\sin\sqrt{x}+C$. **1302.** $2\sqrt{1+\ln x}+2\ln\left|\sqrt{1+\ln x}-1\right|-\ln|\ln x|+C$.

1303. $x-2\sqrt{x}+2\ln\left(\sqrt{x}+1\right)+C$. **1304.** $\ln\left|\frac{\sqrt{e^x+1}-1}{\sqrt{e^x+1}+1}\right|+C$.

1305. $2x+\frac{1}{2}\left(e^{2x}-e^{-2x}\right)+C$. **1306.** $\frac{\sin^4 x}{4}+C$. **1307.** $-\frac{1}{2}\sqrt{1-4x}+C$.

1308. $-\frac{1}{b}\sin(a-bx)+C$. **1309.** $\frac{1}{4}(1+3x)^{\frac{4}{3}}+C$. **1310.** $-\frac{1}{7}\left(1-2x^3\right)^{\frac{7}{6}}+C$.

1311. $\sqrt{1+x^2}+C$. **1312.** $\frac{\sin x-2}{\cos x}+C$. **1313.** $2\ln|\sin x|-\text{ctg}x+C$.

1314. $e^{\sin x}+C$. **1315.** $-\frac{1}{3}\ln\left|1-x^3\right|+C$. **1316.** $\frac{1}{2b(a-bx)^2}+C$. **1317.** $-\frac{3^{\frac{1}{x}}}{\ln 3}+C$. **1318.** $\frac{(\text{arctg}x)^{101}}{101}+C$. **1319.** $\arcsin\frac{e^x}{2}+C$. **1320.** $\frac{1}{4\arccos^4 x}+C$.

1321. $\frac{1}{a}\text{arctg}\frac{x}{a}+C$. **1322.** $\arcsin\frac{x}{a}+C$. **1323.** $\frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|+C$.

1324. $\ln\left|x+\sqrt{x^2+k}+C\right|$. **1325.** $0{,}1\ln\left|\frac{x-5}{x+5}\right|+C$. **1326.** $\frac{1}{3}\text{arctg}\frac{x}{3}+C$.

1327. $\arcsin\frac{x}{2}+C$; **1328.** $\ln\left(x+\sqrt{x^2+5}\right)+C$. **1329.** $\ln\left|x+\sqrt{x^2-4}\right|+C$.

1330. $\frac{1}{\sqrt{3}}\text{arctg}\frac{x}{\sqrt{3}}+C$. **1331.** $\arcsin\frac{x}{\sqrt{5}}+C$. **1332.** $\frac{1}{6}\text{arctg}\frac{x^3}{2}+C$.

1333. $\frac{1}{2}\arcsin\frac{x^2}{\sqrt{3}}+C$. 1334. $\frac{1}{2ab}\ln\left|\frac{bx-a}{bx+a}\right|+C$. **1335.** $\frac{1}{2}\arcsin\frac{2x}{\sqrt{3}}+C$.

1336. $\frac{1}{4}\ln\left(x^4+\sqrt{x^8-1}\right)+C$. **1337.** $2{,}5\ln\left(x^2+4\right)-\text{arctg}\frac{x}{2}+C$. **1338.** $\frac{3}{2}\ln\left|x^2-4\right|-\ln\left|\frac{x-2}{x+2}\right|+C$. **1339.** $\sqrt{x^2+1}+\ln\left(x+\sqrt{x^1+1}\right)+C$. **1340.** $-\sqrt{1-x^2}+\arcsin x+C$. **1341.** $x-\text{arctg}x+C$. **1342.** $\frac{x^3}{3}+3x+\frac{3\sqrt{3}}{2}\times$ $\times\ \ln\left|\frac{x-\sqrt{3}}{x+\sqrt{3}}\right|+C$. **1343.** $\text{arctg}(x+2)+C$. **1344.** $\frac{1}{2}\text{arctg}\frac{x-3}{2}+C$.

1345. $\ln(x+1+\sqrt{x^2+2x+3})+C$. **1346.** $\arcsin\frac{x+1}{\sqrt{2}}+C$. **1347.** $\arcsin\frac{x-2}{2}+C$. **1348.** $\frac{2}{\sqrt{3}}\text{arctg}\frac{2x+3}{\sqrt{3}}+C$. **1349.** $\frac{1}{\sqrt{2}}\arcsin\frac{4x-3}{5}+C$.

1350. $\frac{1}{\sqrt{3}}\ln\left|3x-1+\sqrt{9x^2-6x+3}\right|+C$. **1351.** $\sqrt{3}\left(\text{arctg}\frac{1}{\sqrt{3}}+\ln\left|\frac{x-\sqrt{3}}{x+\sqrt{3}}\right|\right)+C$.

1352. $\arcsin\frac{x}{\sqrt{2}}+\ln\left(x+\sqrt{2+x^2}\right)+C$. **1353.** $2\ln(x^2+5)-\sqrt{5}\operatorname{arctg}\frac{x}{\sqrt{5}}+C$.

1354. $x+\frac{1}{\sqrt{2}}\ln\left|\frac{x-\sqrt{2}}{x+\sqrt{2}}\right|+C$. **1355.** $\frac{x^3}{3}-2x+2\sqrt{2}\operatorname{arctg}\frac{x}{\sqrt{2}}+C$.

1356. $\arcsin(e^x)+C$. **1357.** $\operatorname{arctg}(2x^2)+C$. **1358.** $0{,}2\operatorname{arctg}\frac{x+2}{3}+C$.

1359. $\frac{1}{2}\operatorname{arctg}\frac{x-1}{2}+C$. **1360.** $\arcsin\frac{x+2}{3}+C$. **1361.** $\frac{1}{2}\ln(x^2+x+1)-$
$-\frac{1}{\sqrt{3}}\operatorname{arctg}\frac{2x+1}{\sqrt{3}}+C$. **1362.** $\frac{1}{2}\ln(2x+1+\sqrt{4x^2+4x-3})+C$. **1363.** $x\ln|x|+C$.

1364. $\frac{x^2}{2}\ln|x-1|-\frac{1}{2}\left(\frac{x^2}{2}+x+\ln|x-1|\right)+C$. **1365.** $\frac{1}{2}e^{2x}\left(x-\frac{1}{2}\right)+C$.

1366. $\frac{x^2+1}{2}\operatorname{arctg}x-\frac{x}{2}+C$. **1367.** $x^2\sin x+2x\cos x-2\sin x+C$.

1368. $\frac{1}{2}e^x(\sin x-\cos x)+C$. **1370.** $x\left[(\ln|x|-1)^2+1\right]+C$.

1371. $-x\operatorname{ctg}x+\ln|\sin x|+C$. **1372.** $-\frac{\ln|x|+1}{x}+C$. **1373.** $2\sqrt{1+x}\arcsin x+$
$+4\sqrt{1-x}+C$. **1374.** $x\arcsin x+\sqrt{1-x^2}+C$. **1375.** $-e^{-x}(x^3+3x^2+$
$+6x+6)+C$. **1376.** $x\ln(x^2+1)-2x+2\operatorname{arctg}x+C$. **1377.** $\frac{x}{2}(\cos\ln x+$
$+\sin\ln x)+C$. **1378.** $\frac{2}{5}\sqrt{x^3}\left(\ln|x|-\frac{2}{3}\right)+C$. **1379.** $-2e^{-\frac{x}{2}}\left(x^2+4x+8\right)+C$.

1380. $x\operatorname{arctg}x-\frac{1}{2}\ln(1+x^2)+C$. **1381.** $x\operatorname{tg}x+\ln|\cos x|+C$.

1382. $0{,}5e^x(\sin x+\cos x)+C$. **1383.** $4\sqrt{2+x}-2\sqrt{2-x}\arcsin\frac{x}{2}+C$.

1384. $-\frac{1}{2}\left(\frac{x}{\sin^2 x}+\operatorname{ctg}x\right)+C$. **1385.** $x\operatorname{arctg}\sqrt{2x-1}-\frac{\sqrt{2x-1}}{2}+C$.

1386. $\ln\left|\frac{1-\sqrt{1-x^2}}{x}\right|-\frac{1}{x}\arcsin x+C$. **1387.** $x\ln\left|x+\sqrt{1+x^2}\right|-\sqrt{1+x^2}+C$.

1388. $\frac{1}{2}x^2\operatorname{arctg}\sqrt{x^2-1}-\frac{1}{2}\sqrt{x^2-1}+C$. **1389.** $\frac{x}{4(1+x^2)}+\frac{1}{4}\operatorname{arctg}x-$
$-\frac{1}{2}\frac{\operatorname{arctg}x}{1+x^2}+C$. **1390.** $\frac{x}{2}-\frac{1}{12}\sin 6x+C$. **1391.** $3x+4\sin x+\sin 2x+C$.

1392. $\frac{3x}{2}+\cos 2x-\frac{\sin 4x}{8}+C$. **1393.** $\frac{3x}{8}-\frac{\sin 4x}{128}+\frac{\sin 4x}{32}+C$.

1394. $\frac{x}{8}-\frac{\sin 4x}{32}+C.$ **1395.** $\frac{3x}{128}-\frac{\sin 4x}{128}+\frac{\sin 8x}{1024}+C.$ **1396.** $\frac{x}{16}-\frac{\sin 4x}{64}+\frac{\sin^3 2x}{48}+C.$ **1397.** $-\cos x+\frac{2}{3}\cos^3 x-\frac{\cos^5 x}{5}+C.$ **1398.** $\frac{x}{16}-\frac{\sin 4x}{64}+\frac{\sin^3 2x}{48}+C.$ **1399.** $\frac{1}{4}\sin^4 x-\frac{1}{6}\sin^6 x+C.$ **1400.** $\sin x-\sin^3 x+\frac{3}{5}\sin^5 x-\frac{1}{7}\sin^7 x+C.$ **1401.** $7x+14\sin x+3\sin 2x-\frac{8\sin^3 x}{3}+C.$

1402. $-\frac{1}{\sin x}-\sin x+C.$ **1403.** $\frac{1}{\cos x}+\cos x+C.$ **1404.** $\frac{1}{2}\ln|\mathrm{tg}x|+C.$

1405. $\ln\left|\mathrm{tg}\frac{x}{2}\right|+C.$ **1406.** $\ln\left|\mathrm{tg}\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|+C.$ **1407.** $\frac{1}{2}\left[\ln\left|\mathrm{tg}\frac{x}{2}\right|+\ln\left|\mathrm{tg}\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|\right]+C.$ **1408.** $\int\frac{dx}{\sin x-\cos x}=\int\frac{dx}{\sin x-\sin\left(\frac{\pi}{2}-x\right)}=\frac{1}{\sqrt{2}}\int\frac{dx}{\sin\left(x-\frac{\pi}{4}\right)}=\frac{1}{\sqrt{2}}\ln\left|\mathrm{tg}\left(\frac{x}{2}-\frac{\pi}{8}\right)\right|+C.$ **1409.** $\frac{\mathrm{tg}^2 x}{2}+\ln|\cos x|+C.$ **1410.** $-\frac{\mathrm{ctg}^2 x}{2}-\ln|\sin x|+C.$

1411. $-\frac{1}{8}(\cos 4x+2\cos 2x)+C.$ **1412.** $\frac{1}{2}\left[\frac{\sin(m+n)x}{m+n}+\frac{\sin(m-n)x}{m-n}\right]+C,$

$m\neq n$ da va $\frac{x}{2}+\frac{1}{4m}\sin 2mx+C,$ $m=n$ bo'lganda.

1413. $\frac{1}{4}\sin 2x-\frac{1}{16}\sin 8x+C$. **1414.** $\frac{1}{2}\left[\frac{\sin(m-n)x}{m-n}-\frac{\sin(m+n)x}{m+n}\right]+C,$

$m\neq n$ da va $\frac{x}{2}-\frac{1}{4m}\sin 2mx+C,$ $m=n$ bo'lganda. **1415.** $-\frac{1}{12}\cos 6x-\frac{1}{8}\sin 4x+C$. **1417.** $\frac{5}{16}x-\cos x\left(\frac{\sin^5 x}{6}+\frac{5\sin^3 x}{24}+\frac{5\sin x}{16}\right)+C$.

1419. $\frac{\cos x}{2\sin^2 x}+\frac{1}{2}\ln\left|\mathrm{tg}\frac{x}{2}\right|+C.$ **1420.** $\frac{\sin x}{2\cos^2 x}+\frac{1}{2}\ln\left|\mathrm{tg}\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|+C.$

1421. $\frac{11x}{2}+3\sin 2x+\frac{9}{8}\sin 4x.$ **1422.** $\frac{3}{8}x-\frac{1}{4}\sin 2x+\frac{1}{32}\sin 4x+C.$

1423. $\frac{x}{16}-\frac{\sin 4x}{64}-\frac{\sin^3 2x}{48}+C.$ **1424.** $\sin x-\frac{2\sin^3 x}{3}+\frac{\sin^5 x}{5}+C.$

1425. $\frac{\cos^5 x}{5}-\frac{\cos^3 x}{3}+C.$ **1426.** $7x-14\cos x-3\sin 2x+\frac{8\cos^3 x}{3}+C.$

1427. $\frac{1}{2}\ln|\mathrm{tg}x|-x+C.$ **1428.** $\frac{1}{8}(2\sin 2x-\sin 4x)+C.$

1429. $\frac{1}{\cos x}+\cos x+\operatorname{tg}x+C$. **1430.** $-\frac{1}{4}\cos\left(2x+\frac{\pi}{6}\right)+\frac{1}{4}x+C$.

1431. $\frac{x^3}{3}+x^2+4x+8\ln|x-2|+C$. **1432.** $\frac{x^3}{3}-a^2x+a^3\operatorname{arctg}\frac{x}{a}+C$.

1433. $\frac{x^3}{3}+\frac{a^3}{3}\ln\left|x^3-a^3\right|+C$. **1434.** $\ln\frac{C(x-2)^2}{x-3}$. **1435.** $\ln\left|\frac{(x-1)^3}{x+2}\right|+C$.

1436. $\ln\frac{Cx^3(x-1)}{x+1}$. **1437.** $\frac{x^2}{2}+4x+\ln\frac{(x-1)^8}{|x|}+C$. **1438.** $\frac{1}{x}+\ln\left|\frac{x-2}{x}\right|+C$.

1439. $\frac{1}{a^2}\ln\left|\frac{x-a}{x}\right|+\frac{x-a}{ax^2}+C$. **1440.** $\ln Cx(x-1)+\frac{2}{x-1}$. **1441.** $\ln\left|\frac{x-2}{x+1}\right|-$ $-\frac{2}{x+1}+C$. **1442.** $\frac{5}{2}\ln(x^2+2x-10)-\operatorname{arctg}\frac{x+1}{3}+C$. **1443.** $2\ln(x^2-0,2x+0,17)-5\operatorname{arctg}\frac{10x-1}{4}+C$. **1444.** $\ln|x+1|\sqrt{x^2+4}+C$.

1445. $3\ln\frac{\sqrt{x^2-2x+5}}{|x|}+2\operatorname{arctg}\frac{x-1}{2}+C$. **1446.** $\frac{1}{24}\ln\frac{(x+2)^2}{x^2-2x+4}+\frac{1}{4\sqrt{3}}\operatorname{arctg}\frac{x-1}{\sqrt{3}}+C$.

1447. $\ln\frac{\sqrt{x^2+1}}{|x+1|}-\frac{1}{x+1}+\operatorname{arctg}x+C$. **1448.** 1) $\frac{1}{2b^3}\left(\operatorname{arctg}\frac{x}{b}+\frac{bx}{x^2+b^2}\right)+C$;

2) $\frac{1}{8b^4}\left[\frac{x(5b^2+3x^2)}{(x^2+b^2)^2}+\frac{3}{b}\operatorname{arctg}\frac{x}{b}\right]+C$. **1449.** $-\frac{x+9}{8(x^2+2x+5)}-\frac{1}{16}\operatorname{arctg}\frac{x+1}{2}+C$.

1450. $\frac{1}{8}\left[\frac{(x-3)(3x^2-18x+32)}{(x^2-6x+10)^2}+3\operatorname{arctg}(x-3)\right]+C$. **1451.** $\ln\frac{\sqrt{x^2+1}}{|x+1|}+\frac{x-1}{x^2+1}+C$.

1452. $\frac{x-2}{4(x^2+2)}+\frac{\sqrt{2}}{8}\operatorname{arctg}\frac{x}{\sqrt{2}}+C$. **1453.** $\frac{1}{a}\ln\left|\frac{x}{x+a}\right|+C$. **1454.** $\frac{1}{a-b}\ln\left|\frac{x+b}{x+a}\right|+C$.

1455. $\frac{1}{2}\ln\left|1-\frac{2}{x}\right|+C$. **1456.** $\frac{1}{10\sqrt{3}}\ln\left|\frac{x-\sqrt{3}}{x+\sqrt{3}}\right|-\frac{1}{5\sqrt{2}}\operatorname{arctg}\frac{x}{\sqrt{2}}+C$.

1457. $\frac{1}{x}+\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right|+C$. **1458.** $\frac{1}{4}\ln\frac{|x|}{\sqrt{4+x^2}}+C$. **1459.** $\ln\frac{C(x-2)^3}{x-1}$.

1460. $\ln C(x-1)\sqrt{2x+3}$. **1461.** $\ln\frac{C(x-1)^3}{(x+2)^2(x-2)}$. **1462.** $3\ln\frac{C(x-1)}{(x+2)}-\frac{2}{x+2}$.

1463. $2\ln\frac{C(x-2)}{x}-\frac{1}{x-2}$. **1464.** $\ln\frac{|x|}{\sqrt{x^2-2x+2}}+2\operatorname{arctg}(x-1)+C$.

1465. $\frac{1}{a}\ln\frac{\sqrt{x^2+a^2}}{|x|}+\frac{1}{a}\operatorname{arctg}\frac{x}{a}+C$. **1466.** $\frac{1}{3}\ln\frac{|x+1|}{\sqrt{x^2+2}}+\frac{1}{3\sqrt{2}}\operatorname{arctg}\frac{x}{\sqrt{2}}+C$.

1467. $\frac{1}{24}\ln\frac{(x-2)^2}{x^2+2x+4}-\frac{1}{4\sqrt{3}}\operatorname{arctg}\frac{x+1}{\sqrt{3}}+C$.

1468. $-\frac{1}{2}\left[\frac{x+2}{x^2+2x+2}+\operatorname{arctg}(x+1)\right]+C$. **1469.** $\frac{1}{5}\ln\left|\frac{x}{x+5}\right|+C$. **1470.** $-\frac{1}{3x}-\frac{1}{3\sqrt{3}}\operatorname{arctg}\frac{x}{\sqrt{3}}+C$. **1471.** $\frac{1}{2}\int\frac{x^2+1-(x^2-1)}{(x^2+1)(x^2-1)}dx=\frac{1}{4}\ln\left|\frac{x-1}{x+1}\right|-\frac{1}{2}\operatorname{arctg}x+C$.

1472. $\frac{1}{3}\int\frac{x^2+1-(x^2-2)}{(x^2+1)(x^2-2)}dx = \frac{1}{6\sqrt{2}}\ln\left|\frac{x-\sqrt{2}}{x+\sqrt{2}}\right| - \frac{1}{3}\operatorname{arctg}x + C$.

1473. $\frac{x+2}{5}\sqrt[3]{(3x+1)^2}+C$. **1474.** $\frac{2x+1}{12}(2\sqrt{2x+1}-3)+C$.

1475. $6\left[\frac{\sqrt{x}}{3}-\frac{\sqrt[3]{x}}{2}+\sqrt[6]{x}-\ln\left(1+\sqrt[6]{x}\right)\right]+C$. **1476.** $\frac{2}{15}(3x^2-ax-2a^2)\times\sqrt{a-x}+C$. **1477.** $\frac{3}{4}\left[\frac{\sqrt[3]{(x^4+1)^2}}{2}-\sqrt[3]{x^4+1}+\ln(\sqrt[3]{x^4+1}+1)\right]+C$.

1478. $\frac{(x^2-4)\sqrt{x^2+2}}{3}+C$. **1479.** $\mp\arcsin\frac{1}{x}+C$ ($x>0$ da – va $x<0$ da +).

1480. $\ln\frac{Cx}{x+1+\sqrt{2x^2+2x+1}}$. **1481.** $-\frac{1}{a}\sqrt{\frac{2a-x}{x}}+C$. **1482.** $\ln\frac{C(x+1)}{1+\sqrt{2x^2+2x+2}}$.

1483. $\frac{1}{2}\left[x\sqrt{a^2-x^2}+a^2\arcsin\frac{x}{a}\right]+C$. **1484.** $\frac{x}{4\sqrt{4+x^2}}+C$.

1485. $2\arcsin\frac{x}{2}-\frac{x}{4}(2-x^2)\sqrt{4-x^2}+C$. **1486.** $\frac{x^3}{3a^2\sqrt{(a^2+x^2)^3}}+C$.

1487. $2\arcsin\frac{x-1}{2}-\frac{(x-1)\sqrt{3+2x-x^2}}{2}+C$. **1488.** $\frac{x}{\sqrt{2-x^2}}-\operatorname{arctg}\frac{x}{\sqrt{2}}+C$.

1489. $\frac{1}{2}(x+5)\sqrt{x^2+2x+2}-3{,}5\ln(x+1+\sqrt{x^2+2x+2})+C$.

1490. $-\sqrt{3-2x-x^2}-\arcsin\frac{x+1}{2}+C$. **1492.** $\frac{x-a}{2}\sqrt{2ax-x^2}+\frac{a^2}{2}\arcsin\frac{x-a}{a}+C$. **1493.** $\frac{(x-2)\sqrt{2x-1}}{3}+C$. **1494.** $\frac{(3x+1)^{2/3}}{2}+(3x+1)^{1/3}+$

$+\ln\left|(3x+1)^{1/3}-1\right|+C$. **1495.** $x-2\sqrt{x}+2\ln(\sqrt{x}+1)+C$. **1496.** $-0,3(2x+3a)\times$ $\times\sqrt[3]{(a-x)^2}+C$. **1497.** $2\sqrt{x-2}+\sqrt{2}\operatorname{arctg}\sqrt{\frac{x-2}{2}}+C$. **1498.** $\frac{3(x^2+1)}{2}\left(\frac{\sqrt[3]{(x^2+1)^2}}{5}+\frac{\sqrt[3]{x^2+1}}{4}+\frac{1}{3}\right)+C$. **1499.** $\ln\left(1+\sqrt{1+x^2}\right)+\frac{1}{1+\sqrt{1+x^2}}+C$.

1500. $x^2+\frac{1}{3}\sqrt{(4-x^2)^3}+C$. **1501.** $\mp\sqrt{\frac{x+2}{x}}+C$ ($x>0$ da – va $x<-2$ da +).

1502. $\arccos\frac{1}{x-1}+C$. **1503.** $2\arcsin\frac{x}{2}-\frac{x}{2}\sqrt{4-x^2}+C$. **1504.** $2\arcsin\sqrt{\frac{x}{2}}-\sqrt{2x-x^2}+C$. **1505.** $\frac{2+x}{2}\sqrt{4x+x^2}-2\ln\left|x+2+\sqrt{4x+x^2}\right|+C$.

1506. $-\frac{x+6}{2}\sqrt{5+4x-x^2}+\frac{17}{2}\arcsin\frac{x-2}{3}+C$. **1507.** 1) $-\frac{\sqrt{1+x^2}}{2x^2}+$ $+\frac{1}{2}\ln\frac{\sqrt{1+x^2}+1}{|x|}+C$; 2) $-\frac{\sqrt{1+x^2}}{x}+C$. **1508.** $\frac{1}{2}\ln(e^{2x}+1)-2\operatorname{arctg}(e^x)+C$.

1509. $\frac{1}{3}\operatorname{tg}^3x-\operatorname{tg}x+x+C$. **1510.** $\frac{e^{2x}}{2}-2e^x+4\ln(e^x+2)+C$.

1511. $\ln\left|\operatorname{tg}\frac{x}{2}\right|+C$. **1512.** $\frac{1}{2}\operatorname{arctg}\left(\frac{1}{2}\operatorname{tg}\frac{x}{2}\right)+C$. **1513.** $\frac{1}{5}\ln\left|\frac{2\operatorname{tg}\frac{x}{2}+1}{\operatorname{tg}\frac{x}{2}-2}\right|+C$.

1514. $-\frac{\operatorname{ctg}^3x}{3}-\operatorname{ctg}x+C$. **1515.** $\frac{1}{2}\operatorname{arctg}\left(\frac{\operatorname{tg}x}{2}\right)+C$. **1516.** $e^x+\ln\left|e^x-1\right|+C$.

1517. $\frac{\operatorname{tg}^4x}{4}-\frac{\operatorname{tg}^2x}{2}-\ln|\cos x|+C$. **1518.** $e^x+\frac{1}{2}\ln\left|\frac{e^x-1}{e^x+1}\right|+C$. **1519.** $\frac{1}{\sqrt{2}}\operatorname{arctg}\left(\frac{\operatorname{tg}\frac{x}{2}}{\sqrt{x}}\right)+C$. **1520.** $\frac{\operatorname{tg}^3x}{3}+\operatorname{tg}x+C$. **1521.** $\frac{1}{2}\operatorname{arctg}(2\operatorname{tg}x)+C$. **1522.** $\frac{1}{4}\ln\left|\operatorname{tg}\frac{x}{2}\right|+\frac{1}{8}\operatorname{tg}^2\frac{x}{2}+C$. **1523.** $\frac{1}{4}\ln\left|\operatorname{tg}\frac{x}{2}\right|-\frac{1}{4}\operatorname{ctg}^2\frac{x}{2}+C$. **1524.** $2\ln\left|e^x-1\right|-x+C$.

1525. $\frac{1}{2}(\operatorname{tg}x+\ln|\operatorname{tg}x|)+C$. **1526.** $2\sqrt{x+1}+\ln\left|\frac{x+2-\sqrt{1+x}}{x}\right|+C$. **1527.** $\frac{(\operatorname{arctg}x)^2}{2}+C$. **1528.** $\frac{1}{a^2}\ln\left|\frac{x+a}{x}\right|-\frac{1}{ax}+C$. **1529.** $\operatorname{tg}\left(\frac{x}{2}-\frac{\pi}{4}\right)+C$.

1530. $2\arcsin\sqrt{x}+C$. **1531.** $ab\operatorname{arctg}\left(\frac{b}{a}\operatorname{tg}x\right)+C$.

1532. $\frac{1}{4}\left(x^2+x\sin 2x+\frac{1}{2}\cos 2x\right)+C.$ **1533.** $\ln C(e^x+1)-x-e^{-x}$.

1534. $\arcsin x+\sqrt{1-x^2}+C.$ **1535.** $-\frac{\text{ctg}^3 x}{3}+C.$ **1536.** $x\text{tg}x+\ln|\cos x|-$ $-\frac{x^2}{2}+C.$ **1537.** $\ln\left|\text{tg}\frac{x}{2}\right|+\cos x+C.$ **1538.** $-\frac{1}{b}\text{arctg}\frac{\cos x}{b}+C.$ **1539.** $3x^{1/3}-12x^{1/6}+24\ln(x^{1/6}+2)+C.$ **1540.** $\frac{b-3ax}{6a(ax+b)^3}+C.$ **1541.** $-\frac{1}{x}+\text{arctg}x+C.$

1542. $-\frac{1}{\text{tg}x+1}$ (surat va maxrajni $\cos^2 x$ ga bo'lib, $\text{tg}x=t$ deb belgilash kerak). **1543.** $\frac{2}{b}\sqrt{a+b\ln x}+C.$ **1544.** $\frac{1}{3b(n-1)(a-bx^3)^{n-1}}+C,\ n\neq 1$ da va $3b\ln\left|a-bx^3\right|+C,\ n=1$ da. **1545.** $\frac{x+1}{2}\sqrt{1-2x-x^2}+\arcsin\frac{x+1}{\sqrt{2}}+C.$

1546. $\int\frac{dx}{(1+\sqrt{x})^3}=-\frac{2\sqrt{x}+1}{(\sqrt{x}+1)^2}+C.$ **1547.** $\frac{1}{2}\ln\frac{x^2}{1+x^2}-\frac{\text{arctg}x}{x}+C.$

1548. $\frac{1}{2}\text{arctg}\frac{e^x}{2}-\frac{1}{2}x+\frac{1}{4}\ln(4+e^{2x})+C.$ **1549.** $\ln\left|\frac{c\sqrt{2x+1}}{1+\sqrt{2x+1}}\right|.$

1550. $x+\text{ctg}x-\frac{\text{ctg}^3 x}{3}+C.$ **1551.** $-\frac{\sqrt{4-x^2}}{x}-\arcsin\frac{x}{2}+C.$

1552. 1) $\frac{1}{2\sqrt{3}}\ln\left|\frac{\sqrt{3}+\text{ctg}x}{\sqrt{3}-\text{ctg}x}\right|+C$; 2) $\frac{1}{2\sqrt{3}}\ln\left|\frac{\sqrt{3}+\text{tg}x}{\sqrt{3}-\text{tg}x}\right|+C.$ **1553.** 1) $\frac{2}{3a}\left[(x+a)^{3/2}-x^{3/2}\right]+C$; 2) $\frac{1}{2}\left[x\sqrt{x^2+1}+\ln(x+\sqrt{x^2+1})+x^2\right]+C.$

1554. $\frac{x^2}{2}+x+\frac{1}{x}+\ln\frac{C(x-1)^2}{x}.$ **1555.** $-\frac{1}{3}\left(\frac{x+2}{x}\right)^{3/2}+C.$ **1556.** $\frac{2\text{arctg}\sqrt{x^3-1}}{3}+C.$ **1557.** $0,5(x+\ln|\sin x+\cos x|)+C.$ **1558.** $2(\sqrt{x}\arcsin\sqrt{x}+\sqrt{1-x})+C.$ **1559.** $\text{tg}^2 x+C$ yoki $\frac{1}{\cos^2 x}+C_1$. **1560.** $\text{ctg}x-\frac{\text{ctg}^3 x}{3}+C.$ **1561.** $-\text{ctg}x\ln|\cos x|-x+C.$ **1562.** $e^{-x}+\frac{1}{2}\ln\left|\frac{e^x-1}{e^x+1}\right|+C.$

1563. $\frac{1}{4}\text{tg}^4 x+C.$ **1564.** $\ln|x|-\frac{x+1}{x}\ln|x+1|+C.$ **1565.** $\pm 2\sqrt{1+\sin x}+C.$

1566. $\frac{1}{\sqrt{2}}\text{arctg}(\sqrt{2}\text{tg}x)+C.$ **1567.** $\frac{1}{b}\ln\frac{|x^2-2|}{x^2+1}+C.$ **1568.** $-2e^{-\sqrt{x}}(\sqrt{x}+1)+C.$

1569. $2\sqrt{x}\operatorname{arctg}\sqrt{x} - \ln|1+x| + C$. **1570.** $\sqrt{\operatorname{tg}x} + C$. **1571.** $\ln|x| - \frac{x^2+1}{2x^2}\times$
$\times\ln(x^2+1) + C$. **1572.** $\frac{1}{\ln a}\operatorname{arctg}(a^x) + C$. **1573.** $2(\sqrt{x} + \cos\sqrt{x}) + C$.

1574. $\frac{2(x+7)}{3}\sqrt{x+1} + 2\sqrt{2}\ln\frac{|\sqrt{x+1}-\sqrt{2}|}{\sqrt{x+1}+\sqrt{2}} + C$. **1575.** $x - \sqrt{1-x^2}\arcsin x + C$.

1576. $\frac{\sqrt{x^2-1}}{x}$. **1577.** $-\frac{3x^2+3x+1}{3(x+1)^3} + C$. **1578.** $\sqrt{2ax+x^2} - 2a\ln|x+$
$+a+\sqrt{2ax+x^2}| + C$. **1579.** $\ln\frac{(2x-1)^2}{|x^2+x|} + C$. **1580.** $-\frac{1+\cos x+\sin^2 x}{\sin x} + C$.

1581. $\frac{1}{16}\ln\frac{C(x^2+2x+2)}{x^2-x+2} + \frac{1}{8}\operatorname{arctg}\frac{2x}{2-x^2}$,

$\left(x^4 + 4 = x^4 + 4x^2 + 4 - 4x^2 = (x^2+2)^2 - 4x^2\right)$.

X BOB | ANIQ INTEGRAL

1- §. Aniq integralni hisoblash

$y = f(x)$ funksiya $[a; b]$ kesmada aniqlangan va uzluksiz bo'lsin. $[a; b]$ kesmani ixtiyoriy ravishda $a = x_0, x_1, x_2, \ldots, x_{n-1}, x_n = b$ nuqtalar yordamida n bo'lakka bo'lamiz. Bu bo'laklarning har birining uzunligini Δx_i ($i = 1, 2, \ldots, n$) orqali belgilaylik, ya'ni $\Delta x_1 = x_1 - x_0$; $\Delta x_2 = x_2 - x_1$; ... ; $\Delta x_n = x_n - x_{n-1}$. Bu bo'laklarning har biridan ixtiyoriy $\xi_1, \xi_2, \ldots, \xi_n$ nuqtalarni olamiz va

$$S_n = \sum_{n=1}^{n} f(\xi_i)\Delta x_i$$

yig'indini tuzamiz. S_n yig'indi $f(x)$ funksiyaning $[a; b]$ kesmadagi *integral yig'indisi* deyiladi.

T a ' r i f . *S_n integral yig'indining Δx_i kesmalarining eng kattasi nolga intilgandagi (bunda bo'laklarga bo'lish soni n cheksizlikka intiladi) limiti $f(x)$ funksiyadan $[a; b]$ kesma bo'yicha olingan aniq integral deyiladi*, ya'ni

$$\int_a^b f(x)dx = \lim_{\max \Delta x_i \to 0} \sum_{i=1}^{n} f(\xi_i)\Delta x_i. \qquad (1)$$

Ma'lumki, $f(x)$ $[a; b]$ da uzluksiz bo'lsa, u holda shu kesmada quyidagi aniqmas integral mavjud:

$$\int_a^b f(x)dx = F(x) + C. \qquad (2)$$

Uzluksiz funksiyadan olingan aniq integral esa

$$\int_a^b f(x)dx = F(b) - F(a) \qquad (3)$$

formula bo'yicha hisoblanadi. (3) formula *Nyuton—Leybnis formulasi* deyiladi.

1582. Integral yig'indilarning limiti sifatida hisoblang:

1) $\int_0^a x dx$; 2) $\int_0^a x^2 dx$; 3) $\int_0^a e^x dx$; 4) $\int_0^a \sin x dx$

K o ' r s a t m a . 2) misolni yechishda $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

formuladan, 4) misolni yechishda $\sum_{m=1}^{n} \sin mx = \frac{\sin\frac{nx}{2}\cdot\sin\frac{n+1}{2}x}{\sin\frac{x}{2}}$ formuladan foydalanish kerak.

Nyuton —Leybnis formulasiga ko‘ra hisoblang:

1583. $\int_1^3 x^3 dx.$

1584. $\int_1^2 \left(x^2 + \frac{1}{x^4}\right)dx.$

1585. $\int_1^4 \sqrt{x}dx.$

1586. $\int_0^1 \frac{dx}{\sqrt{4-x^2}}.$

1587. $\int_a^{a\sqrt{3}} \frac{dx}{a^2+x^2}.$

1588. $\int_0^3 e^{\frac{x}{3}}dx.$

1589. $\int_0^1 \frac{dx}{\sqrt{x^2+1}}.$

1590. $\int_0^{\pi/4} \sin 4x dx.$

1591. $\int_4^9 \frac{dx}{\sqrt{x}-1}.$

1592. $\int_{\pi/4}^{\pi/3} \frac{1+\operatorname{tg}^2 x}{(1+\operatorname{tg}x)^2}dx.$

Ko‘rsatma. 1591- misolda $x=t^2$, 1592- misolda $\operatorname{tg}x = t$ almashtirishlardan foydalanish kerak. Bunda integrallarning chegaralari ham mos ravishda o‘zgaradi.

1593. $\int_0^4 \frac{dx}{1+\sqrt{2x+1}}.$

1594. $\int_0^1 \frac{x^2dx}{\sqrt{4-x^2}}.$

1595. $\int_0^1 \frac{dx}{e^x+1}.$

1596. $\int_0^{a/2} \sqrt{\frac{x}{a-x}}dx.$

1597. $\int_0^{\pi/2} \sin x\cos^2 x dx.$

1598. $\int_0^{\sqrt{a}} x^2\sqrt{a-x^2}dx.$

1599. $\int_0^1 \ln(x+1)dx.$

1600. $\int_0^1 \sqrt{1+x^2}dx.$

1601. $\int_1^{\sqrt{3}} \frac{dx}{\sqrt{(1+x^2)^3}}.$

1602. $\int_1^3 \frac{dx}{x+x^2}.$

1603. $\int_0^{\frac{\pi}{2}} \sin^n x dx = \frac{n-1}{n}\int_0^{\frac{\pi}{2}} \sin^{n-2} x dx$ formulani bo‘laklab integrallash usuli yordamida isbotlang va quyidagilarni hisoblang:

1) $\int_0^{\frac{\pi}{2}} \sin^2 x dx$; 2) $\int_0^{\frac{\pi}{2}} \sin^4 x dx$; 3) $\int_0^{\frac{\pi}{2}} \sin^6 x dx$.

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Hisoblang:

1604. $\int_0^a (x^2 - ax)dx$.

1605. $\int_2^3 \frac{dx}{x^2}$.

1606. $\int_0^{\sqrt{3}} \frac{xdx}{\sqrt{4-x^2}}$.

1607. $\int_{\pi/8}^{\pi/6} \frac{dx}{\cos^2 2x}$.

1608. $\int_1^4 \frac{dx}{(1+\sqrt{x})^2}$.

1609. $\int_0^1 \frac{e^x dx}{1+e^{2x}}$.

1610. $\int_1^5 \frac{xdx}{\sqrt{4x}+5}$.

1611. $\int_1^{\sqrt{2}} \sqrt{2-x^2}dx$.

1612. $\int_0^{\pi/2} x \cos x dx$.

1613. $\int_0^{\pi/4} \mathrm{tg}^3 x dx$.

1614. $\int_0^{\pi/2} \cos^n x dx = \frac{n-1}{n} \int_0^{\pi/2} \cos^{n-2} x dx$ ni isbotlang va

1) $\int_0^{\pi/2} \cos^2 x dx$; 2) $\int_0^{\pi/2} \cos^4 x dx$; 3) $\int_0^{\pi/2} \cos^6 x dx$ larni hisoblang.

2- §. Yuzlarni hisoblash

1°. Asosi Ox o‘qida joylashgan, $y = f(x)$ egri chiziq va $x = x_1$, $x = x_2$ to‘g‘ri chiziqlar bilan chegaralangan egri chiziqli trapetsiyaning yuzi:

$$S = \lim_{\Delta x \to 0} \sum y\Delta x = \int_{x_1}^{x_2} ydx. \qquad (1)$$

O‘zgaruvchan yuzning differensiali $ds = ydx$. Agar egri chiziq $x = f(t)$, $y = \varphi(t)$ tenglamalar bilan berilgan bo‘lsa, u holda $ds = \varphi(t) f'(t)dt$.

2°. Asosi Oy o‘qida bo‘lgan egri chiziqli trapetsiyaning yuzi:

$$S=\lim_{\Delta y\to 0}\sum x\Delta y=\int_{y_1}^{y_2} xdy. \quad (2)$$

O‘zgaruvchi yuzning differensiali $ds = xdy$.

3°. $r = f(\varphi)$, $\varphi = \varphi_1$, $\varphi = \varphi_2$ lar bilan chegaralangan sektorning yuzi

$$S=\lim_{\Delta\varphi\to 0}\sum \frac{1}{2}r^2\Delta\varphi=\int_{\varphi_1}^{\varphi_2}\frac{1}{2}r^2 d\varphi \quad (3)$$

formula yordamida hisoblanadi. O‘zgaruvchan yuzning differensiali $ds=\frac{1}{2}r^2 d\varphi$.

Quyidagi chiziqlar bilan chegaralangan shakllarning yuzini toping:

1615. $y = 4 - x^2$, $y = 0$.

1616. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

1617. $y^2 = 2px$, $x = h$.

1618. $y = 3 - 2x - x^2$, $y = 0$.

1619. $xy=4$, $x=1$, $x=4$, $y=0$.

1620. $y=\ln x$, $x=e$, $y=0$.

1621. $y^2=2x+4$, $x=0$.

1622. $y^2=x^3$, $y=8$, $x=0$.

1623. $y^2= (4-x)^3$, $x=0$.

1624. $4(y^2-x^2)+x^3=0$ egri chiziq sirtmog‘ining yuzini toping.

1625. $y = x^2$, $y = 2-x^2$.

1626. $y = x^2 + 4x$, $y = x + 4$.

1627. $a^2y^2 = x^3(2a - x)$.

1628. $(y-x)^2 = x^3$, $x=1$.

1629. $y^2 (2a - x) = x (x - a)^2$ strofoida sirtmog‘ining yuzini toping.

1630. Zanjir chiziq $y=\frac{a}{2}\left(e^{\frac{x}{a}}+e^{-\frac{x}{a}}\right)$, $x =\pm a$ va $y=0$.

1631. $x = a (t - \sin t)$, $y = a (1 - \cos t)$ sikloidaning bir arkasi va Ox o‘qi bilan chegaralangan yuzni hisoblang.

1632. Astroida: $x = a\cos^3 t$, $y = a\sin^3 t$.

1633. Lemniskata: $r^2 = a^2\cos^2\varphi$.

1634. Kardioida: $r = a (1 - \cos\varphi)$.

1635. $r = 3 + \sin 2\varphi$ egri chiziqning eng katta va eng kichik qo'shni radius-vektori orasidagi yuzni hisoblang.

1636. $r = 2 - \cos 3\varphi$ egri chiziqning eng katta va eng kichik qo'shni radiusi-vektori orasidagi yuzni hisoblang.

1637. $r = a\cos 2\varphi$. **1638**. $r = a\sin 3\varphi$.

1639. $r = a(\sin\varphi + \cos\varphi)$. **1640**. $r = \frac{a}{\varphi}$, $\frac{\pi}{4} \le \varphi \le 2\pi$.

1641. $r = \alpha\sin^3\frac{\varphi}{3}$ chiziqnmg qutb o'qidan pastgi qismi bilan chegaralangan yuzni hisoblang.

1642. $x^3 + y^3 - 3axy = 0$ dekart yaprog'i sirtmog'ining yuzini toping. (Qutb koordinatalariga o'ting).

Ko'rsatma. $\int\frac{\sin^2\varphi\cos^2\varphi d\varphi}{(\sin^3\varphi+\cos^3\varphi)^2}$ integralda surat va maxrajni $\cos^6\varphi$ ga bo'lib olib, $\mathrm{tg}\varphi = u$ almashtirish kiriting.

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Quyidagi chiziqlar bilan chegaralangan shakllarning yuzini toping:

1643. $y = 6x - x^2$, $y = 0$. **1644**. $y = x^3$, $y = 8$, $x = 0$.

1645. $y^2 = 1 - x$, $x = -3$. **1646**. $y^2 + x^4 = x^2$.

1647. $x = 0$, $y = 0$ va $y = x^2 + 4x + 5$ ning miniınal ordinatasi bilan chegaralangan yuzni toping.

1648. $y = \sin x$, $y = 0$ $(0 \le x \le \pi)$.

1649. $4y = x^2$, $y^2 = 4x$.

1650. $xy = 6$, $x + y - 7 = 0$.

1651. $x^3 + x^2 - y^2 = 0$ egri chiziq sirtınog'ining yuzini toping.

1652. $r = 3 - \cos 2\varphi$ egri chiziqning eng katta va eng kichik qo'shni radius-vektori orasidagi yuzni hisoblang.

1653. $r = 2 + \sin 3\varphi$ egri chiziqning eng katta va eng kichik qo'shni radius-vektori orasidagi yuzni toping.

1654. $r = a\sin 2\varphi$.

1655. $r = a\cos 3\varphi$.

1656. $r = ae^{\varphi}$, $-\pi \le \varphi \le \pi$.

1657. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ va $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$ ellipslarning umumiy qismi bilan chegaralangan yuzni toping.

1658. $r = a\,(1 + \sin^2 2\varphi)$ va $r = a$.

3- §. Aylanish jismlarining hajmi

1°. Egri chiziqli trapetsiyaning Ox o‘q atrofida aylanishidan hosil bo‘lgan jism hajmi

$$V=\lim_{\Delta x\to 0}\sum \pi y^2\Delta x=\int_{x_1}^{x_2}\pi y^2 dx \qquad (1)$$

formula bilan aniqlanadi. Bu yerda $\overset{\smile}{AB} - f(x)$ egri chiziq yoyi. O‘zgaruvchan hajm differensiali $dv = \pi y^2 dx$.

2°. Asosi Oy o‘qida bo‘lgan egri chiziqli trapetsiyaning Oy o‘q atrofida aylanishidan hosll bo‘lgan jism hajmi

$$V=\lim_{\Delta y\to 0}\sum \pi x^2\Delta y=\int_{y_1}^{y_2}\pi y^2 dx \qquad (2)$$

formula bo‘yicha topiladi, o‘zgaruvchan hajm differensiali $dv = \pi x^2 dy$.

Quyidagi chiziqlar bilan chegaralangan shakllarning aylanishidan hosil bo‘lgan jism hajmini aniqlang:

1659. $y^2 = 2px$, $x = h$; Ox o‘q atrofida.

1660. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, $y=\pm b$; Oy o‘q atrofida.

1661. $xy = 4$, $x = 1$, $x = 4$, $y = 0$; Ox o‘q atrofida.

1662. $y^2 = (x + 4)^3$, $x = 0$; Oy o‘q atrofida.

1663. $x^2 + y^2 = a^2$; $x = b > 0$ to‘g‘ri chiziq atrofida.

K o ‘ r s a t m a . $dv = \pi(b+x)^2dy - \pi(b - x)^2dy = 4\pi bx dy$.

1664. $y = a\,\mathrm{ch}\,\frac{x}{a}$, $x = \pm a$, $y = 0$; Ox o‘q atrofida.

1665. $y^2 = 4 - x$, $x = 0$; Oy o‘q atrofida.

1666. $(y - a)^2 = ax$, $x = 0$, $y = 2a$; Ox o‘q atrofida.

1667. $y = \cos x$, $y = -1$ $(-\pi \le x \le \pi)$; $y = -1$ to‘g‘ri chiziq atrofida.

1668. $y=x\sqrt{-x}$, $x=-4$ va $y= 0$; Oy o‘q atrofida.

1669. $y=\cos(x-\frac{\pi}{3})$, $x=0$, $y=0$ $(x>0)$; Ox o‘q atrofida.

1670. $y=a-\frac{x^2}{a}$, $x+y=a$; Oy o‘q atrofida.

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1671. $y=\sin x$ $(0\le x\le\pi)$, $y=0$; Ox o‘q atrofida.
1672. $x^2-y^2=4$, $y=\pm2$; Oy o‘q atrofida.

1673. $y=\frac{1}{1+x^2}$, $x=\pm1$, $y=0$; Ox o‘q atrofida.

1674. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$; Oy o‘q atrofida.

1675. $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$; Ox o‘q atrofida.

1676. $y=x^3$, $x=0$, $y=8$; Oy o‘q atrofida.
1677. $x^2-y^2=a^2$, $x=\pm2a$; Ox o‘q atrofida.
1678. $y=x^2$, $y=4v$; $x=2$ to‘g‘ri chiziq atrofida.
Korsatma. $dv=\pi(2+x)^2\,dy-\pi(2-x)^2\,dy$.
1679. $x=a(t-\sin t)$, $y=a(1-\cos t)$ sikloida bir arkasining Ox o‘q atrofida.
1680. $(y-3)^2+3x=0$, $x=-3$; Ox o‘q atrofida.

4- §. Tekis egri chiziq yoyi uzunligini hisoblash

1°. $y=f(x)$ egri chiziq $\breve{AB}$ yoyi uzunligi:

$$l=\int_{x_A}^{x_B}\sqrt{1+y'^2}\,dx. \qquad (1)$$

Yoy differensiali: $dl=\sqrt{1+y'^2}\,dx=\sqrt{dx^2+dy^2}$.

2°. $x=f(t)$, $y=\varphi(t)$ egri chiziq $\breve{AB}$ yoyi uzunligi:

$$l=\int_{t_A}^{t_B}\sqrt{x^2+y^2}\,dt. \qquad (2)$$

3°. $r = f(\varphi)$ egri chiziq $\breve{AB}$ yoyi uzunligi:

$$l=\int_{\varphi_A}^{\varphi_B}\sqrt{r^2+r'^2}\,d\varphi. \qquad (3)$$

Quyidagi egri chiziqlar yoyining uzunligini aniqlang:

1681. $y^2 = x^3$ ning $x=\frac{4}{3}$ to'g'ri chiziq bilan kesilgan qismi.

1682. $x^2 + y^2 = a^2$ butun egri chiziq.

1683. $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ butun egri chiziq.

1684. $y^2 = (x + 1)^3$ ning $x = 4$ bilan kesilgan qismi.

1685. $x = a(t - \sin t)$, $y = a(1 - \cos t)$ sikloidaning bir arkasi.

1686. $x=\frac{t^6}{6}$, $y=2-\frac{t^4}{4}$ ning koordinata o'qlari bilan kesishgan nuqtalari orasi.

1687. $y=\frac{x^2}{2}-1$ ning Ox bilan kesilgan qismi.

1688. $y=\frac{a}{2}\left(e^{\frac{x}{a}}+e^{-\frac{x}{a}}\right)=a\,\mathrm{ch}\,\frac{x}{a}$ ning $x = \pm a$ to'g'ri chiziqlar bilan kesilgan qismi.

1689. $y=\ln x$, $\quad \frac{3}{4}<x<\frac{12}{5}$.

K o r s a t m a. $\int\frac{\sqrt{1+x^2}}{x}dx$ integralda $1 + x^2 = t^2$ almashtirish bajarish kerak.

1690. $y = \ln(2\cos x)$ ning Oy va Ox o'qlari bilan kesishgan qo'shni nuqtalari orasidagi qismi.

1691. 1) $9y^2 = x(x - 3)^2$ ning Ox o'q bilan kesishgan nuqtalari orasi.

2) $e^{2y}\mathrm{th}x = 1$, $1 \le x \le 2$.

1692. 1) Kardioida: $r = a(1 - \cos\varphi)$;

2) $r = a\varphi$ spiralning birinchi o'rami yoyining uzunligi.

1693. $r = a\sin^3\frac{\varphi}{3}$ egri chiziq uzunligi.

1694. $y^2 = \frac{4}{9}(2-x)^3$ ning $x = -1$ to‘g‘ri chiziq bilan kesilgan qismi yoyi uzunligi.

1695. $y = \ln(\sin x)$, $\frac{\pi}{3} \le x \le \frac{2\pi}{3}$.

1696. $y = \ln(1-x^2)$, $-\frac{1}{2} \le x \le \frac{1}{2}$.

1697. $y^2 = 2px$ ning $x = \frac{P}{2}$ to‘g‘ri chiziq bilan qirqilgan qismi yoyi uzunligi.

1698. $x = t^2$, $y = \frac{t}{3}(t^2 - 3)$ ning Ox o‘qi bilan kesishgan nuqtalari orasidagi yoyi uzunligi.

5- §. Aylanish sirtining yuzini hisoblash

1°. $y = f(x)$ egri chiziq $\breve{AB}$ yoyining Ox o‘q atrofida aylanishidan hosil bo‘lgan sirt yuzi $p_x = 2\pi\int\limits_{\breve{AB}} y ds$ formula yordamida aniqlanadi, bu yerda $ds = \sqrt{dx^2 + dy^2}$.

2°. $x = \varphi(y)$ egri chiziq $\breve{AB}$ yoyining Oy o‘q atrofida aylanishidan hosil bo‘lgan sirt yuzi $p_y = 2\pi\int\limits_{\breve{AB}} x ds$ formula yordamida aniqlanadi, bu yerda $ds = \sqrt{dx^2 + dy^2}$.

Quyidagi chiziqlarning aylanishidan hosil bo‘lgan sirt yuzini aniqlang.

1699. $x^2 + y^2 = R^2$ ning Ox o‘q atrofida.

1700. $y = \frac{x^2}{2}$ ning $y = 1{,}5$ to‘g‘ri chiziq bilan qirqilgan qismining Oy o‘q atrofida.

1701. $y = a\,\mathrm{ch}\,\frac{x}{a}$ ning $x = \pm a$ lar orasidagi qismining Ox o‘q atrofida.

1702. $4x^2 + y^2 = 4$ ning Oy o‘q atrofida.

K o ‘ r s a t m a. y ni argument desak, $p=\pi\int_0^2\sqrt{16-3y^2}\,dy.$

1703. $y = \sin x$, $0 < x < \pi$ ning Ox o‘q atrofida.

1704. $x = a(t - \sin t)$, $y = a(1 - \cos t)$ sikloida bir arkasining Ox o‘q atrofida.

1705. $x=t^2,\ y=\frac{t}{3}(t^2-3)$ egri chiziq sirtmog‘ining Ox o‘q atrofida.

1706. $x^2 + y^2 = a^2$ ning $x = b > a$ to‘g‘ri chiziq atrofida.

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1707. $y=\frac{x^3}{3},\ -2\le x\le 2$ ning Ox o‘q atrofida.

1708. $y^2 = 4 + x$ ning $x = 2$ bilan qirqilgan qismining Ox o‘q atrofida.

1709. $x = a\cos^3 t$, $y = a\sin^3 t$ egri chiziqning Ox o‘q atrofida.

1710. $x=\frac{t^3}{3},\ y=4-\frac{t^2}{2}$ ning koordinata o‘qlari bilan kesishgan nuqtalari orasidagi qismining Ox o‘q atrofida.

6- §. Fizikaga doir masalalar yechish

1711. Asosi 8 m, balandligi 6 m bo‘lgan to‘g‘ri burchakli vertikal shluzdagi suvning bosim kuchini aniqlang. Shluzning pastki yarmidagi bosim kuchini ham aniqlang.

1712. a asosi suv yuzida joylashgan, balandligi h ga teng uchburchakli vertikal yuzga bo‘lgan suv bosimini aniqlang.

1713. $2R$ diametri suv yuzida joylashgan vertikal yarim doiraga bo‘lgan suv bosimini aniqlang.

1714. To‘g‘on yuqori asosi 20 m, quyi asosi 10 m va balandligi 6 m bo‘lgan trapetsiya shaklida. Suvning to‘g‘onga bo‘lgan bosimini aniqlang.

K o ‘ r s a t m a.

$$\frac{x}{6}=\frac{y-20}{-10}\ \text{yoki}\ y=-\frac{5}{3}x+20.$$

Suv uchun: $\overline{\varpi} = 1$; $P = \int_0^6 \left(20 - \frac{5}{3}x\right)x\, dx = 240$ t.

1715. $x = 0$, $x = a$, $y = 0$ va $y = b$ chiziqlar bilan chegaralangan to'g'ri to'rtburchakning Ox va Oy o'qlarga nisbatan inersiya momentlarini aniqlang.

K o ' r s a t m a . To'g'ri to'rtburchakni gorizontal yuzlarga ajratib, har bir yuzni undan Ox o'qqacha bo'lgan masofa kvadratiga, ya'ni y^2 ga ko'paytiramiz. Ko'paytmalarni qo'shib limitga o'tsak, quyidagini hosil qilamiz:

$$I_x = \lim_{\Delta y \to 0} \sum a\Delta y \cdot y^2 = \int_0^b ay^2\, dy.$$

Shunga o'xshash: $I_y = \int_a^b bx^2 dx.$

1716. $x = 0$, $y = 0$ va $\frac{x}{a} + \frac{y}{b} = 1$ chiziqlar bilan chegaralangan uchburchakning Ox va Oy o'qlarga nisbatan inersiya momentlarini toping.

1717. $x = 2$, $y = x^2$ va $y = 0$ chiziqlar bilan chegaralangan yuzning Oy o'qqa nisbatan inersiya momentini toping.

1718. $x = 0$ va $x + y = a$ chiziqlar bilan chegaralangan uchburchakning Ox va Oy o'qlarga nisbatan statik momenti va og'irlik markazining koordinatalarini toping.

K o ' r s a t m a . Statik momentlar quyidagidan iborat:

$$M_x = \int_0^a xydy; \quad M_y = \int_0^a xydx.$$

Og'irlik markazining koordinatalari:

$$x_c = \frac{M_y}{S}; \quad y_c = \frac{M_x}{S},$$

bunda S — shaklning yuzi.

1719. $a^2y = bx^2$, $x = a$ va $y = 0$ chiziqlar bilan chegaralangan yuzning og'irlik markazini toping.

1720. $x^2 + y^2 = a^2$ aylananing Ox o'q bilan kesishishidan hosil bo'lgan yarim doiraning og'irlik markazini toping.

1721. 1) Asosining radiusi 0,5 m bo'lgan silindrik hovuzdagi suvning boshlang'ich sathi 2,8 m va silindrdagi suv oqib chiqadigan jo'mrakdan 0,2 m quyi bo'lsa, hovuzdagi suvni tortib chiqarish uchun sarf etilgan ishni hisoblang.

2) Radiusi R_m bo'lgan yarim shardagi suvni tortib chiqarish uchun sarf etilgan ishni hisoblang.

1722. Massasi m bo'lgan jismni yerdan h balandlikka ko'tarish uchun sarf etish kerak bo'lgan ishni aniqlang.

Ko'rsatma. Yer markazidan x masofada markazga tortish kuchi F ushbu proporsiyadan aniqlanadi: $F : mg = R^2 : x^2$, bunda R — Yer sharining radiusi.

1723. Qozon asosining radiusi $R = 0,4$ m, chuqurligi $H = 0,5$ m dan iborat aylanma paraboloid shaklida. Suv to'ldirilgan shunday qozondan barcha suvni tortib chiqarish uchun sarf etilgan ishni aniqlang.

1724. Silindrdagi porshen ostida hajmi $V_0 = 0,1$ m^3, elastikligi $P_0 = 10330$ kg/m^2 bo'lgan havo bor. Havo hajmini $V_1 = 0,03$ m^3 ga keltirish uchun, havoni izotermik siqish uchun bajarilgan ishni aniqlang (Boyl—Mariott qonuni boyicha $PV = P_0V_0$).

1725. Uzunligi 1 m, kesim radiusi 2 mm bo'lgan mis simni 0,001 m cho'zish uchun sarf etilgan ishni hisoblang.

Ko'rsatma. Uzunligi l_m, kesimi S mm^2 bo'lgan simni x m cho'zish uchun sarf etiladigan kuch $F = E \cdot \frac{Sx}{l}$ formula bilan aniqlanadi, bunda E — elastiklik moduli. Mis uchun $E \approx 12000$ kg/mm^2 deb olish mumkin.

1726. Asosi $S = 420$ sm^2, balandligi $H = 40$ sm bo'lgan silindrik idishdagi suv silindr tubidagi yuzi $S = 2$ sm^2 bo'lgan teshikdan qancha vaqt ichida oqib tamom bo'ladi?

Ko'rsatma. Yuksakligi x sm balandlikda bo'lgan suvning oqimi tezligi $v = \mu\sqrt{2gx}$ formula bilan hisoblanadi, bunda μ — suyuqlikning yopishqoqligiga, idishning shakliga va teshikning yuziga bog'liq koeffitsiyent. Bu yerda va shuningdek, 1727- masalada $\mu = 0,6$ deb olamiz.

1727. Pastki asosining radiusi $r = 0,3$ sm, yuqori asosining radiusi $R = 6$ sm, balandligi $H = 40$ sm bo'lgan konus shaklidagi voronkadan suv qancha vaqt ichida oqib bo'ladi? (1726- masalaga berilgan ko'rsatmaga qarang).

1728. Balandligi h, asosi a suv yuziga parallel, unga qarshi uchi esa suv yuzida bo'lgan uchburchakli vertikal yuzga bo'lgan suv bosimini aniqlang.

1729. Asosi 4 m ga teng va suv yuziga joylashgan parabolik segmentning uchi 4 m chuqurlikda yotadi. Bu segmentga bo'lgan suv bosimini aniqlang.

1730. Balandligi h ga teng to'g'ri burchakli shluzni shunday x chuqurlikda ikki gorizontal bo'lakka ajratingki, ularga bo'lgan suv bosimi bir xil bo'lsin.

1731. Gorizontal o'qqa ega silindrik idish yarmisigacha yog' (solishtirma og'irligi 0,9) bilan to'ldirilgan. Agar silindr tekis devorlarining radiusi 2 m ga teng bo'lsa, ularning har biriga bo'lgan yog' bosimini aniqlang.

1732. $x = a\cos t$, $y = a\sin t$ doira chorak yuzining Ox o'qqa nisbatan inersiya momentini toping.

1733. $y = 4 - x^2$ va $y = 0$ chiziqlar bilan chegaralangan yuz og'irlik markazining koordinatalarini toping.

1734. Balandligi $H = 2$ m, asosining radiusi $R = 0{,}3$ m ga teng konus shaklidagi chuqurdan (konusning uchi pastga qaragan) barcha suvni tortib chiqarish uchun bajarish kerak bo'lgan ishni hisoblang.

1735. Hajmi $V_0 = 0{,}1$ m^3, elastikligi $P_0 = 10330$ kg/m^2 ga teng havoni $V_1 = 0{,}03$ m^3 hajmgacha adiabatik siqish uchun bajarilgan ishni aniqlang. (Adiabatik siqish Puasson qonuniga bo'ysunadi: $PV^k = P_0V_0^k$, bunda $k \approx 1{,}4$).

1736. Radiusi 40 sm ga teng yarimshar shaklidagi idishni to'lg'izib turgan suv qancha vaqt ichida shar tubidagi yuzi 2 sm^2 bo'lgan teshikdan oqib bo'ladi? (1726- masalaga berilgan ko'rsatmaga qarang. $\mu = 0{,}8$ deb faraz qilamiz).

7- §. Xosmas integrallar

1°. Ta'riflar

I. Agar $\lim\limits_{b\to+\infty}\int\limits_a^b f(x)dx$ mavjud va chekli bo'lsa, u holda $\int\limits_a^{+\infty} f(x)dx$ integral yuqori chegarasi cheksiz integral deb aytiladi. $\int\limits_{-\infty}^{a} f(x)dx$ va $\int\limits_{-\infty}^{+\infty} f(x)dx$ integrallar ham shunga o'xshash ta'riflanadi.

II. Agar $f(x)$ funksiya $[a; b]$ segmentning c nuqtasidan boshqa barcha nuqtalarida uzluksiz bo‘lib, c da II tur uzilishga ega bo‘lsa, u holda $f(x)$ dan a dan b gacha olingan integral deb

$$\lim_{\varepsilon\to 0}\int_a^{c-\varepsilon} f(x)dx+\lim_{\varepsilon\to 0}\int_{c+\varepsilon}^{b} f(x)dx$$

yig‘indiga aytiladi (agar bu limitlar mavjud va chekli bo‘lsa).

Chegaralari cheksiz bo‘lgan hamda uzlukli (chegaralanmagan) funksiyalardan olingan integrallar *xosmas integrallar* deyiladi.

Agar yoqorida keltirilgan limitlar chekli bo‘lsa, umumlashgan integrallar yaqinlashadi, chekli bo‘lmasa — uzoqlashadi deyiladi.

2°. Xosmas integrallarning yaqinlashishi ko‘pincha taqqoslash usuli bilan aniqlanadi: agar $x > a$ bo‘lganda $|f(x)| \le \varphi(x)$ bo‘lib, $\int_a^{+\infty} \varphi(x)dx$ yaqinlashsa, u holda $\int_a^{+\infty} f(x)dx$ ham yaqinlashadi. Shunda o‘xshash yaqinlashish alomatini uziluvchi funksiyadan olingan integral uchun ham keltirish mumkin.

Quyidagi integrallarni hisoblang:

1737. 1) $\int_1^{\infty}\frac{dx}{x^2}$; 2) $\int_1^{\infty}\frac{dx}{x}$; 3) $\int_1^{\infty}\frac{dx}{\sqrt{x}}$; 4) $\int_1^{\infty}\frac{dx}{x^n}$.

1738. 1) $\int_0^{\infty} e^{-x}dx$; 2) $\int_0^{\infty} xe^{-x^2}dx$; 3) $\int_0^{\infty}\frac{dx}{1+x^2}$;

4) $\int_1^{\infty}\frac{dx}{x^2\sqrt{x^2-1}}$; 5) $\int_1^{\infty}\frac{dx}{x^2+x}$; 6) $\int_0^{\infty} x^2 edx$.

1739. 1) $\int_2^{\infty}\frac{dx}{x\sqrt{x^2-1}}$; 2) $\int_1^{\infty}\frac{\operatorname{arctg}x\,dx}{x^2}$; 3) $\int_1^{\infty}\frac{dx}{(x^2+1)^2}$.

1740. 1) $\int_2^{6}\frac{dx}{\sqrt[3]{(4-x)^2}}$; 2) $\int_0^{2}\frac{dx}{(x-1)^2}$; 3) $\int_0^{2}\frac{dx}{\sqrt[3]{(x-1)^2}}$.

Quyidagi integrallarning yaqinlashishini tekshiring:

1741. 1) $\int_0^{\infty}\frac{dx}{\sqrt{1+x^2}}$; 2) $\int_0^{\infty}\frac{dx}{\sqrt[3]{x^3-1}}$; 3) $\int_1^{\infty}\frac{e^{-x}dx}{x}$;

4) $\int_1^{\infty}\frac{\sin x dx}{x^2}$; 5) $\int_2^{\infty}\frac{dx}{\sqrt{x^4+1}}$; 6) $\int_0^{\infty} e^{-x}dx$.

1742. 1) $\int_0^1 \frac{dx}{x^n}$; 2) $\int_a^b \frac{dx}{(b-x)^n}$ ($b > 0$ bo'lganda).

K o ' r s a t m a . $n = 1 - a < 1, n = 1$ va $n = 1 + a > 1$ bo'lgan uch holni ko'rib chiqing.

1743. $y = \frac{1}{1+x^2}$ zulf bilan uning asimptotasi orasidagi yuzni hisoblang.

1744. $y = xe^{-\frac{x^2}{2}}$ egri chiziq bilan uning asimptotasi orasidagi yuzni hisoblang ($x > 0$ bo'lganda).

1745. $y^2 = \frac{x^3}{2a-x}$ sissoida bilan uning asimptotasi orasidagi yuzni hisoblang.

K o ' r s a t m a . $x = 2a\sin^2 t$ deb parametrik tenglamalarga o'tish kerak.

1746. $y^2 = \frac{x^2}{2a-x}$ sissoidaning o'z asimptotasi atrofida aylanishidan hosil bo'lgan jism hajmini aniqlang (1745- masalaga qarang).

1747. $y - e^{-x}$ egri chiziqning x musbat bo'lgandagi cheksiz yoyining Ox o'q atrofida aylanishidan hosil bo'lgan jism hajmini aniqlang.

1748. $y=2\left(\frac{1}{x} - \frac{1}{x^2}\right)$ egri chiziqning $x \geq 1$ bo'lgandagi cheksiz shoxchasining Ox o'q atrofida aylanishidan hosil bo'lgan jismning hajmini toping.

1749. m butun, musbat bo'lganda:

1) $\int_0^\infty e^{-x}x^m dx = m!$, 2) $\int_0^\infty e^{-x^2} x^{2m+1} dx = \frac{m!}{2}$ ekanini ko'rsating

(0! = 1 deb hisoblanadi).

1750. Quyidagi integrallarni hisoblang:

1) $\int_2^\infty \frac{dx}{x^2}$; 2) $\int_0^\infty x^2 e^{-x^3} dx$; 3) $\int_1^\infty \frac{\ln x dx}{x^2}$; 4) $\int_1^e \frac{dx}{x\ln x}$.

K o ' r s a t m a . 3) misolda $\lim\limits_{x\to\infty} \frac{\ln x}{x}$ ni topishda Lopital qoidasini tatbiq eting.

1751. 1) $\int\limits_{1}^{\infty} \frac{dx}{x\sqrt{1+x^2}}$; 2) $\int\limits_{0}^{\infty} \frac{dx}{\sqrt{(1+x)^3}}$; 3) $\int\limits_{1}^{\infty} \frac{dx}{x^2+x^4}$.

1752. $y = e^{-2x}$ egri chiziq va koordinata o‘qlari orasidagi yuzni hisoblang ($x > 0$ bo‘lganda).

1753. $xy = 4$, $y = 1$, $x = 0$ chiziqlar bilan chegaralangan cheksiz uzun yuz Ox atrofida aylanishidan hosil bo‘lgan jismning hajmini toping.

1754. $y=xe^{\frac{x}{2}}$ egri chiziqning ($x > 0$ bo‘lganda) o‘z asimptotasi atrofida aylanishidan hosil bo‘lgan jismning hajmini aniqlang.

8- §. Funksiyaning o‘rta qiymati

O‘rta qiymat haqidagi teorema. *Agar $y = f(x)$ funksiya $[a; b]$ segmentda uzluksiz bo‘lsa, u holda $\int\limits_a^b f(x)dx$ integralning chegaralari orasida shunday $x = c$ nuqta topiladiki,*

$\int\limits_a^b f(x)dx = (b-a)\, f(c)$ *bo‘ladi.*

Funksiyaning

$$y_m = f(c) = \frac{\int\limits_a^b f(x)dx}{b-a}$$

qiymati $y = f(x)$ funksiyaning $[a; b]$ segmentdagi *o‘rta qiymati* deyiladi.

1755. Quyidagi funksiyalarning o‘rta qiymatlarini aniqlang:

1) $y = \sin x$, $[0; \pi]$ segmentda;

2) $y = \mathrm{tg}x$, $[0; \frac{\pi}{3}]$ segmentda;

3) $y = \ln x$, $[1; e]$ segmentda;

4) $y = x^2$, $[a; b]$ segmentda;

5) $y = \frac{1}{1+x^2}$, $[-1; 1]$ segmentda.

Har bir misolda funksiyaning o‘rta qiymatini chizmada ko‘rsating.

9- §. Trapetsiyalar va Simpson formulalari

1°. Trapetsiyalar formulasi:

$$\int_a^b f(x)dx \approx h\left[\frac{y_0 + y_n}{2} + \sum_{i=1}^{n-1} y_i\right], \qquad (1)$$

bunda $h = \frac{b-a}{n}$, $y_0, y_1, y_2, \ldots, y_n$ lar $y = f(x)$ ning $[a; b]$ dagi bir-biridan bir xil masofadagi ordinatalari.

(1) formulaning xatoligi:

$$\varepsilon(h) \le \frac{(b-a)h^2}{12}|y''|_{\max}. \qquad (2)$$

2°. $2n$ teng bo'lakka bo'lingan $[a; b]$ oraliq uchun *Simpson formulasi:*

$$\int_a^b f(x)dx \approx \frac{h}{3}\left[y_0 + y_{2n} + 4\sum_{i=1}^{h} y_{2i-1} + 2\sum_{i=1}^{n-1} y_{2i}\right], \qquad (3)$$

bunda $h=\frac{b-a}{2n}$. (3) formulaning xatoligi:

$$\varepsilon(h) \le \frac{(b-a)h^4}{180}\left|y^{\mathrm{IV}}\right|_{\max}. \qquad (4)$$

1756. Trapetsiyalar formulasiga ko'ra $\ln 2 = \int_1^2 \frac{dx}{x}$ ni hisoblang va (2) formula bo'yicha xatoligini baholang.

1757. Simpson formulasiga ko'ra $\int_1^5 x^3 dx$ va $\int_0^2 x^4 dx$ integrallarni hisoblang. (4) formula bo'yicha xatollgini baholang va integrallarning aniq qiymatlari bilan taqqoslang.

1758. Simpson formulasiga ko'ra quyidagi integrallarni hisoblang:

1) $\int_0^2 \sqrt{1+x^3}\, dx \quad (2n = 4)$; 2) $\int_0^{\pi/2} \sqrt{3-\cos 2x}dx \quad (2n = 6)$;

3) $\int_0^4 \frac{dx}{1+x^4} \quad (2n = 4)$; va (4) formulada $h^4\left|y^{\mathrm{IV}}\right|_{\max} \approx \left|\Delta^4 y\right|_{\max}$ deb xatoliklarni baholang.

1759. Simpson formulasidan foydalanib, piramida va shar hajmi formulalarini keltirib chiqaring.

1760. $\ln 2 = \int_1^2 \frac{dx}{x}$ ni Simpson formulasiga ko‘ra ($2n = 10$) hisoblang va (4) ga ko‘ra xatolikni baholang.

1761. $x = 5\cos t$, $y = 3\sin t$ ellipsning 1- chorakdagi yoyi uzunligini aniqlovchi integralga Simpson formulasini qo‘llab, ellips uzunligini toping.

1762. $\pi = 6\int_0^1 \frac{dx}{\sqrt{4-x^2}}$ ni Simpson formulasi yordamida taqribiy hisoblang.

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1763. $\frac{\pi}{4} = \int_0^1 \frac{dx}{1+x^2}$ integralni Simpson formulasiga ko‘ra ($2n = 10$) hisoblang va (4) da $h^4 \left|y^{IV}\right|_{max} \approx \left|\Delta^4 y\right|_{max}$ deb qabul qilib, xatolikni baholang.

1764. $x^2 + y^2 = 32$ bilan chegaralangan doira yuzining qismi uchun $\int_0^4 \sqrt{32 - x^2}\,dx = 4\pi + 8$ o‘rinli ekanligini ko‘rsating; integralni Simpson ($2n = 4$) formulasi bo‘yicha hisoblab π ni aniqlang.

1765. $y = \sin x$ ning $[0; \pi]$ dagi yoyi uzunligini Simpson formulasiga ko‘ra hisoblang ($2n = 6$).

X bob javoblari

1583. 20. **1584.** $\frac{21}{8}$. **1585.** $\frac{14}{3}$. **1586.** $\frac{\pi}{6}$. **1587.** $\frac{\pi}{12a}$. **1588.** $3(e-1)$. **1589.** $\ln(1+\sqrt{2})$. **1590.** $\frac{1}{2}$. **1591.** $(1+\ln 2)$. **1592.** $\frac{2-\sqrt{3}}{2}$. **1593.** $2-\ln 2$. **1594.** $\frac{\pi}{3} - \frac{\sqrt{3}}{2}$. **1595.** $\ln\frac{2e}{e+1}$. **1596.** $\frac{a(\pi-2)}{4}$. **1597.** $\frac{1}{3}$. **1598.** $\frac{\pi a^2}{16}$. **1599.** $2\ln 2-1$. **1600.** $\frac{\sqrt{2}+\ln(1+\sqrt{2})}{2}$. **1601.** $\frac{\sqrt{3}-\sqrt{2}}{2}$. **1602.** $\ln\frac{3}{2}$. **1603.** 1) $\frac{1}{2}\cdot\frac{\pi}{2}$; 2) $\frac{1\cdot 3}{2\cdot 4}\cdot\frac{\pi}{2}$;

3) $\frac{1\cdot3\cdot5}{2\cdot4\cdot6}\frac{\pi}{2}$; **1604.** $-\frac{a^3}{3}$. **1605.** $\frac{1}{6}$. **1606.** 1. **1607.** $\frac{\sqrt{3}-1}{2}$. **1608.** $2\ln1{,}5-\frac{1}{3}$. **1609.** $\mathrm{arctg}\,e-\frac{\pi}{4}\approx0{,}433$. **1610.** $\frac{17}{6}$. **1611.** $\frac{\pi-2}{4}$. **1612.** $\frac{\pi}{2}-1$.

1613. $\frac{1-\ln2}{6}$. **1614.** 1) $\frac{1}{2}\cdot\frac{\pi}{2}$; 2) $\frac{1\cdot3}{2\cdot4}\cdot\frac{\pi}{2}$; 3) $\frac{1\cdot3\cdot5}{2\cdot4\cdot6}\cdot\frac{\pi}{2}$. **1615.** $\frac{32}{3}$. **1616.** πab.

1617. $\frac{4h}{3}\sqrt{2ph}$. **1618.** $\frac{32}{3}$. **1619.** $8\ln2$. **1620.** 1. **1621.** $\frac{16}{3}$. **1622.** 19,2. **1623.** 25,6. **1624.** $\frac{128}{15}$. **1625.** $\frac{8}{3}$. **1626.** $\frac{125}{6}$. **1627.** πa^2.

1628. 0,8. **1629.** $(4-\pi)\frac{a^2}{2}$. **1630.** $2a^2\mathrm{sh}1=a^2(e-e^{-1})\approx2{,}35a^2$. **1631.** $3\pi a^2$.

1632. $\frac{3\pi a^2}{8}$. **1633.** a^2. **1634.** $\frac{3\pi a^2}{2}$. **1635.** $r_{max}=4$, $\varphi=45°+180°n=45°$, $225°$ da; $r_{min}=2$, $\varphi=-45°+180°n=135°$, $315°$. Qo'shni ekstremal radius-vektorlar 45° va 135° da. Izlanayotgan yuza $\frac{19\pi}{8}$. **1636.** $\frac{3\pi}{4}$. **1637.** $\frac{\pi a^2}{2}$. **1638.** $\frac{\pi a^2}{4}$. **1639.** $r=a(\sin\varphi+\cos\varphi)=a\sqrt{2}\cos\left(\varphi-\frac{\pi}{4}\right)$ $r_{max}=a\sqrt{2}$, $\varphi=\frac{\pi}{4}$; $r_{min}=0$ $\varphi=-\frac{\pi}{4}$ va $\frac{3\pi}{4}$, yuza $\frac{\pi a^2}{2}$. **1640.** $\frac{7a^2}{4\pi}$. **1641.** $\frac{(10\pi+27\sqrt{3})a^2}{64}$. **1642.** $\frac{3a^2}{2}$.

1643. 36. **1644.** 12. **1645.** $\frac{32}{3}$. **1646.** $\frac{4}{3}$. **1647.** $\frac{14}{3}$. **1648.** 2. **1649.** $\frac{16}{3}$. **1650.** $17{,}5-6\ln6$. **1651.** $\frac{8}{15}$. **1652.** $r_{max}=4$, $\varphi=90°+180°n=90°$ yoki 270°; $r_{min}=2$, $\varphi=180°n=0°$ yoki 180°. Yuza $s=\frac{19\pi}{8}$. **1653.** $\frac{3\pi}{4}$. **1654.** $\frac{\pi a^2}{2}$. **1655.** $\frac{\pi a^2}{4}$. **1656.** $\frac{a^2}{4}(e^{2\pi}-e^{-2\pi})$. **1657.** $4ab\,\mathrm{arctg}\frac{b}{a}$. **1658.** $\frac{11}{8}\pi a^2$. **1659.** πph^2. **1660.** $\frac{8\pi a^2b}{3}$. **1661.** 12π. **1662.** $58{,}5\pi$. **1663.** $2\pi^2a^2b$.

1664. $\pi a^3\left(\frac{\mathrm{sh}2}{2}+1\right)$ **1665.** $\frac{512\pi}{15}$. **1666.** $\frac{7}{6}\pi a^3$. **1667.** $3\pi^2$. **1668.** $\frac{512\pi}{7}$.

1669. $\frac{\pi}{4}\left(\frac{5\pi}{3}+\frac{\sqrt{3}}{2}\right)$ **1670.** $\frac{\pi a^3}{6}$. **1671.** $\frac{\pi^2}{6}$. **1672.** $\frac{64\pi}{3}$. **1673.** $\frac{(\pi+2)\pi}{4}$. **1674.** $\frac{4}{3}\pi a^2b$. **1675.** $\frac{32\pi a^3}{105}$. **1676.** $19{,}2\pi$. **1677.** $\frac{8\pi a^3}{3}$. **1678.** $V=\frac{128\pi}{3}$. **1679.** $5\pi^2a^3$. **1680.** 72π. **1681.** $\frac{112}{27}$. **1683.** $6a$. **1684.** $\frac{670}{27}$. **1685.** $8a$.

1686. $t_1=0,\ t_2=\sqrt[4]{8};\quad s=\int\limits_0^{\sqrt[4]{8}}\sqrt{t^2+1}\cdot t^3dt=\frac{13}{3}$. **1687.** $\sqrt{6}+\ln(\sqrt{2}+\sqrt{3})$.

1688. $2a\operatorname{sh}1\approx 2{,}35a$. **1689.** $S=\int\limits_{3/4}^{12/5}\frac{\sqrt{1+x^2}}{x}dx$ da $1+x^2=t^2$ belgilash kiritilsa, $S=\int\limits_{5/4}^{13/5}\frac{t^2dt}{t^2-1}=1{,}35+\ln 2\approx 2{,}043$; **1690.** Koordinata o‘qlari bilan kesishgan qo‘shni nuqtalarda $x_1=0$ va $x_2=\frac{\pi}{3}$; $S=\int\limits_0^{\pi/3}\frac{dx}{\cos x}=\int\limits_0^{\pi/3}\frac{d\sin x}{1-\sin^2x}=$ $=\ln(2+\sqrt{3})\approx 1{,}31$. **1691.** 1) $4\sqrt{3}$; 2) $0{,}5\ln(2\operatorname{ch}2)\approx 1{,}009$. **1692.** 1) $8a$; 2) $\pi a\sqrt{1+4\pi^2}+\frac{a}{2}\ln(2\pi+\sqrt{1+4\pi^2})$. **1693.** $\frac{3\pi a}{2}$. **1694.** $\frac{28}{3}$. **1695.** $\ln 3$.

1696. $2\ln 3-1$. **1697.** $P\left(\sqrt{2}+\ln(1+\sqrt{2})\right)\approx 2{,}29P$. **1698.** $4\sqrt{3}$. **1700.** $\frac{14\pi}{3}$.

1701. $\pi a^2(\operatorname{sh}2+2)$. **1702.** $2\pi(1+\frac{4\pi}{3\sqrt{3}})$. **1703.** $2\pi\left(\sqrt{2}+\ln(1+\sqrt{2})\right)$; **1704.** $\frac{64\pi a^2}{3}$.

1705. 3π. **1706.** $4\pi^2ab$. **1707.** $\frac{34\sqrt{17}-2}{2}\pi$. **1708.** $\frac{62\pi}{3}$. **1709.** $2{,}4\pi a^2$. **1710.** $29{,}6\pi$.

1711. $1{,}44\cdot 10^6$ N; pastki yarmiga bosim kuchi $1{,}08\cdot 10^6$N. **1712.** $\frac{ah^2}{6}$.

1713. $\frac{2}{3}R^3$. **1714.** $2{,}4\cdot 10^6$ N. **1715.** $I_x=\frac{ab^3}{3},\ I_y=\frac{a^3b}{3}$. **1716.** $I_x=\frac{ab^3}{12},\ I_y=\frac{a^3b}{12}$.

1717. 6,4. **1718.** $M_x=M_y=\frac{a^3}{6};\ x_c=y_c=\frac{a}{3}$. **1719.** $M_x=\int\limits_0^a\frac{y}{2}ydx=0{,}1ab^2$, $M_y=\int\limits_0^a xydx=\frac{1}{4}a^2b,\ S=\int\limits_0^a ydx=\frac{ab}{3},\ x_c=\frac{3}{4a},\ y_c=0{,}3b$. **1720.** $x_c=0,\ y_c=$ $=\frac{2\int\limits_0^a\frac{y}{2}ydx}{0{,}5\pi a^2}=\frac{4}{3\pi}a\approx\frac{4}{9}a$. **1721.** 1) $1{,}12\cdot 10^4\pi$ J; 2) $2{,}5\cdot 10^3\pi R^4$ J.

1722. $\int\limits_R^{R+h}\frac{mgR^2}{x^2}dx=\frac{mgRh}{R+h}$. **1723.** $\frac{1000\pi R^2H^2}{6}\approx 210$J. **1724.** 12410 J. **1725.** $0{,}24\pi$ J.

1726. $t=\int\limits_0^H\frac{Sdx}{0{,}6S\sqrt{2gx}}=100$sekund. **1727.** $t=\frac{R^2}{0{,}6r^2H^2\sqrt{2g}}\int\limits_h^{H+h}x\sqrt{x}dx\approx 42$sekund, bunda

$h=2$. **1728.** $\frac{ah^2}{3}$. **1729.** $17\frac{1}{5}$. **1730.** $\frac{h}{\sqrt{2}}$. **1731.** $2{,}4\cdot10^4$ N. **1732.** $I_x=\int_0^a y^2xdy=\int_0^{\pi/2} a^4\sin^2t\cos^2tdt=\frac{\pi a^4}{16}$. **1733.** $x_c=0, y_c=\frac{\int_0^2 y^2dx}{2\int_0^2 ydx}=\frac{8}{5}$. **1734.** $\frac{\pi R^2\cdot1000}{H^2}\int_0^H(H-x)^2xdx\approx300\pi J$. **1735.** $\frac{P_0V_0}{K-1}\left[\left(\frac{V_0}{V_1}\right)^{K-1}-1\right]\approx15980$ J. **1736.** $t=\frac{14\pi R^2}{15\cdot S\cdot0{,}8}\sqrt{\frac{R}{2g}}\approx419$ sekund. **1737.** 1) 1; 2) va 3) uzoqlashuvchi; 4) $\int_1^\infty\frac{dx}{x^n}=\frac{1}{n-1}$, $n>1$; $n<1$ da uzoqlashuvchi. **1738.** 1) 1; 2) 1/2; 3) $\pi/4$; 4) 1; 5) ln2; 6) 16. **1739.** 1) $\pi/6$; 2) $\frac{\pi}{4}+\frac{\ln2}{2}$; 3) $\frac{\pi-2}{8}$. **1740.** 1) $6\sqrt[3]{2}$; 2) uzoqlashuvchi. **1741.** 1) yaqinlashuvchi, chunki $\frac{1}{\sqrt{1+x^3}}<\frac{1}{x^{3/2}}$; 2) uzoqlashuvchi, chunki $\frac{1}{\sqrt[3]{x^3-1}}>\frac{1}{x}$; 3) yaqinlashuvchi, chunki $\frac{e^{-x}}{x}\le e^{-x}$; 4) absolut yaqinlashuvchi, chunki $\frac{|\sin x|}{x^2}\le\frac{1}{x^2}$; 5) uzoqlashuvchi, chunki $\frac{x}{\sqrt{x^4+1}}>\frac{x}{\sqrt{x^4+x^4}}$, $x>1$ da; 6) yaqinlashuvchi, chunki $e^{-x^2}\le e^{-x}, x\ge1$ da. **1742.** 1) $n<1$ da yaqinlashuvchi, $n>1$ da uzoqlashuvchi; 2) $n<1$ da yaqinlashuvchi, $n\ge1$ da uzoqlashuvchi. **1743.** π. **1744.** 2. **1745.** $3\pi a^2$. **1746.** $2\pi^2a^3$. **1747.** $\pi\left(\sqrt{2}+\ln(1+\sqrt{2})\right)$. **1748.** $\frac{4\pi}{3}$. **1750.** 1) $\frac{1}{2}$; 2) $\frac{1}{3}$; 3) 1; 4) uzoqlashuvchi. **1751.** 1) $\ln(1+\sqrt{2})$; 2) 2; 3) $1-\frac{\pi}{4}$. **1752.** $1/2$. **1753.** 16π. **1754.** 2π; **1756.** 1) $2/\pi$; 2) $\frac{3\ln2}{\pi}$; 3) $\frac{1}{e-1}$; 4) $\frac{a^2+ab+b^2}{3}$; 5) $\frac{\pi}{4}$. **1758.** 1) $E(h)=0$; 2) $|E(h)|\le\frac{4}{15}<0{,}3$. **1761.** $\ln2\approx0{,}6932$; $|E(h)|\le\frac{2\cdot10^{-4}}{15}<0{,}001$. **1762.** $8{,}16\pi$. **1763.** Taqriban $1{,}22\pi$ ga teng.

XI BOB | KO'P ARGUMENTLI FUNKSIYALAR

1- §. Ko'p argumentli funksiyalar va ularning geometrik tasvirlari

1°. Biror oraliqda olingan x va y o'zgaruvchilarning bir juft qiymatlariga z o'zgaruvchining aniq bir qiymati mos keltirilgan bo'lsa, z o'zgaruvchiga x va y o'zgaruvchilarning *ikki argumentli funksiyasi* deyiladi va $z=f(x, y)$ deb yoziladi. $z=f(x, y)$ da x va y lar Xoy tekisligida qandaydir nuqtani aniqlaydi, $z=f(x, y)$ esa sirtdagi $M(x; y; z)$ nuqtaning applikatasini aniqlaydi.

$z=f(x, y)$ funksiyaga aniq qiymat beradigan x va y larning qiymatlari to'plamiga uning aniqlanish (mavjudlik) sohasi deyiladi.

2°. $z=f(x, y)$ funksiyaning sath chizig'i deb Xoy tekisligida $f(x, y)=c$ chizig'iga aytiladi. $u=f(x, y, z)$ funksiyaning sath sirti deb $f(x, y)=c$ sirtga aytiladi.

Quyidagi funksiyalarning aniqlanish sohalarini toping va grafiklarda ko'rsating:

1766. $2 \le x \le 6, \quad 1 \le y \le 3.$

1767. $\frac{x^2}{9}+\frac{y^2}{4}<1.$

1768. $4 \le x^2+y^2 \le 9.$

1769. $0<y<x.$

1770. $z=\ln\frac{x^2+y^2}{x^2-y^2}.$

1771. $z=\frac{1}{\sqrt{2z^2-6x^2-3y^2-6}}.$

1772. $z^2+y^2+x^2=9.$

1773. $z=\ln(x+3y).$

1774. $u=\arcsin(x+y).$

1775. $u=\frac{1}{\sqrt{4-x^2-y^2}}.$

1776. $u=\arcsin\frac{x}{y^2}.$

Yuksaklik chizig'i va yuksaklik sirtini toping:

1777. $z=\frac{1}{4x^2+9y^2}.$

1778. $z=2x+y.$

1779. $z=\frac{\sqrt{x}}{y}.$

1780. $z=\sqrt{1-\frac{x^2}{4}-y^2}.$

1781. $z=x^2-y.$

1782. $z=x^2-y^2.$

1783. $u=(x^2+y^2+z^2)^2$. **1784.** $u=x^2-y^2-z^2$.

1785. $u=\frac{x^2}{4}+\frac{y^2}{9}+\frac{z^2}{16}$. **1786.** $u=x^2+z^2-y^2$.

Quyidagi limitlarni hisoblang:

1787. $\lim\limits_{\substack{x\to 0\\ y\to 0}}\frac{3-\sqrt{xy+9}}{xy}$. **1788.** $\lim\limits_{\substack{x\to 0\\ y\to 0}}\frac{\sin xy}{y}$.

1789. $\lim\limits_{\substack{x\to 0\\ y\to 5}}\frac{\sqrt{x^2+(y-5)^2}-1}{x^2+(y-5)^2}$. **1790.** $\lim\limits_{\substack{x\to 0\\ y\to 0}}\frac{x^2y}{x^2+y^2}$.

2- §. Birinchi tartibli xususiy hosilalar

Birinchi tartibli xususiy hosilalarni toping:

1791. $z=x^2+5xy^2-y^3$. **1792.** $u=\frac{x}{y}+\frac{y}{z}-\frac{z}{x}$.

1793. $a) u=\sqrt[x]{e^y}$; $b) u=\frac{\operatorname{tg}x}{y}$.

1794. $z=\ln(x^2-y^2)$, $\frac{\partial z}{\partial x}$ va $\frac{\partial z}{\partial y}$ larning $M(2;\ -1)$ nuqtadagi qiymatlarini toping.

1795. $z=\frac{x}{y}-e^x\cdot\operatorname{arctg}y$. **1796.** $z=\ln(x+\sqrt{x^2+y^2})$.

1797. $z=x^4\cos^2 y$. **1798.** $z=e^{(x^2+y^2)}\cdot xy$.

1799. $z=\operatorname{arctg}\frac{y}{1+x^2}$.

1800. $u=\sin^2(x+y)-\sin^2 x-\sin^2 y$.

1801. $u=\sqrt{x}\cdot\sin\frac{y}{x}$ bo'lsa, $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{z}{2}$ ligini ko'rsating.

1802. $u=\arcsin\frac{x-y}{x+y}$ bo'lsa, $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0$ bo'lishini ko'rsating.

1803. $u=\frac{xy}{x+y}$ bo'lsa, $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=u$ bo'lishini ko'rsating.

1804. $u=\frac{y^2}{\sqrt{xy}}$ bo‘lsa, $x^2\frac{\partial^2 u}{\partial x^2}-y\frac{\partial^2 u}{\partial y^2}=0$ bo‘lishini ko‘rsating.

1805. $u=\sin^2(3x+2y-z)$ bo‘lsa, $\frac{\partial u}{\partial x}$ ning $M(1;\ -1;\ 1)$, $\frac{\partial u}{\partial y}$ ning $N(1;\ 1;\ 4)$, $\frac{\partial u}{\partial z}$ ning $R(-\frac{1}{2};\ 0;\ -1)$ nuqtalardagi qiymatlarini toping.

1806. $u=x\ln\frac{y}{x}$ bo‘lsa, $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=u$ bo‘lishini ko‘rsating.

1807. $u=x+\frac{x-y}{y-z}$ bo‘lsa, $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=1$ bo‘lishini ko‘rsating.

1808. $u=\sqrt{x^2+y^2+z^2}$ bo‘lsa, $\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial u}{\partial z}\right)^2=1$ bo‘lishini ko‘rsating.

1809. $u=e^{\frac{x}{y^2}}$ bo‘lsa, $2x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0$ bo‘lishini ko‘rsating.

1810. $u=\sqrt{x}\cdot\sin\frac{y}{x}$ bo‘lsa, $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{u}{2}$ bo‘lishini ko‘rsating.

3- §. Funksiyaning to‘la orttirmasi va to‘la differensiali

$z=f(x,y)$ funksiya uzluksiz $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$ xususiy hosilalarga ega bo‘lsa, uning to‘la orttirmasi va to‘la differensiali $\Delta z=\frac{\partial z}{\partial x}\Delta x+\frac{\partial z}{\partial y}\Delta y+\alpha_1\Delta x+\alpha_2\Delta y$ va $dz=\frac{\partial z}{\partial x}\Delta x+\frac{\partial z}{\partial y}\Delta y$ bo‘lib, cheksiz kichik Δx, Δy lar uchun $\Delta z\approx dz$ bo‘ladi, shuningdek,

$$f(x+\Delta x,\ y+\Delta y)\approx f(x,y)+\frac{\partial f(x,y)}{\partial x}\Delta x+\frac{\partial f(x,y)}{\partial y}\Delta y$$ bo‘ladi.

Quyidagi funksiyalarning to'la differensiali dz ni toping:

1811. $z=\frac{xy}{x-y}$. **1812.** $z=\ln(x^3+y^3)$. **1813.** $z=\ln\operatorname{tg}\frac{y}{x}$.

1814. $z=\frac{xy}{t}$. **1815.** $z=\sin^2 x\cdot\cos^2 y$. **1816.** $z=e^{xyt}$.

1817. $z=\operatorname{arctg}\frac{x+y}{x-y}$. **1818.** $z=\frac{y}{x}$ funksiyaning $x=2, y=1$, $dx=0{,}1$, $dy=0{,}1$ qiymatlardagi to'la differensialini toping.

1819. $z=\operatorname{arctg}\frac{y}{x}$ funksiyaning $x=1, y=3$, $dx=0{,}01$, $dy=-0{,}05$ qiymatlardagi to'la differensialini toping.

4- §. Funksiyaning to'la differensiali yordamida taqribiy hisoblashlar

Funksiyaning to'la differensiali yordamida taqribiy hisoblash formulasidan foydalanib, quyidagilarni hisoblang:

1820. $z=x^y$ funksiyadan foydalanib, $x=1$, $y=4$ qiymatlarda $1{,}02^{4{,}05}$ ni hisoblang.

1821. $z=\sqrt[3]{x^2+y^2}$ funksiyadan foydalanib, $x=1$, $y=0$ qiymatlarda $\sqrt[3]{1{,}02^2+0{,}05^2}$ ni hisoblang.

1822. $z=\sqrt{5e^x+y^2}$ funksiyadan foydalanib, $x=0$, $y=2$ qiymatlarda $\sqrt{5e^{0{,}02}+2{,}03^2}$ ni hisoblang.

1823. Funksiyaning to'la differensialidan foydalanib, quyidagilarni hisoblang:

a) $\operatorname{arctg}\left(\frac{1{,}97}{1{,}02}-1\right)$ b) $\sqrt{1{,}04^{1{,}99}+\ln 1{,}02}$.

1824. Radiusi R ga, balandligi H ga teng bo'lgan silindr deformatsiyalanganda radiusi R 2 dan 2,05 dm ga, balandligi H esa 10 dm dan 9,8 dm ga o'zgarsa, uning hajmi qanday o'zgaradi?

1825. To'g'ri to'rtburchak tomonlari $a=10$ sm, $b=24$ sm. Agar a tomoni 4 mm ga uzaytirilib, b tomoni 1 mm ga qisqartirilsa, uning diagonali qanday o'zgaradi? Taqribiy qiymatni topib, aniq qiymat bilan solishtiring.

1826. Konusning balandligi $H=60$ sm, radiusi $R=30$ sm ga teng. Agar konus deformatsiyalanib, radiusi 0,1 sm ga oshsa, balandligi esa 0,5 sm ga kamaysa, konusning hajmi qanday o'zgaradi?

5- §. Murakkab funksiya hosilasi

Agar $z=f(u,v)$ bo'lib, o'z navbatida, $u=\varphi(x, y)$, $v=\psi(x, y)$ bo'lsa va $f(u,v)$, $\varphi(x, y)$, $\psi(x, y)$ lar differensiallanuvchi funksiyalar bo'lsa, murakkab funksiyaning hosilasi quyidagicha topiladi:

$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x},$$

$$\frac{\partial z}{\partial y}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}.$$

Agar $z=f(u,v)$ bo'lib, $u=\varphi(t)$, $v=\psi(t)$ bo'lsa,

$$\frac{dz}{dt}=\frac{\partial z}{\partial u}\cdot\frac{du}{dt}+\frac{\partial z}{\partial v}\cdot\frac{dv}{dt}$$ bo'ladi.

Murakkab funksiyaning hosilalarini toping:

1827. $z=u^2e^v$, $u=\sin x$, $v=\cos x$.

1828. $z=u^v$, $u=\ln(x-y)$, $v=e^{\frac{x}{y}}$.

1829. $z=u^2\ln v$, $u=\frac{x}{y}$, $v=3x-2y$.

1830. $z=\mathrm{arctg}\frac{u}{v}$, $u=x\sin y$, $v=x\cos y$.

1831. $z=\ln(u^2+v^2)$, $u=xy$, $v=\frac{x}{y}$.

1832. $z=\arcsin(u+v)$, $u=\sin x\cos y$, $v=\cos x\sin y$.

1833. $z=e^{u-2v}$, $u=\sin x$, $v=x^3$; $\frac{dz}{dx}=?$

1834. $z=u^v$, $u=\sin x$, $v=\cos x$; $\frac{dz}{dx}=?$

1835. $z=u\cdot v\cdot w$, $u=x^2+1$, $v=\ln x$, $w=\mathrm{tg}x$; $\frac{dz}{dx}=?$

1836. $z=\frac{u}{v}$, $u=e^t$, $v=1-e^{2t}$, $\frac{dz}{dt}=?$

1837. $z = xy + xf(u)$, $u=\frac{y}{x}$ bo‘lsa, $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=z+xy$ ekanligini ko‘rsating.

1838. $z=e^{u^2+v^2}$, $u = a\cos t$, $v = a\sin t$ bo‘lsa, $\frac{dz}{dt}=?$

1839. $z=\frac{1}{2}\ln\frac{u}{v}$, $u = \mathrm{tg}^2 x$, $v = \mathrm{ctg}^2 x$ bo‘lsa, $\frac{dz}{dx}=?$

6- §. Oshkormas funksiyaning hosilasi

$F(x,y) = 0$ oshkormas funksiyaning hosilasi $\frac{dy}{dx}=-\frac{\frac{\partial F(x,y)}{\partial x}}{\frac{\partial F(x,y)}{\partial y}}$ formulaga asosan topiladi, bu yerda $\frac{\partial F(x,y)}{\partial y}\neq 0$.

$F(x, y, z) = 0$ oshkormas funksiyaning xususiy hosilalari:

$\frac{\partial z}{\partial x}=-\frac{\frac{\partial F(x,y)}{\partial x}}{\frac{\partial F(x,y)}{\partial z}}$, $\frac{\partial z}{\partial y}=-\frac{\frac{\partial F(x,y)}{\partial y}}{\frac{\partial F(x,y)}{\partial z}}$ bo‘lib, $\frac{\partial F(x,y)}{\partial z}\neq 0$ bo‘ladi.

1840. $F(x, y, z) = x^2 + y^2 + z^2 + 2xz + 1$, $\frac{\partial z}{\partial x}=?$ $\frac{\partial z}{\partial y}=?$

1841. $F(x, y, z) = 4\sin(3x + 2y + 5z) - 3x + 2y + 5z$ bo‘lsa, $\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}+1=0$ bo‘lishini ko‘rsating.

1842. $x + y + z = e^z$, $\frac{\partial z}{\partial x}=?$ $\frac{\partial z}{\partial y}=?$

1843. $x^3 + y^3 + z^3 - 3xyz = 0$, $\frac{\partial z}{\partial x}=?$ $\frac{\partial z}{\partial y}=?$

1844. $z = x^2 + xy + y^2 + \frac{1}{x}+\frac{1}{y}$, $\frac{\partial z}{\partial x}=?$ $\frac{\partial z}{\partial y}=?$

1845. $z = x + \mathrm{arctg}\,\frac{y}{z-x}$, $\frac{\partial z}{\partial x}=?$ $\frac{\partial z}{\partial y}=?$

1846. $xe^y + ye^x + ze^x = a$, $\frac{\partial z}{\partial x} = ?$ $\frac{\partial z}{\partial y} = ?$

7- §. Yuqori tartibli xususiy hosilalar va differensiallar

$z=f(x, y)$ funksiyaning birinchi tartibli hosilalaridan olingan hosilalarga ikkinchi tartibli xususiy hosilalar deyiladi, ya'ni

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)=\frac{\partial^2 z}{\partial x^2}, \quad f''_{xx}(x,y)=(f'_x(x,y))'_x;$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)=\frac{\partial^2 z}{\partial y^2}, \quad f''_{yy}(x,y)=(f'_y(x,y))'_y;$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)=\frac{\partial^2 z}{\partial y\partial x}, \quad f''_{yx}(x,y)=(f'_y(x,y))'_x;$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)=\frac{\partial^2 z}{\partial x\partial y}, \quad f''_{xy}(x,y)=(f'_x(x,y))'_y.$$

Agar $\frac{\partial^2 z}{\partial x\partial y}$ va $\frac{\partial^2 z}{\partial y\partial x}$ lar qaralayotgan nuqtada uzluksiz bo'lsalar,

$\frac{\partial^2 z}{\partial x\partial y}=\frac{\partial^2 z}{\partial y\partial x}$ bo'ladi.

$z=f(x, y)$ funksiyaning ikkinchi tartibli differensiali $d^2z=d(dz)$ quyidagicha aniqlanadi:

$$d^2z=\frac{\partial^2 z}{\partial x^2}dx^2+2\frac{\partial^2 z}{\partial x\partial y}dxdy+\frac{\partial^2 z}{\partial y^2}dy^2; \qquad d^n z=\left(\frac{\partial}{\partial x}dx+\frac{\partial}{\partial y}dy\right)^n z.$$

1847. $z=x^4+x^3y+y^4$ funksiyaning 3- tartibli xususiy hosilalarini toping.

1848. $z=\ln(x-2y)$ funksiya uchun $\frac{\partial^2 z}{\partial x\partial y}=\frac{\partial^2 z}{\partial y\partial x}$ ekanligini ko'rsating.

1849. $z=\operatorname{arctg}\frac{x}{y}$ funksiyaning 2- tartibli xususiy hosilalarini toping.

1850. $z=xy+yz+zx$ funksiyaning 2- tartibli xususiy hosilalarini toping.

1851. $z=y\cdot\ln x$ funksiyaning 2- tartibli xususiy hosilalarini toping.

1852. $z=\ln\operatorname{tg}(x+y)$ funksiyaning $\frac{\partial^2 z}{\partial x^2}$ hosilasini toping.

1853. $z=\frac{x^2}{2y-3}$ funksiyaning 2- tartibli xususiy hosilalarini toping.

1854. $u=\sin xy$ funksiyaning u'''_{xyy} hosilasini toping.

1855. $z=e^{\frac{x}{y}}$ bo'lsa, $\frac{\partial u}{\partial x}-\frac{\partial u}{\partial y}+y\frac{\partial^2 u}{\partial x\partial y}=0$ ekanligini ko'rsating.

1856. $z=\sin x\cdot\sin y$ bo'lsa, d^2z ni toping.

1857. $z=x^3+y^3-x^2y^2-2x+y+5$ bo'lsa, d^2z ni toping.

1858. $z=x\ln\frac{y}{x}$ bo'lsa, d^2z ni toping.

1859. $z=\frac{1}{2}\ln(x^2+y^2)$ bo'lsa, d^2z ni toping.

8- §. Yo'nalish bo'yicha hosila va gradiyent

$u=u(x, y, z)$ funksiyaning $M(x, y, z)$ nuqtadagi $\overline{S}$ vektor yo'nalishi bo'yicha olingan hosilasi

$$\frac{\partial u}{\partial s}=\frac{\partial u}{\partial x}\cos\alpha+\frac{\partial u}{\partial y}\cos\beta+\frac{\partial u}{\partial z}\cos\gamma$$

formulaga asosan topiladi. Bu yerda $\cos\alpha$, $\cos\beta$, $\cos\gamma$ larga $\overline{S}$ vektorning *yo'naltiruvchi kosinuslari* deyiladi. Gradiyent

$$\text{grad}\,\bar{u}=\boldsymbol{i}\frac{\partial u}{\partial x}+\boldsymbol{j}\frac{\partial u}{\partial y}+\boldsymbol{k}\frac{\partial u}{\partial z}$$

formulaga ko'ra aniqlanadi. **i, j, k** larga *ortlar* deyiladi.

1860. $z=x^2y$ funksiyaning $M(1;\ 1)$ dagi gradiyentini toping.

1861. $u=x^2-y^2$ funksiyaning $M(1;\ 1)$ nuqtada Ox o'qi bilan 60° li burchak hosil qiluvchi $\overline{S}$ vektor yo'nalishi bo'yicha hosilasini toping.

1862. $u=xy^2z^3$ funksiyaning $M(3;\ 2;\ 1)$, $N(5;\ 4;\ 2)$ dagi $\overline{MN}$ vektor yo'nalishi bo'yicha hosilasini toping.

1863. $u=\ln(x^2+y^2+z^2)$ funksiyaning $M(1;2;\ 1)$ nuqtadan $\overline{S}=2\boldsymbol{i}+4\boldsymbol{j}+4\boldsymbol{k}$ vektor yo'nalishi bo'yicha hosilasini toping.

1864. $u = x^2 + y^2 + z^2$ funksiyaning $M(1;1;1)$ nuqtadan $\bar{S} = 2\boldsymbol{i} + \boldsymbol{j} + 3\boldsymbol{k}$ vektor yo'nalishi bo'yicha hosilasini toping.

1865. $u = x^3 + y^3 + z^3$ funksiyaning $M(1;1;1)$ nuqtadagi gradiyentini toping.

1866. $u = \text{tg}x - x - 3\sin y - \sin 3y + z + \text{ctg}z$ funksiyaning $M\left(\frac{\pi}{4}; \frac{\pi}{3}; \frac{\pi}{2}\right)$ nuqtadagi gradiyenti va uning yo'nalishini aniqlang.

1867. $u = xyz$ funksiyaning $M(2;\ 1;\ 1)$ nuqtadagi gradiyenti va uning yo'nalishini aniqlang.

1868. $u = x^2 + y^2 - 2z$ funksiyaning Ox o'qining $u = 4$ sirt bilan kesishgan nuqtasidagi gradiyentini toping.

1869. $M = \sqrt{x^2 + y^2 + z^2}$ funksiyaning $M(2;1;4)$ nuqtadagi gradiyentini va uning uzunligini toping.

1870. $u = x^2 + y^2 + z^2$ funksiyaning $M(1;\ 1;\ 1)$ nuqtadagi gradiyenti yo'nalishi bo'yicha hosilasini toping.

9- §. Egri chiziqlar oilasining o'ramasi

$F(x, y, \alpha) = 0$ egri chiziqlar oilasining o'ramasi (agar u mavjud bo'lsa)

$$\begin{cases} f(x, y, a) = 0, \\ f'_\alpha(x, y, a) = 0 \end{cases}$$

dan α parametrni yo'qotish yo'li bilan topiladi.

Egri chiziqlar oilasining o'ramasini toping:

1871. $y = ax + a^2$. **1872.** $y = ax^2 + \frac{1}{a}$.

1873. $(x - a)^2 + y^2 = R^2$. **1874.** $y = \frac{1}{4a}(x - a)^2$.

1875. $(y - 1) = (x - a)^2$.

1876. Markazi Ox o'qida joylashgan bir xil radiusli aylanalar oilasining o'ramasini toping.

1877. $y = ax + \frac{1}{a}$ to'g'ri chiziqlar oilasining o'ramasini toping.

1878. $y = 1 + (x - a)^3$ kubik parabolalar oilasining o‘ramasini toping.

1879. $x\cos\alpha + y\sin\alpha - p = 0$ ($p = \text{const}$, $p > 0$) to‘g‘ri chiziqlar oilasining o‘ramasini toping.

10- §. Sirtga o‘tkazilgan urinma tekislik va normal tenglamalari

Sirt tenglamasi $F(x,y,z) = 0$, unda yotgan $M(x;\ y;\ z)$ nuqta berilgan. $M(x;\ y;\ z)$ nuqtaga o‘tkazilgan *urinma tekislik tenglamasi*:

$$\frac{\partial F}{\partial x}(X-x)+\frac{\partial F}{\partial y}(Y-y)+\frac{\partial F}{\partial z}(Z-z)=0;$$

sirtga normal tenglamasi esa

$$\frac{X-x}{\frac{\partial F}{\partial x}}=\frac{Y-y}{\frac{\partial F}{\partial y}}=\frac{Z-z}{\frac{\partial F}{\partial y}}$$

ko‘rinishga ega bo‘ladi.

Bu yerda X, Y, Z lar urinma tekislik va normal uchun o‘zgaruvchi koordinatalar. Agar biror nuqtada $\frac{\partial F}{\partial x}=0$, $\frac{\partial F}{\partial y}=0$, $\frac{\partial F}{\partial z}=0$ bo‘lsa, u nuqta *maxsus nuqta* deyiladi. Bu nuqtada urinma tekislik ham, sirtga normal ham mavjud emas.

Sirtga o‘tkazilgan urinma tekislik va normal tenglamalarini tuzing:

1880. $F(x,y,z) = x^2 + 2y^2 - z$ ning $M(1;\ 1;\ 3)$ nuqtasida.

1881. $F(x,y,z) = xy - z^2$ ning $M(x_0;\ y_0;\ z_0)$ nuqtasida.

1882. $\frac{x^2}{16}+\frac{y^2}{9}-\frac{z^2}{4}=1$ ning $M(4;\ 3;\ 2)$ nuqtasida.

1883. $F(x,\ y,\ z) = x^2 + y^2 + z^2 - 14$ sirtga $M(1;\ 2;\ 3)$ nuqtadan o‘tkazilgan urinma tekislik va sirtga o‘tkazilgan normalning tenglamasini tuzing.

1884. $\frac{x^2}{a^2}-\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ giperboloidning $M(x_1;\ y_1;\ z_1)$ nuqtasidan o‘tkazilgan urinma tekislik tenglamasini yozing.

1885. $x^2 - 4y^2 + 2z^2 = 6$ sirtning $M(2;\ 2;\ 3)$ nuqtasidan o‘tkazilgan normal tenglamasini yozing.

1886. $z = 2x^2 + 4y^2$ sirtning $M(2;\ 1;\ 12)$ nuqtasidagi urinma tekislik tenglamasini yozing.

1887. $x^2 + y^2 + z^2 - 2rz = 0$ sirtning $M(r\cos\alpha;\ r\sin\alpha;\ r)$ nuqtasidagi urinma tekislik va normali tenglamalarini yozing.

11- §. Ikki o‘zgaruvchili funksiyalar ekstremumi

Agar biror D sohada differensiallanuvchi $z = f(x,\ y)$ funksiya $M_0(x_0;\ y_0)$ nuqtada ekstremumga erishsa, $\frac{\partial f(x_0, y_0)}{\partial x} = 0$, $\frac{\partial f(x_0, y_0)}{\partial y} = 0$ bo‘ladi (ekstremumga erishishning zaruriy sharti). Xususiy hosilalarni nolga aylantiruvchi nuqtalar *kritik nuqtalar* deyiladi, lekin barcha kritik nuqtalarda funksiya ekstremumga erishavermaydi.

Agar $z = f(x,\ y)$ funksiya uchun $M_0(x_0;\ y_0)$ kritik nuqtada

$$A = \frac{\partial^2 f(x_0, y_0)}{\partial x^2},\quad B = \frac{\partial^2 f(x_0, y_0)}{\partial x \partial y},\quad C = \frac{\partial^2 f(x_0, y_0)}{\partial y^2},\quad \Delta = AC - B^2$$

bo‘lib:

a) $\Delta > 0$, $A < 0$ bo‘lsa, funksiya $M_0(x_0;\ y_0)$ nuqtada max ga erishadi;

b) $\Delta > 0$, $A > 0$ bo‘lsa, funksiya $M_0(x_0;\ y_0)$ nuqtada min ga erishadi;

d) $\Delta < 0$ bo‘lsa, funksiya ekstremumga erishmaydi;

e) $\Delta = 0$ bo‘lsa, funksiya ekstremumga ega bo‘lishi ham, bo‘lmasligi ham mumkin (shubhali hol). Bu hol maxsus (masalan, yuqoriroq tartibli Teylor formulasi yordami bilan yoki boshqa biron usul bilan) tekshirishni talab qiladi.

Quyidagi funksiyalar ekstremumlarini toping:

1888. $z = x^2 - xy + y^2 + 9x - 6y + 20.$

1889. $z = x^3 + 8y^3 - 6xy + 1.$

1890. $z = 2xy - 4x - 2y.$

1891. $z = x^2 + xy + y^2 - 6x - 9y.$

1892. $z = x\sqrt{y} - x^2 - y + 6x + 3.$

1893. $z = x^3 + xy^2 + 6xy$.

1894. $z = x^2 + xy + y^2 - 3x - 6y$.

1895. $z = x^3y^2(a - x - y)$.

1896. $z=3x^2-2x\sqrt{y}+y-8x+8$.

12- §. Ikki argumentli funksiyaning eng katta va eng kichik qiymatlari

Biror yopiq D sohada differensiallanuvchi $z=f(x,y)$ funksiya shu sohada o'zining eng katta va eng kichik qiymatlariga ega bo'ladi. Funksiyaning eng katta va eng kichik qiymatlarini topish uchun:

1) funksiyaning mavjud kritik nuqtalardagi qiymatlari topiladi;

2) funksiyaning D sohaning chegaralaridagi qiymatlari topiladi;

3) shu topilgan qiymatlarning eng kattasi (eng kichigi) funksiyaning D sohadagi eng katta (eng kichik) qiymati bo'ladi.

Funksiyalarning eng katta va eng kichik qiymatlarini toping:

1897. $z = x^2 - y^2 + 2a^2$ funksiyaning $x^2 + y^2 \le a^2$ doirada.

1898. $z = 2x^3 + 4x^2 + y^2 - 2xy$ funksiyaning $y = x^2$ va $y = 4$ sohada.

1899. $z = x^3 + y^3 - 9xy + 27$, $0 \le x \le 4$, $0 \le y \le 4$ kvadratda.

1900. $z = 3xy$ funksiyaning $x^2 + y^2 \le 2$ doirada.

XI bob javoblari

1787. $-\frac{1}{6}$. **1788.** 0. **1789.** $\frac{1}{2}$. **1790.** 0. **1820.** 1,08. **1821.** 1,013. **1822.** 3,037. **1823.** a) 0,82; b) 1,05. **1824.** $1{,}2\pi$ dm^2. **1825.** $d = 0{,}62$ sm. **1826.** $-30\pi sm^3$. **1860.** $\text{grad}\, z = 2\mathbf{i} + \mathbf{j}$. **1861.** $\frac{\partial u}{\partial s} = -0{,}7$. **1862.** $\frac{\partial u}{\partial s} = 22\frac{2}{3}$. **1863.** $\left(\frac{\partial u}{\partial s}\right)_M = \frac{7}{9}$. **1864.** $\frac{\partial u}{\partial s} = \frac{12}{\sqrt{14}}$. **1866.** $\text{grad}\, \bar{u} = \boldsymbol{i} + \frac{3}{8}\boldsymbol{j}$, $\cos\alpha = \frac{8}{\sqrt{73}}$, $\cos\beta = \frac{3}{\sqrt{73}}$. **1868.** $\text{grad}\ \bar{u} = 4\underline{\boldsymbol{i}} - 2\underline{\boldsymbol{k}}$. **1870.** $\frac{\partial u}{\partial s} = |grad\bar{u}|$,

$\frac{\partial u}{\partial s} = 2\sqrt{3}$. **1871.** $y = -\frac{x^2}{4}$. **1872.** $y = \pm 2x$. **1873.** $y = \pm R$. **1874.** $y = 0$; $y = -x$. **1875.** $y = 1$. **1876.** $y = \pm R$. **1877.** $y^2 = 4x$. **1878.** $y = 1$. **1879.** $x^2 + y^2 = p^2$. **1880.** $2x + 4y - z = 3$. **1881.** $xx_0 + yy_0 = 2zz_0$. **1882.** $\frac{xx_0}{16} + \frac{yy_0}{9} - \frac{zz_0}{4} = 1$.

1883. $2(x-1) + 4(y-2) + 6(z-3) = 0, \quad x + 2y + 3z - 14 = 0$.

1884. $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} - \frac{zz_1}{c^2} = 1$. **1885.** $y + 4x - 10 = 0, \ 3x - z - 3 = 0$.

1886. $8x + 8y - z = 12$. **1887.** $x \cos x + y \sin\alpha - R = 0$. **1888.** $M(-4;\ 1)$ nuqtada $z_{min} = -1$. **1889.** $M(1; -\frac{1}{2})$ nuqtada $z_{min} = 10$. **1890.** Ekstremum yo‘q. **1891.** $M_0(1;\ 4)$ nuqtada $z_{min} = -21$. **1892.** $M_0(4;\ 4)$ nuqtada $z_{max} = 15$. **1893.** $M_0(\sqrt{3}; -3)$ nuqtada $z_{min} = -6\sqrt{3}$. **1894.** $M_0(0;\ 3)$ nuqtada $z_{min} = -9$. **1895.** $M_0\left(\frac{a}{2}; \frac{a}{3}\right)$ nuqtada $z_{max} = \frac{a^6}{432}$. **1896.** $M_0(2;\ 4)$ nuqtada $z_{min} = 0$. **1897.** $3a^2$, a^2. **1898.** 32,0. **1899.** 91,0. **1900.** 3, −3.

XII BOB DIFFERENSIAL TENGLAMALAR

1- §. Differensial tenglama to'g'risida tushuncha

1°. n-tartibli *oddiy differensial tenglama* deb quyidagi ko'rinishdagi tenglamaga aytiladi:

$$F(x, y, y', y'', \dots, y^{(n)}) = 0. \qquad (1)$$

Agar $y = \varphi(x)$ funksiya va uning mos hosilalarini (1) ga qo'yganimizda uni ayniyatga aylantirsa, bu funksiya uning *yechimi* deyiladi. $y = \varphi(x)$ yoki $\Phi(x, y) = 0$ funksiya differensial tenglamaning *integrali* deyiladi. Har bir integral xOy tekisligida egri chiziq bo'lib, unga differensial tenglamaning *integral egri chiziqlari* deyiladi. Yuqoridagi (1) tenglamaning umumiy integrali x, y va n ta ixtiyoriy o'zgarmasni o'z ichiga olgan

$$\Phi(x, y, C_1, C_2, \dots, C_n) = 0 \qquad (2)$$

ko'rinishdagi tenglama bo'ladi.

Umumiy integraldan ixtiyoriy o'zgarmasning ba'zi qiymatlarida hosil qilingan yechim (integral) *xususiy yechim* (integral) deyiladi.

(2) umumiy integralni x bo'yicha n marta differensiallab, hosil bo'lgan n ta tenglamadan n ta ixtiyoriy o'zgarmasni chiqarib, differensial tenglamani hosil qilamiz.

2°. 1- tartibli differensial tenglamaning umumiy ko'rinishi

$$F(x, y, \frac{dy}{dx}) = 0 \qquad (3)$$

yoki

$$\frac{dy}{dx} = f(x, y) \qquad (4)$$

shaklda bo'ladi.

Agar biror sohada $f(x, y)$ funksiya uzluksiz va chegaralangan xususiy hosila $\frac{\partial f}{\partial y}$ ga ega bo'lsa, bu sohaning har bir ichki nuqtasi $(x_0; y_0)$ dan birgina integral egri chiziq o'tadi.

1901. 1) o‘rniga qo‘yib tekshiring: $y = Cx^3$ funksiya $3y - xy' = 0$ differensial tenglamaning yechimi bo‘ladi.

1) (1; 1/3); 2) (1;1); 3) $(1; -\frac{1}{3})$ nuqtalardan o‘tuvchi integral egri chiziqlarni yasang.

1902. O‘rniga qo‘yib: 1) $y = C_1\cos 2x + C_2\sin 2x$ va 2) $y = C_1 + C_2e^{3x} + C_3e^{-3x}$ umumiy integralning, mos ravishda, 1) $y'' + 4y = 0$ va 2) $y''' - 9y' = 0$ differensial tenglamalarning yechimlari ekanligini tekshiring.

1903. $y = Cx^2$ parabolani $C = 0; \pm 1; \pm 2$ larda chizing va uning differensial tenglamasini tuzing.

1904. 1) $x^2 + y^2 = 2Cx$ aylanalar; 2) $y = x^2 + 2Cx$ parabolalar oilasini chizing va ularning differensial tenglamalarini tuzing.

1905. 1) $y^2 = 2Cx$; 2) $y = Ce^x$; 3) $x^2 + y^2 = C^2$ egri chiziqlar oilasini chizing va ularning differensial tenglamalarini tuzing.

2- §. O‘zgaruvchilari ajraladigan 1- tartibli differensial tenglama. Ortogonal trayektoriyalar

1°. Agar differensial tenglama

$$f(x)\cdot\varphi(y)dx + f_1(x)\cdot\varphi_1(y)\,dy = 0 \qquad (1)$$

ko‘rinishida bo‘lsa, bunday tenglama *o‘zgaruvchilari ajraladigan* differensial tenglama deyiladi.

(1) tenglamaning ikkala tomonini $f_1(x)\cdot\varphi(y)$ ga bo‘lib,

$$\frac{f(x)dx}{f_1(x)} + \frac{\varphi_1(y)dy}{\varphi(y)} = 0 \qquad (2)$$

tenglamani hosil qilamiz.

(1) yoki (2) tenglamaning umumiy integrali

$$\int\frac{f(x)}{f_1(x)}dx + \int\frac{\varphi_1(y)}{\varphi(y)}dy = C \qquad (3)$$

bo‘ladi.

2°. $F(x, y, a) = 0$ egri chiziqlar oilasini to‘g‘ri burchak ostida kesib o‘tuvchi chiziq *ortogonal trayektoriyalar* deyiladi. $F(x, y, a) = 0$ tenglamani x bo‘yicha differensiallab, hosil bo‘lgan va berilgan tenglamadan a ni chiqarib, berilgan oilaning differensial tenglamasi $y'=f(x,y)$ ni hosil qilamiz. Bu holda ortogonal trayektoriyalar tenglamasi $y'=-\frac{1}{f(x,y)}$ bo'ladi.

Quyidagi differensial tenglamalarning: 1) umumiy integralini toping; 2) bir nechta integral egri chiziqlarini yasang; 3) $x=-2$ da $y=4$ boshlang'ich shartga ko'ra xususiy integralini toping.

1906. $xy'-y=0$. **1907.** $xy'+y=0$.

1908. $yy'+x=0$. **1909.** $y'=y$.

Tenglamalarning umumiy integralini toping:

1910. $x^2y'+y=0$. **1911.** $x+xy+y'(y+xy)=0$.

1912. $\varphi^2 dr+(r-a)\,d\varphi=0$. **1913.** $2st^2 ds=(1+t^2)\,dt$.

Quyidagi tenglamalarning umumiy va boshlang'ich shartlariga ko'ra xususiy integrallarini toping:

1914. $2y'\sqrt{x}=y, x=4$ da $y=1$.

1915. $y'=(2y+1)\text{ctg}x,\ x=\frac{\pi}{4}$ da $y=1/2$.

1916. $ay=x^2$ parabolalar oilasi ortogonal trayektoriyasini toping. Ularni yasang.

1917. $yx=C$ giperbolalar oilasi ortogonal trayektoriyasini toping.

1918. $x^2+4y^2=a^2$ ellipslar oilasi ortogonal trayektoriyasini toping.

1919. $x^2-2y^2=a^2$ giperbolalar oilasini toping.

1920. Kuzatishlarga ko'ra t momentdagi bakteriyaning o'sish tezligi $x(t)$ ni 3 ga bo'lganga teng. Shu jarayonning differensial tenglamasini tuzing.

Y e c h i s h . Masala shartiga ko'ra bakteriyaning o'sish tezligi $\frac{x(t)}{3}$ ga teng. Bu holda differensial tenglama $\frac{dx(t)}{dt}=\frac{1}{3}x(t)$ bo'ladi.

1921. Ozish muammolarini tekshirish eksperimentiga ko'ra tekshiriluvchining massasi 30 kunda 140 kg dan 110 kg ga kamaygan. Kuzatishga ko'ra, bir kunlik massaning kamayishi tekshiriluvchi massasiga proporsional. Tekshiriluvchining 15 kun ochlikdan keyingi massasini toping.

3- §. Birinchi tartibli differensial tenglamalar. Bir jinsli, chiziqli tenglamalar. Bernulli tenglamasi

1°. Agar P va Q funksiyalar x va y larning bir xil o'lchovli bir jinsli funksiyalari bo'lsa, $Pdx + Qdy = 0$ tenglama *bir jinsli* deyiladi. Bu tenglama $\frac{dy}{dx} = \varphi\left(\frac{y}{x}\right)$ ko'rinishga keltiriladi va $y = ux$ almashtirish yordamida yechiladi.

2°. $y + P(x)y = Q(x)$ tenglama *chiziqli tenglama* deyiladi. Bu yerda $P(x)$ va $Q(x)$ lar berilgan bo'lib, x ning uzluksiz funksiyalari. Bu tenglama $y=uv$ almashtirish orqali

$$y=e^{-\int P(x)dx}\left[C+\int e^{\int P(x)dx}Q(x)dx\right] \qquad (1)$$

yechimga ega bo'ladi.

3°. Ushbu

$$y'+P(x)y=Q(x)y^{n} \quad (n \neq 0, n \neq 1) \qquad (2)$$

tenglama *Bernulli tenglamasi* deyiladi. Bu tenglama $z=y^{1-n}$ almashtirish orqali chiziqli tenglamaga keltiriladi.

Differensial tenglamalarni integrallang:

1922. $yy' = 2y - x$.

1923. $x^2 + y^2 - 2xyy' = 0$.

1924. $\frac{ds}{dt} = \frac{s}{t} - \frac{t}{s}$.

1925. $xy'\cos\frac{y}{x} = y\cos\frac{y}{x} - x$.

1926. $xy+y^2 = (2x^2 + xy)y'$.

1927. $xy'\sin(y/x) + x = y\sin(y/x)$.

1928. $xyy' = y^2 + 2x^2$.

1929. $y' = y/x + \cos(y/x)$.

1930. $(x^2 + y^2)\,dx - xydy = 0$.

1931. $y' = (x + y) / (x - y)$.

1932. $y' - \frac{3}{x}y = x$.

1933. $y' + \frac{2}{x}y = \frac{e^{x^{-2}}}{x}$.

1934. $y'\cos x - y\sin x = \sin 2x$.

1935. $xy' - y = x^2\cos x$.

1936. $y' + 2xy = xe^{-x^2}$.

1937. $(1+x^2)y' + y = \operatorname{arctg} x$.

1938. $y'\cos x + y = 1 - \sin x$.

1939. $y' - 2xy = 3x^2 - 2x^4$.

1940. $xy' - 2y = 2x^4$.

1941. $y' + y\cos x = e^{-\sin x}$.

1942. $y'x + y = -xy^2$.

1943. $y' - xy = -y^3 e^{-x^2}$.

1944. $y' + \frac{2}{x}y = 3x^2 y^{4/3}$.

1945. $y' - \frac{y}{x-1} = \frac{y^2}{x-1}$.

1946. $y' + \frac{2}{x}y = \frac{2\sqrt{y}}{\cos^2 x}$.

Boshlang'ich shartlarga ko'ra xususiy integrallarni toping:

1947. $y + \sqrt{x^2 + y^2} - xy' = 0$; $x=1$ da $y=0$.

1948. $xy' = y(1 + \ln\frac{y}{x})$; $x=1$ da $y = \frac{1}{\sqrt{e}}$.

1949. $xy' - \frac{y}{x+1} = x$; $x = 1$ da $y = 0$.

1950. $y'\sin x - y\cos x = 1$; $x = \pi/2$ da $y = 0$.

1951. $3y^2 y' + y^3 = x + 1$; $x = 1$ da $y = -1$.

1952. $xy' - y = x\operatorname{tg}(y/x)$; $y(1) = \pi/2$.

1953. $xy' = xe^{y/x} + y$; $y(1) = 0$.

1954. $y' = (y/x) + \cos(y/x)$.

1955. $y' = 4 + y/x + (y/x)^2$.

1956. $(x^4 + 6x^2y^2 + y^4)dx + 4xy(x^2 + y^2)dy = 0$; $y(1) = 0$.

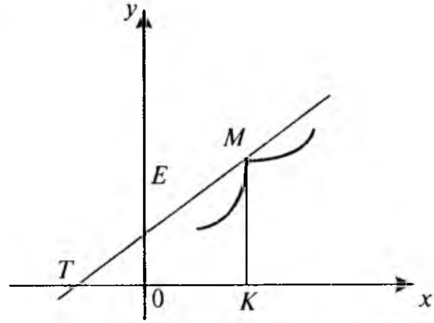
1957. $y'\sqrt{1 + x^2} + y = \arcsin x$; $y(0) = 0$.

1958. $y' - \frac{y}{x\ln x} = x\ln x$; $y(e) = e^2/2$.

1959. $y' + y = e^{\frac{x}{2}}\sqrt{y}$; $y(0) = 9/4$.

1960. $y' + \frac{3x^2}{x^3 + 1}y = y^2(x^3 + 1)\sin x$; $y(0) = 1$.

1961. Shunday chiziqni topingki, uning ixtiyoriy nuqtasidan o‘tkazilgan urunmaning Oy o‘qidan ajratgan kesmasi uzunligi kvadrati urinish nuqtasi koordinatalari ko‘paytmasiga teng bo‘lsin.



34- chizma.

Yechish. To‘g‘ri burchakli koordinatalar sistemasini tekislikda tanlab, bu sistemada chiziq shakli rasmda ko‘rsatilgandek bo‘lsin (33-chizma).

Chiziq ustida ixtiyoriy $M(x;\,y)$ nuqta olib, bu nuqtadan TM urunma o‘tkazamiz. Masala shartiga ko‘ra $|OE|^2 = xy$.

$$\Delta TOE \sim \Delta TKM \text{ va } \left|\frac{OE}{TO}\right| = \left|\frac{MK}{TK}\right|.$$

Demak, $|OE| = \dfrac{|TO|\cdot|MK|}{|TK|}$. Ma’lumki, $|TK|$ urinma osti va $|TK| = \left|\dfrac{y}{y'}\right|$, $|MK| = y$, $|TO| = \left|\dfrac{y}{y'}\right| - x$. Bu holda $|OE| = y - y'x$. Bundan,masala shartiga ko‘ra, topamiz: $(y - y'x)^2 = xy$ yoki $y - y'x = \sqrt{xy}$. Bu bir jinsli differensial tenglamani yechib topamiz: $x = Ce^{2\sqrt{\frac{y}{x}}}$.

1962. Shunday chiziqni topingki, bu chiziq $(\sqrt{3};1)$ nuqtadan o‘tib, uning ixtiyoriy nuqtasidan o‘tkazilgan urinmaning Oy o‘qidan ajratgan kesmasining urinish nuqtasi radius-vektoriga teng bo‘lsin.

1963. Katta o‘lchamli populatsiyada yuqumli kasallik bo‘ladi, bu kasallik odamlar orasida vaqt o‘tishi bilan tarqaladi. Faraz qilaylik, $p(t)$ populatsiyada kasallik paydo bo‘lgandan keyin t yil ichida kasallangan odamlar soni bo‘lsin va kasallikning tarqalish tenglamasi $\dfrac{dp}{dt} + \dfrac{1}{3}p(t) = \dfrac{1}{3}$ bo‘lsa: a) $p(0)=0$, $t > 0$ bo‘lganda $p(t)$ ni toping; b) qancha yilda kasallanish hissasi 90 % bo‘ladi?

Y e c h i s h. Berilgan chiziqli tenglama yechimi har qanday $p(0)$ uchun $p(t) = 1 + [p(0) - 1]e^{-t/3}$ bo'ladi.

a) $p(0) = 0$ bo'lsa, $p(t)=1-e^{-t/3}$, b) $0,9=1-e^{-t/3}$, $-0,1 = -e^{-t/3}$, $0,1 = e^{-t/3}$, bundan $-\ln 10=\ln e^{-t/3}$, $t = 3\ln 10$, $t = 6,9078 \approx 7$. Shunday qilib, bu kasallik bilan kasallanuvchilar soni 7 yilda 90 % ga yetadi.

1964. Agar g'altakning qarshiligi R, induktivlik koeffitsiyenti L, boshlang'ich tok $I_0 = 0$, EYUK $E = E_0\sin\omega t$ bo'lsa, g'altakdagi t momentdagi tok I topilsin.

1965. Ikki modda kimyoviy reaksiyada teng miqdorda qatnashib, uchinchi $s(t)$ birikma modda hosil bo'ladi. γ_1 va γ_2 lar bu moddalarning boshlang'ich ($t = 0$) konsentratsiyalari bo'lsin.

a) birikma moddaning $t > 0$, $s(0) = 0$ dagi konsentratsiyasini toping.

в) birikmaning $\gamma_1 = 20$, $\gamma_2 = 30$ dagi limit konsentratsiyasini toping.

K o ' r s a t m a. Kimyoviy reaksiyaning modeli uchun $\frac{ds}{dt} = k(\gamma_1 - s)(\gamma_2 - s)$ proporsionallik tenglamasi qaraladi, bunda $k > 0$ — koeffitsiyent, $\gamma_1 - s$ va $\gamma_2 - s$ lar birikmadagi, mos ravishda, birinchi va ikkinchi moddalarning komponentlari. $\lim s(t) = \gamma_1$.

1966. Shunday chiziqni topingki, ixtiyoriy nuqtasida o'tkazilgan normalning Ox o'qidan ajratgan kesmasi $y^2/3x$ ga teng bo'lsin.

K o ' r s a t m a. $yy' + x = \frac{y^2}{3x}$ tenglamani integrallang ($y^2 = z$ deb oling).

4- §. Birinchi tartibli to'la differensialli tenglamalar. Integrallovchi ko'paytuvchi

1°. Agar $Pdx + Qdy = 0$ tenglamada $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ bo'lsa, uni $du = 0$ ko'rinishiga keltirib, umumiy yechimini $u = C$ ko'rinishida ifodalash mumkin.

2°. Agar $\frac{\partial P}{\partial y} \neq \frac{\partial Q}{\partial x}$ bo‘lsa, shunday $\mu(x; y)$ funksiya mavjudki, $\mu Pdx + \mu Qdy = du$ bo‘ladi. Bu $\mu(x; y)$ funksiyaga *integrallovchi ko‘paytuvchi* deyiladi. Integrallovchi ko‘paytuvchini quyidagi hollarda topish mumkin:

1) $\dfrac{\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}}{Q} = \Phi(x)$ bo‘lsa, $\ln\mu = \int \Phi(x)dx$;

2) $\dfrac{\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}}{P} = \Phi(y)$ bo‘lsa, $\ln\mu = \int \Phi_1(y)dy$.

Quyidagi to‘la differensialli tenglamalarni yeching:

1967. $\left(4 - \frac{y^2}{x^2}\right)dx + \frac{2y}{x}dy = 0$.

1968. $3x^2e^y dx + (x^3e^y - 1)dy = 0$.

1969. $e^{-y}dx + (1 - xe^{-y})dy = 0$.

1970. $2x\cos^2 y dx + (2y - x^2 \sin 2y)\, dy = 0$.

1971. $(3x^2 + 2y)dx + (2x - 3)dy = 0$.

1972. $(3x^2y - 4xy^2)dx + (x^3 - 4x^2y + 12y^3)dy = 0$.

1973. $(x\cos 2y + 1)dx - x^2 \sin 2y dy = 0$.

Integrallovchi ko‘paytuvchilarni topib, differensial tenglamalarni yeching:

1974. $(x^2 - y)\, dx + xdy = 0$.

1975. $2x\,\mathrm{tg}y dx + (x^2 - 2\sin y)dy = 0$.

1976. $(e^{2x} - y^2)dx + ydy = 0$.

1977. $(1 + 3x^2 \sin y)dx - x\,\mathrm{ctg}y dy = 0$.

1978. $y^2dx + (yx - 1)dy = 0$.

1979. $(x^2 - 3y^2)dx + 2xydy = 0$.

1980. $(\sin x + e^y)dx + \cos x dy = 0$.

1981. $(x\sin y + y)dx + (x^2\cos y + x\ln x)dy = 0$.

5- §. Tartibini pasaytirish mumkin bo'lgan yuqori tartibli differensial tenglamalar

1°. $y^{(n)} = f(x)$ ko'rinishdagi tenglama o'ng tomonini n karra ketma-ket integrallash natijasida yechiladi. Har bir integrallashda bitta ixtiyoriy o'zgarmas, oxirgi natijada n ta o'zgarmas qatnashadi.

2°. $F(x, y', y'') = 0$ tenglamada tarkibida y oshkor holda qatnashmaydi. Bu tenglama $y' = p,\ y'' = \frac{dp}{dx}$ almashtirish orqali $F(x, p, \frac{dp}{dx}) = 0$ ko'rinishga keltiriladi.

3°. $F(y, y', y'') = 0$ tenglamada x oshkor holda qatnashmaydi va bu tenglama $y' = p,\ y'' = \frac{dp}{dx} = \frac{dp}{dy} \cdot \frac{dy}{dx} = p\frac{dp}{dy}$ almashtirishga ko'ra $F(y, p, p\frac{dp}{dy}) = 0$ ko'rinishga keltirib yechiladi.

Tenglamalarni yeching:

1982. $y''' = \frac{6}{x^3}$; boshlang'ich shartlar: $x = 1$ da $y = 2,\ y' = 1,\ y'' = 1$.

1983. $y'' = 4\cos 2x$, $x = 0$ da $y = 0,\ y' = 0$.

1984. $y'' = \frac{1}{1+x^2}$.

1985. $x^3y'' + x^2y' = 1$.

1986. $y'' + y'\,\mathrm{tg}x = \sin 2x$.

1987. $y''\,x\ln x = y'$.

1988. $xy'' - y' = e^x x^2$.

1989. $(1 + x^2)y'' + 2xy' = x^3$.

1990. $yy'' + y'^2 = 0$.

1991. $y'' + 2y(y')^3 = 0$.

1992. $y''\mathrm{tg}y = 2(y')^2$.

1993. $2yy'' = y'^2$.

1994. $y''y^3 = 1$.

1995. $(y'')^2 = y'$.

1996. $(y')^2 + 2yy'' = 0$.

1997. Massasi m bo'lgan moddiy nuqta $F = F_0 \cdot \cos\omega t$, bu yerda F_0 va ω — o'zgarmas nuqtalar, kuch ta'sirida to'g'ri chiziqli harakat qiladi. Boshlang'ich momentdagi nuqta tezligi $x_0 = v_0$. Nuqta harakatining tenglamasini toping.

Y e c h i s h. To'g'ri chiziqli harakatdagi m massali moddiy nuqtaga F kuch ta'sir qiladi. Nyutonning ikkinchi qonunini tatbiq qilib, topamiz: $\frac{d^2x}{dt^2} = \frac{F_0}{m}\cos\omega t$. Bu tenglamani integrallab va $t =$ $= 0$ da $x = 0$, $x_0 = v_0$ boshlang'ich shartlardan foydalanib, $C_1 = v_0$, $C_2 = \frac{F_0}{m\omega^2}$ larni hosil qilamiz.

Yechim quyidagicha bo'ladi:

1998. Juda katta masofadan, tinch holatdan to'g'ri chiziqli harakatlanib Yerga uriluvchi meteorning tezligini toping (uning Yerga tomon harakatidagi tezlanishi Yer markazi bilan meteor orasidagi masofa kvadratiga teskari proporsional).

K o ' r s a t m a . Meteorning harakat tenglamasi: $\frac{d^2r}{dt^2} = \frac{k}{r^2}$, bu yerda r — meteor bilan Yer markazi orasidagi masofa; $k = -gR^2$, $R = 6{,}377\cdot10^6$ m — Yer radiusi.

1999. 10 000 t sig'imga ega bo'lgan (suv joylashadigan) kema 16 m/s tezlik bilan harakatlanmoqda. Suvning qarshiligi kema tezligi kvadratiga proporsional bo'lib, 1 m/s tezlikda 30 t ga to'g'ri keladi. Kema tezligi 4 m/s bo'lganga qadar u qancha masofani bosib o'tadi?

Y e c h i s h . Kemaga suv qarshiligi kuchi va inersiya kuchi ta'sir qiladi. Nyutonning ikkinchi qonuniga ko'ra $m\dot{x} + k\dot{x}^2 = 0$ harakat tenglamasini hosil qilamiz, bu yerda $k=30\ \frac{s^2}{m^2}$. Bu tenglama o'zgaruvchi y ga ega emas, shu sababli $\dot{x} = p, \ddot{x} = \dot{p}$ deb olib, yuqoridagi tenglamani $\frac{dp}{dt} + \frac{k}{m}p^2 = 0$ yoki $\frac{dp}{p^2} = -\frac{k}{m}dt$ ko'rinishga keltiramiz. Buni integrallab, topamiz: $-\frac{1}{p} = -\frac{k}{m}t + C$, $p = \frac{m}{k(t-C_1)}x$ da $x(0) = p = v_0 = 16$ m/s. Shartga ko'ra $C_1 = -\frac{m}{kv_0}$ ni hosil qilib, topamiz: $p = \frac{m/k}{t + m/kv_0}$ yoki $\frac{dx}{dt} = \frac{m/k}{t + m/kv_0}$, buni yana bir marta integrallaymiz: $x(t) = \frac{m}{k}\ln C_2(l + \frac{m}{kv_0})$.

Boshlang'ich shartga asosan $t=0$ da $x=0$ dan $C_2 = kv_0/m$ ni hosil qilamiz. Demak, xususiy yechim $x(t) = \frac{m}{k}\ln(\frac{kv_0}{m}t+1)$ bo'ladi.

2000. Og'irligi 0,3 t ga teng bo'lgan motorli kema 40 m/s boshlang'ich tezlik bilan to'g'ri chiziqli harakat qiladi. Suv qarshillgi tezlikka proporsional bo'lib, 1 m/s tezlikda 0,01 m ga teng. Qancha vaqtdan keyin tezlik 8 m/s bo'ladi?

K o ' r s a t m a . Harakat tenglamasi: $0,3\frac{d^2x}{dt^2} = -0,01v$.

6- §. O'zgarmas koeffitsiyentli chiziqli bir jinsli differensial tenglamalar

Bir jinsli chiziqli differensial tenglama

$$y^{(n)} + P_1 y^{(n-1)} + \ldots + P_n y = 0 \qquad (1)$$

ko'rinishga ega bo'lib, bu yerda $P_i - x$ ning funksiyasi, uning umumiy yechimi

$$y = C_1y_1 + C_2y_2 + \ldots + C_ny_n \qquad (2)$$

bo'ladi, $y_1, y_2, \ldots, y_n$ lar (1) tenglamanimg chiziqli bog'liq bo'lmagan xususiy yechimlari, $C_1, C_2, \ldots, C_n$ — ixtiyoriy o'zgarmaslar.

Agarda $P_1, P_2, \ldots, P_n$ koeffitsiyentlar o'zgarmaslar bo'lsa, $y_1, y_2, \ldots, y_n$ xususiy yechimlar

$$r^n + P_1r^{n-1} + \ldots + P_n = 0 \qquad (3)$$

xarakteristik tenglama yordamida topiladi.

1) (3) tenglamaning har bir m karrali $r = a$ haqiqiy ildiziga m ta $e^{ax}, x^{ax}, \ldots, x^{m-1}e^{ax}$ xususiy yechimlar mos keladi;

2) har bir juft m karrali kompleks $r=\alpha\pm\beta i$ ildizga m juft xususiy yechimlar:

$$\begin{cases} e^{ax}\cos\beta x, xe^{ax}\cos\beta x, \ldots, x^{m-1}e^{ax}\cos\beta x, \\ e^{ax}\sin\beta x, xe^{ax}\sin\beta x, \ldots, x^{m-1}e^{ax}\sin\beta x \end{cases}$$

mos keladi.

Tenglamalarni yeching:

2001. $y'' - 4y' + 3y = 0$. **2002**. $y'' - 4y' + 4y = 0$.

2003. $y'' - 4y' + 13y = 0$. **2004**. $y'' - 4y = 0$.

2005. $y'' + 4y = 0.$ **2006.** $y'' + 4y' = 0.$

2007. $\frac{d^2x}{dt^2} + 3\frac{dx}{dt} - 4x = 0.$ **2008.** $4\frac{d^2p}{d\varphi^2} + p = 0.$

2009. $\frac{d^2s}{dt^2} + 2\frac{ds}{dt} + 2s = 0,\ t = 0$ da $s=1,\ s'=1.$

2010. $y''' - 5y'' + 8y' - 4y = 0.$

2011. $y^{IV} - 16y = 0.$

2012. $y''' - 8y = 0.$

2013. $y''' + 3ay'' + 3a^2y' + a^3y = 0.$

2014. $y^{IV} + 4y = 0.$

2015. $4y^{IV} - 3y'' - y = 0.$

7- §. O'zgarmas koeffitsiyentli chiziqli bir jinsli bo'lmagan differensial tenglamalar

1°. Asosiy xossalari. Faraz qilaylik, quyidagi tenglamalar berilgan bo'lsin:

$$y^{(n)} + P_1 y^{(n-1)} + \ldots + P_n y = f(x), \tag{1}$$

$$y^{(n)} + P_1 y^{(n-1)} + \ldots + P_n y = 0. \tag{2}$$

(1) va (2) tenglamalar, mos ravishda, *bir jinsli bo'lmagan* va *bir jinsli* differensial tenglamalar deyiladi. Agar (2) tenglamaning umumiy yechimi u, (1) tenglamaning xususiy yechimi y_1 bo'lsa,

$$y = u + y_1 \tag{3}$$

(1) tenglamaning *umumiy yechimi* bo'ladi.

2°. O'zgarmas koeffitsiyentlar usuli. $P_1,\ P_2,\ \ldots,\ P_n$ lar o'zgarmas koeffitsiyentlar usuli bo'yicha quyidagi hollarda topiladi:

1) $f(x)$ — ko'phaq;

2) $f(x) = e^{mx}(a\cos nx + b\sin nx)$;

3) $f(x)$ — oldingi funksiyalar yig'indisi yoki ko'paytmasi.

Bu hollarda y_1 ning ko'rinishi $f(x)$ kabi bo'lib, faqat koeffitsiyentlari bilan farq qiladi.

M a x s u s h o l l a r: 1) $f(x)$ — ko'phad, $r = 0$ xarakteristik tenglamaning k karrali ildizi; 2) $f(x) = e^{mx}$ $(a \cos nx + b \sin nx)$, $r = m \pm$ ni xarakteristik tenglamaning k karrali ildizi bo'lsa, y_1 xususiy yechim $f(x)$ dan koeffitsiyenti bilangina emas, x^k ko'paytuvchisi bilan ham farq qiladi.

3°. O'zgarmas sonni variatsiyalash usuli. Bu usul umumiy bo'lib, quyidagicha tahlil qilinadi. Agar y_1 va y_2 lar $y'' + py' + qy = 0$ tenglamaning chiziqli bog'liqsiz xususiy yechimlari bo'lsa, $y'' + py' + qy = f(x)$ tenglamaning umumiy yechimi $y = Ay_1 + By_2$ ko'rinishida bo'ladi, bu yerda A va B lar x ning funksiyasi bo'lib,

$$\begin{cases} A'y_1 + B'y_2 = 0, \\ A'y'_1 + B'y'_2 = f(x) \end{cases}$$

sistemani qanoatlantiradi.

Bundan $A' = -\frac{y_2 f(x)}{W}$; $B' = \frac{y_1 f(x)}{W}$; $W = \begin{vmatrix} y_1 & y_2 \\ y'_1 & y'_2 \end{vmatrix}$.

Tenglamalarni yeching:

2016. $y'' - 2y' + y = e^{2x}$.

2017. $y'' - 4y = 8x^3$.

2018. $y'' + 3y' + 2y = \sin 2x + 2\cos 2x$.

2019. $y'' + y = x + 2e^x$.

2020. $y'' + 3y' = 9x$.

2021. $y'' + 4y' + 5y = 5x^2 - 32x + 5$.

2022. $y'' - 3y' + 2y = e^x$.

2023. $\ddot{x} + k^2 x = 2k \sin kt$.

2024. $y'' - 2y = xe^{-x}$.

2025. $y''' + y'' = 6x + e^{-x}$.

2026. $y^{IV} - 81y = 27e^{-3x}$.

2027. $\ddot{x} + \dot{x} = 3t^2$.

2028. $y'' + 4y = \frac{1}{\sin 2x}$.

2029. $y'' + 4y' + 4y = \frac{e^{-2x}}{x^3}$.

2030. $y''-2y'+y=\frac{e^x}{\sqrt{4-x^2}}$.

2031. $y''+y'-2y=6x^2$.

2032. $y''-5y'+6y=13\sin 3x$.

2033. $y''+2y'+y=e^x$.

2034. $y''+y'+2{,}5y=25\cos 2x$.

2035. $4y''-y=x^3-24x$.

2036. $y''-y=e^{-x}$.

2037. $\ddot{s}+2\dot{s}+2s=2t^3-2$.

2038. $y''-2my+m^2y=\sin mx$.

2039. $y'''-3y''+3y'-y=e^x$.

2040. $y''+4y'+4y=e^{-2x}\ln x$.

2041. $y''+y=\frac{1}{\cos^3 x}$.

2042. $y''+4y=\frac{1}{\cos^2 x}$.

8- §. O'zgarmas koeffitsiyentli chiziqli differensial tenglamalar sistemasi

1°. Chiziqli bir jinsli o'zgarmas koeffitsiyentli differensial tenglamalar sistemasi umumiy ko'rinishda quyidagicha yoziladi:

$$\begin{cases} \frac{dx}{dt}=a_{11}x+a_{12}y, \\ \frac{dy}{dt}=a_{21}x+a_{22}y. \end{cases} \tag{1}$$

Bu sistemaning birorta tenglamasini t bo'yicha differensiallanuvchi ikkunchi tartibli chiziqli o'zgarmas koeffitsiyentli tenglamaga keltirib yechiladi.

2°. (1) sistemani ikkinchi tartibli bitta bir noma'lumli differensial tenglamaga keltirmasdan, ushbu

$$\begin{vmatrix} a_{11}-k & a_{12} \\ a_{21} & a_{22}-k \end{vmatrix}=0 \qquad (2)$$

xarakteristik tenglama ildizlariga asoslanib yechish ham mumkin.

Tenglamalar sistemalarini 1° ga ko‘ra yeching:

2043. $\begin{cases} \frac{dx}{dt}+y=0, \\ \frac{dx}{dt}-\frac{dy}{dt}=3x+y. \end{cases}$ **2044.** $\begin{cases} \frac{dx}{dt}+x-y=e^t, \\ \frac{dx}{dt}-x+y=e^t. \end{cases}$

2045. $\begin{cases} 5\frac{dx}{dt}-2\frac{dy}{dt}+4x-y=e^{-t}, \\ \frac{dx}{dt}+8x-3y=5e^{-t}. \end{cases}$ **2046.** $\begin{cases} \frac{dx}{dt}-2x+4y=4t, \\ \frac{dy}{dt}+x-y=t^2. \end{cases}$

2047. $\begin{cases} \frac{dx}{dt}=y, \\ \frac{dy}{dt}=-x. \end{cases}$

Tenglamalar sistemalarini 2° ga ko‘ra yeching:

2048. $\begin{cases} \dot{x}=x-y, \\ \dot{y}=y-4x. \end{cases}$ **2049.** $\begin{cases} \dot{x}+3x+y=0, \\ \dot{y}+x+y=0. \end{cases}$ **2050.** $\begin{cases} \frac{dy}{dx}=y+5z, \\ \frac{dz}{dx}=-y-3z. \end{cases}$

2051. $\begin{cases} \frac{dy}{dx}+z=0, \\ \frac{dz}{dx}+4y=0. \end{cases}$ **2052.** $\begin{cases} \frac{dy}{dx}+2y+z=\sin x, \\ \frac{dz}{dx}+4y=0. \end{cases}$

XII bob javoblari

1901. 1) $y=\frac{x^3}{3}$; 2) $y=x^3$; 3) $y=-\frac{x^3}{3}$. **1903.** $xy'=2y$. **1904.** $2xyy'=y^2-x^2$; $xy'=x^2+y$. **1905.** $2xy'=y$; $y'=y$; $yy'+x=0$. **1906.** $y=Cx$; $y=-2x$. **1907.** $xy=C$; $xy=-8$. **1908.** $x^2+y^2=C$; $x^2+y^2=20$. **1909.** $y=Ce^x$; $y=4e^{x+2}$. **1910.** $y=Ce^{\frac{1}{x}}$. **1911.** $x+y=\ln C(x+1)(y+1)$.

1912. $r=Ce^{\frac{1}{\varphi}}+a$. **1913.** $S^2=(t^2+Ct-1)/t$. **1914.** $y=Ce^{\sqrt{x}}$; $y=e^{\sqrt{x}-2}$.
1915. $y=(C\sin^2 x-1)/2$; $y=2\sin^2 x-\frac{1}{2}$. **1916.** $x^2+2y^2=C^2$. **1917.** y^2-
$-x^2=C$. **1918.** $y=Cx^4$. **1919.** $x^2y=C$. **1920.** $x=\frac{1}{3}x$. **1921.** $x=124$ kg.
1922. $y-x=Ce^{\frac{x}{y-x}}$. **1923.** $x^2-y^2=Cx$. **1924.** $S^2=2t^2\ln\frac{C}{t}$.
1925. $\sin\frac{y}{x}+\ln x=C$. **1926.** $y^2=Cxe^{-y/x}$. **1927.** $Cx=e^{\cos(y/x)}$. **1928.**
$y^2=4x^2\ln Cx$. **1929.** $1+\sin(y/x)=Cx\cos(y/x)$. **1930.** $y^2=x^2\ln Cx^2$.
1931. $\mathrm{arctg}(y/x)=\ln c\sqrt{x^2+y^2}$. **1932.** $y=Cx^3-x^2$. **1933.** $y=\frac{C-e^{-x^2}}{2x^2}$.
1934. $y=\frac{C-\cos 2x}{2\cos x}$. **1935.** $y=x(\sin x+C)$. **1936.** $y=e^{-x^2}(\frac{x^2}{2}+C)$.
1937. $y=\mathrm{arctg}x-1+Ce^{-\mathrm{arctg}x}$. **1938.** $\cos x(x+C)/(1+\sin x)=y$.
1939. $y=Ce^{x^2}+x^3$. **1940.** $y=Cx^2+x^4$. **1941.** $y=(x+C)e^{-\sin x}$.
1942. $y=\frac{1}{x\ln Cx}$. **1943.** $y^2=e^{\frac{x^2}{2x+C}}$. **1944.** $y^{-1/3}=Cx^{2/3}-(7/3)x^2$.
1945. $y=(x-1)/(C-x)$. **1946.** $y^{-1/2}-\mathrm{tg}x=(\ln\cos x+C)/x$. **1947.**
$y=\frac{x^2-1}{2}$. **1948.** $y=xe^{Cx}$; $y=xe^{-\frac{x}{2}}$. **1949.** $y=\frac{x}{x+1}(x-1+\ln|x|)$. **1950.**
$y=-\cos x$. **1951.** $y^3=x+Ce^{-x}$; $y^3=x-2e$; $y^3=x-2e$. **1952.** $y=x\arcsin x$.
1953. $y=-x\ln|1-\ln x|$. **1954.** $1+\sin(y/x)=Cx\cos(y/x)$. **1955.** $\mathrm{arctg}(0{,}5\,y/x)=$
$=2\ln|x|=\frac{\pi}{4}$. **1956.** $x^5+10x^3y^2+5xy^4=1$. **1957.** $y=e^{-\arcsin x}+\arcsin x-1$.
1958. $y=\frac{1}{2}x^2\ln x$. **1959.** $y=e^{-x}\left[\frac{1}{2}e^x+1\right]^2$. **1960.** $y=\sec x/(x^3+1)$.
1961. $x=Ce^{+2\sqrt{y/x}}$. **1962.** $x^2=3(3-2y)$. **1963.** a) $p(t)=1-e^{-t/3}$; b) 7yil.
1964. $I=\frac{E}{R^2+L^2\omega^2}\left[L\omega e^{-\frac{Rt}{L}}+R\sin\omega t-L\omega\cos\omega t\right]$.
1965. $s(t)=\frac{\gamma_1\gamma_2[1-e^{-(\gamma_1\gamma_2)kt}]}{\gamma_1-\gamma_2 e^{(\gamma_1-\gamma_2)kt}}$; $\lim S(t)=\gamma_1$. **1966.** $y^2=C^3\sqrt{x^2}-\frac{3}{2}x^2$. **1967.**
$4x^2+y^2=Cx$. **1968.** $x^3e^y-y=C$. **1969.** $y+xe^{-y}=C$. **1970.** $x^2\cos^2 y+y^2=C$.

1971. $x^3 + 2xy - 3y = C$. **1972.** $x^3 - 2x^2y^2 + 3y^4 = C$. **1973.** $x^2\cos 2y + 2x = C$.

1974. $\mu = \frac{1}{x^2}$; $x + \frac{y}{x} = C$. **1975.** $\ln\mu = \ln\cos y$; $x^2\sin y + \frac{1}{2}\cos 2y = C$. **1976.** $\mu = e^{-2x}$; $y^2 = (C - 2x)e^{2x}$. **1977.** $\mu = \frac{1}{\sin\mu}$; $\frac{x}{\sin y} + x^3 = C$. **1978.** $\mu = \frac{1}{y}$; $xy - \ln y = C$. **1979.** $\mu = \frac{1}{x^4}$; $y^2 = Cx^3 + x^2$. **1980.** $\mu = e^{-y}$; $e^y\cos x = x + C$. **1981.** $\ln\mu = -\ln x$; $\mu = \frac{1}{x}$; $x\sin y + y\ln x = C$. **1982.** $y = 3\ln x + 2x^2 - 6x + 6$. **1983.** $y = 1 - \cos 2x$. **1984.** $y = C_1x + C_2 + x\,\mathrm{arctg}x - \ln\sqrt{1 + x^2}$.

1985. $y = \frac{1}{x} + C_1\ln x + C_2$. **1986.** $y = C_1\sin x - x - \frac{1}{2}\sin 2x + C_2$. **1987.** $y = C_1x(\ln x - 1) + C_2$. **1988.** $y = e^x(x - 1) + C_1x^2 + C_2$. **1989.** $y = \frac{x^3}{12} - \frac{x}{4} + C_1\mathrm{arctg}x + C_2$. **1990.** $y^2 = C_1x + C_2$. **1991.** $y^3 + C_1y + C_2 = 3x$. **1992.** $\mathrm{ctg}y = C_2 - C_1x$. **1993.** $y = (C_1x + C_2)^2$. **1994.** $C_1y^2 = (C_1x + C_2)^2 + 1$.

1995. $y = \frac{1}{12}(x + C_1)^3 + C_2$. **1996.** $y = C_1(x + C_2)^{2/3}$. **1997.** $x(t) = \frac{F_0}{m\omega^2}(1 - \cos\omega t) + v_0t$. **1998.** $v = \sqrt{\frac{2gR^2}{R}} = \sqrt{2gR} = 11$km/s. **1999.** $x(t) = \frac{m}{k}\ln\left(\frac{kv_0}{m}t + 1\right)$. **2000.** $t = 6{,}45$s. **2001.** $y = C_1e^x + C_2e^{3x}$. **2002.** $y = (C_1 + C_2x)e^{2x}$ $(A\cos 3x + B\sin 3x)$. **2004.** $y = C_1e^{2x} + C_2e^{-2x}$. **2005.** $y = a\sin(2x + j) = A\cos 2x + B\sin 2x$; **2006.** $y = C_1 + C_2e^{-4x}$. **2007.** $x = C_1e^t + C_2e^{-4t}$.

2008. $p = A\cos\frac{\varphi}{2} + B\sin\frac{\varphi}{2}$. **2009.** $s = e^t(A\cos t + B\sin t)$; $s = e -$ $-t(\cos t + 2\sin t)$. **2010.** $y = C_1e^x + (C_2 + C_3x)e^{2x}$. **2011.** $y = C_1\mathrm{ch}2x + C_2\mathrm{sh}2x + C_3\cos 2x + C_4\sin 2x$. **2012.** $y = C_1e^{2x} + e^{-x}(C_2\cos\sqrt{3}x + C_3\sin\sqrt{3}x)$. **2013.** $y = (C_1 + C_2x + C_3x^2)e^{-ax}$. **2014.** $y = (A\sin x + C\cos x)\mathrm{sh}x + (B\sin x + 2\cos x)\mathrm{ch}x$. **2015.** $y = A\,\mathrm{ch}x + B\,\mathrm{sh}x + C\cos\frac{x}{2} + D\sin\frac{x}{2}$. **2016.** $y = e^x(C_1 + C_2x) + e^{2x}$. **2017.** $y = C_1e^{2x} + C_2e^{-2x} - 3x - 2x^3$. **2018.** $y = C_1e^{-x} + C_2e^{-2x} + 1{,}25\sqrt{2}\cos(\frac{\pi}{4} - 2x)$. **2019.** $y = C_1\cos x + C_2\sin x + x + e^x$. **2020.** $y = C_1 + C_2e^{-3x} + \frac{3}{2}x^2 - x$. **2021.** $y = e^{-2x}(C_1\cos x + C_2\sin x) +$

$+x^2-8x+7$. **2022.** $y=C_1e^{2x}+(C_1-x)e^x$. **2023.** $x=A\sin k(t-t_0)-t\cos kt$.

2024. $y=C_1e^{\sqrt{2x}}+C_2e^{-\sqrt{2x}}-(x-2)e^{-x}$. **2025.** $y=C_1+C_2x+((C_3+x)e^{-x}$

$+x^3-3x^2$. **2026.** $y=C_1e^{(C_2-\frac{x}{4})}+e^{-3x}+C_3\cos 3x+C_4\sin 3x$. **2027.** $x=C_1+$

$+C_2\cos t+C_3\sin t+t^3-6t$. **2028.** $y=C_1\cos 2x+C_2\sin 2x-\frac{x}{2}\cos 2x+\frac{\sin 2x}{4}$

$\ln|\sin 2x|-\frac{x}{2}\cos 2x+\frac{1}{4}\sin 2x\ln\sin 2x$. **2029.** $y=e^{-2x}(C_1+C_2x+\frac{1}{2x})$.

2030. $y=(C_1+\sqrt{4-x^2}+x\arcsin\frac{x}{2}+C_2x-e^x$. **2031.** $y=C_1e^x+C_2e^{-2x}-$

$-3(x^2+x+1,5)$. **2032.** $y=C_1e^{2x}+C_2e^{3x}+\frac{1}{6}(5\cos 3x-\sin 3x)$. **2033.**

$y=(C_1x+C_2)e^{-x}+\frac{1}{4}e^x$. **2034.** $y=e^{-\frac{x}{2}}\left(C_1\cos\frac{3x}{2}+C_2\sin\frac{3x}{2}\right)-6\cos 2x+$

$+8\sin 2x$. **2035.** $y=C_1e^{\frac{x}{2}}+C_2e^{-\frac{x}{2}}-x^3$. **2036.** $y=C_1e^x+\left(C_2-\frac{x}{2}\right)e^{-x}$.

2037. $s=e^{-t}(C_1\cos t+C_2\sin t)+(t-1)^3$. **2038.** $y=e^{mx}(C_1+C_2x)+\frac{\cos mx}{2m^2}$.

2039. $y=\left(C_1+C_2x+C_3x^2+\frac{x^3}{6}\right)e^x$. **2040.** $y=\left(\frac{x^2\ln x}{2}-\frac{3x^2}{4}+C_1+C_2x\right)e^{-2x}$.

2041. $y=C_1\sin x+C_2\cos x+\frac{1}{2\cos x}$. **2042.** $y=(C_1-\ln|\sin x|\cos 2x)+$

$+\left(C_2-x-\frac{1}{2}\operatorname{ctg}x\right)\sin 2x$. **2043.** $x=C_1e^t+C_2e^{-3t}$; $y=-x=-C_1e^t+3e^{-3t}$.

2044. $x=e^t+C_1+C_2e^{-2t}$; $y=e^t+C_1-C_2e^{-2t}$. **2045.** $x=2e^{-t}+C_1e^t+C_2e^{-2t}$;

$y=3e^{-t}+3C_2e^t+2C_2e^{-2t}$. **2046.** $x=C_1e^{2t}+C_2e^{-3t}+\frac{2}{3}t^2+\frac{8}{9}t-\frac{8}{27}$;

$y=-C_1e^{2t}+\frac{C_2}{4}e^{-3t}-\frac{t^2}{3}+\frac{2}{9}t-\frac{2}{27}$. **2047.** $x=C_1\cos t+C_2\sin t$; $y=-C_1\sin t+$

$+C_2\cos t$. **2048.** $x=C_1e^{-t}+C_2e^{3t}$; $y=2C_1e^{-t}-2C_2e^{3t}$. **2049.** $x=e^{-2t}(1-2t)$.

2050. $y=e^{-x}(C_1\cos x+C_2\sin x)$; $z=\frac{1}{5}e^{-x}\left[(C_2-2C_1)\cos x-(C_1+2C_2)\sin x\right]$.

2051. $y=C_1e^{2x}+C_2e^{-2x}$; $z=-2(C_1e^{2x}-C_2e^{-2x})$. **2052.** $y=C_1+C_2x+2\sin x$;

$z=-2C_1-C_2(2x-1)-3\sin x-2\cos x$.

XIII BOB | QATORLAR

1- §. Sonli qatorlar

Cheksiz sonlar ketma-ketligi u_1, u_2, u_3, u_4, ..., u_n, ... lardan (u_n — musbat sonlar) tuzilgan $u_1 + u_2 + u_3 + ... + u_n + ...$ ifoda *sonli qator* deyiladi.

u_1, u_2, u_3, ..., u_n lar qatorning *hadlari*, u_n esa qatorning *umumiy hadi* deyiladi. Ushbu

$$u_1 + u_2 + u_3 + ... + u_n + ... = \sum_{n=1}^{\infty} u_n \qquad (1)$$

qatorning n ta hadi yig'indisi, ya'ni

$$S(x) = u_1 + u_2 + u_3 + ... + u_n$$

yig'indi qatorning *xususiy yig'indisi* deyiladi.

Agar $\lim_{n \to \infty} S_n = S$ bo'lib, S chekli son bo'lsa, qator *yaqinlashuvchi* va S uning *yig'indisi* deyiladi. Cheksiz kamayuvchi geometrik progressiya hadlaridan tuzilgan

$$a + aq + aq^2 + aq^3 + ... aq^{n-1} + ...$$

qator $|q| < 1$ bo'lganda yaqinlashuvchi va uning yig'indisi $S = \frac{a}{1-q}$ ga teng bo'ladi.

$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + ... + \frac{1}{n} + ...$ qator *garmonik qator* deyiladi va u uzoqlashuvchi qator bo'ladi.

Agar (1) qator yaqinlashuvchi bo'lsa, $\lim_{n \to \infty} u_n = 0$ bo'ladi. (Qatorning yaqinlashuvchi bo'lishligining zaruriy sharti.)

Agar (1) qator va

$$\upsilon_1 + \upsilon_2 + ... + \upsilon_n + ... \qquad (2)$$

uchun $u_n \le \upsilon_n$ bo'lib, (2) qator yaqinlashuvchi bo'lsa, (1) qator ham yaqinlashuvchi bo'ladi.

Agar $u_n \le \upsilon_n$ bo'lib, (1) qator uzoqlashuvchi bo'lsa, (2) qator ham uzoqlashuvchi bo'ladi.

Dalamber alomati. Agar (1) qator uchun

$$\lim_{n\to\infty}\frac{u_{n+1}}{u_n}=D$$

bo‘lib, $D<1$ bo‘lsa, (1) qator yaqinlashuvchi, $D>1$ bo‘lsa, uzoqlashuvchi bo‘ladi; $D=1$ bo‘lsa, noaniq hol bo‘lib, (1) qator yaqinlashuvchi ham, uzoqlashuvchi ham bo‘lishi mumkin.

Koshi alomati. Agar (1) qator uchun

$$\lim_{n\to\infty}\sqrt[n]{u_n}=K$$

bo‘lib, $K<1$ bo‘lsa, qator yaqinlashuvchi; $K>1$ bo‘lsa, qator uzoqlashuvchi bo‘ladi; $K=1$ bo‘lsa, noaniq hol bo‘lib, qator yaqinlashuvchi ham, uzoqlashuvchi ham bo‘lishi mumkin.

Koshining integral alomati. Agar (1) qatorning hadlari musbat va

$$u_1\geq u_2\geq u_3\geq\ldots+\geq u_n\geq\ \ldots$$

shart bajarilib, shunday o‘smaydigan *f(x)* funksiya mavjud bo‘lib, $f(1)=u_1,\ f(2)=u_2,\ldots,\ f(n)=u_n,\ldots$ bo‘lsa, $\int\limits_1^{\infty}f(x)dx$ xosmas integralning yaqinlashuvchi bo‘lishidan (1) qatorning yaqinlashuvchi bo‘lishi, uzoqlashuvchi bo‘lishidan (1) qatorning uzoqlashuvchi bo‘lishi kelib chiqadi.

Qatorning *n*- hadi (umumiy hadi)ni toping.

2053. $\frac{1}{2}+\frac{1}{2^2}+\frac{5}{2^3}+\frac{7}{2^4}\ldots$

2054. $\frac{2}{3}+\left(\frac{3}{7}\right)^2+\left(\frac{4}{11}\right)^3+\left(\frac{5}{15}\right)^4+\ldots$

2055. $\frac{1}{2}+\frac{4}{4}+\frac{9}{8}+\frac{16}{16}+\frac{25}{32}+\ldots$

2056. $1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{4}}+\ldots$

2057. $\frac{1}{2}+\frac{3}{4}+\frac{5}{8}+\frac{7}{16}+\frac{9}{32}+\ldots$

2058. $\frac{1}{2}+\frac{2}{5}+\frac{3}{8}+\frac{4}{11}+\ldots$

2059. $\frac{1}{\sqrt{3}}+\frac{2}{3}+\frac{3}{3\sqrt{3}}+\frac{4}{9}+\frac{5}{9\sqrt{3}}+\ldots$

2060. $\frac{1}{3}+\left(\frac{2}{5}\right)^2+\left(\frac{3}{7}\right)^3+\left(\frac{4}{9}\right)^4+\ldots$

2061. $1+\frac{3}{4}+\frac{5}{9}+\frac{7}{16}+\ldots\ldots$

Qatorning yig'indisini toping:

2062. $\frac{1}{1\cdot3}+\frac{1}{3\cdot5}+\frac{1}{5\cdot7}+\frac{1}{7\cdot9}+\ldots$

2063. $\frac{2}{3}+\frac{1}{3}+\frac{1}{6}+\frac{1}{12}+\frac{1}{24}+\ldots$

2064. $\frac{1}{1\cdot2\cdot3}+\frac{1}{2\cdot3\cdot4}+\frac{1}{3\cdot4\cdot5}+\ldots$

2065. $\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\frac{1}{81}+\ldots$

2066. $\frac{1}{1\cdot4}+\frac{1}{4\cdot7}+\frac{1}{7\cdot10}+\frac{1}{10\cdot13}+\ldots$

Qator yaqinlashishligining zaruriy sharti bajarilishini tekshiring:

2067. $\frac{1}{2}+\frac{2}{5}+\frac{3}{8}+\frac{4}{11}+\ldots$

2068. $\frac{1}{5}+\frac{3}{8}+\frac{5}{11}+\ldots$

2069. $1+\frac{4}{5}+\frac{6}{10}+\frac{8}{17}+\frac{10}{26}+\ldots$

2070. $1+\frac{3}{4}+\frac{5}{9}+\frac{7}{16}+\ldots$

2071. $\frac{1}{3}+\frac{1}{5}+\frac{1}{10}+\frac{1}{17}+\ldots$

2072. $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\frac{5}{6}+\ldots$

Solishtirish yordamida qatorlar xarakterini aniqlang:

2073. $1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\ldots+\frac{1}{\sqrt{n}}+\ldots=\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}.$

2074. $\sum_{n=1}^{\infty}\frac{1}{n^n}.$

2075. $\sum_{n=1}^{\infty}\frac{1}{(n+1)\cdot3^n}.$

2076. $1+\frac{1}{2\cdot5}+\frac{1}{3\cdot5^2}+\frac{1}{4\cdot5^3}+\ldots$

2077. $1+\frac{3}{4}+\frac{5}{9}+\frac{7}{16}+\ldots$

2078. $\sum_{n=1}^{\infty}\frac{1}{n^2+1}.$

Qatorlarni Dalamber alomati yordamida yaqinlashishga tekshiring:

2079. $\frac{1}{2}+\frac{2^5}{2^2}+\frac{3^5}{2^3}+\frac{4^5}{2^4}+\ldots$ **2080.** $\sum_{n=0}^{\infty}\frac{3^{2n+1}}{2^{3n-1}}$.

2081. $\sum_{n=1}^{\infty}\frac{n!}{5^n}$. **2082.** $\sum_{n=3}^{\infty}\frac{7^{3n}}{(2n-5)!}$.

2083. $\frac{2}{3}+\frac{4}{9}+\frac{6}{27}+\frac{8}{81}+\ldots$ **2084.** $1+\frac{3}{2\cdot 3}+\frac{3^2}{2^2\cdot 5}+\frac{3^3}{2^3\cdot 7}+\ldots$

2085. $\frac{2}{1}+\frac{2^2}{2^{10}}+\frac{2^3}{3^{10}}+\frac{2^4}{4^{10}}+\ldots+\frac{2^n}{n^{10}}+\ldots$

2086. $\frac{10}{1}+\frac{10^2}{2!}+\frac{10^3}{3!}+\frac{10^4}{4!}+\ldots$

Qatorlarni Koshi alomati yordamida yaqinlashishga tekshiring:

2087. $\frac{1}{3}+\left(\frac{2}{5}\right)^2+\left(\frac{3}{7}\right)^3+\left(\frac{4}{9}\right)^4+\left(\frac{5}{11}\right)^5+\ldots$

2088. $\sum_{n=1}^{\infty}\frac{1}{2^n}\left(1+\frac{1}{n}\right)^n$.

2089. $\operatorname{arctg}1+4\operatorname{arctg}^2\frac{1}{\sqrt{2}}+9\operatorname{arctg}^3\frac{1}{\sqrt{3}}+\ldots+n^2\operatorname{arctg}^n\frac{1}{\sqrt{n}}+\ldots$

2090. $\sum_{n=1}^{\infty}\left(\frac{3n^2-5n+2}{4n^2-n+6}\right)^n$.

2091. $\sum_{n=1}^{\infty}\frac{4^n}{3^n(3n+1)}$.

Koshining integral alomati yordamida qatorlarning xarakterini tekshiring:

2092. $1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\ldots$ **2093.** $\frac{1}{2\ln 2}+\frac{1}{3\ln 3}+\frac{1}{4\ln 4}+\ldots$

2094. $\sum_{n=2}^{\infty}\frac{1}{n^2-1}$. **2095.** $\sum_{n=2}^{\infty}\frac{1}{\sqrt[3]{(2n-3)^2}}$.

2096. $\sum_{n=1}^{\infty}\frac{1}{(n+1)\ln^2(n+1)}$. **2097.** $\frac{1}{1+1^2}+\frac{1}{1+2^2}+\frac{1}{1+3^2}+\frac{1}{1+4^2}+\ldots$

2098. $\frac{1}{1+1^2}+\frac{2}{1+2^2}+\frac{3}{1+3^2}+\ldots$

Quyidagi qatorlar xarakterlarini aniqlang:

2099. $1+\frac{1}{3\sqrt{3}}+\frac{1}{5\sqrt{5}}+\frac{1}{7\sqrt{7}}+\ldots$

2100. $1+\frac{1}{101}+\frac{1}{201}+\frac{1}{301}+\frac{1}{401}+\ldots$

2101. $\frac{1}{1+1^4}+\frac{2}{1+2^4}+\frac{3}{1+3^4}+\ldots\frac{n}{1+n^4}+\ldots$

2102. $1+\frac{3}{4}+\frac{5}{9}+\frac{7}{16}+\ldots$

2103. $\frac{2}{1}+\frac{4}{3!}+\frac{6}{5!}+\frac{8}{7!}+\ldots$

2- §. Ishoralari almashinuvchi va o'zgaruvchi qatorlarning xarakterlarini aniqlash

Leybnis teoremasi. *Agar ishoralari almashinuvchi*

$$u_1-u_2+u_3-u_4+u_5-\ldots \quad (u_n>0) \qquad (3)$$

qatorda $u_1>u_2>u_3>\ldots$ *bo'lib,* $\lim\limits_{n\to\infty} u_n=0$ *bo'lsa, qator yaqinlashuvchi va qatorning yig'indisi musbat bo'lib,* u_1 *dan kichik bo'ladi.*

Agar (3) qatordan tuzilgan

$$|u_1|+|u_2|+|u_3|+\ldots|u_n|+\ldots \qquad (4)$$

qator yaqinlashuvchi bo'lsa, (3) qator *absolut yaqinlashuvchi* qator deyiladi. Agar (4) qator uzoqlashuvchi bo'lib, (3) qator yaqinlashuvchi bo'lsa, unga *shartli yaqinlashuvchi qator* deyiladi.

Qatorlarni yaqinlashuvchi yoki uzoqlashuvchi ekanligini aniqlang:

2104. $\sum\limits_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}$. **2105.** $\sum\limits_{n=1}^{\infty}\frac{(-1)^n}{n(n+1)}$.

2106. $\sum\limits_{n=0}^{\infty}\frac{\cos n\alpha}{2^n}$. **2107.** $\sum\limits_{n=1}^{\infty}\sin\frac{n\pi}{3}$.

2108. $1-\frac{1}{3^2}+\frac{1}{5^2}-\frac{1}{7^2}+\ldots$ **2109.** $\frac{1}{2\ln 2}-\frac{1}{3\ln 3}+\frac{1}{4\ln 4}-\frac{1}{5\ln 5}+\ldots$

2110. $\sum\limits_{n=1}^{\infty}(-1)^{n-1}\frac{n^2}{3n^2+n}$. **2111.** $\sum\limits_{n=1}^{\infty}(-1)^{n-1}\operatorname{tg}\frac{1}{\sqrt[3]{n^2}}$.

2112. $1-\left(\frac{2}{3}\right)^2-\left(\frac{3}{5}\right)^3+\left(\frac{4}{7}\right)^4+...(-1)^{\frac{n(n-1)}{2}}\left(\frac{n}{2n-1}\right)^n+...$

2113. $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{2n+1}{n(n+1)}$.

2114. $\frac{1}{2}-\frac{2}{2^2+1}+\frac{3}{3^2+1}-\frac{4}{4^2+1}+...$

2115. $1,1-1,01+1,001-1,0001+...$

3-§. Funksional qatorlar

Ushbu

$$u_1(x)+u_2(x)+u_3(x)+\ ...\ +u_n(x)+... \qquad (5)$$

funksional qator yaqinlashuvchi bo'ladigan x o'zgaruvchining qiymatlari to'plamiga uning *yaqinlashish sohasi* deyiladi.

$S(x)=S_n(x)+R_n(x)$ da $\lim\limits_{n\to\infty}S_n=S$ ga (5) *qatorning yig'indisi,* $R_n(x)$ ga *qoldig'i* deyiladi. Agar yaqinlashuvchi sonli qator

$$\alpha_1+\alpha_2+\alpha_3+...+\alpha_n+... \qquad (6)$$

mavjud bo'lib, $|u_n(x)|<\alpha_n$ $(a<x<b)$ bo'lsa, (5) qator $[a;\ b]$ kesmada *absolut* va *tekis yaqinlashuvchi* bo'ladi.

2116. $\frac{4-x}{7x+2}+\frac{1}{3}\cdot\left(\frac{4-x}{7x+2}\right)^2+\frac{1}{5}\left(\frac{4-x}{7x+2}\right)^3+...$ funksional qatorning $x=0$ va $x=1$ qiymatlarda yaqinlashuvchiligini tekshiring.

2117. $\frac{1}{1+x^2}+\frac{1}{1+x^4}+\frac{1}{1+x^6}+...$ funksional qatorning yaqinlashish sohasini toping ($|x|<1$, $|x|=1$, $|x|>1$ hollar uchun tekshiring).

2118. $\frac{1}{1+x^2}-\frac{1}{2+x^2}+\frac{1}{3+x^2}-\frac{1}{4+x^2}+...$ funksional qatorning x ning barcha qiymatlarida tekis yaqinlashuvchi ekanligini ko'rsating.

2119. $\cos x+\frac{1}{2^2}\cos^2 2x+\frac{1}{3^2}\cos^3 3x+\frac{1}{4^2}\cos^4 4x+...$ qatorning $(-\infty;\ +\infty)$ da tekis yaqinlashuvchi ekanligini ko'rsating.

2120. $\frac{x^3}{1}+\frac{x^3}{1+x^3}+\frac{x^3}{(1+x^3)^2}+\frac{x^3}{(1+x^3)^4}+...$ qatorning $x \geq 1$ da tekis yaqinlashuvchi ekanligini ko'rsating.

2121. $\sum_{n=1}^{\infty} \frac{(-x)^n}{2^{n-1}\sqrt{n}}$ qatorning yaqinlashish intervallarini toping.

2122. $\sum_{n=1}^{\infty} \frac{3^n \cdot n!}{(3n)!} x^{2n}$ qatorning yaqinlashish intervallarini toping.

2123. $\sum_{n=1}^{\infty} \frac{(x+8)^{3n}}{n^2}$ qatorning yaqinlashish intervallarini toping.

2124. $\sum_{n=1}^{\infty} 10^{2n}(2x-3)^{2n-1}$ qatorning yaqinlashish intervallarini toping.

2125. $\frac{x+1}{1}+\frac{(x+1)^2}{2\cdot 4}+\frac{(x+1)^3}{3\cdot 4^2}+\frac{(x+1)^4}{4\cdot 4^3}+...$ qatorning yaqinlashish intervallarini toping.

2126. $\frac{2x-3}{1}+\frac{(2x-3)^2}{3}+\frac{(2x-3)^3}{5}+\frac{(2x-3)^4}{7}+...$ qatorning yaqinlashish intervallarini toping.

2127. $(x-3)\cdot 1!+(x-3)^2\cdot 2!+(x-3)^3\cdot 3!+(x-3)^4\cdot 4!+...$ qatorning yaqinlashish intervallarini toping.

2128. $1+\frac{x^3}{5}+\frac{x^6}{25}+\frac{x^9}{125}+...$ qatorning yaqinlashish intervallarini toping.

4- §. Teylor va Makloren qatorlari. Binomial qatorlar

$y=f(x)$ funksiya uchun binoınial, Teylor va Makloren qatorlari quyidagi ko'rinishga ega:

$$(1+x)^m=1+\frac{m}{1!}x+\frac{m(m-1)}{2!}x^2+\frac{m(m-1)(m-2)}{3!}x^3+...+x^m.$$

$$f(x)=f(a)+\frac{f'(a)}{1!}(x-a)+\frac{f''(a)}{2!}(x-a)^2+\frac{f'''(a)}{3!}(x-a)^3+...$$

$$\lim_{n\to\infty} R_n=0.$$

$$f(x)=f(0)+\frac{f'(0)}{1}x+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3+\ldots$$

Misol va masalalar yechishda ko‘pincha quyidagi formulalardan foydalaniladi:

$$e^x=1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\ldots \qquad -\infty<x<+\infty.$$

$$\sin x=\frac{x}{1!}-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\ldots \qquad -\infty<x<+\infty.$$

$$\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\ldots \qquad -\infty<x<+\infty.$$

$$\ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\ldots \qquad -1<x\le 1.$$

$$\operatorname{arctg} x=x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\ldots \qquad -1\le x\le 1.$$

$$\ln\frac{1+x}{1-x}=2\left[\frac{1}{2n+1}+\frac{1}{3\cdot(2n+1)^3}+\frac{1}{5(2n+1)^5}+\ldots\right].$$

2129. Quyidagi funksiyalarni darajali qatorlarga yoying:

a) $f(x)=3^x$; b) $f(x)=e^{-3x}$; d) $f(x)=\sin^2 x$;

e) $f(x)=\ln(x+a),\ a>0$; f) $f(x)=\cos x$ funksiyani $x-\frac{\pi}{4}$ ning darajasi bo‘yicha yoying; g) $f(x)=\frac{1}{x}$ funksiyani $(x+2)$ ning darajasi bo‘yicha darajali qatorga yoying.

2130. Makloren qatoridan foydalanib quyidagi funksiyalarni x ning darajasi bo‘yicha yoyilmasini toping:

a) $f(x)=(1+x)e^x$; b) $f(x)=\frac{x-3}{(x+1)^2}$;

d) $f(x)=e^{-x}\cdot\sin x$; e) $f(x)=\ln(x^2-x-2)$;

f) $f(x)=\sqrt{1+x}\cdot\ln(1+x)$; g) $f(x)=(x-1)\operatorname{arctg} x$;

h) $f(x)=\sqrt[3]{x}\cdot\sqrt{1-x^3}$; i) $f(x)=\frac{1}{x}\arcsin x$.

5-§. Qatorlarning taqribiy hisoblashlarga tatbiqi

Darajali qatorlar yordamida funksiyalar qiymatlarini taqribiy hisoblash

2131. $\sqrt{e}$ ni 0,00001 aniqlikda hisoblang.

2132. $\frac{1}{\sqrt[3]{e}}$ ni 0,0001 aniqlikda hisoblang.

2133. $\cos 18^\circ$ ni 0,0001 aniqlikda hisoblang.

2134. $\ln 1{,}2$ ni 0,0001 aniqlikda hisoblang.

2135. To'g'ri burchakli uchburchakning katetlari 1 sm va 5 sm ga teng. Kichik katet qarshisidagi o'tkir burchakni 0,001 aniqlikda hisoblang.

2136. $\sqrt[3]{1{,}06}$ ni 0,0001 aniqlikda hisoblang.

2137. $\sqrt{27}$ ni 0,001 aniqlikda hisoblang.

2138. $\ln 1{,}3$ ni 0,0001 aniqlikda hisoblang.

2139. Binomial qatorning ikkita hadi bilan chegaralanib, quyidagilarni hisoblang va xatolarni toping:

a) $\sqrt{1{,}05}$; b) $\sqrt[3]{1{,}012}$; d) $\sqrt{0{,}993}$;

e) $\sqrt[3]{0{,}997}$; f) $\sqrt[3]{70}$; g) $\sqrt[3]{130}$.

Darajali qatorlar yordamida aniq integrallarni hisoblang:

2140. $\int_0^{1/2} \frac{1-\cos x}{x^2}\,dx$ ni 0,0001 aniqlikda hisoblang.

2141. $\int_0^{0,1} \frac{e^x-1}{x}\,dx$ ni 0,001 aniqlikda hisoblang.

2142. $\int_0^{0,5} \sqrt{1+x^3}\,dx$ ni 0,001 aniqlikda hisoblang.

2143. $\int_0^{1/2} \frac{dx}{\sqrt{1+x^4}}$ ni 0,0001 aniqlikda hisoblang.

2144. $\int_0^1 \cos\sqrt{x}\,dx$ ni 0,001 aniqlikda hisoblang.

2145. $\int_0^{1,5} \frac{1}{x} \operatorname{arctg} \frac{x}{4} dx$ ni 0,001 aniqlikda hisoblang.

2146. $\int_0^{0,25} \ln(1+\sqrt{x})dx$ ni 0,001 aniqlikda hisoblang.

2147. $\int_0^{0,8} \sqrt[3]{x} \cdot \cos^2 x\, dx$ ni 0,001 aniqlikda hisoblang.

2148. $\int_0^{\frac{\pi}{4}} \sin(x^2)dx$ ni 0,0001 aniqlikda hisoblang.

2149. $\int_0^1 \frac{\ln(1-x)}{x} dx$ ni 0,0001 aniqlikda hisoblang.

2150. $\int_0^1 e^{-\frac{x^2}{4}} dx$ ni 0,0001 aniqlikda hisoblang.

Darajali qatorlar yordamida differensial tenglamalarni taqribiy yechish va limitlarni hisoblash

2151. $y''=-xy$ tenglamaning $x=0$, $y=1$, $y'=0$ boshlang'ich shartlarni qanoatlantiruvchi yechimini toping.

2152. $y'+y^2=e^x$ tenglamaning $x=0$, $y=0$ boshlang'ich shartlarni qanoatlantiruvchi yechimini toping (yoyilmada 3 ta had oling).

2153. $y''+xy'+y=0$ tenglamaning $x=0$, $y=0$, $y'=1$ boshlang'ich shartlarni qanoatlantiruvchi yechimini toping.

2154. $y'=x^2-y^2$ tenglamaning $x=0$, $y=0$ boshlang'ich shartlarni qanoatlantiruvchi yechimini toping.

Limitlarni hisoblang:

2155. $\lim\limits_{x\to 0} \frac{2e^x-2-2x-x^2}{x-\sin x}$.

2156. $\lim\limits_{x\to 0} \frac{\sin x - \operatorname{arctg} x}{x^3}$.

2157. $\lim\limits_{x\to 0} \frac{x-\operatorname{arctg} x}{x^3}$.

2158. $\lim\limits_{x\to 0} \frac{1-\cos x}{e^x-1-x}$.

6- §. Furye qatori

$[-\pi;\ \pi]$ da aniqlangan *f(x)* funksiya uchun Furye qatori quyidagi ko'rinishga ega:

$$\frac{a_0}{2}+\sum_{m=1}^{\infty}(a_m\cos mx+b_m\sin mx),$$

bu yerda:

$$a_m=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\cos mxdx \qquad (m=0,1,2,...),$$

$$b_m=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\sin mxdx \qquad (m=1,2,3,...).$$

Agar $f(x)$ juft funksiya bo'lib, $[-l;\ +l]$ da berilgan bo'lsa,

$$f(x)=\frac{a_0}{2}+\sum_{m=1}^{\infty}a_m\cos\frac{m\pi x}{l}\quad ,\quad m=1,2,...$$

bo'lib,

$$a_m=\frac{2}{l}\int_0^l f(x)\cos\frac{m\pi x}{l}dx\quad ,\quad m=0,1,2,...$$

bo'ladi.

Agar $f(x)$ toq funksiya bo'lib, $[-l;\ +l]$ da berilgan bo'lsa,

$$f(x)=\sum_{m=1}^{\infty}b_m\sin\frac{m\pi x}{l}$$

bo'lib,

$$b_m=\frac{2}{l}\int_0^l(x)\sin\frac{m\pi x}{l}dx$$

bo'ladi.

2159. $f(x)=x+\pi$ funksiyani $[-\pi;\ \pi]$ oraliqda Furye qatoriga yoying.

2160. $f(x)=x^2$ funksiyani $[-1;\ 1]$ oraliqda Furye qatoriga yoying.

2161. $f(x)=|x|$ funksiyani $[-1;\ 1]$ oraliqda Furye qatoriga yoying.

2162. $f(x)=1$ ni $0<x<\pi$ va $f(-x)=-f(x)$ ni hisobga olib, Furye qatoriga yoying va $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+...=\frac{\pi}{4}$ ekanligini ko'rsating.

2163. $f(x)=x$ ni $0\le x\le\pi$ va $f(-x)=f(x)$ ni hisobga olib, Furye qatoriga yoying va $1+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+...=\frac{\pi^2}{8}$ ekanligini ko'rsating.

2164. $\begin{cases} f(x)=0, & -l \le x \le 0, \\ f(x)=x, & 0 \le x \le \frac{l}{2}, \\ f(x)=\frac{l}{2}, & \frac{l}{2} \le x < l \end{cases}$

funksiyani $[-l;\ l]$ oraliqda Furye qatoriga yoying.

2165. $f(x)=x^3$ funksiyani $[-\pi;\ \pi]$ oraliqda Furye qatoriga yoying.

2166. $\begin{cases} f(x)=-2x, & -\pi \le x \le 0, \\ f(x)=3x, & 0 \le x \le \pi \end{cases}$

funksiyani $[-\pi;\ \pi]$ oraliqda Furye qatoriga yoying.

2167. $f(x)=\begin{cases} x, & 0 < x \le 1, \\ 2-x, & 0 < x \le 2 \end{cases}$

funksiyani [0; 2] oraliqda sinuslar bo‘yicha qatorga yoying.

2168. $f(x)=\begin{cases} -\frac{(\pi+x)}{2}, & -\pi \le x < 0, \\ \frac{1}{2}(\pi-x), & 0 \le x < \pi \end{cases}$

funksiyani $[-\pi;\ \pi]$ oraliqda kosinuslar bo‘yicha qatoriga yoying.

XIII bob javoblari

2053. $u_n=\frac{2n-1}{2^n}$. **2054.** $u_n=\left(\frac{n+1}{4n-3}\right)^n$. **2055.** $u_n=\frac{n^2}{2^n}$. **2056.** $u_n=\frac{1}{\sqrt{n}}$.
2057. $u_n=\frac{2n-1}{2^n}$. **2058.** $u_n=\frac{n}{3n-1}$. **2059.** $u_n=\frac{n}{3^{n/2}}$. **2060.** $u_n=\left(\frac{n}{2n+1}\right)^n$.
2061. $u_n=\frac{2n-1}{n^2}$. **2062.** $S=\frac{1}{2}$. **2063.** $S=\frac{4}{3}$. **2064.** $S=\frac{1}{4}$. **2065.** $S=\frac{1}{2}$.
2066. $S=\frac{1}{3}$. **2067.** Bajarilmaydi. **2068.** Bajarilmaydi. **2069.** Bajariladi.
2070. Bajariladi. **2071.** Bajariladi. **2072.** Bajarilmaydi. **2073.** Uzoqlashuvchi.

2074. Yaqinlashuvchi. **2075.** Yaqinlashuvchi. **2076.** Yaqinlashuvchi. **2077.** Uzoqlashuvchi. **2078.** Yaqinlashuvchi. **2079.** Yaqinlashuvchi. **2080.** Uzoqlashuvchi. **2081.** Uzoqlashuvchi. **2082.** Yaqinlashuvchi. **2083.** Yaqinlashuvchi. **2084.** Uzoqlashuvchi. **2085.** Uzoqlashuvchi. **2086.** Yaqinlashuvchi. **2087.** Yaqinlashuvchi. **2088.** Uzoqlashuvchi. **2089.** Yaqinlashuvchi. **2090.** Yaqinlashuvchi. **2091.** Yaqinlashuvchi. **2092.** Yaqinlashuvchi. **2093.** Uzoqlashuvchi. **2094.** Yaqinlashuvchi. **2095.** Uzoqlashuvchi. **2096.** Yaqinlashuvchi. **2097.** Yaqinlashuvchi. **2098.** Uzoqlashuvchi. **2099.** Yaqinlashuvchi. **2100.** Uzoqlashuvchi. **2101.** Yaqinlashuvchi. **2102.** Uzoqlashuvchi. **2103.** Yaqinlashuvchi. **2104.** Nisbiy yaqinlashuvchi. **2105.** Absolut yaqinlashuvchi. **2106.** Absolut yaqinlashuvchi. **2107.** Uzoqlashuvchi. **2108.** Absolut yaqinlashuvchi. **2109.** Nisbiy yaqinlashuvchi. **2110.** Uzoqlashuvchi. **2111.** Nisbiy yaqinlashuvchi. **2112.** Absolut yaqinlashuvchi. **2113.** Nisbiy yaqinlashuvchi. **2114.** Yaqinlashuvchi. **2115.** Yaqinlashuvchi. **2117.** $|x| < 1$, $x = 1$ da uzoqlashuvchi, $|x| > 1$ da yaqinlashuvchi. **2121.** $-2 < x \le 2$. **2123.** $-9 \le x < -7$. **2124.** $1{,}45 < x < 1{,}55$. **2125.** $-5 \le x < 3$. **2126.** $1 < x \le 2$. **2128.** $-\sqrt[3]{5} < x < \sqrt[3]{5}$.

2159. $f(x) = \pi + 2\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin nx$. **2160.** $f(x) = \frac{1}{3} + \frac{4}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos n\pi x$.

2164. $f(x) = 1\left[\frac{3}{16} + \left(-\frac{1}{\pi^2}\cos\frac{\pi x}{1} + \frac{2+\pi}{2\pi^2}\sin\frac{\pi}{x}\right)\right] +$

$+\left[-\frac{1}{2\pi^2}\cos\frac{2\pi x}{1} - \frac{1}{4\pi}\sin\frac{2\pi x}{1}\right] + \left[-\frac{1}{9\pi^2}\cos\frac{3\pi x}{1} + \frac{3\pi-2}{18\pi^2}\sin\frac{3\pi x}{1}\right] + \ldots$

2165. $f(x) = \sum_{m=1}^{\infty} (-1)^m \left[\frac{12}{m^3} - \frac{2\pi^2}{m}\right] \sin mx$.

2166. $f(x) = \frac{5\pi}{4} - \frac{10}{\pi}\left[\frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + \ldots\right] + \left[\frac{\sin x}{1} - \frac{\sin 2x}{2} + \ldots\right]$.

2167. $f(x) = \frac{1}{2} - \frac{4}{\pi^2} \sum_{m=0}^{\infty} \frac{\cos(2m+1)\pi x}{(2m+1)^2}$. **2168.** $f(x) = \sum_{n=1}^{\infty} \frac{\sin x}{n}$.

O'TILGAN MAVZULARNING O'ZLASHTIRILISHINI TEKSHIRISH UCHUN SAVOLLAR

Talabalar o'quv qo'llanma bo'yicha mavzularni va ularga oid masalalar yechish malakasini o'zlashtirganlaridan so'ng, o'zlashtirilgan ta'riflar, formulalar va teoremalar isbotini takrorlash tavsiya etiladi. Ushbu savollar o'zlashtirilgan materiallarni takrorlashda talabalarga yordam berish maqsadida keltirilgan.

I. Chiziqli algebra elementlari

1. Determinant deb nimaga aytiladi? Uning asosiy xossalarini keltiring?
2. Determinantning minori va algebraik to'ldiruvchilari deganda nimani tushunasiz?
3. Determinantlarni hisoblash usullarini bilasizmi?
4. Matritsa deganda nimani tushunasiz? Matritsalar ustidagi chiziqli amallar qanday bajariladi? Ularning asosiy xossalarini ayting?
5. Birlik matritsa deb qanday matritsaga aytiladi?
6. Teskari matritsa deb qanday matritsaga aytiladi va u qanday topiladi?
7. Chiziqli tenglamalar sistemasining yechimlari deganda nimani tushunasiz?
8. Tenglamalar sistemasini yechishdagi Kramer formulasi va uni qanday hollarda qo'llab bo'ladi?
9. Qanday shart bajarilganda chiziqli tenglamalar sistemasi yagona yechimga ega bo'ladi?
10. Agar asosiy determinant 0 ga teng bo'lsa, chiziqli tenglamalar sistemasi haqida nima deyish mumkin?
11. Qanday shart bajarilganda bir jinsli tenglamalar sistemasi noldan farqli yechimga ega bo'ladi?
12. Chiziqli tenglamalar sistemasini yechishda Gauss usulining ma'nosi nimadan iborat?
13. Tenglamalar sistemasini matritsa usuli bilan yechish.

II. Tekislikdagi analitik geometriya

1. Chiziqning tenglamasini qanday tuzish mumkin?
2. To‘g‘ri chiziqning burchak koeffitsiyenti deb nimaga aytiladi?
3. To‘g‘ri chiziqning burchak koeffitsiyentli va umumiy tenglamalarini yozing.
4. To‘g‘ri chiziqning kesmalardagi tenglamasi qanday ko‘rinishda bo‘ladi?
5. To‘g‘ri chiziqlar dastasining tenglamasi. Ikki to‘g‘ri chiziq orasidagi burchaklar bissektrisalarining tenglamalarini yozing.
6. Ikki nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasini qanday hosil qilasiz?
7. To‘g‘ri chiziqning normal tenglamasini va umumiy tenglamasini normal ko‘rinishga qanday keltiriladi?
8. Berilgan nuqtadan to‘g‘ri chiziqqacha bo‘lgan masofa qanday aniqlanadi?
9. Ikki to‘g‘ri chiziq orasidagi burchak qanday hisoblanadi?
10. Aylana deb qanday egri chiziqqa aytiladi? Uning tenglamalarini yozing.
11. Ellips deb qanday egri chiziqqa aytiladi? Ellipsning fokuslari va ekssentrisiteti qanday aniqlanadi?
12. Giperbola deb qanday nuqtalarning geometrik o‘rniga aytiladi?
13. Parabola deb qanday nuqtalarning geometrik o‘rniga aytiladi?
14. Ikkinchi tartibli egri chiziqlarning qutb koordinatalaridagi tenglamalarini yozing.

III. Vektorlar algebrasi

1. Vektor va uning moduli deb nimaga aytiladi?
2. Qanday vektorlarga kollinear, komplanar, teng vektorlar deyiladi?
3. Modullari teng bo‘lgan ikki vektor o‘zaro teng bo‘lmasligi mumkinmi? Agar teng bo‘lmasa, farqi nimada?
4. Vektorlar ustida qanday algebraik amallar bajarish mumkin? Nol vektor deb qanday vektorga aytiladi? Vektorlar ustida kiritilgan amallar uchun qanday xossalar o‘rinli?
5. Tekislikda, fazoda bazis deb qanday vektorlarga aytiladi? Qanday bazisga ortonormal bazis deyiladi?
6. Qanday vektorlarga chiziqli bog‘liq vektorlar deyiladi?

7. Dekart koordinatalar sistemasi qanday tanlanadi?

8. Vektorning komponentalari, uning boshlang'ich va oxirgi nuqtalarining koordinatalari orqali qanday ifodalanadi?

9. Kesmani berilgan nisbatda bo'lishni ko'rsating.

10. Uchburchak og'irlik markazining koordinatalarini uning uchlarining koordinatalari orqali ifodalang.

11. Nuqtaning va kesmaning o'qdagi proyeksiyasi deb nimaga aytiladi?

12. Ikki vektorning skalar ko'paytmasi deb nimaga aytiladi? Uning xossalari. Proyeksiyalari bilan berilgan ikki vektorning skalar ko'paytmasini qanday topasiz?

13. Vektorning uzunligini skalar ko'paytma orqali ifodalang.

14. Ikki vektorning vektor ko'paytmasi deb nimaga aytiladi? Uning xossalari va berilgan vektorlarning proyeksiyalari orqali ifodasi.

15. Uchta vektorning aralash ko'paytmasi deb nimaga aytiladi? Uning xossalari va geometrik ma'nosini aytib bering.

16. Uchta vektorning komplanarlik shartini ifodalang.

IV. Fazodagi analitik geometriya

1. Qanday parametrlar berilganda fazoda tekislikning o'rni aniqlangan bo'ladi?

2. Tekislik tenglamalarini (normal, umumiy, kesmalar bo'yicha; berilgan bitta nuqtadan, uchta nuqtadan o'tuvchi) yozing.

3. Ikki tekislik orasidagi burchakni qanday aniqlaysiz? Ikki tekislikning parallellik va perpendikularlik shartlarini yozing.

4. Berilgan nuqtadan berilgan tekislikkacha bo'lgan masofa qanday topiladi?

5. Fazoda ikki tekislik kesishish chizig'idan o'tuvchi tekisliklar dastasining tenglamasini yozing. To'g'ri chiziqning proyeksiyalar bo'yicha tenglamalarini yozing.

6. To'g'ri chiziqning yo'naltiruvchi vektori deb qanday vektorga aytiladi? To'g'ri chiziqning kanonik va parametrik tenglamalarini yozing. Berilgan ikki nuqtadan o'tuvchi to'g'ri chiziq tenglamasini yozing.

7. To'g'ri chiziq bilan tekislik orasidagi burchak deb qanday burchakka aytiladi va u qanday aniqlanadi? To'g'ri chiziq bilan tekislikning parallellik va perpendikularlik shartlarini yozing.

8. To'g'ri chiziq bilan tekislikning kesishish nuqtasini qanday topasiz?

9. Ikki to‘g‘ri chiziqning bir tekislikda yotish shartini yozing.
10. Sfera tenglamasini yozing.
11. Yasovchisi Oz o‘qiga parallel silindrik sirt tenglamasini yozing.
12. Aylanish sirtini qanday hosil qilasiz? Konus sirtlar tenglamasini yozing.

V. Matematik analiz

1. Funksiyaga ta’rif bering. Funksiyaning aniqlanish sohasi deb nimaga aytiladi?
2. Qanday funksiyaga davriy funksiya deyiladi? Misol bilan tushuntiring. Monoton funksiyalar, chegaralangan, chegaralanmagan funksiyalar .
3. Murakkab funksiya deb qanday funksiyaga aytiladi?
4. Qanday fuknsiyalarga elementar funksiyalar deyiladi?
5. Funksiyaning limiti deb nimaga aytiladi?
6. Funksiyaning chap va o‘ng limiti deganda nimani tushunasiz?
7. Chegaralangan funksiya ta’rifini ayting. Qanday funksiyalar cheksiz kichik, qanday funksiyalar cheksiz katta deyiladi?
8. Funksiya limiti haqidagi asosiy teoremalarni ayting va birini isbotlang.
9. Birinchi ajoyib limitni isbotlang.
10. *e* soni. (Ikkinchi ajoyib limit).
11. Funksiyaning nuqtada uzluksizligini ta’riflang.
12. Kesmada uzluksiz funksiyalar xossalarini ayting.
13. Uzilish nuqtasi deb qanday nuqtaga aytiladi?
14. Cheksiz kichik miqdorga ta’rif bering, misol keltiring.
15. Funksiya hosilasi ta’rifini ayting. Uning fizik va geometrik ma’nosi nimadan iborat?
16. Yigindi, ko‘paytma va bo‘linmaning hosilalari qanday topiladi? Misol keltiring.
17. Murakkab funksiyaning hosilasi qanday topiladi?
18. Trigonometrik va logarifmik funksiyalarning hosilasi qanday topiladi?
19. Darajali va ko‘rsatkichli funksiyalar hosilasi. Murakkab ko‘rsatkichli funksiyaga misol keltiring.
20. Teskari funksiya va teskari trigonometrik funksiyalar hosilasini qanday topasiz. Parametrik tenglamalari bilan berilgan funksiya hosilasini qanday topasiz?

21. Funksiya differensiali deb nimaga aytiladi? Uning geometrik ma'nosi nimadan iborat?

22. Yuqori tartibli hosila va differensialni qanday topasiz?

23. Roll teoremasini isbotlang. Uning geometrik ma'nosi nimadan iborat?

24. Lagranj teoremasining geometrik ma'nosini tushuntiring.

25. Qanday ko'rinishdagi aniqmasliklar uchun Lopital qoidasi qo'llaniladi? Misollar keltiring.

VI. Funksiyani hosila yordamida tekshirish

1. O'suvchi funksiya hosilasi kesmada musbat bo'lishini tushuntiring.

2. Funksiya ekstremumining zaruriy sharti nimadan iborat?

3. Funksiyani birinchi va ikkinchi tartibli hosilalar yordamida ekstremumga tekshirishni ko'rsating.

4. Funksiya grafigining qavariq yoki botiqligini ikkinchi tartibli hosila yordamida izohlang.

5. $y=f(x)$ tenglama bilan berilgan chiziq uchun vertikal va og'ma asimptotalar qanday aniqlanadi?

6. Funksiyani to'la tekshirish sxemasi va grafigini chizishni bayon qiling.

VII. Kompleks sonlar

1. Qanday ifodaga kompleks son deyiladi?

2. Kompleks sonning trigonometrik shaklini yozing. Uning moduli va argumenti deb nimaga aytiladi?

3. Kompleks sonlar ustida qo'shish, ayirish, ko'paytirish va ildiz chiqarish amallari qanday bajariladi? Muavr formulasini yozing. Misol keltiring.

4. Haqiqiy sonni trigonometrik shaklda qanday tasvirlash mumkin?

VIII. Aniqmas integral

1. Boshlang'ich funksiya deb qanday funksiyaga aytiladi? Misol keltiring.

2. Biror funksiyaning aniqmas integrali deb nimaga aytiladi? Uning geometrik ma'nosi.

3. Aniqmas integralning hosilasi va u nimaga teng? Misollar keltiring.

4. Asosiy integrallar jadvalini yozing.

5. Aniqmas integralning xossalari.

6. Aniqmas integralni o'zgaruvchini almashtirish yoki o'rniga qo'yish usuli bilan integrallash qanday bajariladi?

7. Bo'laklab integrallash usuli formulasini yozing. Qaysi turdagi integrallarni bo'laklab integrallash qulaylik tug'diradi?

8. Kvadrat uchhad qatnashgan funksiyalar qanday integrallanadi?

9. Eng sodda ratsional kasrlarning birinchi, ikkinchi va uchunchi turlarini integrallash qanday bajariladi va qanday funksiyalarni beradi?

10. Ratsional kasr maxrajining ildizi haqiqiy karrali va kompleks bo'lganda qanday eng sodda kasrlar yigindisi etib yoziladi?

11. Trigonometrik funksiyalarni integrallash qanday usul bilan ratsional funksiyalarni integrallashga keltiriladi? Misol keltiring.

12. Irratsional funksiyalar qanday integrallanadi?

IX. Aniq integral

1. Quyi va yuqori integral yigindilar deb qanday yigindiga aytiladi?

2. [*a*; *b*] kesmada funksiyaning aniq integrali deb nimaga aytiladi? Aniq integralning geometrik ma'nosini izohlang.

3. Aniq integralning xossalarini ayting.

4. [*a*; *b*] kesmada juft va toq funksiyalarning integrali. Misol keltiring.

5. Aniq integralni hisoblash. Nyuton—Leybnis formulasini yozing.

6. Aniq integralda o'zgaruvchini almashtirish qanday bajariladi?

7. Aniq integralni bo'laklab integrallash formulasini yozing.

8. Aniq integralni taqribiy hisoblash formulalarini yozing.

9. Jismning hajmini parallel kesimlar yuzlari bo'yicha qanday hisoblash mumkin? Aylanish jismining hajmini-chi?

10. Aylanish jismining sirtini hisoblash formulasini yozing.

11. Aniq integral yordamida ishni qanday hisoblaysiz?

12. Tekis shaklning og'irlik markazi qanday aniqlanadi?

13. Qanday integralga xosmas integral deyiladi? Qachon xosmas integral mavjud yoki yaqinlashuvchi deyiladi?

X. Ko‘p o‘zgaruvchili funksiya

1. Ko‘p o‘zgaruvchili funksiyaning berilish usullari.

2. Ko‘p o‘zgaruvchili funksiyaning aniqlanish sohasi deb nimaga aytiladi? Ochiq va yopiq sohaga misollar keltiring.

3. Skalar maydonning sath chiziqlari deb nimaga aytiladi? Skalar maydonda funksiya grafigi ma’lum bo‘lsa, sath chiziqlarini qanday hosil qilasiz? Sath chiziqlari kesishadimi?

4. Qanday shart bajarilganda $M_0(x_0; y_0)$ nuqtada $z=f(x,y)$ funksiya uzluksiz deyiladi?

5. $z=f(x,y)$ funksiyaning xususiy hosilalari qanday topiladi? Geometrik ma’nosi.

6. Qachon $z=f(x,y)$ funksiya berilgan nuqtada differensiallanuvchi deyiladi? Berilgan nuqtada funksiyaning to‘liq differensiali deb nimaga aytiladi? To‘liq differensial taqribiy hisoblashda qanday qo‘llaniladi?

7. Murakkab $z=f(u,v)$, $u=\varphi(x,y)$, $v=\theta(y,x)$ funksiyaning xususiy hosilalari qanday topiladi.

8. $z=f(u,v)$, $u=u(x)$, $v=v(x)$ bo‘lganda hosilani qanday topasiz?

9. Funksiya $F(x,y)=0$ tenglama bilan oshkormas shaklda berilganda hosila qanday topiladi?

10. Yuqori tartibli xususiy hosilalar qanday topiladi? Ikki o‘zgaruvchili funksiyaning aralash hosilalari.

11. $u=u(x,y,z)$ funksiyaning nuqtada vektor yo‘nalishi bo‘yicha hosilasi deb nimaga aytiladi?

12. Berilgan $M(x,y,z)$ nuqtada $u=u(x,y,z)$ skalar maydonning gradienti deb nimaga aytiladi?

13. Ikki o‘zgaruvchili funksiya ekstremumga ega bo‘lishining zaruriy va yetarli shartlari nimadan iborat? Minimaks yoki ekstremum nuqtasi deb qanday nuqtaga aytiladi?

14. $z=f(x,y)$ funksiyaning shartli ekstremumi deb nimaga aytiladi va u qanday topiladi?

XI. Differensial tenglamalar

1. Differensial tenglama deb qanday tenglamaga aytiladi? Birinchi tartibli differensial tenglama umumiy ko‘rinishda qanday yoziladi?

2. Differensial tenglamaning yechimi deb nimaga aytiladi? Integral egri chiziq nimani bildiradi?

3. Birinchi tartibli differensial tenglamaning umumiy yechimi deb nimaga aytiladi? Qanday qilib umumiy yechimdan xususiy yechim topiladi?

4. Boshlang'ich shart nimani bildiradi va uning geometrik ma'nosi nimadan iborat? Koshi masalasi nimadan iborat va qanday yechiladi?

5. Qanday birinchi tartibli differensial tenglamalar o'zgaruvchilari ajralgan va ajralmagan differensial tenglamalar deyiladi?

6. Qachon funksiya x va y o'zgaruvchilarga nisbatan o'lchovli bir jinsli funksiya deyiladi? Misol keltiring.

7. Bir jinsli differensial tenglama va uni yechish usuli.

8. Birinchi tartibli chiziqli differensial tenglamalar va ularni yechish usuli.

9. Bernulli tenglamasi qanday yechiladi?

10. To'liq differensialli tenglama va uni yechish usuli.

11. Tartibini pasaytirish mumkin bo'lgan $y^{(n)}=f(x)$ tenglamaning yechimi qanday topiladi?

12. Noma'lum y funksiyani oshkor holda o'z ichiga olmagan ikkinchi tartibli differensial tenglama qanday yechiladi?

13. x erkli o'zgaruvchini oshkor holda o'z ichiga olmagan ikkinchi tartibli differensial tenglama yechimi qanday aniqlanadi?

14. Ikkinchi kosmik tezlik haqidagi masala qanday yechiladi?

15. Ikkinchi tartibli chiziqli differensial tenglamaning umumiy ko'rinishi (bir jinsli bo'lmagan va bir jinsli bo'lgan).

16. Qachon ikkita funksiya o'zaro chiziqli bog'liq va qachon chiziqli funksiyalar deyiladi?

17. Ikkinchi tartibli chiziqli differensial tenglamaning umumiy yechimi qanday aniqlanadi?

18. O'zgarmas koeffitsiyentli ikkinchi tartibli chiziqli bir jinsli differensial tenglamaning umumiy ko'rinishini yozing. Uning xarakteristik tenglamasi deb qanday tenglamaga aytiladi?

19. O'zgarmas koeffitsiyentli ikkinchi tartibli, chiziqli, bir jinsli differensial tenglamaning umumiy yechimini yozing:

1) xarakteristik tenglamaning yechimlari— a) haqiqiy, b) kompleks son bo'lganda;

2) xarakteristik tenglamaning yechimlari haqiqiy karrali bo'lgan holda.

20. Chiziqli, bir jinsli bo'lmagan ikkinchi tartibli differensial tenglamaning umumiy yechimi nimadan iborat?

21. Bir jinsli bo'lmagan differensial tenglamaning xususiy yechimini topishning aniqmas koeffitsiyentlar usuli va ixtiyoriy o'zgarmasni variatsiyalash usuli nimadan iborat?

22. Erkin va majburiy tebranishlar tenglamasini yozing.

23. O'zgarmas koeffitsiyentli, chiziqli differensial tenglamalar sistemasining xarakteristik tenglamasi qanday tuziladi?

24. Differensial tenglamalarning normal sistemasi va uni integrallash qanday bajariladi?

XII. Qatorlar

1. Sonli qator deb nimaga aytiladi?
2. Qatorning xususiy yig'indisi deb qanday yig'indiga aytiladi?
3. Qatorning yig'indisi deb nimaga aytiladi?
4. Sonli qatorning yaqinlashuvchiligi va uzoqlashuvchiligi deganda nimani tushunasiz?
5. Yaqinlashuvchi qatorning xossalarini ayting.
6. Qator yaqinlashuvchiligining zaruriy sharti nimadan iborat?
7. Garmonik qator deb qanday qatorga aytiladi?
8. Musbat hadli qatorlarni taqqoslash deganda nimani tushunasiz?
9. Musbat hadli qatorlar uchun Dalamber alomati nimadan iborat?
10. Musbat hadli qatorlar uchun Koshi alomati qanday?
11. Musbat hadli qatorlar uchun Koshining integral alomati nimadan iborat?
12. Ishoralari almashinuvchi qator deb qanday qatorga aytiladi?
13. Leybnis teoremasini isbotlang.
14. O'zgaruvchan ishorali qator deb qanday qatorga aytiladi?
15. Qanday qatorga absolut va shartli yaqinlashuvchi qator deyiladi?
16. Funksional qator deb qanday qatorga aytiladi?
17. Funksional qatorning yaqinlashish sohasi qanday aniqlanadi?
18. Funksional qatorning yigindisi va uning qoldiq hadi nimadan iborat?

19. Funksional qatorning kesmada tekis yaqinlashish shartlari nimadan iborat?

20. Qator yig'indisining uzluksizligini ayting.

21. Qanday funksional qatorni integrallash va differensiallash mumkin?

22. Darajali qator deb qanday qatorga aytiladi?

23. Teylor qatorini yozing.

24. Makloren qatorini yozing.

25. sin$x$, cos$x$, exp$x$ funksiyalarni Makloren qatoriga yoying.

26. Aniq integrallarni qator yordamida hisoblang.

27. Differensial tenglamalarini qator yordamida yechish.

28. Furye qatori (trigonometrik qator) deb qanday qatorga aytiladi?

29. Furye koeffitsiyentlarini yozing.

30. Juft va toq funksiyalarni Furye qatoriga yoyilmasini yozing.

31. Davri $2l$ bo'lgan funksiyalar uchun Furye qatorini yozing.

32. Davriy bo'lmagan funksiyalarning Furye qatoriga yoyilmasini yozing.

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MUNDARIJA

IV bob. VEKTORLAR ALGEBRASI

V bob. FAZODAGI ANALITIK GEOMETRIYA

VI bob. ANALIZGA KIRISH

XI bob. KO'P ARGUMENTLI FUNKSIYALAR

XII bob. DIFFERENSIAL TENGLAMALAR

XIII bob. QATORLAR

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SULEYMENOV ZINADDIN ISKANDAROVICH
XUDOYOROV BAXTIYOR ALIMOVICH

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